TOPICAL
WORKED SOLUTIONS

O LEVEL
EXAM PAPERS

Mathematics

- Actual Exam Papers
- Thinking Process
- Teacher's Comments

COMPILED FOR 'O' LEVELS

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REVISED SYLLABUS

O LEVEL MATHEMATICS 4024/4029

All candidates will study the following themes or topics

Theme or topic	Theme or topic
1. Number	22. Sequences
2. Set language and notation	23. Variation
3. Squares, square roots, cubes and cube roots	24. Graphs in practical situations
4. Directed numbers	25. Graphs of functions
5. Vulgar and decimal fractions and percentages	26. Function notation
6. Ordering	27. Coordinate geometry
7. Standard form	28. Geometrical terms
8. The four operations	29. Geometrical constructions
9. Estimation	30. Similarity and congruence
10. Limits of accuracy	31. Symmetry
11. Ratio, proportion, rate	32. Angles
12. Percentages	33. Loci
13. Use of an electronic calculator	34. Measures
14. Time	35. Mensuration
15. Money	36. Trigonometry
16. Personal and small business finance	37. Vectors in two dimensions
17. Algebraic representation and formulae	38. Matrices
18. Algebraic manipulation	39. Transformations
19. Indices	40. Probability
20. Solutions of equations and inequalities	41. Categorical, numerical and grouped data
21. Graphical representation of inequalities	42. Statistical diagrams

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All candidates take two papers: Paper 1 and Paper 2.

Each paper may contain questions on any part of the syllabus and questions may assess more than one topic.

Paper 1	**2 hours**
Paper 1 has approximately 25 short answer questions. Candidates should show all working in the spaces provided on the question paper. Essential working must be shown for full marks to be awarded. **No calculators are allowed for this paper.** 80 marks This paper will be weighted at 50% of the total qualification.	

Paper 2	**2 hours 30 minutes**
Paper 2 has approximately 11 structured questions. Candidates should answer **all** questions. **Electronic calculators may be used and candidates should have access to a calculator for this paper.** Candidates should show all working in the spaces provided on the question paper. Essential working must be shown for full marks to be awarded. 100 marks This paper will be weighted at 50% of the total qualification.	

Syllabus Aims

The aims are to enable candidates to:

- increase intellectual curiosity, develop mathematical language as a means of communication and investigation and explore mathematical ways of reasoning
- acquire and apply skills and knowledge relating to number, measure and space in mathematical situations that they will meet in life
- acquire a foundation appropriate to their further study of mathematics and of other disciplines
- appreciate the pattern, structure and power of mathematics and derive satisfaction, enjoyment and confidence from the understanding of concepts and the mastery of skills.

Assessment Objectives

The two assessment objectives in Cambridge O Level Mathematics are:

AO1 Mathematical techniques

AO2 Applying mathematical techniques to solve problems

AO1: Mathematical techniques

Candidates should be able to:

- recognise the appropriate mathematical procedures for a given situation
- perform calculations by suitable methods, with and without a calculator
- understand systems of measurement in everyday use and make use of them in the solution of problems
- estimate, approximate and work to degrees of accuracy appropriate to the context and convert between equivalent numerical forms
- organise, interpret and present information accurately in written, tabular, graphical and diagrammatic forms
- use mathematical and other instruments to measure and to draw to an acceptable degree of accuracy
- recognise and use spatial relationships in two and three dimensions, particularly when solving problems
- interpret, transform and make appropriate use of mathematical statements expressed in words or symbols
- recall, apply and interpret mathematical knowledge in the context of everyday situations.

AO2: Applying mathematical techniques to solve problems

In questions which are set in context and/or which require a sequence of steps to solve, candidates should be able to:

- recognise patterns and structures in a variety of situations and form and justify generalisations
- make logical deductions from given mathematical data
- respond to a problem relating to a relatively unstructured situation by translating it into an appropriately structured form
- analyse a problem, select a suitable strategy and apply an appropriate technique to obtain its solution
- apply combinations of mathematical skills and techniques in problem solving
- set out mathematical work, including the solution of problems, in a logical and clear form using appropriate symbols and terminology.

Relationship between Assessment Objectives and Components

The weightings allocated to each of the assessment objectives (AOs) are summarised below.

The table shows the assessment objectives as an approximate percentage of each component and as an approximate percentage of the overall Cambridge O Level Mathematics qualification.

Component	AO1 (%)	AO2 (%)	Weighting of component in overall qualification (%)
Paper 1	55 – 65	35 – 45	50
Paper 2	28 – 38	62 – 72	50
Weighting of AO in overall qualification	**40 – 50**	**50 – 60**	

Syllabus Content

Theme or topic	Subject content	Notes/examples
1. Number	*Candidates should be able to:* • identify and use natural numbers, integers (positive, negative and zero), prime numbers, square numbers, cube numbers, common factors and common multiples, rational and irrational numbers (e.g. π, $\sqrt{2}$), real numbers	Includes expressing numbers as a product of prime factors, finding the Lowest Common Multiple (LCM) and Highest Common Factor (HCF) of two or more numbers.
2. Set language and notation	• use language, notation and Venn diagrams to describe sets and represent relationships between sets Definition of sets: e.g. $A = \{x : x \text{ is a natural number}\}$ $B = \{(x, y): y = mx + c\}$ $C = \{x : a \leqslant x \leqslant b\}$ $D = \{a, b, c\ldots\}$	Includes using Venn diagrams to solve problems. Notation: Number of elements in set A — $n(A)$ "… is an element of …" — $\in$ "… is not an element of …" — $\notin$ Complement of set A — A' The empty set — $\varnothing$ Universal set — $\mathscr{E}$ A is a subset of B — $A \subseteq B$ A is a proper subset of B — $A \subset B$ A is not a subset of B — $A \nsubseteq B$ A is not a proper subset of B — $A \not\subset B$ Union of A and B — $A \cup B$ Intersection of A and B — $A \cap B$
3. Squares, square roots, cubes and cube roots	• calculate squares, square roots, cubes and cube roots of numbers	Includes recall of squares and their corresponding roots from 1 to 15 and cubes and their corresponding roots from 1 to 10.
4. Directed numbers	• use directed numbers in practical situations	e.g. temperature changes or flood levels
5. Vulgar and decimal fractions and percentages	• use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts • recognise equivalence and convert between these forms	
6. Ordering	• order quantities by magnitude and demonstrate familiarity with the symbols $=, \neq, >, <, \geqslant, \leqslant$	

Theme or topic	Subject content	Notes/examples
7. Standard form	• use the standard form $A \times 10^n$ where n is a positive or negative integer, and $1 \leqslant A < 10$	Convert numbers into and out of standard form. Calculate with values in standard form.
8. The four operations	• use the four operations for calculations with whole numbers, decimals and vulgar (and mixed) fractions, including correct ordering of operations and use of brackets	
9. Estimation	• make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem	e.g. by writing each number correct to one significant figure, estimate the value of $\frac{41.3}{9.79 \times 0.765}$
10. Limits of accuracy	• give appropriate upper and lower bounds for data given to a specified accuracy • obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy	e.g. measured lengths e.g. the calculation of the perimeter or the area of a rectangle
11. Ratio, proportion, rate	• demonstrate an understanding of ratio and proportion • increase and decrease a quantity by a given ratio • use common measures of rate • solve problems involving average speed	Divide a quantity in a given ratio. Direct and inverse proportion. Use scales in practical situations. Interpreting the ratio as old quantity : new quantity, e.g. decrease $240 in the ratio 5 : 3. e.g. hourly rate of pay or flow rates
12. Percentages	• calculate a given percentage of a quantity • express one quantity as a percentage of another • calculate percentage increase or decrease • carry out calculations involving reverse percentages	e.g. finding the cost price given the selling price and the percentage profit
13. Use of an electronic calculator	• use a calculator efficiently • apply appropriate checks of accuracy • enter a range of measures including 'time' • interpret the calculator display appropriately	e.g. enter 2 hours 30 minutes as 2.5 hours e.g. in money 4.8 means $4.80; in time 3.25 means 3 hours 15 minutes

Theme or topic	Subject content	Notes/examples
14. Time	• calculate times in terms of the 24-hour and 12-hour clock • read clocks, dials and timetables	Includes problems involving time zones.
15. Money	• solve problems involving money and convert from one currency to another	
16. Personal and small business finance	• use given data to solve problems on personal and small business finance involving earnings, simple interest and compound interest • extract data from tables and charts	Includes discount, and profit and loss (as an amount or a percentage). Knowledge of compound interest formula given below is required: Value of investment = $P\left(1+\frac{r}{100}\right)^n$ where P is the amount invested, r is the percentage rate of interest and n is the number of years of compound interest.
17. Algebraic representation and formulae	• use letters to express generalised numbers and express arithmetic processes algebraically • substitute numbers for words and letters in formulae • construct and transform formulae and equations	e.g. transform formulae where the subject appears twice or where a power of the subject appears e.g. construct equations from numerical and geometrical problems.
18. Algebraic manipulation	• manipulate directed numbers • use brackets and extract common factors • expand products of algebraic expressions • factorise where possible expressions of the form: $ax + bx + kay + kby$ $a^2x^2 - b^2y^2$ $a^2 + 2ab + b^2$ $ax^2 + bx + c$ • manipulate algebraic fractions • factorise and simplify rational expressions	e.g. factorise $9x^2 + 15xy$ e.g. expand $3x(2x - 4y)$, $(x + 4)(x - 7)$ e.g. $\frac{x}{3}+\frac{x-4}{2}$, $\frac{2x}{3}-\frac{3(x-5)}{2}$, $\frac{3a}{4}\times\frac{9a}{10}$, $\frac{3a}{4}\div\frac{9a}{10}$, $\frac{1}{x-2}+\frac{2}{x-3}$ e.g. $\frac{x^2-2x}{x^2-5x+6}$

Theme or topic	Subject content	Notes/examples
19. Indices	• understand and use the rules of indices • use and interpret positive, negative, fractional and zero indices	e.g. work out $2^{-3} \times 2^4$ e.g. simplify $3x^{-4} \times \frac{2}{3}x^{\frac{1}{2}}$, $\frac{2}{5}x^{\frac{1}{2}} \div 2x^{-2}$ and $\left(\frac{2x^5}{3}\right)^3$ e.g. $5^{\frac{1}{2}} = \sqrt{5}$ e.g. evaluate 2^5, 4^0, 5^{-2}, $100^{\frac{1}{2}}$, $8^{-\frac{1}{3}}$ e.g. solve $32^x = 2$
20. Solutions of equations and inequalities	• solve simple linear equations in one unknown • solve fractional equations with numerical and linear algebraic denominators • solve simultaneous linear equations in two unknowns • solve quadratic equations by factorisation, completing the square or by use of the formula • solve simple linear inequalities	Includes writing a quadratic expression in completed square form.
21. Graphical representation of inequalities	• represent linear inequalities graphically	Linear programming problems are not included.
22. Sequences	• continue a given number sequence • recognise patterns in sequences and relationships between different sequences • generalise sequences as simple algebraic statements	Includes linear sequences, quadratic and cubic sequences, exponential sequences and simple combinations of these. Including expressions for the *n*th term.
23. Variation	• express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities	Includes linear, square, square root and cubic variation (direct and inverse). e.g. y is inversely proportional to the square of x. Given that $y = 2$ when $x = 6$, find the value of y when $x = 2$
24. Graphs in practical situations	• interpret and use graphs in practical situations including travel graphs and conversion graphs • draw graphs from given data • apply the idea of rate of change to easy kinematics involving distance–time and speed–time graphs, acceleration and deceleration • calculate distance travelled as area under a linear speed–time graph	

Theme or topic	Subject content	Notes/examples
25. Graphs of functions	• construct tables of values and draw graphs for functions of the form ax^n where a is a rational constant, and $n = -2, -1, 0, 1, 2, 3$, and simple sums of not more than three of these and for functions of the form ka^x where a is a positive integer • interpret graphs of linear, quadratic, cubic, reciprocal and exponential functions • solve associated equations approximately by graphical methods • estimate gradients of curves by drawing tangents	
26. Function notation	• use function notation, e.g. $f(x) = 3x - 5$, $f: x \mapsto 3x - 5$, to describe simple functions • find inverse functions $f^{-1}(x)$	
27. Coordinate geometry	• demonstrate familiarity with Cartesian coordinates in two dimensions • find the gradient of a straight line • calculate the gradient of a straight line from the coordinates of two points on it • calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points • interpret and obtain the equation of a straight line graph in the form $y = mx + c$ • determine the equation of a straight line parallel to a given line • find the gradient of parallel and perpendicular lines	e.g. find the equation of a line parallel to $y = 4x - 1$ that passes through $(0, -3)$ e.g. find the gradient of a line perpendicular to $y = 3x + 1$ e.g. find the equation of a line perpendicular to one passing through the coordinates $(1, 3)$ and $(-2, -9)$

Theme or topic	Subject content	Notes/examples
28. Geometrical terms	• use and interpret the geometrical terms: point; line; plane; parallel; perpendicular; bearing; right angle, acute, obtuse and reflex angles; interior and exterior angles; similarity and congruence • use and interpret vocabulary of triangles, special quadrilaterals, circles, polygons and simple solid figures • understand and use the terms: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment	Includes the following terms: Triangles: equilateral, isosceles and scalene (including right-angled triangles). Quadrilaterals: square, rectangle, kite, rhombus, parallelogram, trapezium. Polygons: Regular and irregular polygons; pentagon, hexagon, octagon, decagon. Simple solid figures: cube, cuboid, prism, cylinder, pyramid, cone, sphere; face, surface, edge, vertex and net.
29. Geometrical constructions	• measure lines and angles • construct a triangle, given the three sides, using a ruler and pair of compasses only • construct other simple geometrical figures from given data, using a ruler and protractor as necessary • construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary • read and make scale drawings • use and interpret nets	
30. Similarity and congruence	• solve problems and give simple explanations involving similarity and congruence • calculate lengths of similar figures • use the relationships between areas of similar triangles, with corresponding results for similar figures, and extension to volumes and surface areas of similar solids	Includes showing that two triangles are similar or showing that two triangles are congruent (using correct congruence condition SSS, SAS, ASA, RHS). Includes use of scale factor.

Theme or topic	Subject content	Notes/examples
31. Symmetry	• recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions • recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone) • use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length	Includes properties of triangles, quadrilaterals and circles directly related to their symmetries.
32. Angles	• calculate unknown angles and give simple explanations using the following geometrical properties: (a) angles at a point (b) angles at a point on a straight line and intersecting straight lines (c) angles formed within parallel lines (d) angle properties of triangles and quadrilaterals (e) angle properties of regular and irregular polygons (f) angle in a semi-circle (g) angle between tangent and radius of a circle (h) angle at the centre of a circle is twice the angle at the circumference (i) angles in the same segment are equal (j) angles in opposite segments are supplementary	Candidates will be expected to use the correct geometrical terminology when giving reasons for answers. Angle properties of polygons includes angle sum.

Theme or topic	Subject content	Notes/examples
33. Loci	• use the following loci and the method of intersecting loci for sets of points in two dimensions which are: (a) at a given distance from a given point (b) at a given distance from a given straight line (c) equidistant from two given points (d) equidistant from two given intersecting straight lines	
34. Measures	• use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units	Convert between units including units of area and volume. e.g. between mm^2 and cm^2 or between cm^3, m^3 and litres
35. Mensuration	• solve problems involving: (a) the perimeter and area of a rectangle and triangle (b) the perimeter and area of a parallelogram and a trapezium (c) the circumference and area of a circle (d) arc length and sector area as fractions of the circumference and area of a circle (e) the surface area and volume of a cuboid, cylinder, prism, sphere, pyramid and cone (f) the areas and volumes of compound shapes	Formulae will be given for the surface area and volume of the sphere, pyramid and cone.

Theme or topic	Subject content	Notes/examples
36. Trigonometry	• interpret and use three-figure bearings • apply Pythagoras' theorem and the sine, cosine and tangent ratios for acute angles to the calculation of a side or of an angle of a right-angled triangle • solve trigonometrical problems in two dimensions involving angles of elevation and depression • extend sine and cosine functions to angles between 90° and 180° • solve problems using the sine and cosine rules for any triangle and the formula area of triangle = $\frac{1}{2}$ $ab \sin C$ • solve simple trigonometrical problems in three dimensions	Measured clockwise from the north, i.e. 000°–360°. e.g. Find the bearing of *A* from *B* if the bearing of *B* from *A* is 125° Angles will be quoted in, and answers required in, degrees and decimals of a degree to one decimal place. Calculations of the angle between two planes or of the angle between a straight line and plane will not be required.
37. Vectors in two dimensions	• describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$ or **a** • add and subtract vectors • multiply a vector by a scalar • calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2+y^2}$ • represent vectors by directed line segments • use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors • use position vectors	Vectors will be printed as $\overrightarrow{AB}$ or **a** and their magnitudes denoted by modulus signs, e.g. $\lvert\overrightarrow{AB}\rvert$ or |**a**|. In their answers to questions candidates are expected to indicate **a** in some definite way, e.g. by an arrow $\overrightarrow{AB}$ or by underlining as follows $\underline{a}$.

Theme or topic	Subject content	Notes/examples
38. Matrices	• display information in the form of a matrix of any order • solve problems involving the calculation of the sum and product (where appropriate) of two matrices, and interpret the results • calculate the product of a matrix and a scalar quantity • use the algebra of 2 × 2 matrices including the zero and identity 2 × 2 matrices • calculate the determinant $\|\mathbf{A}\|$ and inverse $\mathbf{A}^{-1}$ of a non-singular matrix **A**	
39. Transformations	• use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations • identify and give precise descriptions of transformations connecting given figures • describe transformations using coordinates and matrices	If M(a) = b and R(b) = c the notation RM(a) = c will be used. Invariants under these transformations may be assumed. Singular matrices are excluded.
40. Probability	• calculate the probability of a single event as either a fraction or a decimal • understand that the probability of an event occurring = 1 – the probability of the event not occurring • understand relative frequency as an estimate of probability • calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate	Probabilities should not be given as ratios. Problems could be set involving extracting information from tables or graphs. e.g. P(blue) = 0.8, find P(not blue) e.g. use results of experiments with a spinner to estimate the probability of a given outcome e.g. use probability to estimate from a population In possibility diagrams outcomes will be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of the branches.

Theme or topic	Subject content	Notes/examples
41. Categorical, numerical and grouped data	• collect, classify and tabulate statistical data • read, interpret and draw simple inferences from tables and statistical diagrams • calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used • calculate an estimate of the mean for grouped and continuous data • identify the modal class from a grouped frequency distribution	
42. Statistical diagrams	• construct and interpret bar charts, pie charts, pictograms, simple frequency distributions, frequency polygons, histograms with equal and unequal intervals and scatter diagrams • construct and use cumulative frequency diagrams • estimate and interpret the median, percentiles, quartiles and interquartile range for cumulative frequency diagrams • calculate with frequency density • understand what is meant by positive, negative and zero correlation with reference to a scatter diagram • draw a straight line of best fit by eye.	For unequal intervals on histograms, areas are proportional to frequencies and the vertical axis is labelled 'Frequency density'.

Topic 1

Numbers

1 *(J2007/P1/Q1)*

(a) Evaluate 3 + 25 ÷ 2. [1]

(b) Express $17\frac{1}{2}\%$ as a decimal. [1]

Thinking Process

(a) Do the division of 25 by 2 first and then add.

(b) Write $17\frac{1}{2}$ as 17.5 and divide it by 100.

Solution

(a) $3+25\div2$

$=3+\frac{25}{2}$

$=\frac{6+25}{2}$

$=\frac{31}{2}=15\frac{1}{2}$ Ans

(b) $17\frac{1}{2}\%$

$=17.5\times\frac{1}{100}=0.175$ Ans

2 *(J2007/P1/Q2)*

Evaluate

(a) $\frac{1}{4}+\frac{1}{7}$, [1]

(b) $1\frac{7}{8}\div\frac{3}{16}$. [1]

Thinking Process

(a) Make common denominator. Add.

(b) Write $1\frac{7}{8}$ as improper fraction. Multiply and reduce.

Solution

(a) $\frac{1}{4}+\frac{1}{7}$

$=\frac{7+4}{28}=\frac{11}{28}$ Ans.

(b) $1\frac{7}{8}\div\frac{3}{16}$

$=\frac{15}{8}\div\frac{3}{16}$

$=\frac{15}{8}\times\frac{16}{3}=10$ Ans.

3 *(J2007/P1/Q3)*

It is given that $\frac{2}{3}$, $\frac{8}{d}$ and $\frac{n}{39}$ are equivalent fractions. [1]

Find the value of d and the value of n. [1]

Thinking Process

It is given that all three fractions are equal. Take two fractions at a time and solve for the values of d and n.

Solution

Consider $\frac{2}{3}$ and $\frac{8}{d}$.

As the fractions are equal

$\therefore\ \frac{2}{3}=\frac{8}{d}$

$2d=24$

$d=\frac{24}{2}=12$ Ans.

Now consider $\frac{2}{3}$ and $\frac{n}{39}$

$\frac{2}{3}=\frac{n}{39}$

$78=3n$

$n=\frac{78}{3}=26$ Ans.

4 *(J2007/P1/Q5)*

(a) Write the following in order of size, starting with the smallest.

$\frac{66}{100}$ 0.6 0.67 $\frac{666}{1000}$

[1]

(b) The distance of Saturn from the Sun is 1507 million kilometres.
Express 1507 million in standard form. [1]

Thinking Process

(a) Express the fractions as decimals.

Solution

(a) Writing in decimal form we have

0.66 0.6 0.67 0.666

∴ starting from the smallest

0.6, $\frac{66}{100}$, $\frac{666}{1000}$, 0.67 Ans.

(b) $1507 \text{ million} = 1507000000$

$= 1.507 \times 10^9$ **Ans.**

5 *(J2007/P1/Q6)*

(a) Express 154 as the product of its prime factors. [1]

(b) Find the lowest common multiple of 154 and 49. [1]

Thinking Process

(a) To express as a product of prime factors ✐ think through the different ways prime numbers may be multiplied to give 154.

(b) To find the lowest common multiple (LCM) ✐ list down all the prime factors of 154 and 49.

Solution

(a) $154 = 2 \times 7 \times 11$ **Ans.**

(b) $154 = 2 \times 7 \times 11$, $49 = 7 \times 7$

$\Rightarrow$ The lowest common multiple $= 2 \times 7 \times 7 \times 11$

$= 1078$ **Ans.**

6 *(N2007/P1/Q1)*

(a) Express $22\frac{1}{2}$ % as a fraction in its lowest terms. [1]

(b) Evaluate 0.9×0.02. [1]

Thinking Process

(a) Multiply the value by $\frac{1}{100}$ ✐ Express the value as fraction

(b) evaluate 9 x 2 and count the number of decimal places.

Solution

(a) $22\frac{1}{2}$ %

$= \frac{45}{2} \times \frac{1}{100}$

$= \frac{45}{200} = \frac{9}{40}$ **Ans**

(b) $9 \times 2 = 18$

$\therefore$ $0.9 \times 0.02 = 0.018$ **Ans**

7 *(N2007/P1/Q2)*

Express as a single fraction in its lowest terms

(a) $3\frac{5}{9} - 2\frac{2}{3}$. [1]

(b) $\frac{3}{8} \div 2\frac{1}{4}$. [1]

Thinking Process

(a) Find LCM and simplify ✐ Express both fractions as an improper fraction.

(b) Express $2\frac{1}{4}$ as an improper fraction and evaluate.

Solution

(a) $3\frac{5}{9} - 2\frac{2}{3}$

$= \frac{32}{9} - \frac{8}{3}$

$= \frac{32 - 24}{9} = \frac{8}{9}$ **Ans**

(b) $\frac{3}{8} \div 2\frac{1}{4}$

$= \frac{3}{8} \div \frac{9}{4}$

$= \frac{3}{8} \times \frac{4}{9} = \frac{1}{6}$ **Ans**

8 *(N2007/P1/Q3)*

(a) Add 620 grams to 3.7 kilograms. Give your answer in kilograms. [1]

(b) Write the following numbers in order of size, starting with the smallest.

3^1 3^{-1} $(-1)^3$ 3^0 [1]

Thinking Process

(a) Express 3.7 kilograms in grams and add.

(b) Arrange the numbers in increasing order ✐ Rewrite the numbers in simpler form.

Solution

(a) $3.7 \text{ kg} = 3.7 \times 1000 \text{ g} = 3700 \text{ g}$

$\therefore$ $620 + 3700 = 4320 \text{ g}$

$\frac{4320}{1000} = 4.32 \text{ kg}$ **Ans**

(b) $3^1 = 3$

$3^{-1} = \frac{1}{3} = 0.333...$

$(-1)^3 = -1$

$3^0 = 1$

$\therefore$ starting with the smallest number, we have

$(-1)^3$, 3^{-1}, 3^0, 3^1 **Ans**

9 *(N2007/P1/Q19)*

(a) Estimate the value, correct to one significant figure, of $\frac{4.03^2 \times 29.88}{\sqrt{150}}$. [2]

(b) Sam ran 100 metres in 12 seconds. Calculate his average speed in kilometres per hour. [2]

Thinking Process

(a) Round 4.03 & 29.88 to one significant figure. For $\sqrt{150}$ ✎ look for a number whose square is close to 150.

(b) ✎ Note that 1 km = 1000 m & 1 hour = 3600 seconds.

Solution

(a) $\frac{4.03^2 \times 29.88}{\sqrt{150}} = \frac{4.03 \times 4.03 \times 29.88}{\sqrt{150}}$

$= \frac{4 \times 4 \times 30}{\sqrt{144}}$

$= \frac{4 \times 4 \times 30}{12} = 40$ **Ans**

(b) $\text{distance} = 100\text{m} = \frac{100}{1000}\text{ km} = \frac{1}{10}\text{ km}$

$\text{time} = 12\text{sec} = \frac{12}{3600}\text{ hr} = \frac{1}{300}\text{ hr}$

$\therefore$ $\text{Average speed} = \frac{\text{distance}}{\text{time}}$

$= \frac{\frac{1}{10}}{\frac{1}{300}}$

$= \frac{1}{10} \times \frac{300}{1} = 30\text{ km/h}$ **Ans**

10 *(J2008/P1/Q1)*

Evaluate

(a) $\frac{1}{2} - \frac{3}{7}$, [1]

(b) $2\frac{2}{3} \times 1\frac{3}{4}$. [1]

Thinking Process

(a) Take LCM and simplify.

(b) Express the mixed numbers as improper fractions and evaluate.

Solution

(a) $\frac{1}{2} - \frac{3}{7}$

$= \frac{7-6}{14} = \frac{1}{14}$ **Ans**

(b) $2\frac{2}{3} \times 1\frac{3}{4}$

$= \frac{8}{3} \times \frac{7}{4}$

$= \frac{14}{3} = 4\frac{2}{3}$ **Ans.**

11 *(J2008/P1/Q2)*

Evaluate

(a) $25 - 18.3$, [1]

(b) 1.7×0.03. [1]

Thinking Process

(a) Evaluate 25.0–18.3

(b) Multiply 17 by 3 and count the number of decimal places.

Solution

(a)
$$\begin{array}{r} 25.0 \\ -18.3 \\ \hline 6.7 \\ \hline \end{array}$$

$\therefore$ $25 - 18.3 = 6.7$ **Ans.**

(b) $17 \times 3 = 51$

$\therefore$ $1.7 \times 0.03 = 0.051$ **Ans.**

12 *(J2008/P1/Q5)*

It is given that $68.2 \times 0.235 = 16.027$.
Hence evaluate

(a) 0.0682×2350, [1]

(b) $160.27 \div 0.0235$. [1]

Thinking Process

(a) To find 0.0682 x 2350 ✎ use the fact that 68.2 x 0.235 = 16.027 and count the number of decimal places.

(b) Express 160.27 ÷ 0.0235 in terms of 16.027 ÷ 0.235. ✎ Use the fact that 16.027 ÷ 0.235 = 68.2

Solution

(a) 0.0682×2350

$= \left(68.2 \times 10^{-3}\right) \times \left(0.235 \times 10^{4}\right)$

$= (68.2 \times 0.235) \times 10^{4-3}$

$= 16.027 \times 10 = 160.27$ **Ans.**

(b) $160.27 \div 0.0235$

$= \frac{160.27}{0.0235}$

$= \frac{16.027 \times 10}{0.235 \times \frac{1}{10}}$

$= \frac{16.027}{0.235} \times 10 \times \frac{10}{1}$

$= 68.2 \times 100 = 6820$ **Ans.**

13 *(N2008/P1/Q1)*

Evaluate

(a) 0.3×0.06, [1]

(b) $0.4 + 0.3 \times 5$. [1]

Thinking Process

(a) Multiply 3 by 6 and count the number of decimal places.

(b) ✎ Do multiplication first, and then addition.

Solution

(a) $3 \times 6 = 18$

$\therefore\ 0.3 \times 0.06 = 0.018$ **Ans.**

(b) $0.4 + 0.3 \times 5$

$= 0.4 + 1.5 = 1.9$ **Ans.**

14 *(N2008/P1/Q2)*

(a) Express 0.45 as a fraction, giving your answer in its lowest terms. [1]

(b) Express $\frac{13}{40}$ as a percentage [1]

Thinking Process

(a) Write 0.45 as $\frac{45}{100}$ and reduce the fraction.
(b) Multiply the fraction by 100.

Solution

(a) $0.45 = \frac{45}{100} = \frac{9}{20}$ **Ans.**

(b) $\frac{13}{40} \times 100$

$= \frac{65}{2} = 32\frac{1}{2}\%$ **Ans.**

15 *(N2008/P1/Q3)*

Evaluate

(a) $3\frac{1}{5} - 2\frac{2}{3}$. [1]

(b) $4^{\frac{3}{2}}$. [1]

Thinking Process

(a) Express the mixed numbers as improper fractions and evaluate.
(b) Rewrite 4 as 2^2.

Solution

(a) $3\frac{1}{5} - 2\frac{2}{3}$

$= \frac{16}{5} - \frac{8}{3}$

$= \frac{48 - 40}{15} = \frac{8}{15}$ **Ans.**

(b) $4^{\frac{3}{2}}$

$= (2^2)^{\frac{3}{2}} = 2^3 = 8$ **Ans.**

16 *(J2009/P1/Q1)*

(a) Evaluate $17 - 5 \times 3 + 1$. [1]

(b) Express 0.82 as a percentage. [1]

Thinking Process

(a) Recall BODMAS rules.
(b) Multiply by 100.

Solution

(a) $17 - 5 \times 3 + 1$

$= 17 - 15 + 1$

$= 3$ **Ans.**

(b) 0.82×100

$= \frac{82}{100} \times 100 = 82\%$ **Ans.**

17 *(J2009/P1/Q2)*

Express as a single fraction in its lowest terms,

(a) $\frac{8}{9} \times \frac{3}{4}$, [1]

(b) $\frac{3}{4} - \frac{2}{3}$. [1]

Thinking Process

(a) Evaluate the given expression.
(b) Take LCM and simplify.

Solution

(a) $\frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$ **Ans.**

(b) $\frac{3}{4} - \frac{2}{3}$

$= \frac{9 - 8}{12} = \frac{1}{12}$ **Ans.**

18 *(J2009/P1/Q3)*

(a) Write down the two cube numbers between 10 and 100. [1]

(b) Write down the two prime numbers between 30 and 40. [1]

Thinking Process

(a) Think of numbers between 10 and 100 whose cube roots are whole numbers.
(b) Recall, prime numbers are whole numbers that cannot be exactly divided by any number except 1 and themselves.

Solution

(a) The two cube numbers are 27 and 64 **Ans.**

(b) The two prime numbers are 31 and 37 **Ans.**

19 (J2009/P1/Q5)

(a) Evaluate 0.5×0.007. [1]

(b) Evaluate $\frac{1}{1.25}$ as a decimal. [1]

Thinking Process

(a) To find 0.5×0.007 use the fact that $5 \times 7 = 35$ and count the number of decimal places.
(b) Remove the decimal from 1.25 and simplify the resulting fraction.

Solution

(a) $5 \times 7 = 35$

$\therefore\ 0.5 \times 0.007 = 0.0035$ **Ans.**

(b) $\frac{1}{1.25} \times \frac{100}{100}$

$= \frac{100}{125} = \frac{4}{5} = 0.8$ **Ans.**

20 (J2009/P1/Q6)

(a) Write down all the factors of 18. [1]
(b) Write 392 as the product of its prime factors. [1]

Thinking Process

(a) List all the numbers which have 18 as their multiple.
(b) Think of different ways prime numbers may be multiplied to give 392.

Solution

(a) Factors of 18 = 1, 2, 3, 6, 9, 18 **Ans.**

(b) $392 = 2 \times 2 \times 2 \times 7 \times 7 = 2^3 \times 7^2$ **Ans.**

21 (N2009/P1/Q1)

(a) Evaluate $\frac{2}{3} - \frac{4}{7}$. [1]

(b) Evaluate $1\frac{1}{3} \times \frac{5}{8}$, giving your answer in its simplest form. [1]

Thinking Process

(a) Find common denominator.
(b) Express $1\frac{1}{3}$ as improper fraction.

Solution

(a) $\frac{2}{3} - \frac{4}{7}$

$= \frac{14-12}{21} = \frac{2}{21}$ **Ans.**

(b) $1\frac{1}{3} \times \frac{5}{8}$

$= \frac{4}{3} \times \frac{5}{8} = \frac{5}{6}$ **Ans.**

22 (N2009/P1/Q2)

(a) Add brackets to the equation in the answer space to make it correct.

$$4 + 6 \times 7 - 5 = 16$$ [1]

(b) Find the value of 27×0.002. [1]

Thinking Process

(a) Multiply 6 to the result of 7 − 5 and add.
(b) Multiply 27 by 2 and count the number of decimal places.

Solution

(a) $4 + \{6 \times (7 - 5)\} = 16$ **Ans.**

(b) $27 \times 2 = 54$

$\therefore\ 27 \times 0.002 = 0.054$ **Ans.**

23 (N2009/P1/Q3)

Arrange these values in order of size, starting with the smallest

$\frac{9}{20}$ 0.39 46% $\frac{2}{5}$ [2]

Thinking Process

Express $\frac{9}{20}$ and 46% as decimals.

Solution

$\frac{9}{20}$	0.39	46%	$\frac{2}{5}$
= 0.45	0.39	0.46	0.4

$\therefore$ starting with the smallest, the numbers are rearranged as 0.39, $\frac{2}{5}$, $\frac{9}{20}$, 46% **Ans.**

24 (N2009/P1/Q4)

The numbers 294 and 784, written as the product of their prime factors, are

$294 = 2 \times 3 \times 7^2$, $784 = 2^4 \times 7^2$

Find
(a) the largest integer which is a factor of both 294 and 784, [1]
(b) $\sqrt{784}$. [1]

Thinking Process

(a) Find the highest common factor, i.e. the product of common factors.
(b) Break 784 into its prime factors.

Solution

(a) Largest common factor $= 2 \times 7^2 = 98$ **Ans.**

(b) $\sqrt{784} = \sqrt{2^4 \times 7^2}$

$= \sqrt{(2^2)^2 \times 7^2}$

$= 2^2 \times 7 = 28$ **Ans.**

25 (J2010/P1/Q1)

Evaluate

(a) $1.5 - 0.2 \times 4$, [1]

(b) $4.2 \div 0.07$. [1]

Thinking Process

(a) Multiply first. Observe BODMAS rules.
(b) Multiply both decimals by 100

Solution

(a) $1.5 - 0.2 \times 4$

$= 1.5 - 0.8$

$= 0.7$ **Ans.**

(b) $4.2 \div 0.07$

$= \frac{4.2}{0.07} \times \frac{100}{100}$

$= \frac{420}{7} = 60$ **Ans.**

26 (J2010/P1/Q2)

Express as a single fraction

(a) $\frac{5}{7} - \frac{2}{5}$, [1]

(b) $1\frac{1}{5} \div 2\frac{1}{3}$. [1]

Thinking Process

(a) Find common denominator.
(b) Change both mixed numbers into improper fractions.

Solution

(a) $\frac{5}{7} - \frac{2}{5}$

$= \frac{25 - 14}{35}$

$= \frac{11}{35}$ **Ans.**

(b) $1\frac{1}{5} \div 2\frac{1}{3}$

$= \frac{6}{5} \div \frac{7}{3}$

$= \frac{6}{5} \times \frac{3}{7}$

$= \frac{18}{35}$ **Ans.**

27 (J2010/P1/Q5)

By writing each number correct to 1 significant figure, estimate the value of

$$\frac{48.9 \times 0.207^2}{3.94}.$$ [2]

Thinking Process

Round off each number to 1 sig.fig.

Solution

$\frac{48.9 \times 0.207^2}{3.94} = \frac{50 \times 0.2^2}{4}$

$= \frac{50 \times \left(\frac{2}{10}\right)^2}{4}$

$= \frac{50 \times \frac{4}{100}}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$ **Ans.**

28 (J2010/P1/Q9)

Written as a product of prime factors, $168 = 2^3 \times 3 \times 7$.

(a) Express 140 as a product of its prime factors. [1]

(b) Find the highest common factor of 168 and 140. [1]

(c) Find the smallest positive integer, n, such that $168n$ is a square number. [1]

Thinking Process

(a) Break 140 into its prime factors.
(b) HCF of 140 &168 is the product of common prime factors of the two numbers.
(c) Make all the prime factors of 168 a perfect square.

Solution

(a) $140 = 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$ **Ans.**

(b) $140 = 2^2 \times 5 \times 7$

$168 = 2^3 \times 3 \times 7$

$\therefore$ Highest common factor $= 2^2 \times 7$

$= 28$ **Ans.**

(c) $168 = 2^3 \times 3 \times 7$

$= 2^2 \times 2 \times 3 \times 7$

$168n = 2^2 \times 2 \times 3 \times 7 \times (2 \times 3 \times 7)$

$\therefore$ $n = 42$ **Ans.**

29 *(N2010/P1/Q1)*

(a) Evaluate $3\frac{1}{7}-2\frac{1}{3}$. [1]

(b) Evaluate $\frac{2}{9}\times 1\frac{7}{8}$, giving your answer as a fraction in its lowest terms. [1]

Thinking Process

(a) ✐ Express both mixed numbers into improper fractions.

(b) ✐ Express $1\frac{7}{8}$ into improper fraction. Multiply the two fractions by cancelling any common factors.

Solution

(a) $3\frac{1}{7}-2\frac{1}{3}$

$=\frac{22}{7}-\frac{7}{3}$

$=\frac{66-49}{21}$

$=\frac{17}{21}$ **Ans.**

(b) $\frac{2}{9}\times 1\frac{7}{8}$

$=\frac{2}{9}\times\frac{15}{8}$

$=\frac{5}{12}$ **Ans.**

30 *(N2010/P1/Q2)*

(a) Evaluate $6.3 \div 0.09$. [1]

(b) Find the decimal number that is exactly halfway between 3.8 and 4.3. [1]

Thinking Process

(a) ✐ Rewrite each decimal as fraction.

(b) Find the average of the two numbers.

Solution

(a) $6.3\div 0.09$

$=\frac{63}{10}\div\frac{9}{100}$

$=\frac{63}{10}\times\frac{100}{9}$

$=70$ **Ans.**

(b) $\frac{3.8+4.3}{2}$

$=\frac{8.1}{2}$

$=4.05$ **Ans.**

31 *(N2010/P1/Q22)*

(a) Express, correct to two significant figures,

(i) 15823.769, [1]

(ii) 0.0030489. [1]

(b) Use your answers to part (a) to estimate, correct to one significant figure, the value of

$15823.769 \times 0.0030489$. [2]

Thinking Process

(a) ✐ Round off (i) & (ii) to 2 sig. fig.

(b) Make use of the answer found in (i) & (ii) to find the product.

Solution

(a) (i) $15823.769 \approx 16000$ **Ans.**

(ii) $0.0030489 \approx 0.0030$ **Ans.**

(b) $15823.769 \times 0.0030489 \approx 16000 \times 0.0030$

$=16000\times\frac{30}{10000}$

$=48 \approx 50$ (1 sf) **Ans.**

32 *(J2011/P1/Q1)*

(a) Evaluate $12+6\div 2-8$. [1]

(b) Evaluate 2.6×0.2. [1]

Thinking Process

(a) ✐ Divide first. Observe BODMAS rules.

(b) ✐ Change decimals into fractions and evaluate.

Solution

(a) $12+6\div 2-8$.

$=12+3-8$

$=7$ **Ans.**

(b) 2.6×0.2

$=\frac{26}{10}\times\frac{2}{10}$

$=\frac{52}{100}=0.52$ **Ans.**

33 *(J2011/P1 Q2)*

(a) It is given that $\frac{1}{5}<n<\frac{1}{4}$.

Write down a decimal value of n that satisfies this inequality. [1]

(b) Express $\frac{48}{60}$ as a percentage. [1]

Thinking Process

(a) ✐ Write any decimal value that falls in the given range.

(b) To express in percentage ✐ multiply by 100.

Solution with TEACHER's COMMENT

(a) $\frac{1}{5} < n < \frac{1}{4} \Rightarrow 0.2 < n < 0.25$

$\therefore \quad n = 0.22$ **Ans.**

Note that n can have any value between 0.21 to 0.24

(b) $\frac{48}{60} \times 100 = 80\%$ **Ans.**

34 (J2011/P1/Q3)

(a) Evaluate $\frac{2}{3} - \frac{3}{8}$. [1]

(b) Evaluate $1\frac{3}{4} \times \frac{2}{9}$, giving your answer as a fraction in its lowest terms. [1]

Thinking Process

(a) Take LCM. Simplify.
(b) Change the mixed number into improper fraction and solve.

Solution

(a) $\frac{2}{3} - \frac{3}{8}$

$= \frac{16-9}{24}$

$= \frac{7}{24}$ **Ans.**

(b) $1\frac{3}{4} \times \frac{2}{9}$

$= \frac{7}{4} \times \frac{2}{9}$

$= \frac{7}{18}$ **Ans.**

35 (J2011/P1/Q14)

(a) Express 108 as a product of its prime factors. [1]

(b) Written as products of their prime factors, $N = 2^p \times 5^q \times 7^r$ and $500 = 2^2 \times 5^3$.
The highest common factor of N and 500 is $2^2 \times 5^2$.
The lowest common multiple of N and 500 is $2^3 \times 5^3 \times 7$.
Find p, q and r. [2]

Thinking Process

(a) Perform prime factorisation of 108.
(b) use your knowledge of LCM and HCF to find the values of p, q and r.

Solution

(a) $108 = 2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^3$ **Ans.**

(b) $N = 2^p \times 5^q \times 7^r, \quad 500 = 2^2 \times 5^3$

HCF of N and $500 = 2^2 \times 5^2$

LCM of N and $500 = 2^3 \times 5^3 \times 7$

$\Rightarrow \quad p = 3, \quad q = 2, \quad r = 1$ **Ans.**

36 (J2011/P1/Q19)

(a) Express 0.047 852 correct to two decimal places. [1]

(b) Estimate the value of $\sqrt{200}$, giving your answer correct to two significant figures. [1]

(c) By writing each number correct to one significant figure, estimate the value of
$\frac{212 \times 1.97^2}{0.763}$. [2]

Thinking Process

(a) Round off to 1 significant figure.
(b) Write $\sqrt{200}$ as $\sqrt{196}$.
(c) Round off each number to one sig. fig.

Solution

(a) $0.047852 = 0.05$ **Ans.**

(b) $\sqrt{200} \approx \sqrt{196} = 14$ **Ans.**

(c) $\frac{212 \times 1.97^2}{0.763} \approx \frac{200 \times 2^2}{0.8}$

$= \frac{800}{0.8} \times \frac{10}{10}$

$= \frac{8000}{8} = 1000$ **Ans.**

37 (N2011/P1/Q1)

(a) Evaluate $2\frac{3}{4} - 1\frac{7}{9}$. [1]

(b) Evaluate $0.7 - 0.1 \times 3$. [1]

Thinking Process

(a) Find common denominator change the mixed fractions into improper fractions.
(b) Multiply first. Observe BODMAS rules.

Solution

(a) $2\frac{3}{4} - 1\frac{7}{9}$

$= \frac{11}{4} - \frac{16}{9}$

$= \frac{99-64}{36}$

$= \frac{35}{36}$ **Ans.**

(b) $0.7 - 0.1 \times 3$

$= 0.7 - 0.3$

$= 0.4$ **Ans.**

38 (N2011/P1/Q11)

By writing each number correct to two significant figures, estimate, correct to one significant figure, the value of

$$\sqrt{110.94 - 0.2034 \times 368.62}.$$ [2]

Thinking Process

Round off each number to 2 significant figure. Solve.

Solution

$\sqrt{110.94 - 0.2034 \times 368.62}$

$= \sqrt{110 - 0.20 \times 370}$

$= \sqrt{110 - (\frac{20}{100} \times 370)}$

$= \sqrt{110 - (2 \times 37)}$

$= \sqrt{110 - 74}$

$= \sqrt{36} = \pm 6$ **Ans.**

39 (J2012/P1/Q1)

(a) Express 72% as a fraction in its lowest terms. [1]

(b) Write down two fractions that are equivalent to 0.4. [1]

Thinking Process

(a) Divide 72 by 100.
(b) Express 0.4 as a fraction. Multiply the numerator and denominator by 2, 3, 4 ......

Solution *with* TEACHER's COMMENTS

(a) $72\% = \frac{72}{100} = \frac{18}{25}$ **Ans.**

(b) $0.4 = \frac{4}{10} = \frac{2}{5}$

∴ two equivalent fractions are.

$\frac{6}{15}$ and $\frac{8}{20}$ **Ans.**

Equivalent fractions are fractions that have the same value. To make an equivalent fraction, multiply or divide the numerator and denominator by the same number.

40 (J2012/P1/Q9)

(a) Evaluate $\frac{2}{5} + \frac{3}{8}$. [1]

(b) Evaluate $1\frac{2}{3} \times 2\frac{1}{4}$, giving your answer as a mixed number in its lowest terms. [2]

Thinking Process

(a) Make common denominator. Add.
(b) Express both mixed fractions as improper fractions.

Solution

(a) $\frac{2}{5} + \frac{3}{8}$

$= \frac{16+15}{40} = \frac{31}{40}$ **Ans.**

(b) $1\frac{2}{3} \times 2\frac{1}{4}$

$= \frac{5}{3} \times \frac{9}{4}$

$= \frac{15}{4} = 3\frac{3}{4}$ **Ans.**

41 (J2012/P1/Q10)

(a) Evaluate $6 \times 3 + 8 \div 2$. [1]

(b) By writing each number correct to 1 significant figure, estimate the value of

$$\frac{19.2 \times 9.09}{0.583}.$$ [2]

Thinking Process

(a) Do the multiplication and division first. Observe BODMAS rule.
(b) Round each number to 1 significant figure.

Solution

(a) $6 \times 3 + 8 \div 2$

$= 18 + 4$

$= 22$ **Ans.**

(b) $\frac{19.2 \times 9.09}{0.583}$

$= \frac{20 \times 9}{0.6}$

$= 20 \times 9 \times \frac{10}{6}$

$= 300$ **Ans.**

42 (N2012/P1/Q1)

(a) Evaluate $8 + 2 \times 1.3$. [1]

(b) Express 0.06 as a fraction, giving your answer in its lowest terms. [1]

Thinking Process

(a) Multiply first, then add 8.
(b) Express 0.06 as a fraction.

Solution

(a) $8 + 2 \times 1.3$

$= 8 + 2.6$

$= 10.6$ **Ans.**

(b) $0.06 = \frac{6}{100}$

$= \frac{3}{50}$ **Ans.**

43 *(N2012/P1/Q1)*

(a) Evaluate $\frac{2}{3} + 2\frac{1}{4}$. [1]

(b) Evaluate $3^0 + 3^1$. [1]

Thinking Process

(a) Change the mixed number to improper fraction.

(b) Note that $3^0 = 1$

Solution

(a) $\frac{2}{3} + 2\frac{1}{4}$

$= \frac{2}{3} + \frac{9}{4}$

$= \frac{8+27}{12}$

$= \frac{35}{12} = 2\frac{11}{12}$ **Ans.**

(b) $3^0 + 3^1 = 1 + 3$

$= 4$ **Ans.**

44 *(N2012/P1/Q5)*

Arrange these numbers in order, starting with the smallest.

$\frac{3}{4} \quad 0 \quad -1 \quad -\frac{17}{20} \quad -\frac{4}{5}$ [2]

Thinking Process

Express the fractions as decimals.

Solution

$\frac{3}{4} \quad 0 \quad -1 \quad -\frac{17}{20} \quad -\frac{4}{5}$

Writing in decimal form.

$0.75 \quad 0 \quad -1 \quad -0.85 \quad -0.8$

$\therefore$ starting with the smallest,

$-1 \quad -\frac{17}{20} \quad -\frac{4}{5} \quad 0 \quad \frac{3}{4}$ **Ans.**

45 *(N2012/P1/Q10)*

(a) Express 180 as the product of its prime factors. [1]

(b) $\sqrt{180}$ can be expressed in the form $p\sqrt{q}$, where p and q are integers.
Find the smallest value of $p + q$. [1]

Thinking Process

(a) Think of different ways prime numbers may be multiplied to give 180.

(b) Use answer to (a) and express $\sqrt{180}$ in the required form.

Solution

(a) $180 = 2 \times 2 \times 3 \times 3 \times 5$

$= 2^2 \times 3^2 \times 5$ **Ans.**

(b) $\sqrt{180} = \sqrt{2^2 \times 3^2 \times 5}$

$= 2 \times 3 \times \sqrt{5}$

$= 6\sqrt{5}$

smallest value of $p + q = 6 + 5 = 11$ **Ans.**

46 *(J2013/P1/Q1)*

Evaluate

(a) $\frac{4}{7} - \frac{2}{5}$ [1]

(b) $\frac{5}{8} \div \frac{2}{3}$ [1]

Thinking Process

(a) Find common denominator.

(b) Rewrite $\frac{5}{8} \div \frac{2}{3}$ as $\frac{5}{8} \times \frac{3}{2}$.

Solution

(a) $\frac{4}{7} - \frac{2}{5}$

$= \frac{20-14}{35} = \frac{6}{35}$ **Ans.**

(b) $\frac{5}{8} \div \frac{2}{3}$

$= \frac{5}{8} \times \frac{3}{2} = \frac{15}{16}$ **Ans.**

47 *(J2013/P1/Q2)*

A bag contains red counters and blue counters.
On each counter there is either an odd or an even number.
The table shows the number of counters of each type.

	Odd	Even
Red	6	9
Blue	5	3

(a) Find the fraction of the counters that are blue. [1]

(b) Find the ratio of odd to even numbers. [1]

Thinking Process

(a) Express the number of blue counters as a fraction of total number of counters.

(b) To find the ratio find the total number of odd and even numbers.

Solution

(a) Total number of counters $= 6+9+5+3 = 23$

fraction of blue counters $= \frac{5+3}{23} = \frac{8}{23}$ **Ans.**

(b) Total odd numbers $= 6+5 = 11$

total even numbers $= 9+3 = 12$

$\therefore$ ratio of odd to even numbers $= 11:12$ **Ans.**

48 *(J2013/P1/Q3)*

(a) Write these lengths in order of size, starting with the shortest.

500 m 5 cm 50 km 500 mm [1]

(b) Convert 41.6cm^2 to mm^2. [1]

Thinking Process

(a) Express all lengths in cm. Compare.
(b) Note that $1\text{ cm}^2 = 100\text{ mm}^2$

Solution

(a)

500 m	5 cm	50 km	500 mm
= 50000 cm	5 cm	5000000 cm	50 cm

$\therefore$ starting with the smallest.

5 cm 500 mm 500 m 50 km **Ans.**

(b) 41.6cm^2

$= 41.6 \times 100$

$= 4160\text{ mm}^2$ **Ans.**

Note that:
$1\text{cm} = 10\text{mm}$
$(1\text{cm})^2 = (10\text{mm})^2$
$1\text{cm}^2 = 100\text{mm}^2$

49 *(J2013/P1/Q8)*

(a) James thinks of a two-digit number.
It is a cube number.
It is an even number.
What is his number? [1]

(b) Omar thinks of a two-digit number.
It is a factor of 78.
It is a prime number.
What is his number? [1]

(c) Write down an irrational number between 1 and 2. [1]

Thinking Process

(a) A cube number results by multiplying an integer by itself three times.
An even number is an integer that can be divided exactly by 2

(b) To find the number break 78 into its prime factors.

(c) Irrational numbers are numbers that cannot be written as a ratio of two integers

Solution

(a) 64. **Ans.**

(b) $78 = 2 \times 3 \times 13$

$\therefore$ the number is 13. **Ans.**

(c) $\sqrt{2}$ **Ans.**

50 *(J2013/P1/Q9)*

(a) Write 0.0040751 correct to two significant figures. [1]

(b) $\sqrt{131}$ lies between two consecutive integers. Complete the inequality below with these integers.

$\ldots\ldots\ldots < \sqrt{131} < \ldots\ldots\ldots$ [1]

(c) Add brackets to the statement below to make it correct.

$3 \times 2 + 1^2 = 49$ [1]

Thinking Process

(a) Round off to 2 significant figures.
(b) Look for two integers whose square is close to 131.
(c) Take note that $7^2 = 49$

Solution

(a) 0.0041 **Ans.**

(b) $11 < \sqrt{131} < 12$ **Ans.**

(c) $(3 \times 2 + 1)^2 = 49$ **Ans.**

51 *(N2013/P1/Q2)*

(a) Evaluate $3\frac{1}{4} - 1\frac{4}{5}$. [1]

(b) Evaluate 3.01×0.02. [1]

Thinking Process

(a) Express the mixed numbers as improper fractions and evaluate.
(b) Rewrite each decimal as fraction. Multiply and convert back to decimal.

Solution

(a) $3\frac{1}{4} - 1\frac{4}{5}$

$= \frac{13}{4} - \frac{9}{5}$

$= \frac{65-36}{20}$

$= \frac{29}{20} = 1\frac{9}{20}$ **Ans.**

(b) 3.01×0.02

$=\frac{301}{100}\times\frac{2}{100}$

$=\frac{602}{10000}$

$=0.0602$ **Ans.**

52 *(N2013/P1/Q9)*

By making suitable approximations, estimate the value of $\frac{\sqrt{35.78}\times\sqrt[3]{1005}}{0.3012}$.

Show clearly the approximate values you use. [2]

Thinking Process

Round off 0.3012 to one significant figure. Look for numbers whose square root and cube root are close to 35.78 and 1005 respectively.

Solution

$\frac{\sqrt{35.78}\times\sqrt[3]{1005}}{0.3012}$

$=\frac{\sqrt{36}\times\sqrt[3]{1000}}{0.3}$

$=\frac{6\times 10}{\frac{3}{10}}$

$=60\times\frac{10}{3}=200$ **Ans.**

53 *(J2014/P1/Q1)*

(a) Evaluate $12+8\div(9-5)$. [1]

(b) Evaluate $0.018\div 0.06$. [1]

Thinking Process

(a) Evaluate the bracket first. Observe BODMAS rules.

(b) You may change each decimal to fraction first.

Solution

(a) $12+8\div(9-5)$

$=12+8\div(4)$

$=12+\frac{8}{4}$

$=12+2$

$=14$ **Ans.**

(b) $0.018\div 0.06$

$=\frac{18}{1000}\div\frac{6}{100}$

$=\frac{18}{1000}\times\frac{100}{6}$

$=\frac{3}{10}=0.3$ **Ans.**

54 *(J2014/P1/Q2)*

Tasnim records the temperature, in °C, at 6 a.m. every day for 10 days.

−6 −3 0 −2 −1 −7 −5 2 −1 −3

(a) Find the difference between the highest and the lowest temperatures. [1]

(b) Find the median temperature. [1]

Thinking Process

(a) ✎ Subtract the lowest temperature from the highest temperature.

(b) Find the middle temperature ✎ Write the temperatures in increasing order.

Solution

(a) $2-(-7)$

$=2+7=9$ °C **Ans.**

(b) Writing the temperatures in increasing order.

−7 −6 −5 −3 −3 −2 −1 −1 0 2

median temperature $=\frac{-3-2}{2}$

$=-\frac{5}{2}=-2.5$ °C **Ans.**

55 *(J2014/P1/Q9)*

(a) Evaluate $\frac{1}{7}+\frac{3}{4}$. [1]

(b) Evaluate $5\frac{1}{3}\div 1\frac{3}{5}$, giving your answer as a mixed number in its lowest terms. [2]

Thinking Process

(a) Make common denominator. Add.

(b) ✎ Change both mixed numbers into improper fractions. Multiply and reduce.

Solution

(a) $\frac{1}{7}+\frac{3}{4}$

$=\frac{4+21}{28}=\frac{25}{28}$ **Ans.**

(b) $5\frac{1}{3}\div 1\frac{3}{5}$

$=\frac{16}{3}\div\frac{8}{5}$

$=\frac{16}{3}\times\frac{5}{8}$

$=\frac{10}{3}=3\frac{1}{3}$ **Ans.**

56 *(J2014/P1/Q10)*

(a) Write 405 917 628 correct to three significant figures. [1]

(b) By writing each number correct to one significant figure, estimate the value of

$$\frac{41.3}{9.79 \times 0.765}.$$ [2]

Thinking Process

(a) Round off to 3 significant figures.

(b) Round off each number to one significant figure.

Solution

(a) 405 917 628 = 406 000 000 (3 sig.fig.) **Ans.**

(b) $\frac{41.3}{9.79 \times 0.765}$

$= \frac{40}{10 \times 0.8}$

$= \frac{40}{8} = 5$ **Ans.**

57 *(N2014/P1/Q5)*

(a) Write the value of 1234.567, correct to 2 significant figures. [1]

(b) Write down an estimate for the value of $\sqrt{\frac{28}{\pi}}$. [1]

Thinking Process

(a) Round off to 2 significant figures.

(b) Round off 28 to 27 and π to 3.

Solution

(a) $1234.567 = 1200$ **Ans.**

(b) $\sqrt{\frac{28}{\pi}}$

$= \sqrt{\frac{27}{3}}$

$= \sqrt{9} = 3$ **Ans.**

58 *(J2015/P1/Q1)*

(a) Evaluate $\frac{1.3 + 2.9}{0.2}$. [1]

(b) Evaluate $2\frac{1}{4} \times \frac{1}{5}$. [1]

Thinking Process

(a) Remove the decimal from the fraction and simplify.

(b) Write $2\frac{1}{4}$ as improper fraction and simplify.

Solution

(a) $\frac{1.3 + 2.9}{0.2}$

$= \frac{4.2}{0.2} \times \frac{10}{10}$

$= \frac{42}{2} = 21$ **Ans.**

(b) $2\frac{1}{4} \times \frac{1}{5}$

$= \frac{9}{4} \times \frac{1}{5} = \frac{9}{20}$ **Ans.**

59 *(J2015/P1/Q2)*

Write these numbers in order of size, starting with the smallest.

$$\frac{13}{20} \quad 0.7 \quad \frac{7}{12} \quad 0.64 \quad \frac{5}{8}$$ [2]

Thinking Process

Express the fractions as decimals.

Solution

$\frac{13}{20} \quad 0.7 \quad \frac{7}{12} \quad 0.64 \quad \frac{5}{8}$

$= 0.65 \quad 0.7 \quad 0.583 \quad 0.64 \quad 0.625$

∴ starting from the smallest.

$\frac{7}{12}, \frac{5}{8}, 0.64, \frac{13}{20}, 0.7$ **Ans.**

60 *(J2015 P1 Q6)*

By writing each number correct to one significant figure, estimate the value of

$$\frac{29.3^2}{2.04 \times 0.874}.$$ [2]

Thinking Process

Round off each number to 1 significant figure.

Solution

$\frac{29.3^2}{2.04 \times 0.874} \approx \frac{30^2}{2 \times 0.9}$

$= \frac{900}{1.8} \times \frac{10}{10}$

$= \frac{9000}{18}$

$= 500$ **Ans.**

61 *(J2015 P1 Q13)*

(a) Express 60 as a product of its prime factors. [1]

(b) Find the smallest possible integer m such that $60m$ is a square number. [1]

(c) The lowest number that is a multiple of both 60 and the integer n is 180 .
Find the smallest possible value of n. [1]

Thinking Process

(a) Perform prime factorisation of 60.
(b) Make all the prime factors perfect square.
(c) Write the prime factors of 60 and 180. Note that 180 is the LCM of 60 and n.

Solution

(a)

2	60
2	30
3	15
5	5
	1

$\therefore\ 60 = 2^2 \times 3 \times 5$ **Ans.**

(b) $60m = 2^2 \times 3 \times 5 \times (3 \times 5)$

$\therefore\ m = 15$ **Ans.**

(c) $60 = 2 \times 2 \times 3 \times 5$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

$\therefore$ smallest value of $n = 3 \times 3$

$= 9$ **Ans.**

Possible values of n are:
$3 \times 3 = 9$
$3 \times 3 \times 2 = 18$
$3 \times 3 \times 5 = 45$
$3 \times 3 \times 2 \times 5 = 90$
$3 \times 3 \times 2 \times 2 \times 5 = 180$
$\therefore$ the least value of n is 9

62 (N2015/P1/Q1)

(a) Evaluate 0.03×0.3. [1]

(b) Evaluate $5 - 2(3 - 1.4)$. [1]

Thinking Process

(a) Rewrite each decimal as fraction.
(b) Evaluate the bracket first

Solution

(a) 0.03×0.3

$= \frac{3}{100} \times \frac{3}{10}$

$= \frac{9}{1000} = 0.009$ **Ans.**

(b) $5 - 2(3 - 1.4)$

$= 5 - 2(1.6)$

$= 5 - 3.2 = 1.8$ **Ans.**

63 (N2015/P1/Q5)

(a) Write the number 0.050 462 correct to 3 significant figures. [1]

(b) By writing each number correct to 1 significant figure, estimate the value of

$$\frac{8.94 \times 0.201}{28.8}.$$ [1]

Thinking Process

(a) Round off to 3 significant figures.
(b) Round off each number to 1 significant figure.

Solution

(a) $0.050\,462 = 0.0505$ **Ans.**

(b) $\frac{8.94 \times 0.201}{28.8}$

$= \frac{9 \times 0.2}{30}$

$= \frac{9 \times 2}{30 \times 10}$

$= \frac{3}{50} = 0.06$ **Ans.**

64 (N2015/P1/Q12)

(a) Express 198 as the product of its prime factors. [1]

(b) $M = 2^2 \times 3 \times 5^2$ $\quad N = 2^3 \times 3^2 \times 7$

(i) Find the largest number that divides exactly into M and N. [1]

(ii) Find the smallest value of k, such that $M \times k$ is a cube number. [1]

Thinking Process

(a) Break 198 into its prime factors.
(b) (i) Find the G.C.D or H.C.F by comparing the prime factors of M and N.
(ii) Make all the prime factors perfect cube.

Solution

(a)

2	198
3	99
3	33
11	11
	1

$\therefore\ 198 = 2 \times 3^2 \times 11$ **Ans.**

(b) (i) $M = 2^2 \times 3 \times 5^2$

$N = 2^3 \times 3^2 \times 7$

$\therefore$ GCD of M and $N = 2^2 \times 3$

$= 12$ **Ans.**

GCD = greatest common divisor.

(ii) $M \times k = 2^2 \times 3 \times 5^2 \times k$

$= 2^2 \times 3 \times 5^2 \times (2 \times 3^2 \times 5)$

$\therefore\ k = 2 \times 3^2 \times 5$

$= 90$ **Ans.**

65 (J2016 P1 Q1)

(a) Evaluate $(2.05+1.4)\times 0.2$. [1]

(b) Evaluate $1\frac{1}{3}-\frac{4}{5}$. [1]

Thinking Process

(a) Evaluate the bracket first.

(b) Find common denominator. ✐ Change $1\frac{1}{3}$ into improper fraction.

Solution

(a) $(2.05+1.4)\times 0.2$

$=3.45\times 0.2=0.69$ **Ans.**

(b) $1\frac{1}{3}-\frac{4}{5}$

$=\frac{4}{3}-\frac{4}{5}$

$=\frac{20-12}{15}=\frac{8}{15}$ **Ans.**

66 (J2016 P1 Q4)

Complete the table.

Fraction		Decimal		Percentage
$\frac{1}{2}$	=	0.5	=	50%
$\frac{3}{20}$	=	....	=	....
....	=	....	=	62.5%

[2]

Thinking Process

In 2nd row, write the fraction as a decimal, then multiply by 100 to express it as a percentage.

In 3rd row, divide 62.5% by 100 to change it into decimal, then express the decimal as a fraction

Solution

Fraction		Decimal		Percentage
$\frac{1}{2}$	=	0.5	=	50%
$\frac{3}{20}$	=	0.15	=	15%
$\frac{5}{8}$	=	0.625	=	62.5%

67 (J2016 P1 Q6)

(a) Express 96 as a product of its prime factors. [1]

(b) 24 is a common factor of 96 and the integer n. Given that n is less than 96, find the largest possible value of n. [1]

Thinking Process

(a) Write the prime factors of 96.

(b) To find n ✐ find the largest multiple of 24 that is less than 96.

Solution

(a) $96=2\times 2\times 2\times 2\times 2\times 3$

$=2^5\times 3$ **Ans.**

(b) Integer n is a multiple of 24.

∴ largest value of n that is less than $96=24\times 3$

$=72$ **Ans.**

68 (J2016 P1 Q13)

(a) Write these values in order of size, starting with the smallest.

$2^5 \quad 5^2 \quad \sqrt[3]{1000} \quad 27^0$ [1]

(b) Write down one possible value of x that satisfies each inequality.

(i) $2<\sqrt{x}<3$ [1]

(ii) $-1<x^3<0$ [1]

Thinking Process

(a) Arrange the numbers in increasing order ✐ Rewrite the numbers in simpler form.

(b) Simplify the inequality and write down one value of x that lie within the given range.

Solution

(a) $2^5 \quad 5^2 \quad \sqrt[3]{1000} \quad 27^0$

$= 32 \quad 25 \quad 10 \quad 1$

∴ starting with the smallest, the numbers are: 27^0, $\sqrt[3]{1000}$, 5^2, 2^5 **Ans.**

(b) (i) $2<\sqrt{x}<3$

$=(2)^2<(\sqrt{x})^2<(3)^2$

$=4<x<9$

∴ one possible value of $x=6$ **Ans.**

(ii) $-1<x^3<0$

$=(-1)^{\frac{1}{3}}<(x^3)^{\frac{1}{3}}<(0)^{\frac{1}{3}}$

$=-1<x<0$

∴ one possible value of $x=-0.5$ **Ans.**

69 *(N2016/P1/Q1)*

(a) Evaluate $9.03 - (4.273 + 2.3)$. [1]

(b) Evaluate $\frac{8}{9} - \frac{6}{7}$. [1]

Thinking Process

(a) Evaluate the bracket first

(b) Find common denominator.

Solution

(a) $9.03-(4.273+2.3)$

$= 9.03 - 6.573$

$= 2.457$ **Ans.**

(b) $\frac{8}{9}-\frac{6}{7}$

$= \frac{56-54}{63} = \frac{2}{63}$ **Ans.**

70 *(N2016/P1/Q2)*

Given that $192 \times 64.3 = 12345.6$, write down the values of

(a) 0.192×643, [1]

(b) $\frac{12.3456}{192}$. [1]

Thinking Process

(a) Express 0.192×643 in the form 192×64.3.

(b) To evaluate Use the fact that $\frac{12345.6}{192} = 64.3$

Solution

(a) 0.192×643,

$= \frac{192}{1000} \times (64.3 \times 10)$

$= \frac{192 \times 64.3}{100}$

$= \frac{12345.6}{100} = 123.456$ **Ans.**

(b) $\frac{12.3456}{192}$

$= \frac{12345.6}{192} \times \frac{1}{1000}$

$= 64.3 \times \frac{1}{1000}$

$= 0.0643$ **Ans.**

71 *(N2016/P1/Q4)*

(a) The total cost of 3 pencils is \$1.23 .
Find the total cost of 5 pencils. [1]

(b) Arrange the following in order, starting with the smallest.

74% $\quad -0.7 \quad 0.7 \quad -\frac{3}{4}$ [1]

Thinking Process

(a) Using ratio concepts, find the cost of 5 pencils.

(b) Rewrite the numbers in simpler form, then arrange them in increasing order.

Solution

(a) Cost of 3 pencils = \$1.23

cost of 5 pencils $= \$\frac{1.23}{3} \times 5$

$= \$2.05$ **Ans.**

(b) 74% $\quad -0.7 \quad 0.7 \quad -\frac{3}{4}$

$= 0.74 \quad -0.7 \quad 0.7 \quad -0.75$

$\therefore$ starting with the smallest, the numbers are :

$-\frac{3}{4}$, -0.7, 0.7, 74% **Ans.**

72 *(N2016/P1/Q6)*

By making suitable approximations, estimate the value of $\frac{\sqrt{3.98} \times 602.3}{2.987}$.

Show clearly the approximations you use. [2]

Thinking Process

Round off each number to one significant figure and simplify.

Solution

$\frac{\sqrt{3.98} \times 602.3}{2.987}$

$= \frac{\sqrt{4} \times 600}{3}$

$= \frac{2 \times 600}{3} = 400$ **Ans.**

73 *(J2017/P1/Q1)*

(a) Evaluate $\frac{4}{5} - \frac{1}{3}$. [1]

(b) Evaluate 0.2×0.006. [1]

Thinking Process

(a) Find common denominator.

(b) Multiply 2 by 6 and count the number of decimal places.

Solution

(a) $\frac{4}{5}-\frac{1}{3}$

$= \frac{12-5}{15} = \frac{7}{15}$ **Ans.**

(b) $2 \times 6 = 12$

$\therefore \quad 0.2 \times 0.006 = 0.0012$ **Ans.**

74 *(J2017 P1 Q3)*

By writing each number correct to one significant figure, estimate the value of

$$\frac{58.7\times 4.08}{19.7^3}.$$ [2]

Thinking Process

Round off each number to 1 sig. fig.

Solution

$\frac{58.7\times 4.08}{19.7^3}$

writing each number correct to one significant figure.

$=\frac{60\times 4}{20^3}$

$=\frac{60\times 4}{8000}=\frac{3}{100}=0.03$

75 *(J2017 P1 Q20)*

(a) (i) Write 54 as the product of its prime factors. [1]

(ii) Find the smallest possible integer m such that $54m$ is a cube number. [1]

(b) Find the value of k in each of the following.

(i) $\sqrt{27}=3^k$ [1]

(ii) $\left(\frac{1}{4}\right)^{-3}=2^k$ [1]

Thinking Process

(a) (i) Break 54 into its prime factors.

(ii) Make all the prime factors perfect cube.

(b) (i) Rewrite 27 as base of 3.

Solution

(a) (i) $54=2\times 3\times 3\times 3$ **Ans.**

(ii) $54m=2\times 3^3\times(2^2)$

$\therefore\ m=4$ **Ans.**

(b) (i) $\sqrt{27}=3^k$

$(3^3)^{\frac{1}{2}}=3^k$

$(3)^{\frac{3}{2}}=3^k \Rightarrow k=\frac{3}{2}$ **Ans.**

(ii) $\left(\frac{1}{4}\right)^{-3}=2^k$

$(4)^3=2^k$

$(2^2)^3=2^k$

$(2)^6=2^k \Rightarrow k=6$ **Ans.**

76 *(N2017 P1 Q1)*

(a) Evaluate $\frac{6}{7}-\frac{3}{5}$. [1]

(b) Evaluate $\frac{90}{0.45}$. [1]

Thinking Process

(a) Make common denominator. Add.

(b) Multiply both numerator and denominator by 100.

Solution

(a) $\frac{6}{7}-\frac{3}{5}$

$=\frac{30-21}{35}=\frac{9}{35}$ **Ans.**

(b) $\frac{90}{0.45}$

$=\frac{90}{0.45}\times\frac{100}{100}$

$=\frac{90\times 100}{45}=200$ **Ans.**

77 *(N2017 P1 Q10)*

By making suitable approximations, calculate an estimate for $\frac{40.32\times\sqrt{35.7}}{2980}$.

Show clearly the approximations you use and give your answer correct to 1 significant figure. [2]

Thinking Process

Round off each number to the nearest whole number and simplify.

Solution

$\frac{40.32\times\sqrt{35.7}}{2980}$

$=\frac{40\times\sqrt{36}}{3000}$

$=\frac{40\times 6}{3000}$

$=\frac{2}{25}=0.08$ **Ans.**

78 *(J2018 P1 Q1)*

(a) Evaluate $\frac{4}{11}-\frac{2}{7}$. [1]

(b) Evaluate 0.9×0.011. [1]

Thinking Process

(a) Make common denominator. Subtract.

(b) Rewrite each decimal as fraction. Multiply and convert back to decimal.

Solution

(a) $\frac{4}{11}-\frac{2}{7}$

$\frac{28-22}{77}=\frac{6}{77}$ Ans.

(b) 0.9×0.011

$\frac{9}{10}\times\frac{11}{1000}$

$\frac{99}{10000}=0.0099$ Ans.

79 *(J2018/P1/Q3)*

0.05 −0.3 1.3 −1.2 0.2

(a) Arrange the five numbers in order, starting with the smallest. [1]

(b) For the five numbers, find

(i) the mean, [1]

(ii) the range. [1]

Thinking Process

(a) Arrange the numbers in increasing order.

(b) (i) To find mean divide the sum of all numbers by 5.

(ii) To find the range subtract the least number from the largest number.

Solution

(a) Starting with the smallest, the numbers are:

−1.2 −0.3 0.05 0.2 1.3

(b) (i) Mean $=\frac{-1.2+(-0.3)+0.05+0.2+1.3}{5}$

$=\frac{-1.5+1.55}{5}=\frac{0.05}{5}=0.01$

(ii) Range: $1.3-(-1.2)$

$=1.3+1.2=2.5$

80 *(J2018/P1/Q10)*

By writing each number correct to 2 significant figures, calculate an estimate of

$$\frac{596\times\sqrt{16.12}}{0.2984}.$$ [2]

Thinking Process

Round off each number to 2 significant figures.

Solution

$$\frac{596\times\sqrt{16.12}}{0.2984}=\frac{600\times\sqrt{16}}{0.30}$$

$$=\frac{600\times4}{0.30}$$

$$=(600\times4)\times\frac{10}{3}=8000$$ Ans.

81 *(N2018/P1/Q1)*

(a) Evaluate $\frac{1}{8}+\frac{2}{5}$. [1]

(b) Evaluate 0.04×0.11. [1]

Thinking Process

(a) Make common denominator. Add.

(b) evaluate 4 x 11 and count the number of decimal places.

Solution

(a) $\frac{1}{8}+\frac{2}{5}$

$\frac{5+16}{40}=\frac{21}{40}$ Ans.

(b) $4\times11=44$

$\therefore\ 0.04\times0.11=0.0044$ Ans.

82 *(N2018/P1/Q4)*

Arrange these fractions in order, starting with the smallest.

$$\frac{4}{5}\quad\frac{9}{10}\quad\frac{17}{20}\quad\frac{21}{25}\quad\frac{41}{50}$$

Show your working. [2]

Thinking Process

Express the fractions as decimals.

Solution

Writing in decimal form we have.

0.8 0.9 0.85 0.84 0.82

∴ starting from the smallest.

$$\frac{4}{5}\quad\frac{41}{50}\quad\frac{21}{25}\quad\frac{17}{20}\quad\frac{9}{10}$$ Ans.

83 *(N2018/P1/Q10)*

By writing each number correct to 2 significant figures, estimate the value of

$$\frac{1212.3}{299.35\times\sqrt{24.73}}.$$ [2]

Thinking Process

Round off each number to two significant figures and simplify.

Solution

$$\frac{1212.3}{299.35\times\sqrt{24.73}}$$

$$=\frac{1200}{300\times\sqrt{25}}=\frac{4}{5}$$ Ans.

Topic 1a

Everyday Mathematics

1 (J2007/P1/Q4)

(a) A car decelerates uniformly from 20m/s to 5m/s in 25 seconds.
Calculate the retardation. [1]

(b) Express 20 metres per second in kilometres per hour. [1]

Thinking Process

(a) retardation = $\frac{\text{change in speed}}{\text{time taken}}$.

(b) 1 km = 1000m, and 1 hr = 3600 seconds.

Solution with TEACHER's COMMENTS

(a) $\frac{5-20}{25} = -\frac{15}{25} = -\frac{3}{5} = -0.6$

$\therefore$ retardation $= 0.6\,m/s^2$ **Ans.**

Note that retardation means negative acceleration.

(b) $20\,m/s = 20 \times \frac{3600}{1000} = 72$ km/h **Ans.**

2 (J2007/P1/Q18)

(a) Calculate 5% of $280 000. [1]

(b) A single carton of juice costs $4.20.
A special offer pack of 3 cartons costs $9.45.
Ali bought a special offer pack instead of 3 single cartons.
Calculate his percentage saving. [2]

Thinking Process

(b) To find his percentage savings ✐ find the cost of one carton from special offer pack. Subtract it from the given cost of one carton and find the savings. Express the savings as a percentage of the normal cost.

Solution

(a) 5% of $280000 = \frac{5}{100} \times 280000 = \$14000$ **Ans.**

(b) Cost of 3 cartons altogether = $9.45

$\therefore$ cost of one carton $= \frac{9.45}{3} = \$3.15$

Given cost of single carton = $4.20

savings $= 4.20 - 3.15 = \$1.05$

percentage savings $= \frac{1.05}{4.20} \times 100 = 25\%$ **Ans.**

3 (J2007/P2/Q1)

(a) The table shows the fares charged by a taxi company.

$1.20 per kilometre for the first 10 km
then
80 cents for each additional kilometre after the first 10 km

(i) Calculate the fare for a journey of
(a) 8 km, [1]
(b) 24 km. [1]

(ii) Find the length of the journey for which the fare was $16. [2]

(b) The table gives the times of high tides at a harbour.

Date	May 5	May 6	May 7
Times	10 00	11 20	00 36
	2256		1250

(i) Calculate, in hours and minutes, the length of time between the high tide on May 6 and the morning high tide on May 7. [1]

(ii) Given that low tides occurred midway between high tides, calculate the time of the low tide on the afternoon of May 5. [2]

(c) The height of a mountain is 1800 metres.
It is suggested that this mountain has been worn away at an average rate of 0.15mm per year.
Assuming that the suggestion is correct, calculate the height of the mountain 20 million years ago. [2]

Thinking Process

(a) (i) (a) Note that the taxi is charging $1.20 for the first 10 km.

(b) First calculate the fare for 10 km. Then calculate the fare for additional kilometres.

(ii) Subtract the fare for 10 km from the given amount. Divide your answer by 80 cents to get the number of kilometres.

(b) (i) Subtract 1120 from 0036 ✐ add 2400 to 0036.

(ii) Find the time that is exactly between 1000 and 2256.

(c) To find the height 20 million years ago ✐ find the height that has been worn away in 20 million years. Then add it to the present height.

Solution with TEACHER'S COMMENTS

(a) (i) (a) Fare for 1 km = $1.20

Fare for 8 km = 1.20 × 8 = $9.60 **Ans**

(b) Fare for the first 10 km = 1.20 × 10

= $12.0

additional kilometres = 24 − 10 = 14

fare for the additional 14 km

$= 14 \times \frac{80}{100} = \$11.20$

∴ total fare = 12.0 + 11.20

= $23.20 **Ans**

> Note that 80 cents = 0.8 dollars

(ii) Fare for the first 10 km = $12.0

remaining amount = $16 − $12 = $4

No. of additional kilometres $= \frac{4}{0.8} = 5$

∴ length of journey = 10 + 5 = 15 km **Ans**

(b) (i) High tide on May 7: 0036 or 2436

High tide on May 6: − 1120

1316

∴ length of time = 13 hours, 16 min. **Ans**

> Note that adding 2400 to 0036 is to facilitate the subtraction.

(ii) Mid-time between two high tides

$= \frac{1000 + 2256}{2} = 1628$

∴ time of low tide on May 5 afternoon

= 1628 **Ans**

(c) Height decrease in 1 year = 0.15 mm

∴ height decrease in 20 million years

= 0.15 × 20000000 = 3000000 mm

$= \frac{3000000}{1000} = 3000\text{ m}$

Height of the mountain 20 million years ago

was 1800 + 3000 = 4800 m **Ans**

4 *(N2007/P1/Q5)*

(a) The rate of exchange between dollars and euros was $0.8 to 1 euro.
Calculate the number of euros received in exchange for $300. [1]

(b) Find the simple interest on $450 for 18 months at 4% per year. [1]

Thinking Process

(a) To find the number of euros divide $300 by $0.8.

(b) Use $I = \frac{PRT}{100}$ to calculate the simple interest.

Solution

(a) $0.8 = 1 euro

$\$1 = \frac{1}{0.8}$ euros

$\$300 = \frac{1}{0.8} \times 300$ euros

= 375 euros **Ans**

(b) $I = \frac{PRT}{100}$

$= \frac{450 \times 4 \times \frac{18}{12}}{100}$

$= \frac{450 \times 6}{100} = 27$

∴ simple interest = $27 **Ans**

> Note that time must be expressed in years.

5 *(N2007/P1/Q11)*

The mass of a marble is given as 5.4 grams, correct to the nearest tenth of a gram.
The mass of a box is given as 85 grams, correct to the nearest 5 grams.

(a) Complete the table in the answer space.

	Lower bound	Upper bound
Mass of 1 marble	.... g	.... g
Mass of the box	.... g	.... g

[2]

(b) Find the lower bound for the total mass of the box and 20 identical marbles. [1]

Thinking Process

(a) To find the lower and upper bound for the mass of one marble, ✐ consider the error (0.1 ÷ 2 = 0.05) in the measurement.
To find the lower and upper bound for the mass of one box ✐ consider the error (5 ÷ 2 = 2.5) in the measurement.

(b) ✐ Find the least possible mass of one box and 20 marbles.

Solution

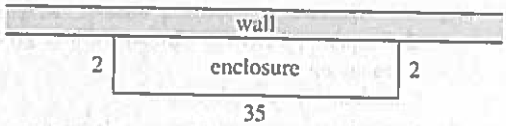
(a)

	Lower bound	Upper bound
Mass of 1 marble	5.35 g	5.45 g
Mass of the box	82.5 g	87.5 g

(b) 82.5 + 20(5.35)

= 82.5 + 107 = 189.5 g **Ans**

6 *(N2007/P1 Q15)*



A farmer wishes to build a rectangular enclosure against a straight wall.
He has 39 identical fence panels, each 1 metre long.
One possible arrangement, which encloses an area of 70 m^2, is shown in the diagram and recorded in the table below.
Find the length of the enclosure which would contain the largest area.
Write down this length and the largest area.
Record all your trials in the table.
Marks will be awarded for clear, appropriate working.

Width (m)	2				
Length (m)	35				
Area (m^2)	70				

[3]

Thinking Process

Use the given table, and record the area of the enclosure with different widths. Look for the width that gives maximum area.

Solution

Width (m)	2	3	4	5	6	7	8	9	10	11	12
Length (m)	35	33	31	29	27	25	23	21	19	17	15
Area (m^2)	70	99	124	145	162	175	184	189	190	187	180

Observing the table we see that the area of the enclosure keeps on increasing for widths 2 m to 10 m. When the width is 11 m (or greater), the area starts to decrease.

$\therefore$ Length of enclosure which gives largest area = 19 m **Ans**

Largest area = 190 m^2 **Ans**

7 *(N2007/P1 Q23)*

The foot of a mountain is at sea level.
The temperature at the foot of the mountain was 16 °C.
The temperature at a height of 3000 m on the mountain was –4 °C.

(a) Find the difference between these temperatures. [1]

(b) Given that the temperature fell at a constant rate, find

(i) the temperature at a height of 1800 m, [1]

(ii) the height at which the temperature was 0 °C, [1]

(iii) an expression, in terms of x, for the temperature, in °C, at a height of x metres. [2]

Thinking Process

(b) (i) To find the temperature at a height of 1800m ⇒ Find the temperature change for one metre.

(ii) To find the required height ⇒ consider the rate at which the temperature is falling.

(iii) To find an expression in x ⇒ find the temperature change for 1m first.

Solution

(a) Difference = $16-(-4) = 20$ °C **Ans**

(b) (i) Temperature change in 3000 m = 20 °C

temperature change in 1 m = $\frac{20}{3000}$ °C

temperature change in 1800 m = $\frac{20}{3000}\times 1800$

= 12 °C

temperature at sea level is 16 °C

$\therefore$ temperature at 1800 m is: $16-12=4$°C

(ii) 20°C —— 3000 m

1°C —— $\frac{3000}{20}=150$ m

temperature falls by 1°C after every 150 m

temperature at sea level is 16 °C

$\therefore$ req. height at 0°C is = 16×150

= 2400 m **Ans**

(iii) Temperature change in 3000 m = 20 °C

temperature change in 1 m = $\frac{20}{3000}=\frac{1}{150}$ °C

temperature change in x m = $\frac{x}{150}$ °C

temperature at sea level is 16 °C

$\therefore$ temperature at x m is: $\left(16-\frac{x}{150}\right)$ °C **Ans**

8 *(N2007/P2 Q2)*

It is given that $x = a+\sqrt{a^2+b^2}$.

(a) (i) Calculate x when $a = 0.73$ and $b = 1.84$.
Give your answer correct to 2 decimal places. [2]

(ii) Express b in terms of x and a. [3]

(b) A shopkeeper sells pens and pencils.
Each pen costs \$5 and each pencil costs \$3.
One day he sold x pens.
On the same day he sold 9 more pens than pencils.

(i) Write down an expression, in terms of x, for his total income from the sale of these pens and pencils. [2]

(ii) This total income was less than \$300.
Form an inequality in x and solve it. [2]

(iii) Hence write down the maximum number of pens that he sold. [1]

Thinking Process

(a) (i) Substitute the values of a and b in the given equation

(ii) make b the subject.

(b) (i) Read the question carefully and form an expression. Note that he sold x pens which are 9 more than pencils.

(iii) Figure out your answer from part (b) (ii)

Solution

(a) (i) $x = a + \sqrt{a^2 + b^2}$

$= 0.73 + \sqrt{(0.73)^2 + (1.84)^2}$

$= 0.73 + \sqrt{0.5329 + 3.3856}$

$= 0.73 + \sqrt{3.9185}$

$= 0.73 + 1.9795$

$= 2.7095 \approx 2.71$ (3 sf) **Ans**

(ii) $x = a + \sqrt{a^2 + b^2}$

$x - a = \sqrt{a^2 + b^2}$

squaring both sides

$(x-a)^2 = \left(\sqrt{a^2+b^2}\right)^2$

$(x-a)^2 = a^2 + b^2$

$b^2 = (x-a)^2 - a^2$

$a^2 - b^2 = (a+b)(a-b)$

$b^2 = (x-a+a)(x-a-a)$

$b^2 = (x)(x-2a)$

$b^2 = x^2 - 2ax$

$b = \pm\sqrt{x^2 - 2ax}$ **Ans**

(b) (i) Cost of one pen = \$5

cost of one pencil = \$3

no. of pens sold = x

as he sold 9 more pens than pencils

$\Rightarrow$ no. of pencils sold = $x - 9$

$\therefore$ Total income = \$ $(5x + 3(x-9))$

= \$ $(5x + 3x - 27)$

= \$ $(8x - 27)$ **Ans**

(ii) $8x - 27 < 300$

$8x < 327$

$x < \frac{327}{8}$

$x < 40.875$ **Ans**

(iii) From part (b) (ii), we see that

$x < 40.875$

$\therefore$ max. number of pens sold = 40 **Ans**

9 *(N2007/P2/Q4)*

(a) In 2005, the cost of posting a letter was 28 cents. A company posted 1200 letters and was given 4% discount on the cost.
Calculate the total discount. [1]

(b) In 2006, the cost of posting a letter was increased from 28 cents to 35 cents.
Calculate the percentage increase in the cost of posting a letter. [2]

(c) After the price increase to 35 cents, the cost to the company of posting 1200 letters was \$399.
Calculate the percentage discount that the company was given in 2006. [2]

(d) In 2006, it cost \$4.60 to post a parcel.
This was an increase of 15% on the cost of posting the parcel in 2005.
Calculate the cost of posting this parcel in 2005. [3]

Thinking Process

(a) Calculate 4% of the total cost ✐ Find the total cost of posting 1200 letters.

(b) To calculate the percentage increase ✐ calculate the increase.

(c) Find the actual cost of posting 1200 letters. Subtract \$399 from it and calculate the percentage discount.

(d) ✐ \$4.60 is equivalent to 115%. Hence find 100% of the amount.

Solution

(a) Total cost of posting 1200 letters = 1200×28

= 33600 cents

= \$336

Total discount = $\frac{4}{100} \times 336 = \$13.44$ **Ans**

(b) Cost increase = $35 - 28 = 7$ cents

percentage increase = $\frac{7}{28} \times 100 = 25\%$ **Ans**

(c) Total cost of posting 1200 letters = 1200×35

= 42000 cents

= \$420

discount given = $420 - 399 = \$21$

percentage discount = $\frac{21}{420} \times 100 = 5\%$ **Ans**

% discount = $\frac{\text{discount}}{\text{amount on which discount was given}}$

(d) 115% —— \$4.60

100% —— $\frac{4.60}{115} \times 100 = \$4$

$\therefore$ cost of posting the parcel in 2005 is \$4.00 **Ans**

10 (J2008/P1/Q14)

(a) A jar contained 370 g of jam.
Usman ate 30% of the jam.
What mass of jam remained in the jar? [1]

(b) In 2006 the population of a town was 30 000.
This was 5000 more than the population in 1999.
Calculate the percentage increase in population. [2]

Thinking Process

(a) To find the remaining mass ✐ find 70% of 370 g.
(b) ✐ Express the increase in population as a percentage of the original population.

Solution

(a) Mass of remaining jam = 70% of 370

$= \frac{70}{100} \times 370 = 259$ g **Ans.**

(b) Population in 1999 = 30000 − 5000 = 25000

$\therefore$ percentage increase $= \frac{5000}{25000} \times 100 = 20\%$ **Ans.**

11 (J2008/P2/Q2)

(a) Anne's digital camera stores its images on a memory card.
The memory card has 128 units of storage space.
When 50 images were stored, there were 40 units of **unused** storage space on the memory card.
(i) Calculate the percentage of **unused** storage space on the memory card. [1]
(ii) Calculate the average amount of storage space used by each image. [2]

(b) Shop A charged 60 cents for each photograph.
Shop B charged 63 cents for each photograph and gave a discount of $1 on all purchases more than $10.
(i) Anne bought 24 photographs from Shop A and paid with a $20 note.
Calculate the change she received. [1]
(ii) Find how much cheaper it was to buy 24 photographs from Shop B than from Shop A. [2]
(iii) Find the smallest number of photographs for which it was cheaper to use Shop B. [2]

Thinking Process

(a) (ii) Note that 128 − 40 = 88 units store 50 images.
(b) (i) Calculate the cost of 24 photographs and subtract it from $20.
(ii) Calculate the cost of 24 photographs from shop B. Remember the discount of $1 on purchases above $10.
(iii) Note that shop B is expensive if the purchases are less than $10. Therefore compare the charges of both shops for above $10.

Solution with TEACHER's COMMENTS

(a) (i) Percentage $= \frac{40}{128} \times 100 = 31.25\%$

$\approx 31.3\%$ **Ans.**

(ii) Used storage space = 128 − 40 = 88 units.

50 images were stored in 88 units

$\therefore$ storage space per image $= \frac{88}{50}$

$= 1.76$ **Ans.**

(b) (i) Cost of 24 photographs $= 24 \times 0.60 = \$\,14.40$

change received $= 20 - 14.40 = \$\,5.60$ **Ans.**

(ii) Shop B:
cost of 24 photographs $= 24 \times 0.63 = \$15.12$
discount = $1
total amount payable $= 15.12 - 1 = \$\,14.12$
difference from shop A $= 14.40 - 14.12$
$= \$0.28$

$\therefore$ shop B is cheaper by $0.28 or 28 cents **Ans.**

(iii) Let x be the smallest number of photographs.
To avail the discount, purchases must be above $10

$\Rightarrow \quad 0.63x > 10$

$x > \frac{10}{0.63}$

$x > 15.8$

$\therefore$ smallest number of photographs for which shop B is cheaper to use is 16 **Ans.**

> Note:
> Shop B is expensive to use if the total amount is less than $10. For 15 photographs, shop A costs $15 \times 0.60 = \$9.00$, whereas shop B costs $15 \times 0.63 = \$9.45$ which is less than $10, therefore no discount is given.
>
> 16 photographs from shop A cost: $16 \times 0.60 = \$9.60$
> 16 photographs from shop B cost: $16 \times 0.63 = \$10.08$
> Amount payable to shop B after discount
> $= \$10.08 - \$1 = \$9.08$
>
> Therefore shop B is cheaper to use for a minimum of 16 photographs

12 (J2008 P2 Q3)

(a) On average, Jim's heart beats 75 times per minute. Calculate the number of times his heart beats during 50 weeks.
Give your answer in standard form. [2]

(b) After an exercise, Ali and Ben measured their heart rates.
The ratio of their heart rates was 15:17.
Ben's heart beat 18 times per minute more than Ali's.
Calculate Ali's heart rate. [2]

(c) The recommended maximum heart rate, H, for a man during exercise, is given by the formula

$$H = \frac{4}{5}(220 - n).$$

where n years is the age of the man.
(i) Calculate H when $n = 25$. [1]
(ii) Calculate n when $H = 144$. [1]
(iii) Make n the subject of this formula. [2]

Thinking Process

(a) Calculate the heart beat for one hour, for one day, for one week and then for 50 weeks.
(b) 17 units represent 18 beats more than Ali.
(c) (i) & (ii) Substitute the values of n and H in the given formula.

Solution

(a) 1 min —— 75 beats
1 hr —— $75 \times 60 = 4500$ beats
1 day —— $4500 \times 24 = 108000$ beats
1 week —— $108000 \times 7 = 756000$ beats
50 weeks —— $756000 \times 50 = 37800000$
$= 3.78 \times 10^7$ beats

$\therefore$ Jim's heart beats 3.78×10^7 times. **Ans.**

(b)

Ali	Ben
15	17
x	$x+18$

$\therefore \frac{15}{x} = \frac{17}{x+18}$

$15(x+18) = 17x$

$15x + 270 = 17x$

$2x = 270$

$x = \frac{270}{2} = 135$

$\therefore$ Ali's heart rate = 135 beats per minute **Ans.**

(c) (i) when $n = 25$

$H = \frac{4}{5}(220 - 25)$

$= \frac{4}{5}(195) = 156$ **Ans.**

(ii) when $H = 144$

$144 = \frac{4}{5}(220 - n)$

$5(144) = 4(220 - n)$

$720 = 880 - 4n$

$4n = 880 - 720$

$4n = 160$

$n = 40$ **Ans.**

(iii) $H = \frac{4}{5}(220 - n)$

$\frac{5}{4}H = 220 - n$

$n = 220 - \frac{5}{4}H$ **Ans.**

13 (J2008 P2 Q5a)

(a) Mary has 50 counters.
Some of the counters are square, the remainder are round.
There are 11 square counters that are green.
There are 15 square counters that are not green.
Of the round counters, the number that are not green is double the number that are green.
By drawing a Venn diagram, or otherwise, find the number of counters that are
(i) round, [1]
(ii) round and green, [1]
(iii) not green. [1]

Thinking Process

(a) (i) Subtract the total number of square counters from 50.
(ii) Make an equation according to the given instructions and solve.
(iii) Total number of counters that are not green = 'not green' square counters + 'not green' round counters.

Solution with TEACHER'S COMMENTS

(a) (i) Total no. of square counters = $11 + 15 = 26$

$\therefore$ Total no. of round counters = $50 - 26$
$= 24$ **Ans.**

(ii) Let the no. of green round counters = x
then, the no. of not green counters = $24 - x$
according to the given condition

$24 - x = 2x$

$3x = 24$ or $x = 8$

$\therefore$ no. of green round counters = 8 **Ans.**

(iii) Number of 'not green' round counters

$= 24 - 8 = 16$

Number of 'not green' square counters $= 15$

total number of 'not green' counters

$= 16 + 15 = 31$ **Ans.**

14 *(N2008 P1 Q4)*

A basketball stadium has 13492 seats.
During a season a basketball team played 26 matches and every seat was sold for each match.
At each match a seat costs \$18.80.

By writing each value correct to 1 significant figure, estimate the total amount of money paid to watch these matches during the season. [2]

Thinking Process

Write each given value to one significant figure and then do the multiplication to obtain an estimation of the total amount.

Solution

$13492 = 10000$ (to one significant figure)

$26 = 30$ (to one significant figure)

$\$18.80 = \$20$ (to one significant figure)

amount received during each match $= 10000 \times \$20$

$= \$\,200000$

$\therefore$ total amount received from 26 matches

$= \$200000 \times 30 = \$6000000$ **Ans.**

15 *(N2008 P1 Q6)*

A wooden plank is cut into three pieces in the ratio 2 : 5 : 1. The length of the longest piece is 125 cm.
Find

(a) the length, in centimetres, of the shortest piece, [1]

(b) the total length, in metres, of the plank. [1]

Thinking Process

(a) 5 units represent 125cm. Find the length of the plank that represents 1 unit.

(b) Find the length of the plank that represents 8 units.

Solution

(a) 5 units —— 125 cm

1 unit —— $\frac{125}{5} = 25$ cm

$\therefore$ length of shortest peice $= 25$ cm **Ans.**

(b) $2 + 5 + 1 = 8$

5 units —— 125 cm

8 units —— $\frac{125}{5} \times 8 = 200$ cm

$\therefore$ Total length of the plank $= 200$ cm

$= 2$ m **Ans.**

16 *(N2008 P1 Q11)*

A rectangular box has dimensions 30cm by 10cm by 5cm.
A container holds exactly 100 of these boxes.

(a) Calculate the total volume, in cubic meters, of the 100 boxes. [1]

(b) Each box has a mass of 1.5kg to the nearest 0.1kg. The empty container has a mass of 6kg to the nearest 0.5kg.
Calculate the greatest possible total mass of the container and 100 boxes. [2]

Thinking Process

(a) Find the volume of one box. Multiply the answer by 100. Change the units into metres.

(b) For greatest possible mass of one box add 0.05 to 1.5kg. For greatest possible mass of the container add 0.25 to 6kg.

Solution

(a) Volume of one box $= 30 \times 10 \times 5$

$= 1500 \text{ cm}^3$

$= \frac{1500}{1000000}$

$= 0.0015 \text{ m}^3$

$\therefore$ Volume of 100 boxes $= 0.0015 \times 100$

$= 0.15 \text{ m}^3$ **Ans.**

> Note that:
> $1 \text{ m} = 100 \text{ cm}$
> $1 \text{ m}^3 = 1000000 \text{ cm}^3$

(b) Greatest mass of one box $= 1.5 + (0.1 \div 2)$

$= 1.5 + 0.05 = 1.55$ kg

$\therefore$ greatest mass of 100 boxes $= 1.55 \times 100$

$= 155$ kg

Greatest mass of the container $= 6 + (0.5 \div 2)$

$= 6 + 0.25$

$= 6.25$ kg

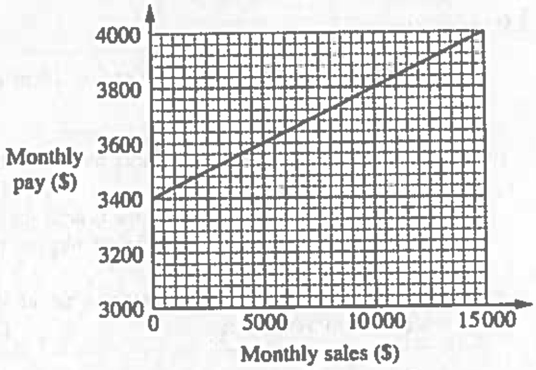
$\therefore$ greatest possible total mass $= 155 + 6.25$

$= 161.25$ kg **Ans.**

17 *(N2008 P1 Q19)*

Every month a salesman's pay is made up of a fixed amount plus a bonus.
The bonus is a percentage of his monthly sales.

(a) In 2006 the bonus paid was $m\%$ of his monthly sales.
The graph shows how the salesman's monthly pay varied with his monthly sales.



Use the graph to find

(i) the fixed amount, [1]

(ii) the value of *m*. [2]

(b) In 2007 the fixed amount was $3500 per month and the bonus was 5% of his monthly sales.
In July his sales were $12 000.
Calculate the salesman's pay for July. [2]

Thinking Process

(a) (i) Fixed amount is the fixed monthly salary. It is the point on the graph where the line meets the *y*-axis.

(ii) Use the graph to find the increase in monthly pay and hence the bonus, m%.

(b) Salesman pay = fixed salary + 5% of sales.

Solution

(a) (i) Fixed amount = $3400 **Ans.**

(ii) From graph, we see that when monthly sales = $5000,
salesman's monthly pay = $3600

$\Rightarrow$ bonus = 3600 − 3400 = $200

$\therefore$ %age bonus (or m%) $= \frac{200}{5000} \times 100$

$= 4\%$ **Ans.**

(b) Salesman's pay = fixed salary + 5% of monthly sales

$= 3500 + \frac{5}{100} \times 12000$

$= 3500 + 600 = \$4100$ **Ans.**

18 *(N2008/P2/Q1)*

(a) In 2006 the cost of fuel was 91.8 cents per litre.

(i) Calculate the maximum number of whole litres that could be bought for $15. [1]

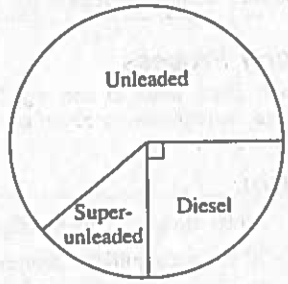
(ii) In 2007 the cost of fuel was increased by 4 cents per litre.

(a) Calculate the percentage increase in the cost of fuel in 2007. [2]

(b) On average, a car travelled 21 km on 1 litre of fuel.
During 2006 this car travelled 19200 km.
In 2007 the car travelled the same distance.
Calculate the extra cost for fuel in 2007.
Give your answer to the nearest dollar. [2]

(iii) In 2006 the cost of fuel was 10% less than the cost in 2005.
Calculate the cost, in cents, per litre in 2005. [2]

(b) A service station sells unleaded, super-unleaded and diesel fuel.
The pie chart represents the amounts of these fuels sold during one week.



The total amount of fuel sold during this week was 54000 litres.

(i) How many litres of diesel were sold? [1]

(ii) The amount of unleaded fuel sold was $\frac{2}{3}$ of the total for the week.
How many litres of super-unleaded fuel were sold? [2]

Thinking Process

(a) (i) Divide $15 by 91.8 cents & convert dollars into cents.

(ii) (a) Percentage increase $= \frac{\text{increase}}{\text{actual value}} \times 100$

(b) Extra cost per litre in 2007 is 4 cents. Multiply 4 cents by the number of litres obtained. Give your answer in dollars.

(iii) Note that 91.8 cents represents 90% of the cost in 2005. Hence find the cost that represents 100%.

(b) (ii) Find the number of litres of unleaded and diesel fuel first. Subtract your answer from the total number of litres.

Solution

(a) (i) $15 = 1500 cents

Number of litres $= \frac{1500}{91.8} = 16.34$

$\therefore$ number of whole litres = 16 **Ans.**

(ii) (a) percentage increase $= \frac{4}{91.8} \times 100$

$= 4.36\%$ **Ans.**

(b) Number of litres consumed $= \frac{19200}{21}$

$= 914.286$

In 2007, the cost increased by 4 cents

$\therefore$ extra cost of fuel in 2007

$= 914.286 \times 4 = 3657.144$ cents

$= 36.57 \approx \$37$ **Ans.**

(iii) Let x be the cost of fuel in 2005

$$x - \frac{10}{100}x = 91.8$$

$$\frac{90}{100}x = 91.8$$

$$x = 91.8 \times \frac{100}{90} = 102$$

$\therefore$ cost per litre in 2005 = 102 cents **Ans.**

(b) (i) $\frac{90}{360} \times 54000 = 13500$

$\therefore$ 13500 litres of diesel were sold. **Ans.**

(ii) Number of litres of unleaded fuel sold

$= \frac{2}{3} \times 54000 = 36000$

total number of litres of unleaded and deisel fuel $= 36000 + 13500 = 49500$

$\therefore$ number of litres of super-unleaded fuel

$= 54000 - 49500 = 4500$ litres **Ans.**

19 *(J2009/P1/Q15)*

The times taken for a bus to travel between five stops *A*, *B*, *C*, *D* and *E* are shown below.

A ——— *B* ——— *C* ——— *D* ——— *E*

4 minutes | $1\frac{1}{2}$ minutes | 75 seconds | 2 minutes 35 seconds

Expressing each answer in minutes and seconds, find

(a) the total time for the journey from *A* to *E*, [1]

(b) the mean time taken between the stops, [2]

(c) the range of times taken between the stops.

Thinking Process

(a) Add the given times. Change the minutes into seconds.

(b) Use mean $= \frac{\text{total time}}{\text{total number of intervals}}$

(c) Arrange all the given times in ascending order. Subtract the least time from the highest time.

Solution with TEACHER's COMMENT

(a) Total time $= 240 + 90 + 75 + 155$

$= 560$ seconds

$= 9$ minutes, 20 seconds. **Ans.**

> Changing minutes into seconds makes the calculation easier.

(b) Mean time $= \frac{\text{total time}}{\text{number of intervals}}$

$= \frac{560}{4} = 140$ seconds

$= 2$ minutes 20 seconds. **Ans.**

(c) Ordering the times (in seconds) from least to greatest, we have

75, 90, 155, 240

Range of times = highest time − lowest time.

$= 240 - 75$

$= 165$ seconds

$= 2$ minutes, 45 seconds. **Ans.**

> Note: The range of a set of data is the difference between the highest and lowest values of the set.

20 *(J2009/P1/Q10)*

Five clocks at a hotel reception desk show the local times in five different cities at the same moment.

LONDON	MOSCOW	SYDNEY	TOKYO	NEW YORK
07 38	10 38	16 38	15 38	02 38

(a) Rosidah has breakfast at 08 00 in Moscow. What is the local time in Sydney? [1]

(b) Elias catches a plane in London and flies to New York. He leaves London at 11 30 local time.
The flight time is 8 hours 10 minutes.
What is the local time in New York when he lands? [2]

Thinking Process

(a) Consider the time difference in the two cities.
(b) Find the time difference between the two cities. Find the arrival time in London local time first and then find the arrival time in New York local time.

Solution

(a) $1638 - 1038 = 0600$

$\therefore$ Sydney is 6 hours ahead of Moscow.

When it is 0800 in Moscow,

the local time in Sydney is $= 0800 + 0600$

$= 1400$ **Ans.**

(b) $0738 - 0238 = 0500$

$\therefore$ London is 5 hours ahead of New York.

Arrival time as per London time $= 1130 + 0810$
$= 1940$

$\therefore$ Arrival time as per New York Local time

$= 1940 - 0500 = 1440$ **Ans.**

21 *(N2009/P1/Q5)*

(a) The local time in Singapore is 7 hours ahead of the local time in London.
A flight to London leaves Singapore at 03 00 local time. The flight takes 12 hours and 45 minutes. What is the local time in London when it arrives? [1]

(b) Mai changes £250 into dollars.
The exchange rate is £1 = $ 3.10.
How many dollars does she receive? [1]

Thinking Process

(a) Compute the arrival time in Singapore local time and then subtract the time difference.
(b) To find the amount of dollars, multiply 3.10 by 250.

Solution

(a) $0300 + 1245 = 1545$

Singapore local time at arrival $= 1545$

$1545 - 0700 = 0845$

$\therefore$ London local time at arrival $= 0845$ **Ans.**

(b) $£1 = \$3.10$

$£250 = \$(3.10 \times 250)$

$= \$775$ **Ans.**

22 *(J2009/P2/Q2)*

(a) During a 20 week period in 2007, a bank made a profit of $378 million.
(i) Calculate the average profit it made each second. [2]
(ii) During the same 20 week period in 2008, the profit was $945 million.
For this 20 week period, calculate the percentage increase in the profit from 2007 to 2008. [2]
(iii) Find the ratio of $378 million to $945 million.
Give your answer in the form $m : n$, where m and n are the smallest possible integers. [2]

(b) Mary changed 480 euros into dollars.
The exchange rate was $1 = 0.6 euros.
The bank took, as commission, 2% of the amount that had been changed.
Calculate the number of dollars the bank took as commission. [2]

Thinking Process

(a) (i) To find the average profit per second convert 20 weeks into seconds.
(ii) Find the increment in profit.
(iii) Reduce the given ratio into smallest integers.
(b) Convert 480 euros into dollars first and then find 2% commission.

Solution

(a) (i) $20 \text{ weeks} = 20 \times 7 = 140 \text{ days}$

$140 \text{ days} \times 24 = 3360 \text{ hours}$

$3360 \text{ hours} \times 3600 = 12096000 \text{ seconds}$

$\$378 \text{ million} = \$\,378000000$

$\therefore$ profit in one second $= \dfrac{378000000}{12096000}$

$= \$\,31.25$ **Ans.**

> Note that, $1 \text{ hour} = 60 \text{ minutes} = 60 \times 60 = 3600 \text{ seconds}$

(ii) Percentage increase

$= \dfrac{945000000 - 378000000}{378000000} \times 100$

$= \dfrac{567000000}{378000000} \times 100 = 150\%$ **Ans.**

(iii) $378000000 : 945000000$

$378 : 945$

$2 : 5$ **Ans.**

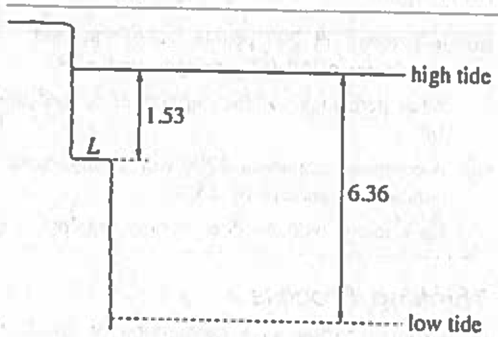
(b) $0.6 \text{ euros} = \$\,1$

$480 \text{ euros} = \dfrac{1}{0.6} \times 480 = \$800$

Bank's commission $= 2\%$ of $\$800$

$= \dfrac{2}{100} \times 800 = \$16$ **Ans.**

23 *(N2009 P1 Q10)*



The sea level at high tide is 1.53 m above a ledge, L, on a cliff.
At low tide the sea level is 6.36 m below the sea level at high tide.

(a) How far below L is the sea level at low tide? [1]

(b) On a certain day, high tide is at 07 32.
After 2 hours and 34 minutes, the sea level has dropped $\frac{1}{3}$ of the distance between high tide and low tide.

(i) At what time does the sea reach this level? [1]

(ii) How far below L is the sea level at this time? [1]

Thinking Process

(a) Subtract 1.53 from the total distance between high and low tides.

(b) (ii) Calculate $\frac{1}{3}$ of 6.36. To find the distance below L ✐ subtract 1.53 from your result.

Solution

(a) $6.36 - 1.53 = 4.83$

∴ Low tide is 4.83 m below L. **Ans.**

$$\begin{array}{r} 6.36 \\ -1.53 \\ \hline 4.83 \end{array}$$

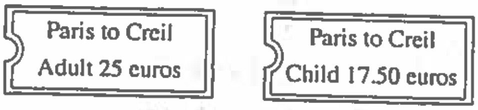
(b) (i) $0732 + 0234 = 0966$
$= 1006$ **Ans.**

(ii) $\frac{1}{3} \times 6.36 = 2.12$

distance of sea below L $= 2.12 - 1.53$
$= 0.59$ m **Ans.**

24 *(N2009 P1 Q23)*

(a)

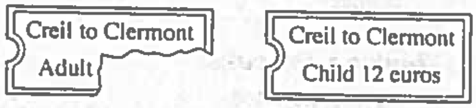


During a visit to France, a family took a train from Paris to Creil.
The cost of an adult ticket was 25 euros and the cost of a child ticket was 17.50 euros.

(i) How much did it cost for a family of 2 adults and 3 children? [1]

(ii) Express the cost of a child ticket as a percentage of the cost of an adult ticket. [2]

(b)



At Creil the family changed trains and travelled to Clermont.
The cost of a child ticket was 12 euros.
The cost of a child ticket was 60% of the cost of an adult ticket.
What was the cost of an adult ticket? [2]

Thinking Process

(a) (i) Add together the total cost of 2 adult and 3 child tickets.

(ii) To find percentage ✐ divide cost of child ticket by the cost of an adult ticket.

(b) Note that 60% represents 12 euros. Find what 100% represents.

Solution

(a) (i) $2(25) + 3(17.50)$
$= 50 + 52.50$
$= 102.50$ euros **Ans.**

(ii) Percentage $= \frac{17.50}{25} \times 100$
$= 70\%$ **Ans.**

(b) 60% of the cost of an adult ticket = 12 euros

100% of the cost of an adult ticket $= \frac{12}{60} \times 100$
$= 20$ euros

∴ cost of an adult ticket = 20 euros **Ans.**

25 *(N2009 P2 Q6)*

(a) 100 g of spaghetti contains 3.6 g of fibre.
Express

mass of fibre : mass of spaghetti

as the ratio of two integers in its simplest form. [1]

(b) A tin contains 210 g of beans.

(i) 100 g of beans contains 4.5 g of protein.
Calculate the mass of protein in the tin. [1]

(ii) 100 g of beans contains 0.3 g of fat.

(a) What percentage of the beans is fat? [1]

(b) The recommended daily amount of fat is 70 g.
Calculate what percentage of the recommended daily amount is in the tin. [3]

(iii) The mass of salt in 100 g of beans is 1.0 g, correct to 1 decimal place.
Calculate an upper bound for the mass of salt contained in the tin. [2]

(c) A tin of soup contains 166 calories.
This is 8.3% of the recommended daily number of calories.
Calculate the recommended daily number of calories. [2]

Thinking Process

(a) Reduce 3.6 : 100
(b) (i) Find the mass of protein in 210g of beans.
(ii) (a) To find percentage ✎ Divide 0.3g by 100g.
(b) Calculate the mass of fat in 210g of beans and express it as a percentage of 70g.
(iii) To find the upper bound ✎ add 0.05g to 1.0g.
(c) 8.3% represents 166 calories. Find 100%.

Solution

(a) Mass of fibre : Mass of spaghetti

3.6 : 100

36 : 1000

9 : 250 **Ans.**

(b) (i) 100 g of beans contain protein = 4.5 g

210 g of beans contain protein $= \frac{4.5}{100} \times 210$

$= 9.45$ g

∴ mass of protein in the tin = 9.45 g **Ans.**

(ii) (a) $\frac{0.3}{100} \times 100 = 0.3\%$

∴ percentage of fat = 0.3% **Ans.**

(b) 100 g of beans contains fat = 0.3 g

210 g of beans contains fat $= \frac{0.3}{100} \times 210$

$= 0.63$ g

∴ mass of fat in the tin = 0.63 g

percentage $= \frac{0.63}{70} \times 100 = 0.9\%$

∴ the tin contains 0.9% fat of the recommended daily amount. **Ans.**

(iii) Upper bound of mass of salt in 100g of beans = 1.0 + 0.05 = 1.05 g

Upper bound of mass of salt in 210 g of beans $= \frac{1.05}{100} \times 210 = 2.205$ g

∴ Upper bound of mass of salt in the tin = 2.205 g **Ans.**

(c) 8.3% —— 166 calories

100% —— $\frac{166}{8.3} \times 100$ calories

= 2000 calories

∴ recommended daily number of calories = 2000 **Ans.**

26 *(J2010/P1/Q3)*

(a) In a town, 11 000 people out of the total population of 50 000 are aged under 18.
What percentage of the population is aged under 18? [1]

(b) A company employing 1200 workers increased the number of workers by 15%.
How many workers does it now employ? [1]

Thinking Process

(a) Express 11000 as a percentage of 50000 and simplify.
(b) ✎ Find 15% of 1200.

Solution

(a) $\frac{11000}{50000} \times 100$

$= \frac{110}{5} = 22\%$ **Ans.**

(b) 15% of 1200

$= \frac{15}{100} \times 1200 = 180$

Number of workers now employed = 1200 + 180

= 1380 **Ans.**

27 *(J2010/P1/Q10)*

(a) Jane and Ken share some money in the ratio 5 : 3.
Ken's share is $16 less than Jane's share.
Find each person's share. [2]

(b) The scale of a map is 1 : 25 000.
The distance between two villages is 10 cm on the map.
Find the actual distance, in kilometres, between the villages. [1]

Thinking Process

(a) To find the share of each person ✎ note that the difference in the ratio (i.e. 5 − 3 = 2) represents $16.
(b) To find actual length in km ✎ find actual length in cm.

Solution

(a) Difference in ratio = 5 − 3 = 2 parts

Difference in share = $16

⇒ 2 —— $16

3 —— $\frac{16}{2} \times 3 = \$24$

5 —— $\frac{16}{2} \times 5 = \$40$

∴ Ken's share = $24 **Ans.**
Jane's share = $40 **Ans.**

(b)

$$1 : 25000$$
$$\Rightarrow 1 \text{ cm} : 25000 \text{ cm}$$
$$1 \text{ cm} : \frac{25000}{100000} \text{ km}$$
$$1 \text{ cm} : 0.25 \text{ km}$$
$$\therefore 10 \text{ cm} : 10 \times 0.25 = 2.5 \text{ km} \quad \textbf{Ans.}$$

28 *(J2010/P1/Q8)*

The table shows the record minimum monthly temperatures, in °C, in Vostok and London.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Vostok	–36	–47	–64	–70	–71	–71	–74	–75	–72	–61	–45	–35
London	–10	–9	–8	–2	–1	5	7	6	3	–4	–5	–7

Find

(a) the difference between the temperatures in Vostok and London in July, [1]

(b) the difference between the temperatures in Vostok in February and June. [1]

Thinking Process

(a) ✎ 7–(–74).

(b) Subtract the June temperature from February temperature.

Solution

(a) $7°C-(-74°C)=81°C$ **Ans.**

(b) $-47°C-(-71°C)=-47°C+71°C$

$= 24°C$ **Ans.**

29 *(J2010/P1/Q22)*

(a) A box has volume 2.5 m^3.
Express this volume in cm^3. [1]

(b) John has a length of string.
The string is 4m long, correct to the nearest 10 cm.

(i) Write down the lower bound of the length of the string.
Give your answer in centimetres. [1]

(ii) John cuts off ten pieces of string.
Each piece is 5 cm long, correct to the nearest centimetre.
Find the minimum possible length of string remaining.
Give your answer in centimetres. [2]

Thinking Process

(a) ✎ Recall: $1 \text{ m}^3 = 1000000 \text{ cm}^3$

(b) (i) ✎ Subtract 5cm (10 ÷ 2) from the given length.

(ii) To find the minimum length of remaining string ✎ Find the upper bound of length of all the ten pieces.

Solution

(a) $2.5 \text{ m}^3 = 2.5 \times 1000000$

$= 2500000 \text{ cm}^3$ **Ans.**

> Note that
> $1 \text{ m} = 100 \text{ cm}$
> $1 \text{ m}^3 = 100 \times 100 \times 100 \text{ cm}^3$
> or $1 \text{ m}^3 = 1000000 \text{ cm}^3$

(b) (i) $4 \text{ m} = 400 \text{ cm}$

Upper bound of length $= 400 - 5$

$= 395 \text{ cm}$ **Ans.**

(ii) $1 \text{ cm} \div 2 = 0.5 \text{ cm}$

Max. possible length of one piece
$= 5 + 0.5 = 5.5 \text{ cm}$

Max. possible length of 10 pieces
$= 5.5 \times 10 = 55 \text{ cm}$

Minimum possible length of remaining string $= 395 - 55$

$= 340 \text{ cm}$ **Ans.**

30 *(J2010/P2/Q1)*

(a) Sarah bought some soup, apples and mushrooms from her local shop.
The table shows some of the amounts and prices.

Items	Price ($)
p cans of soup at 90 cents per can	6.30
1.5 kilograms of apples at $ q per kilogram	4.35
r kilograms of mushrooms at $6.40 per kilogram	1.60

(i) Find the values of p, q and r. [2]

(ii) Sarah gives the shopkeeper $20.00 to pay for all these items.
How much change does she receive? [1]

(b)

> **Washing Machine**
> **$980**

> <u>Finance offer</u>
> Pay a 20% deposit
> <u>and</u>
> 24 monthly payments of
> $36 each

Lavin decides to buy this washing machine.
How much more would it cost Lavin if he paid for the washing machine using the finance offer instead of paying the $980 immediately? [2]

(c) Asif deposits $650 into a bank paying simple interest.
He leaves the money there for 5 years.
At the end of the 5 years, the amount in the bank is $763.75.
Calculate the percentage rate of interest the bank paid per year. [3]

Thinking Process

(a) (i) To find p ✎ divide \$6.30 by \$0.90.
To find q ✎ divide \$4.35 by 1.5 kg
To find r ✎ divide \$1.60 by \$6.40
(ii) Subtract the total amount of all the items from \$20.00.

(b) Find 20% of the total cost of machine.

(c) $I=\frac{PRT}{100}$

Solution

(a) (i) $p=\frac{6.30}{0.90}=7$ **Ans.**

$q=\frac{4.35}{1.5}=2.90$ **Ans.**

$r=\frac{1.60}{6.40}=0.25$ **Ans.**

(ii) Total cost of all the items

$=\$6.30+\$4.35+\$1.60=\$12.25$

Change received $=\$20.00-\$12.25$

$=\$7.75$ **Ans.**

(b) Initial deposit = 20% of \$980

$=\frac{20}{100}\times 980=\$196$

24 monthly installment costs $=24\times 36$

$=\$864$

Total cost under finance offer $=\$196+\$864$

$=\$1060$

difference from actual cost $=\$1060-\$980=\$80$

∴ Lavin would pay \$80 more if he uses the finance offer. **Ans.**

(c) Simple interest $=\$763.75-\$650=\$113.75$

$I=\frac{PRT}{100}$

$113.75=\frac{650\times R\times 5}{100}$

$11375=3250\times R$

$R=3.5$

∴ percentage rate = 3.5% **Ans.**

31 *(N2010 P1 Q3)*

(a) Express 0.0000706 in standard form. [1]

(b) A house was bought for \$20000 and sold for \$50000.
Calculate the percentage profit. [1]

Thinking Process

(a) Standard form is written in the form $A\times 10^n$ where A is the number between 1 and 10.

(b) Find the profit and express the amount as a percentage of the original cost.

Solution

(a) $0.0000706=7.06\times 10^{-5}$ **Ans.**

(b) Profit $=50000-20000=30000$

∴ Percentage profit $=\frac{30000}{20000}\times 100$

$=150\%$ **Ans.**

32 *(N2010 P1 Q4)*

The temperatures, in °C, at midnight on 10 consecutive days were

4, 1, 0, −2, −1, −3, 1, −2, 3, −1.

(a) Find the difference between the highest and the lowest temperature. [1]

(b) How many of these temperatures are within 2.5°C of 1°C? [1]

Thinking Process

(a) Subtract the lowest temperature from the highest temperature.

(b) Count the temperatures that are within the range of 2.5°C of 1°C.

Solution

(a) Difference $=4-(-3)=7$°C **Ans.**

(b) Writing the temperatures in ascending order.

−3, −2, −2, −1, −1, 0, 1, 1, 3, 4

The temperatures within 2.5°C of 1°C are:

−1, −1, 0, 1, 1, 3

∴ required number of temperatures = 6 **Ans.**

33 *(N2010 P1 Q5)*

(a) The mass of a container and its contents is 2.4 kg.
The mass of the contents is 750 g.
Calculate the mass, in kilograms, of the container. [1]

(b) Express the ratio 24 cm to 3 m in its lowest terms.
Give your answer in the form $p:q$, where p and q are integers. [1]

Thinking Process

(a) ✎ Convert 750g into kg. Subtract.

(b) Express 3 m into cm and subsequently reduce the ratio to its simplest form.

Solution

(a) Mass of the container $=2.4-0.750$

$=1.65$ kg **Ans.**

(b) 24 cm : 300 cm

4 : 50

2 : 25 **Ans.**

34 (N2010/P1/Q15)

In a sale, a shopkeeper reduced the marked price of his goods by 20%.

(a) The marked price of a book was $20. Calculate its price in the sale. [1]

(b) The price of a camera in the sale was $60. Calculate its marked price. [2]

Thinking Process

(a) find 80% of $20.

(b) 80% represent $60. Find 100%.

Solution

(a) 80% of $20

$= \frac{80}{100} \times \$20$

$= \$16$ **Ans.**

(b) 80% — $60

100% — $\$\frac{60 \times 100}{80}$

$= \$75$ **Ans.**

35 (N2010/P1/Q21)

ABC is a triangle.
Angle *A* is 62°, correct to the nearest degree.
Angle *B* is 53.4°, correct to the nearest tenth of a degree.

(a) Write down the lower bound for angle *B*. [1]

(b) Calculate the upper bound for angle *C*. [2]

Thinking Process

(a) To find the lower bound subtract 0.05° (i.e. 0.1÷2) from angle *B*.

(b) To find the upper bound for angle *C* subtract the lower bounds of angle *A* and angle *B* from 180°.

Solution

(a) Lower bound for $\angle B = 53.4° - 0.05°$

$= 53.35°$ **Ans.**

(b) Upper bound for $\angle C$

$= 180° -$ lower bound for $\hat{A} -$ lower bound for $\hat{B}$

$= 180° - 61.5° - 53.35°$

$= 65.15°$ **Ans.**

36 (N2010/P2/Q2)

(a) The rate of exchange between pounds (£) and dollars ($) is £1 = $1.87.
The rate of exchange between pounds (£) and euros (€) is £1 = €1.21.

(i) Catherine changes £500 into dollars. Calculate how many dollars she receives. [1]

(ii) Esther changes €726 into pounds. Calculate how many pounds she receives. [1]

(iii) Rose changes $850 into euros. Calculate how many euros she receives. [2]

(b) Matthew changes $770 into rupees.
He receives 40 000 rupees.
How many rupees did he receive for each dollar? [2]

(c) (i) Lily bought a car for $13 500.
She paid for it in 36 equal monthly payments.
Calculate the amount she paid each month. [1]

(ii) George bought a car for $27 000.
He borrowed the $27 000 at 15% per year simple interest for 3 years.
He repaid the total amount in 36 equal monthly payments.
Calculate the amount he paid each month. [3]

Thinking Process

(a) (i) Multiply £500 by 1.87.
(ii) Divide €726 by 1.21.
(iii) Note that $1.87 = €1.21

(b) Divide 40000 by 770 to get 1$ worth of rupees.

(c) (i) Divide $13500 by 36.

(ii) Use $I = \frac{PRT}{100}$ to find the interest. Add it to the principal amount to get the total amount payable.

Solution

(a) (i) $£1 = \$1.87$

$£500 = 1.87 \times 500$

$= \$935$

∴ Catherine receives $935. **Ans.**

(ii) $€1.21 = £1$

$€726 = £\frac{726}{1.21}$

$= £600$

∴ Esther receives £600 **Ans.**

(iii) From the information given, we see that,

$\$1.87 = €1.21$

$\Rightarrow \$850 = €\left(\frac{1.21}{1.87} \times 850\right)$

$= €550$

∴ Rose receives €550 **Ans.**

(b) $\$770 = 40000$ rupees

$\$1 = \frac{40000}{770}$

$= 51.9481 \approx 51.9$ rupees

∴ Matthew gets 51.9 rupees for each dollar. **Ans.**

(c) (i) Amount paid each month $= \$\frac{13500}{36}$

$= \$375$ Ans.

(ii) Interest for 3 years $= \$\frac{27000 \times 15 \times 3}{100}$

$= \$12150$

Total amount payable $= \$27000 + \$12150$

$= \$39150$

∴ the amount George paid each month

$= \$\frac{39150}{36} = \$1087.50$ Ans.

37 (J2011/P1/Q7)

(a) The ratio of boys to girls in a class is 4 : 5.
What fraction of the class are boys? [1]

(b) The ratio of boys to girls in a school is 3 : 4.
There are 120 more girls than boys.
How many students are in the school? [1]

Thinking Process

(a) To find fraction ✐ find the total number of parts of the class.
(b) using ratio concepts, find the number of boys and girls in the school. Form an equation using the fact that number of girls is 120 more than boys. Solve the equation.

Solution

(a) $4 + 5 = 9$

∴ fraction of boys in the class $= \frac{4}{9}$ Ans.

(b) Total parts $= 3 + 4 = 7$

let x be the total number of students.

∴ total number of boys $= \frac{3}{7}(x)$

total number of girls $= \frac{4}{7}(x)$

there are 120 more girls than boys,

$\Rightarrow \frac{4}{7}(x) - \frac{3}{7}(x) = 120$

$\Rightarrow \frac{1}{7}x = 120 \Rightarrow x = 840$

∴ Number of students in school $= 840$ Ans.

38 (J2011/P1/Q17)

The table shows the height, in metres, above sea level of the highest and lowest points in some continents. A negative value indicates a point below sea level.

	Asia	Africa	Europe	South America
Highest point (m)	8850	5963	5633	6959
Lowest point (m)	−409	−156	−28	−40

(a) What is the height above sea level of the highest point in Africa?
Give your answer in **kilometres**. [1]

(b) In South America, how much higher is the highest point than the lowest point?
Give your answer in metres. [1]

(c) How much higher is the lowest point in Europe than the lowest point in Asia?
Give your answer in metres. [1]

Thinking Process

(a) ✐ Write the height in kilometres.
(b) ✐ Subtract the lowest point from the highest point.
(c) ✐ Subtract the lowest point in Asia from the highest point in Europe.

Solution

(a) From table, the height of the highest point in Africa $= 5963$ m

$= 5.963$ km Ans.

(b) $6959 - (-40) = 6999$ m

∴ highest point in South America is 6999 m higher than its lowest point. Ans.

(c) $-28 - (-409)$

$= -28 + 409 = 381$ m

∴ lowest point in Europe is 381 m higher than the lowest point in Asia. Ans.

39 (J2011/P1/Q23)

(a) Imran is paid $16 per hour.

(i) One week he works 35 hours. Calculate the amount he is paid for the week. [1]

(ii) Imran is paid 20% extra per hour for working at weekends.
Work out the total amount Imran is paid for working 4 hours at the weekend. [2]

(b) The exchange rate between pounds and dollars is £1 = $1.80. Anna converts $270 into pounds.
Calculate the number of pounds Anna receives. [2]

Thinking Process

(a) (i) ✐ Multiply $16 by 35 hours.
(ii) To find the total amount ✐ find 20% of $16.
(b) ✐ Divide $270 by $1.80

Solution

(a) (i) $\$16 \times 35 = \$560$

∴ amount paid for the week $= \$560$ Ans.

(ii) Extra amount paid for one hour

$= \$16 + 20\%$ of $\$16$

$= \$16 + \frac{20}{100} \times \$16 = 19.20$

∴ extra amount paid for working 4 hours

$= 4 \times \$19.20 = \$76.80$ Ans.

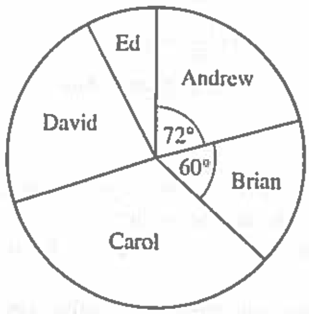
(b) $\$1.80 = £1$

$\$270 = £\frac{270}{1.80}$

$= £\frac{27000}{180} = £150$ **Ans.**

40 *(J2011/P2 Q6)*

The pie chart, not drawn accurately, represents the weekly income of the five employees in a small British company in 2009.



Andrew's weekly income is represented by a sector with an angle of 72°.
Brian's weekly income is represented by a sector with an angle of 60°.

(a) Andrew's weekly income was £270. Find the total weekly income of the five employees. [1]

(b) Calculate Brian's weekly income. [1]

(c) Carol's weekly income was £405. Calculate the angle of the sector representing Carol's weekly income. [1]

(d) David's weekly income was twice as much as Ed's weekly income. Calculate David's weekly income. [2]

(e) Andrew paid 20% of his weekly income of £270 as tax. He also paid 6% of his weekly income of £270 towards his pension. How much of his weekly income did he have left after paying tax and pension? [2]

(f) Carol paid 20% of her weekly income of £405 as tax. She also paid x% of her weekly income towards her pension. She then had £287.55 of her weekly income left. Find x. [3]

(g) Andrew's weekly income of £270 in 2009 was 8% more than his weekly income in 2008. Find his weekly income in 2008. [2]

Thinking Process

(a) 72° represents £280. Find what 360° represents.

(b) Use answer to (a) and find what 60° represents.

(c) Use answer to (a) and find the angle represented by £405.

(d) Let David's income be x. Use the given information to form an equation of income. Solve the equation to find David's income.

(e) Find 74% of £270.

(f) Find 20% of £405 and x% of £405 separately. Form an equation in x and solve.

(g) £270 is equivalent to 108%. Hence find 100% of the amount.

Solution

(a) 72° —— £270

360° —— $£\frac{270}{72} \times 360$

—— £1350

∴ total weekly income = £1350 **Ans.**

(b) 360° represents £1350

60° represents $£\frac{1350}{360} \times 60 = £225$

∴ Brian's weekly income = £225 **Ans.**

(c) £1350 represents 360°

£405 represents $\frac{360}{1350} \times 405 = 108°$

∴ angle of sector that represents Carol's weekly income = 108° **Ans.**

(d) Let David's weekly income be £ x.
David's income is twice as much as Ed's.

∴ Ed's weekly income = $£\frac{x}{2}$

Total income of five employees = £1350

$\Rightarrow \quad 270 + 225 + 405 + x + \frac{x}{2} = 1350$

$900 + \frac{3x}{2} = 1350$

$\frac{3x}{2} = 450$

$x = 450 \times \frac{2}{3} = 300$

∴ David's weekly income = £300 **Ans.**

(e) Andrew paid 20% + 6% = 26% as tax & pension.

$(100 - 26)\%$ of $270 = \frac{74}{100} \times 270$

$= 199.8$

∴ amount left with Andrew = £199.80 **Ans.**

(f) Tax = 20% of $405 = \frac{20}{100} \times 405$

$= £81$

Pension = x% of $405 = \frac{x}{100} \times 405$

$= £4.05x$

given that she had left with £287.55

$\Rightarrow \quad 405 - 81 - 4.05x = 287.55$

$324 - 4.05x = 287.55$

$4.05x = 324 - 287.55$

$x = \frac{36.45}{4.05} = 9\%$ **Ans.**

(g) 108% —— £270

100% —— $£\frac{270}{108} \times 100 = £250$

∴ weekly income in 2008 = £250 **Ans.**

41 *(N2011/P1/Q2)*

(a) Ali and Ben share $30 such that

Ali's share : Ben's share = 3 : 2.

Calculate Ali's share. [1]

(b) Write the following times in order of size, starting with the smallest.

6500 seconds 110 minutes $1\frac{3}{4}$ hours [1]

Thinking Process

(a) $30 represent 5 units. Find 3 units.
(b) Write each time in seconds before comparing.

Solution

(a) $3 + 2 = 5$

Ali's share $= \frac{3}{5} \times 30$

$= \$18$ **Ans.**

(b) 110 minutes $= 110 \times 60 = 6600$ seconds

$1\frac{3}{4}$ hours $= \frac{7}{4} \times 60 \times 60 = 6300$ seconds

∴ starting with the smallest, the numbers are,

$1\frac{3}{4}$ hours 6500 seconds 110 minutes **Ans.**

42 *(N2011/P1/Q3)*

Exactly 9 litres of liquid filled 60 identical bottles.

(a) How many litres filled 40 of these bottles? [1]

(b) How many of these bottles are filled using 750 ml of liquid? [1]

Thinking Process

(a) Using ratio concepts, find the number of litres.
(b) Change 9 litres into millilitres. Use ratio concept to find the number of bottles.

Solution

(a) 60 bottles —— 9 litres

40 bottles —— $(\frac{9}{60} \times 40)$ litres

= 6 litres **Ans.**

(b) 9 litres = 9000 ml

∴ 9000 ml —— 60 bottles

750 ml —— $(\frac{60}{9000} \times 750)$ bottles

= 5 bottles **Ans.**

43 *(N2011/P1/Q12)*

The length of a rectangle is 8 cm.
It is increased by 150%.
Calculate the new length. [2]

Thinking Process

To find the new length ✎ Find 8 + 150% of 8 cm.

Solution

New length $= 8 + 150\%$ of 8

$= 8 + (\frac{150}{100} \times 8)$

$= 8 + 12 = 20$ cm **Ans.**

44 *(N2011/P1/Q17)*

The length of a side of a square is given as 57 mm, correct to the nearest millimetre.

(a) Write down the upper bound for the length of a side. [1]

(b) Giving your answer in **centimetres**, calculate the upper bound for the perimeter of the square. [2]

Thinking Process

(a) ✎ Add 0.5 mm in the length.
(b) ✎ Perimeter = 4 (length)

Solution

(a) Upper bound for the length $= 57 + 0.5$

$= 57.5$ mm **Ans.**

(b) Upper bound for the perimeter $= 4(57.5)$

$= 230$ mm

$= 23$ cm **Ans.**

45 *(N2011/P1/Q18)*

Renata went on a journey that took $7\frac{1}{2}$ hours.

(a) The journey started at 22 48 on Monday.
At what time on Tuesday did it finish? [1]

(b) In the first part of the journey Renata travelled 150 km in 5 hours.
She travelled at an average speed of 20 km/h for the rest of the journey.
Calculate her average speed for the whole journey. [2]

Thinking Process

(a) Add 0730 hours to 2248. Convert it to 24-hour clock format.
(b) Find the distance travelled after first 5 hours.

Then apply, average speed $= \frac{\text{distance travelled}}{\text{time taken}}$

Solution

(a) $22\,48 + 07\,30 = 29\,78$

$= 30\,18$

$30\,18 - 24\,00 = 06\,18$

∴ the journey finished at 0618 **Ans.**

(b) Distance covered in 5 hours = 150 km

remaining hours: $7\frac{1}{2} - 5 = \frac{15}{2} - 5$

$= \frac{5}{2}$ hours

distance covered in $\frac{5}{2}$ hours = speed × time

$= 20 \times \frac{5}{2} = 50$ km

∴ average speed $= \dfrac{150 + 50}{\frac{15}{2}}$

$= 200 \times \frac{2}{15}$

$= \frac{80}{3} = 26\frac{2}{3}$ km/h **Ans.**

46 *(N2011/P2 Q4)*

(a) A shopkeeper buys some plates from a manufacturer for \$10 each.

(i) (a) The shopkeeper sells a plate for \$12. Calculate the percentage profit. [1]

(b) The shopkeeper buys 240 plates and sells 180 at \$12 each. The rest were sold to a café for a total of \$540. Calculate the percentage discount given to the café. [2]

(ii) The manufacturer made a profit of 60% when he sold each plate for \$10. Calculate the cost of manufacturing each plate. [2]

(b) Another shopkeeper bought 100 pans at \$5 each. He sold 63 at \$6 each and x at \$4 each. He did not sell all the pans nor enough to make an overall profit.

(i) Form an inequality in x. [1]

(ii) Hence find the greatest possible number of pans that were sold. [2]

(c) One day, the rate of exchange between American dollars (\$) and British pounds (£) was \$1.45 = £1.

(i) Alan changed £300 into dollars. Calculate how many dollars he received. [1]

(ii) On the same day, the rate of exchange between South African rands (R) and pounds was R10.44 = £1. Calculate the number of rands received in exchange for one dollar. [2]

Thinking Process

(a) (i) (a) Express the profit as a percentage of the original cost.

(b) To find the percentage discount ✎ find the selling price of one of the plates sold to cafe.

(ii) To find the cost ✎ find the incremental increase in the cost.

(b) (i) Note that the total sales must be less than or equal to the total cost.

(ii) ✎ Solve the inequality formed in (b) (i).

(c) (i) ✎ multiply £300 by £1.45.

(ii) Divide 10.44 rands by \$1.45 to find what \$1 worth in South African rands.

Solution

(a) (i) (a) Profit percentage $= \frac{12 - 10}{10} \times 100$

$= 20\%$ **Ans.**

(b) Amount received from 180 plates

$= 180 \times \$12 = \$2160$

remaining plates: $240 - 180 = 60$

60 plates were sold for \$540.

∴ selling price of one plate $= \frac{540}{60} = \$9$

percentage discount $= \frac{12 - 9}{12} \times 100$

$= 25\%$ **Ans.**

(ii) Let x be the cost of manufacturing one plate.

∴ $x + \frac{60}{100}(x) = \$10$

$x + 0.6x = \$10$

$x = \frac{10}{1.6} = \$6.25$ **Ans.**

(b) (i) $63 \times \$6 = \$378$

$x \times \$4 = \$4x$

total amount received from sales

$= \$(378 + 4x)$

Total cost of 100 pans $= 100 \times \$5 = \$500$

as the shopkeeper could not make an overall profit, therefore,

$378 + 4x \le 500$

$\Rightarrow$ $189 + 2x \le 250$ **Ans.**

(ii) From (i), $189 + 2x \le 250$

$2x \le 61$

$x \le 30.5$

$\Rightarrow$ greatest value of $x = 30$

∴ max. number of pans sold $= 63 + 30$

$= 93$ **Ans.**

(c) (i) £1 = \$1.45

∴ £300 = \$1.45 × 300

= \$435 **Ans.**

(ii) £1 = \$1.45, and £1 = R 10.44

$\Rightarrow$ \$1.45 = R 10.44

$\$1 = \frac{10.44}{1.45} = R\,7.2$

∴ \$1 is equivalent to R 7.2 **Ans.**

47 *(J2012 P1 Q2)*

The temperature in a freezer is −18 °C.

The outside temperature is 24 °C.

(a) Find the difference between the outside temperature and the freezer temperature. [1]

(b) The temperature in a fridge is 22 °C warmer than the freezer temperature.
Find the temperature in the fridge. [1]

Thinking Process

(a) ✐ Subtract −18 °C from 24 °C.

(b) To find the temperature in the fridge ✐ add 22 °C to −18 °C.

Solution

(a) $24-(-18)=42$ °C **Ans.**

(b) $22+(-18)=4$ °C **Ans.**

48 *(J2012 P1 Q4)*

(a) A bag contains red and blue counters in the ratio 3 : 8.
There are 24 blue counters in the bag.
How many red counters are there? [1]

(b) Amy and Ben share $360 in the ratio 3 : 2.
How much is Ben's share? [1]

Thinking Process

(a) 8 parts represent 24 blue counters. Find the number of counters that represents 3 parts.

(b) ✐ 5 parts represents $360. Find 2 parts worth of amount.

Solution

(a) 8 parts —— 24 counters

3 parts —— $\frac{24}{8}\times 3=9$ counters

∴ number of red counters = 9 **Ans.**

(b) Sum of ratio = 3 + 2 = 5

Ben's share $=\frac{2}{5}\times \$360=\$144$ **Ans.**

49 *(J2012 P1 Q8)*

The length of a rectangular rug is given as 0.9 m, correct to the nearest ten centimetres.
The width of the rug is given as 0.6 m, correct to the nearest ten centimetres.

(a) Write down the upper bound, in metres, of the length of the rug. [1]

(b) Find the lower bound, in metres, of the perimeter of the rug. [1]

Thinking Process

(a) To find the upper bound of the length ✐ consider the error $(0.1\div 2=0.05$ m) in the measurement.

(b) To find the lower bound for the perimeter ✐ subtract 0.05 m from length and width of the rectangle.

Solution

(a) 10 cm = 0.1 m

0.1 m $\div 2=0.05$ m

Upper bound of length = 0.9 + 0.05
= 0.95 m **Ans.**

(b) Lower bound of length = 0.9 − 0.05 = 0.85 m

lower bound of width = 0.6 − 0.05 = 0.55 m

∴ lower bound of the perimeter
= 2(length + width)
= 2(0.85 + 0.55) = 2.8 m **Ans.**

50 *(J2012 P1 Q17)*

(a) A carton contains 2.5 litres of juice.
Carlos drinks 650 ml of the juice.
How much juice is left in the carton?
Give your answer in litres. [1]

(b) The time in Chennai is $4\frac{1}{2}$ hours ahead of the time in London.

(i) What time is it in London when it is 14 45 in Chennai? [1]

(ii) A flight leaves London at 13 25 local time.
It arrives in Chennai at 04 00 local time the next day.
Work out, in hours and minutes, the length of the flight. [2]

Thinking Process

(a) ✐ Express 2.5 litres as milliliters. Subtract.

(b) (i) ✐ Subtract 4.5 hours from 14 45.

(ii) To find the length of flight ✐ Find the departure time in Chennai local time.

Solution

(a) 2.5 litres = 2.5 × 1000 = 2500 ml

∴ juice left = 2500 − 650
= 1850 ml
= 1.85 litres **Ans.**

(b) (i) 14 45 − 04 30 = 10 15

∴ time in London = 10 15 **Ans.**

(ii) When it is 13 25 in London.

the local time in Chennai = 13 25 + 04 30
= 17 55

the flight arrives at 04 00 the next day.

⇒ 04 00 − 17 55
= 27 60 − 17 55 = 10 05

∴ length of flight = 10 hours, 5 minutes. **Ans.**

51 (J2012/P1/Q24)

(a) The price of a television is \$350.
In a sale, its price is reduced by 30%.
Calculate the sale price of the television. [1]

(b) The exchange rate between dollars and euros is \$1 = €0.80.
Ben changes \$275 into euros.
Calculate the number of euros Ben receives. [1]

(c) Aisha buys a new car.

Cash price	Credit terms
\$4500	Deposit: 25% of cash price + 12 monthly payments of \$320

She buys the car using the credit terms.
How much more than the cash price will she pay overall for the car? [3]

Thinking Process

(a) Find 30% of \$350. Subtract it from \$350.
(b) Multiply \$275 by €0.80.
(c) Calculate the total cost on credit terms and then subtract cash price from it.

Solution

(a) Sale price $= \$350 - \$350 \times \frac{30}{100}$
$= \$350 - \$105$
$= \$245$ **Ans.**

(b) $\$1 = €0.80$
$\$275 = €0.80 \times 275$
$= €220$ **Ans.**

(c) Deposit $= \frac{25}{100} \times 4500 = \$1125$

cost of 12 monthly payments $= 12 \times \$320$
$= \$3840$

total amount payable on credit terms
$= \$1125 + \$3840 = \$4965$

difference $= \$4965 - \$4500 = \$465$

∴ Aisha will pay \$465 more than the cash price. **Ans.**

52 (N2012/P1/Q6)

A car travelled from A to B and then continued to C.
It travelled from A to B at an average speed of 30 km/h. The distance from A to B is 90 km.

(a) How many hours did the journey from A to B take? [1]

(b) The distance from B to C is 50 km and took 1 hour.
Calculate the average speed of the whole journey from A to C. [1]

Thinking Process

(a) Apply: time taken $= \frac{\text{distance covered}}{\text{average speed}}$

(b) Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

Solution

(a) Time taken $= \frac{\text{distance}}{\text{speed}}$
$= \frac{90}{30} = 3$ hour **Ans.**

(b) Average speed $= \frac{\text{total distance}}{\text{total time}}$
$= \frac{90+50}{3+1}$
$= \frac{140}{4} = 35$ km/h **Ans.**

53 (N2012/P1/Q13)

Sam and Tom ran 60 m.
Sam took 9.4 seconds, correct to the nearest tenth of a second.
Tom took 8 seconds, correct to the nearest second.

(a) Write down the upper bound for the time taken by Sam. [1]

(b) Calculate the greatest possible difference between the time taken by Sam and the time taken by Tom. [1]

Thinking Process

(a) To find the upper bound for time subtract 0.05 $(0.1 \div 2 = 0.05)$ from the time taken.

(b) For greatest possible difference, add 0.05 to 9.4 seconds and subtract 0.5 from 8 seconds.

Solution

(a) Upper bound for the time taken
$= 9.4 + 0.05$
$= 9.45$ seconds **Ans.**

(b) Greatest possible difference between the time taken $= (9.4 + 0.05) - (8 - 0.5)$
$= 9.45 - 7.5$
$= 1.95$ seconds **Ans.**

54 (N2012/P2/Q2 d)

(d) A shopkeeper sells cartons of milk and bottles of water.
Each carton of milk costs \$2.40, and each bottle of water costs \$0.80.
One day he sells x cartons of milk.
On the same day, he sells 20 more bottles of water than cartons of milk.

(i) Write down an expression, in terms of x, for the number of dollars he receives from the sale of these cartons and bottles.
Simplify your answer. [2]

(ii) The total amount he receives that day from the sale of these cartons and bottles is greater than $250.
Form an inequality in x and solve it. [2]

(iii) Hence write down the least number of cartons of milk that he sells that day. [1]

Thinking Process

(d) (i) Multiply x cartons by $2.40. Multiply $(x + 20)$ bottles by $0.80. Form an expression.

(iii) Figure out your answer from part (d) (ii).

Solution

(d) (i) Cost of x cartons = 2.40x$

No. of bottles sold = $20 + x$

∴ cost of bottles = $$0.80(20+x)$ = $$(16+0.8x)$

total amount received = $$(2.40x+16+0.8x)$ = $$(3.2x+16)$ **Ans.**

(ii) $3.2x + 16 > 250$

$3.2x > 234$

$x > \frac{234}{3.2}$

$x > 73.125$ **Ans.**

(iii) From part (d) (ii),

least number of cartons sold = 74 **Ans.**

55 *(N2012/P2/Q3)*

(a) In 2009 the cost of posting a letter was 36 cents.

(i) A company posted 3000 letters and was given a discount of 4%.
Calculate the total discount given.
Give your answer in dollars. [1]

(ii) In 2010, the cost of posting a letter was increased from 36 cents to 45 cents.
Calculate the percentage increase. [2]

(iii) After the price increase to 45 cents, the cost to the company of posting 3000 letters was $1302.75.
Calculate the new percentage discount given. [2]

(b) In 2010, it cost $5.40 to post a parcel.
This was an increase of $12\frac{1}{2}$% on the cost of posting the parcel in 2009.
Calculate the increase in the cost of posting this type of parcel in 2010 compared to 2009. [3]

Thinking Process

(a) (i) Calculate 4% of the total cost ⇒ Find the total cost of posting 3000 letters.

(ii) To find the percentage increase ⇒ calculate the increase.

(iii) Find the actual cost of posting 3000 letters. Subtract $1302.75 from it and compute the percentage discount.

(b) To find the increase ⇒ find the cost of posting in 2009. Note that $5.40 represents 112.5% of the cost in 2009. Find 100% of the amount.

Solution

(a) (i) Cost of 3000 letters = 3000×36 = 108000 cents = $1080

discount given = $$1080 \times \frac{4}{100}$ = $43.2 **Ans.**

(ii) Increase in cost = $45 - 36 = 9$ cents

percentage increase = $\frac{9}{36} \times 100 = 25\%$ **Ans.**

(iii) Total actual cost of posting 3000 letters
= 3000×45
= 135000 cents
= $1350

discount given = $1350 − $1302.75 = $47.25

∴ percentage discount = $\frac{47.25}{1350} \times 100$ = 3.5% **Ans.**

(b) 112.5% of the cost in 2009 = $5.40

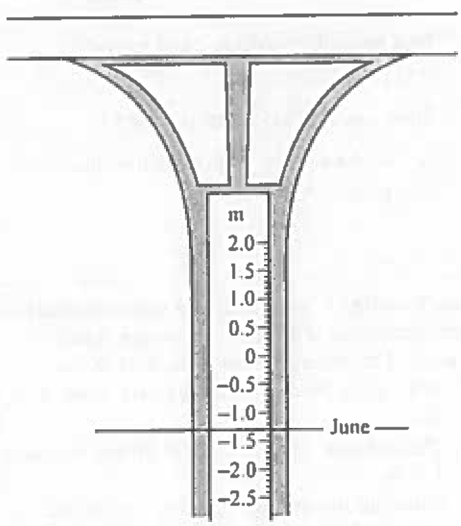
100% of the cost in 2009 = $$(\frac{5.40}{112.5} \times 100)$ = $4.80

∴ cost of posting a parcel in 2009 = $4.80

increase in the cost = $5.40 − $4.80 = $0.60 **Ans.**

56 *(J2013/P1/Q7)*

The diagram shows a scale used to measure the water level in a river.



The table shows the reading, in metres, at the beginning of each month.

Month	January	February	March	April	May	June	July
Reading (m)	0.8	1.2	1.3	0.5	−0.1		−1.9

(a) The diagram shows the water level at the beginning of June.
Complete the table with the June reading. [1]

(b) Work out the difference between the highest and lowest levels shown in the table. [1]

(c) The August reading was 0.4 m higher than the July reading.
Work out the reading in August. [1]

Thinking Process

(b) Subtract the lowest temperature from the highest temperature.

(c) Add 0.4 to −1.9.

Solution

(a) − 1.3 m. **Ans.**

(b) Difference $= 1.3 - (-1.9)$
$= 1.3 + 1.9 = 3.2$ m **Ans.**

(c) August reading $= -1.9 + 0.4$
$= -1.5$ m **Ans.**

57 *(J2013/P1/Q11)*

A photo is 10 cm long.
It is enlarged so that all dimensions are increased by 20%.

(a) Find the length of the enlarged photo. [1]

(b) Find the ratio of the area of the enlarged photo to the area of the original photo.
Give your answer in the form $k : 1$. [2]

Thinking Process

(a) Find 20% of 10 cm.

(b) Apply rule of similar figures, i.e. ratio of area of similar figures = ratio of squares of corresponding lengths.

Solution

(a) Enlarged length $= 10 + \frac{20}{100} \times 10$
$= 10 + 2$
$= 12$ cm **Ans.**

(b) $\frac{\text{Area of enlarged photo}}{\text{Area of original photo}} = \left(\frac{12}{10}\right)^2$
$= \frac{144}{100}$
$= 1.44$

∴ required ratio = 1.44 : 1 **Ans.**

58 *(J2013/P1/Q14)*

(a) Sofia earns \$7.60 for each hour she works.
She starts work at 7.45 a.m. and finishes at 4.30 p.m.
She stops work for half an hour for lunch.
How much does she earn for the day? [2]

(b) Marlon earns \$1500 each month.
He pays rent of \$525 each month.
Find the amount he pays in rent as a percentage of his earnings. [1]

Thinking Process

(a) Convert the given times to 24-hour clock and find the time difference in hours. Compute the number of working hours and multiply it by \$7.60.

(b) Express the rent as a percentage of the earnings.

Solution

(a) 4.30 pm = 1630
= 1590

> Note that writing 1630 as 1590 is to facilitate the subtraction.

1590 − 0745 = 0845

∴ length of time = 8 hours 45 minutes.
= 8.75 hours

lunch break = 0.5 hours.

∴ Total working hours for the day = 8.75 − 0.5
= 8.25 hours.

Earnings for the day = 8.25 × \$7.60
= \$62.70 **Ans.**

(b) $\frac{525}{1500} \times 100 = 35\%$ **Ans.**

59 *(J2013/P2/Q1)*

(a) (i)

> Exchange rate
> £1 = \$2.06
> £1 = 72 rupees

Manraj changes 25 200 rupees into dollars (\$).
Calculate how many dollars he receives. [2]

(ii) Misja changes 380 euros into dollars (\$).
He receives \$551.
How many dollars does he receive for each euro? [1]

(b)

Account	Simple interest per year
Super Saver	3.4%
Extra Saver	3.5%

On 31 March 2011, Lydia and Simone each had \$8000 in an account.
Lydia's money is in a Super Saver Account.
Simone's money is in an Extra Saver Account.

(i) How much money did Lydia have in her account on 31 March 2012 after the interest had been added? [2]

(ii) On 31 March 2012, Lydia transferred this money to an Extra Saver Account.

How much money did she have in this account on 31 March 2013 after the interest had been added? [1]

(iii) Simone kept her money for the two years in the Extra Saver Account, which earned simple interest of 3.5% per year.
After all interest had been added, who had more money in their account on 31 March 2013 and by how much? [2]

Thinking Process

(a) (i) To find number of dollars ✐ find the rate of exchange between rupees and dollars (\$).
(ii) 380 euros = \$551. Find what 1 euro is worth in dollars.

(b) (i) ✐ Use $I = \frac{PRT}{100}$ to calculate simple interest. Add it to the principal amount.
(ii) Apply $I = \frac{PRT}{100}$. Use the answer found in (b)(i) as the new principal amount.
(iii) Calculate simple interest, add it to the principal amount to get the total amount. Compare it with Lydia's amount found in (b)(ii).

Solution

(a) (i) 72 rupees = £1

$\Rightarrow$ 72 rupees = \$2.06

$\Rightarrow$ 25200 rupees $= \$\,\frac{2.06}{72} \times 25200 = \$\,721$

$\therefore$ Manraj receives \$721 **Ans.**

(ii) 380 euros = \$551

$1 \text{ euro} = \$\,\frac{551}{380} = \$\,1.45$

$\therefore$ Misja receives \$1.45 for each euro. **Ans.**

(b) (i) $I = \frac{PRT}{100}$

$= \frac{8000 \times 3.4 \times 1}{100} = \$272$

$\therefore$ Total amount in Lydia's account $= \$8000 + \$272 = \$8272$ **Ans.**

(ii) Lydia's new principal amount = \$8272

$I = \frac{PRT}{100} = \frac{8272 \times 3.5 \times 1}{100} = \$289.52$

$\therefore$ Total amount in Lydia's account $= \$8272 + \$289.52 = \$8561.52$ **Ans.**

(iii) $I = \frac{PRT}{100}$

$= \frac{8000 \times 3.5 \times 2}{100} = \$560$

Total amount in Simone's account

$= \$8000 + \$560 = \$8560$

After two years. Total amount in Lydia's account = \$8561.52

$\$8561.52 - \$8560 = \$1.52$

$\therefore$ Lydia had \$1.52 more. **Ans.**

60 *(N2013/P1/Q1)*

(a) Amy buys 3 drinks at \$1.86 each and 1 drink for \$2.04.
She pays for the 4 drinks with a \$10 note.
How much change should she receive? [1]

(b) \$180 is shared between Ali and Ben so that Ali's share : Ben's share = 4 : 5.
Find Ali's share. [1]

Thinking Process

(a) ✐ Find how much did she spent on four drinks.
(b) ✐ 9 parts represents \$180. Find 4 parts

Solution

(a) Total cost of 4 drinks $= (3 \times 1.86) + 2.04 = \$7.62$

Change received $= \$10 - \$7.62 = \$2.38$ **Ans.**

(b) $4 + 5 = 9$

Ali's share $= \frac{4}{9} \times \$180 = \$80$ **Ans.**

61 *(N2013/P1/Q4)*

(a) A journey started at 07 44 and finished at 11 32.
How long, in hours and minutes, did the journey take? [1]

(b) Arrange these values in order, starting with the smallest.

$\frac{4}{9}$ $\frac{2}{5}$ 44% [1]

Thinking Process

(a) Subtract 0744 from 1132 ✐ Express 1132 as 1092.
(b) ✐ Express all values as decimals.

Solution

(a) $11\ 32 = 10\ 92$

$10\ 92 - 07\ 44 = 03\ 48$

∴ the journey takes 3 hours 48 minutes **Ans.**

> Note that writing 1132 as 1092 is to facilitate the subtraction.

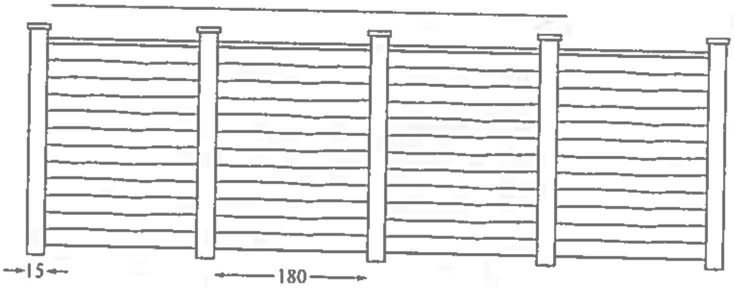
(b) $\frac{4}{9}$ $\frac{2}{5}$ 44%

= 0.444 0.4 0.44

∴ starting with the smallest, the values are,

$\frac{2}{5}$ 44% $\frac{4}{9}$ **Ans.**

62 (J2013/P2 Q5)



Mr Chan wants a fence along the side of his garden which is 8 metres long.
He buys 4 fence panels and 5 posts.
Each fence panel is 180 cm wide, correct to the nearest centimetre.
Each post is 15 cm wide, correct to the nearest centimetre.

(a) If there are no gaps between the panels and the posts, is it possible for the fence to be longer than 8 metres? Show your working. [2]

(b) A shop buys the posts from a manufacturer and sells them at a profit of 30%.
The shop sells each post for $35.10.

(i) How much does each post cost from the manufacturer? [2]

(ii)

Fence panels	$50.70 each
Posts	$35.10 each

Mr Chan buys 4 fence panels and 5 posts.
He hires a builder to put up the fence.
The builder charges 220% of the total cost of the fence panels and posts to do the work.
What is the total amount Mr Chan pays for his fence? [3]

Thinking Process

(a) To check the possibility ✐ find the maximum possible length of the fence ✐ find the upper bound of length for the 4 fence panels and 5 posts.

(b) (i) ✐ $35.10 is equivalent to 130%. Hence find 100% of the amount.

(ii) Calculate the total cost of 4 fence and 5 posts, then add builder's charges to it.

Solution

(a) Upper bound of length of 4 fence panels
$= 4(180 + 0.5) = 722$ cm.

Upper bound of length of 5 posts $= 5(15 + 0.5)$
$= 77.5$ cm.

Maximum possible length of the fence
$= 722 + 77.5$
$= 799.5$ cm $= 7.995$ m.

∴ No, it is not possible for the fence to be longer than 8 metres. **Ans.**

(b) (i) 130% — $35.10

100% — $\$\frac{35.10}{130} \times 100 = \$27$

∴ cost price of each post
$= \$27$. **Ans.**

(ii) Cost of 4 fence panels $= 4 \times \$50.70$
$= \$202.80$

cost of 5 posts $= 5 \times \$35.10$
$= \$175.50$

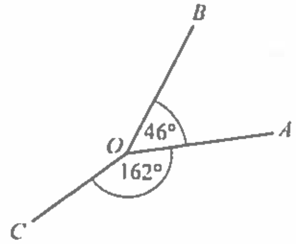
total cost of the fence and posts
$= \$202.80 + \$175.50$
$= \$378.30$

builder's charges $= 220\%$ of $\$378.30$
$= \frac{220}{100} \times 378.30 = \$832.26$

∴ total amount that Mr Chan pays
$= \$378.30 + \$832.26$
$= \$1210.56$. **Ans.**

63 (N2013/P1 Q11)

In the diagram,
$A\hat{O}B = 46°$, correct to the nearest degree,
$A\hat{O}C = 162°$, correct to the nearest degree.



(a) Write down the lower bound for $A\hat{O}B$. [1]

(b) Find the lower bound for $B\hat{O}C$. [2]

Thinking Process

(a) To find the lower bound → Subtract 0.5° from 46°.

(b) To find the lower bound for $B\hat{O}C$ → find the upper bound for 162° and 46°.

Solution

(a) Lower bound for $A\hat{O}B = 46° - 0.5°$
$= 45.5°$ **Ans.**

(b) Upper bound for $A\hat{O}B = 46° + 0.5°$
$= 46.5°$

Upper bound for $A\hat{O}C = 162° + 0.5°$
$= 162.5°$

$\therefore$ Lower bound for $B\hat{O}C = 360° - 46.5° - 162.5°$
$= 151°$ **Ans.**

64 *(N2013/P2/Q6)*

(a) (i) The cost price of bicycle A is \$620. The shopkeeper sells it and makes a profit of 45%. Calculate the selling price. [1]

(ii) In a sale, the price of bicycle B is reduced from \$2400 to \$1596. Calculate the percentage reduction given. [2]

(iii) Tax on the original price of bicycle C is charged at 20% of the original price. After tax has been included, Matthew pays \$1080 for this bicycle. Calculate the original price. [2]

(b) Ada invests \$600 in an account that earns simple interest. At the end of 3 years, the investment is worth \$681. Calculate the rate of simple interest per year. [3]

Thinking Process

(a) (i) → Selling price = cost price + profit.

(ii) To find percentage reduction → find the decrease in price.

(iii) \$1080 is equivalent to 120%. Hence find 100% of the amount.

(b) Apply $I = \frac{PRT}{100}$.

Solution

(a) (i) Selling price $= \$620 + \frac{45}{100}(\$620)$
$= \$620 + \$279$
$= \$899$ **Ans.**

(ii) $\$2400 - \$1596 = \$804$

$\therefore$ percentage reduction $= \frac{804}{2400} \times 100$
$= 33.5\%$ **Ans.**

(iii) 120% —— \$1080

100% —— $\$\frac{1080}{120} \times 100$

—— \$900

$\therefore$ original price of bicycle = \$900 **Ans.**

(b) Simple interest = \$681 − \$600 = \$81

$$I = \frac{PRT}{100}$$
$$81 = \frac{600 \times R \times 3}{100}$$
$$8100 = 1800 \times R$$
$$R = \frac{8100}{1800} = 4.5$$

$\therefore$ rate of simple interest per year = 4.5% **Ans.**

65 *(J2014/P1/Q4)*

Here is part of a bus timetable.

Bus station	09 56	10 26	10 56	11 26	11 56
City Hall	10 03	10 33	11 03	11 33	12 03
Railway station	10 17	10 47	11 17	11 47	12 17
Hospital	10 28	10 58	11 28	11 58	12 28
Airport	10 43	11 13	11 43	12 13	12 43

(a) How long does the bus take to get from the bus station to the airport? [1]

(b) Chris has a flight from the airport at 14 05. He must check in at the airport 2 hours before the flight. He will take a bus to the airport from the City Hall. Write down the latest time that Chris can take a bus from the City Hall to be at the airport in time. [1]

Thinking Process

(a) Use second column. Subtract 10 26 from 11 13 → Express 11 13 as 10 73.

(b) Find the length of time from City Hall to airport. Subtract this time and 2 hours from 14 05.

Solution *with* TEACHER'S COMMENTS

(a) Consider the second column.

11 13 = 10 73

10 73 − 10 26 = 47

$\therefore$ bus takes 47 minutes from the bus station to the airport **Ans.**

> Note that writing 11 13 as 10 73 is to facilitate the subtraction.

(b) Time of flight = 1405

check-in time at the airport = 1405 − 0200

= 1205

Time taken from the City Hall to Airport

= 1043 − 1003 = 40 minutes

1205 − 0040

= 1165 − 0040 = 1125

∴ latest bus that Chris can take from the City Hall is at 1103 **Ans.**

66 *(J2014/P1/Q14)*

A rectangular garden has length 35 metres and width 25 metres.
These distances are measured correct to the nearest metre.

(a) Write down the upper bound of the length of the garden. [1]

(b) Work out the lower bound of the perimeter of the garden. [2]

Thinking Process

(a) Divide 1 m by 2. Add the value to the length of garden.

(b) To find the lower bound of the perimeter ✐ subtract 0.5 m from length and width.

Solution

(a) Upper bound of the length = 35 + 0.5

= 35.5 m

(b) Lower bound of the perimeter

= 2(L + B)

= 2[(35 − 0.5) + (25 − 0.5)]

= 2(34.5 + 24.5)

= 2(59)

= 118 m **Ans.**

67 *(J2014/P1/Q16)*

(a) Dwayne buys a camera for $90.
He sells the camera for $126.
Calculate his percentage profit. [1]

(b) The price of a computer was $375.
In a sale, the price was reduced by 15%.
Calculate the reduction in the price of the computer. [1]

(c) The exchange rate between euros and dollars is €1 = $1.25 .

(i) Convert €180 to dollars. [1]

(ii) Convert $500 to euros. [1]

Thinking Process

(a) ✐ Find the profit earned and express it as a percentage of original cost price.

(b) ✐ Find 15% of $375.

(c) (i) To get €180 worth of dollars ✐ multiply €180 by 1.25.

(ii) Divide $500 by 1.25

Solution

(a) Profit = $126 − $90 = $36

percentage profit = $\frac{36}{90} \times 100 = 40\%$ **Ans.**

(b) Reduction in the price = 15% of $375

$= \frac{15}{100} \times \$375$

$= \$\frac{225}{4} = \$56.25$ **Ans.**

(c) (i) €1 = $1.25

€180 = $1.25 × 180

= $225 **Ans.**

(ii) $1.25 = €1

$\$500 = €(\frac{1}{1.25} \times 500)$

$= €(\frac{100}{125} \times 500) = €400$ **Ans.**

68 *(J2014/P2/Q3)*

(a) Mariam works in a shop.
She earns $5.20 per hour.
She also earns a bonus of 15% of the value of the items she sells in a week.

(i) In one week she works for 32 hours and sells items with a value of £2450.
Calculate Mariam's total earnings for the week. [2]

(ii) In another week, Mariam worked for 28 hours and earned a total of $409.60.
Calculate the value of the items she sold that week. [3]

(b) (i) Jack opens a bank account paying simple interest.
He pays in $800 and leaves it in the account for 4 years.
At the end of 4 years he closes the account and receives $920.
Calculate the percentage rate of simple interest paid per year. [2]

(ii) Jack uses some of the $920 to pay for a holiday and a computer.
He saves the remainder.
The money is divided between the holiday, computer and savings in the ratio 4 : 5 : 7.
Calculate the amount he saves. [2]

Thinking Process

(a) (i) ✐ Find the basic earning. Find the bonus. Add them together.

(ii) Find the basic earnings for 28 hours and subtract it from $409.60 to find bonus. This bonus is equal to 15% of Mariam's weekly total sale. Thus, form an equation and solve it to find total sales.

(b) (i) Find simple interest earned and then use $I = \frac{PRT}{100}$ to calculate the percentage rate.

(ii) To find his savings ✐ find 7 units worth of amount.

Solution

(a) (i) Basic earning = $5.20 × 32
= $166.40

bonus = 15% × $2450 = $367.50

∴ total earnings for the week
= $166.40 + $367.50
= $533.90 **Ans.**

(ii) Basic earning for 28 hours = $5.20 × 28
= $145.60

⇒ bonus = $409.60 − $145.60
= $264.00

let the value of items she sold be $x

∴ 15% of x = $264.00

$\frac{15}{100} \times x = \$264.00$

$x = \$264.00 \times \frac{100}{15}$

= $1760 **Ans.**

(b) (i) Simple Interest earned = $920 − $800
= $120

$I = \frac{PRT}{100}$

$\Rightarrow 120 = \frac{800 \times R \times 4}{100}$

$120 = 32R$

$R = \frac{120}{32} = 3.75\%$

∴ rate of simple interest per year
= 3.75% **Ans.**

(ii) 4 + 5 + 7 = 16

∴ amount he saves = $\frac{7}{16} \times \$920$
= $402.50 **Ans.**

69 (N2014/P1/Q1)

Mavis went to a cafe to meet some friends.

(a) She bought 3 drinks at $1.42 each and 1 cake for 85 cents.
How much did she spend altogether? [1]

(b) She left home at 10.45 a.m. and returned at 1.20 p.m.
How long, in hours and minutes, was she away from home? [1]

Thinking Process

(a) Find the total cost of 3 drinks and 1 cake. ✐ change 85 cents into dollars.

(b) ✐ Convert 1.20 p.m to 24-hour clock reading.

Solution with TEACHER'S COMMENTS

(a) Total amount spent = (3 × $1.42) + $0.85
= $5.11 **Ans.**

(b) 1.20 p.m = 1320
= 1280

1280 − 1045 = 0235

∴ she was away from home for 2 hours and 35 minutes. **Ans.**

> Note that writing 1320 as 1280 is to facilitate the subtraction.

70 (N2014/P1/Q2)

A cookery book states that the time it takes to cook some meat is
13 minutes for every 500 grams of meat + 20 minutes.

(a) Calculate the number of minutes it takes to cook 1.5 kg of meat. [1]

(b) It takes T minutes to cook M grams of meat.
Find a formula for T. [1]

Thinking Process

(a) 500 grams take 13 minutes to cook. Find how long would 1.5 kg take. ✐ Convert 1.5 kg into grams.

(b) Form a formula according to the given statement.

Solution

(a) 1.5 kg = 1.5 × 1000 = 1500 grams.

500 grams —— 13 minutes

1500 grams —— $\frac{13}{500} \times 1500 = 39$ minutes

∴ total cooking time = 39 + 20
= 59 minutes **Ans.**

(b) $T = \frac{13}{500}M + 20$ **Ans.**

71 (N2014/P1/Q7)

A car travels at 90 km/h.
How many metres does it travel in 1 second? [2]

Thinking Process

✐ Express the speed in m/s.

Solution

> Note that,
> 1 km = 1000 metres.
> 1 hour = 60 × 60
> = 3600 seconds

90 km/h

$= 90 \frac{\text{km}}{\text{h}}$

$= 90 \times \frac{1000 \text{ m}}{3600 \text{ s}} = 25$ m/s

∴ the car travels 25 metres in 1 second **Ans.**

72 *(N2014 P1 Q9)*

The time taken to run a race is given as 54.3 seconds, correct to the nearest 0.1 of a second.

(a) Find the lower bound for the time taken. [1]

(b) The distance run is given as d metres, correct to the nearest metre.
Write down an expression, in terms of d, for the maximum possible average speed, in metres per second. [1]

Thinking Process

(a) Divide 0.1 by 2. Subtract the value from 54.3.

(b) For greatest possible speed, increase the numerator by 0.5 m and decrease the denominator by 0.05 seconds.

Solution

(a) Lower bound for the time taken $= 54.3 - 0.05$

$= 54.25$ s. **Ans.**

(b) Maximum possible average speed

$= \dfrac{\text{maximum distance}}{\text{minimum time}}$

$= \dfrac{d + 0.5}{54.25}$ m/s **Ans.**

73 *(N2014 P2 Q1)*

(a) In 2013, Mary worked for Company A.
Her salary for the year was $18750.

(i) $5625 of her salary was not taxed.
What percentage of her salary was not taxed? [2]

(ii) The remaining $13125 of Mary's salary was taxed.
22% of this amount was deducted for tax.
Mary's take-home pay was the amount remaining from $18750 after tax had been deducted.
She received this in 52 equal amounts as a weekly wage.
Calculate Mary's weekly wage. [3]

(iii) In 2012 Mary had worked for Company B.
When she moved from Company B to Company A, her salary increased by 25% to $18750.
Calculate her salary when she worked for Company B. [2]

(b) The rate of exchange between pounds (£) and Indian rupees (R) is £1 = R87.21.
The rate of exchange between pounds (£) and Swiss francs (F) is £1 = F1.53.

(i) Mavis changed £750 into Indian rupees.
How many rupees did she receive? [1]

(ii) David changed F450 into pounds. How many pounds did he receive? [1]

(iii) Brian changed R50 000 into Swiss francs.
How many Swiss francs did he receive? [2]

Thinking Process

(a) (i) Express $5625 as a percentage of $18750 and simplify.

(ii) Calculate the tax and subtract it from $13125. Add the remaining amount to $5625 to find take-home salary. Divide this amount by 52 to find weekly wage.

(iii) 125% represent $18750. Find 100% of the amount.

(b) (i) Multiply £750 by 87.21

(ii) Divide F450 by 1.53

(iii) Note that R87.21 = F1.53

Solution

(a) (i) $\dfrac{5625}{18750} \times 100 = 30\%$ **Ans.**

(ii) Amount of tax deducted $= \dfrac{22}{100} \times \$13125$

$= \$2887.5$

remaining amount $= \$13125 - \$2887.5$

$= \$10237.5$

total take-home pay $= \$5625 + \$10237.5$

$= \$15862.5$

$\therefore$ weekly wage $= \dfrac{15862.5}{52} = \$305.05$

$\approx \$305$ **Ans.**

(iii) 125% —— $18750

125% —— $\$\dfrac{18750}{125} \times 100 = \$15000$

$\therefore$ salary in company B = $15000 **Ans.**

(b) (i) £1 —— R87.21

£750 —— R87.21 × 750

= R65407.50 **Ans.**

(ii) 1.53F —— £1

450F —— £ $\dfrac{1}{1.53} \times 450$

$= £294.118 \approx £294$ **Ans.**

(iii) From the given information, we see that,

R87.21 —— F1.53

$\Rightarrow$ R50000 —— F$\left(\dfrac{1.53}{87.21} \times 50000\right)$

$= F877.193 \approx F877$ **Ans.**

74 *(J2015 P1 Q5)*

Fariza travels from London to Astana.
The time in Astana is 5 hours ahead of the time in London, so when it is 10 00 in London the local time in Astana is 15 00 .
She flies from London to Moscow and then from Moscow to Astana.
The flight leaves London at 12 25 and takes 4 hours to reach Moscow.

Fariza waits $4\frac{1}{2}$ hours in Moscow for the flight to Astana.
She arrives in Astana at 05 25 local time.
How long did the flight from Moscow to Astana take? [2]

Thinking Process

To find the length of flight ✐ Find the departure time from Moscow in London local time.
Find the arrival time in Astana as per London local time and then subtract the time difference.

Solution

Flight duration from London to Moscow = 4 hours
waiting time in Moscow = 4 hours 30 minutes.

∴ Time of departure from Moscow as per London local time = 1225 + 0400 + 0430
= 2055

When it is 0525 in Astana, the local time in London is = 0525 − 0500
= 0025 or 2425

length of flight from Moscow to Astana.
= 2425 − 2055
= 2385 − 2055 = 0330

∴ the flight takes 3 hours 30 minutes from Moscow to Astana. **Ans.**

75 *(J2015/P1/Q19)*

(a) Luis works in an office.
For normal time he is paid $8 per hour.
For overtime he is paid the same rate as normal time plus an extra 50%.
One month he works 140 hours normal time and 10 hours overtime.
Work out how much he is paid for that month's work. [2]

(b) Sara invests $240 in an account that pays 3% per year simple interest.
She leaves the money in the account for 5 years.
Work out how much money Sara has at the end of 5 years. [2]

Thinking Process

(a) Find the normal time payment and overtime payment separately and then add them to find the payment for the month.

(b) Use $I = \frac{PRT}{100}$ to calculate the interest earned and then add the interest earned to the principal amount invested.

Solution

(a) Normal time payment per hour = $8

Overtime payment per hour = $8 + $\frac{50}{100}$ × $8
= $8 + $4 = $12

Payment for 140 hours normal time = $8 × 140
= $1120

Payment for 10 hours over time = $12 × 10
= $120

Total payment made = $1120 + $120
= $1240 **Ans.**

(b) Simple interest $= \frac{PRT}{100}$

$= \frac{240 \times 3 \times 5}{100}$ = $36

∴ total money in Sara's account = $240 + $36
= $276 **Ans.**

76 *(J2015/P1/Q22)*

The scale of a map is 1 : 25 000 .

(a) The scale can be written as 1 cm : d km .
Find d. [1]

(b) The distance between two villages is 8 km.
Find the distance, in centimetres, between the two villages on the map. [1]

(c) The distance between the peaks of two mountains is measured on the map as 76 mm.
Calculate the distance, in kilometres, between the two peaks. [2]

Thinking Process

(a) Express 25000 cm as km.
(b) Use the given scale to find the distance in cm.
(c) Express 76 mm in terms of cm and use the scale to find the distance.

Solution

(a) 1 : 25000
⇒ 1 cm : 25000 cm
1 cm : $\frac{25000}{100000}$ km
1 cm : 0.25 km
∴ d = 0.25 km **Ans.**

(b) 0.25 km —— 1 cm
8 km —— $\frac{1}{0.25} \times 8$
$= \frac{100}{25} \times 8$
= 32 cm **Ans.**

(c) 76 mm = 7.6 cm

given scale is:

1 cm —— 0.25 km

∴ 7.6 cm —— 0.25×7.6

$= \frac{25}{100} \times \frac{76}{10}$

$= \frac{19}{10} = 1.9$ km **Ans.**

77 *(J2015/P2/Q6)*

(a) Yuvraj and Sachin travel to England.
Yuvraj exchanges 20500 rupees and receives £250.
Sachin exchanges 26650 rupees into pounds (£) at the same exchange rate.
How many pounds does Sachin receive? [2]

(b) Dan goes to a bank to exchange some pounds (£) for euros (€).
He has £400 which he wants to exchange.
The bank only gives euros in multiples of 5 euros.
The exchange rate is £1= €1.17.
Find the number of euros he receives and his change from £400. [3]

(c) Kristianne buys a fridge and a freezer in a sale.
The sale offers 15% off everything and she pays a total of $357.
Before the sale, the freezer cost $250.
What was the cost of the fridge before the sale? [3]

Thinking Process

(a) 20500 rupees = £250. Using ratio concepts, find what 26650 rupees are worth in pounds.

(b) Multiply £400 by €1.17.

(c) Calculate the cost of freezer in the sale. Subtract it from $357 to find the cost of fridge in the sale. This amount is equivalent to 85% of the actual cost of fridge. Hence find 100% of the amount.

Solution

(a) 20500 rupees = £250

26650 rupees = $£(\frac{250}{20500} \times 26650)$

= £325 **Ans.**

(b) £1 = €1.17

£400 = €(1.17×400)

= €468

Given that the bank only gives euros in multiples of 5 euros

∴ Dan receives €465 **Ans.**

€465 = $£(\frac{465}{1.17})$

= £397.44

∴ His change is: £400 − £397.44

= £2.56 **Ans.**

(c) 85% of $250

$= \frac{85}{100} \times \$250$

= $212.50

∴ price of freezer in the sale = $212.50

⇒ price of fridge in the sale = $357 − $212.50

= $144.50

$144.50 is equivalent to 85% of the actual cost of fridge,

85% —— $144.50

100% —— $\$\ \frac{144.50 \times 100}{85}$

= $ 170

∴ cost of fridge before the sale = $170 **Ans.**

78 *(N2015/P1/Q2)*

(a) A trader buys 7 items for $4.10 each and 5 items for $6.40 each.
He sells all of them for $10 each.
Calculate his profit. [1]

(b) Find the simple interest on $450 for 5 years at 4% per annum. [1]

Thinking Process

(a) To find the profit ✎ calculate the total cost of 12 items.

(b) Simple interest = $\frac{PRT}{100}$

Solution

(a) Cost of 7 items = $7 \times \$4.10 = \$28.70$

Cost of 5 items = $5 \times \$6.40 = \$32$

total cost price of 12 items = $28.70 + $32

= $60.70

total selling price of 12 items = $12 \times \$10 = \$120$

∴ profit = $120 − $60.70

= $59.30 **Ans.**

(b) Simple interest = $\frac{PRT}{100}$

$= \frac{450 \times 4 \times 5}{100}$

$= \frac{9000}{100} = \$90$ **Ans.**

79 *(N2015/P1/Q9)*

At an athletics event, Dave and Ed each threw a javelin.
Dave threw 60 m, correct to the nearest 10 metres.
Ed threw 61 m, correct to the nearest metre.

(a) Write down the lower bound for the distance thrown by Dave. [1]

(b) Calculate the greatest possible difference between the distance thrown by Dave and the distance thrown by Ed. [1]

Thinking Process

(a) Divide 10 by 2. Subtract the value from 60.
(b) For greatest possible difference, add 0.5 to 61 metres and subtract 5 from 60 metres.

Solution *with* TEACHER's COMMENTS

(a) $10 \text{ m} \div 2 = 5 \text{ m}$

$\therefore$ Lower bound for the distance $= 60 - 5$

$= 55 \text{ m}$ **Ans.**

(b) Greatest possible difference in the distances

= greatest distance thrown by Ed − smallest distance thrown by Dave

$= (61 + 0.5) - (60 - 5)$

$= 61.5 - 55$

$= 6.5 \text{ m}$ **Ans.**

> Distance for Dave is measured correct to the nearest 10 m, therefore the error is of ± 5 m. Similarly distance for Ed is measured correct to nearest one metre, therefore the error is of ± 0.5 m.

80 (N2015/P1/Q14)

Meeraa went on a journey from P to Q to R.
The first part of the journey, from P to Q, took 4 hours to travel 80 km.

(a) Find the average speed for the journey from P to Q. [1]

(b) In the second part of the journey, from Q to R, she travelled 45 km.
Her average speed for both parts of the whole journey from P to Q to R was 25 km/h.
Find the time taken for the second part of the journey, from Q to R. [2]

Thinking Process

(a) & (b) Average speed $= \dfrac{\text{total distance}}{\text{total time}}$

Solution

(a) Average speed $= \dfrac{\text{distance}}{\text{time}}$

$= \dfrac{80}{4} = 20 \text{ km/h}$ **Ans.**

(b) Let t be the time taken from Q to R

Average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$25 = \dfrac{80 + 45}{4 + t}$

$25(4 + t) = 125$

$100 + 25t = 125$

$25t = 25$

$t = 1$ hour. **Ans.**

81 (N2015/P2/Q1)

(a) Fatima and Mohammed buy new bikes.

(i) Fatima buys a city bike costing \$360.
She pays 60% of the cost then pays \$15 per month for 12 months.

(a) How much does Fatima pay altogether? [2]

(b) Express this amount as a percentage of the original cost. [1]

(ii) Mohammed pays \$569.80 for a mountain bike in a sale.
The original price had been reduced by 26%.
Calculate the original price of the mountain bike. [2]

(b) The rate of exchange between pounds (£) and dollars is £1 = \$1.87.
The rate of exchange between pounds (£) and euros (€) is £1 = € x.
Rose changed \$850 and received €550.
Calculate x. [3]

Thinking Process

(a) (i) (a) Calculate 60% of \$360. Multiply \$15 by 2 months to find the total monthly payments. Add the two answers.

(ii) 74% represent \$569.80. Find 100% of the amount.

(b) Divide 1.87 by x and make an expression for €1. Divide 850 by 550 to get €1 worth of dollars. Equate to find x.

Solution

(a) (i) (a) Total payments

$=$ initial payment + total monthly payments

$= (60\% \times \$360) + (\$15 \times 12)$

$= (\frac{60}{100} \times \$360) + \$180$

$= \$216 + \$180$

$= \$396$ **Ans.**

(b) $\frac{396}{360} \times 100 = 110\%$ **Ans.**

(ii) $100\% - 26\% = 74\%$

74% —— \$569.80

100% —— $\$ \frac{569.80}{74} \times 100$

—— \$770

$\therefore$ original price of bike = \$770 **Ans.**

(b) Given that, £1 = \$1.87 and £1 = € x

$\Rightarrow$ € x = \$1.87

€1 = $\$ \frac{1.87}{x}$ ....(1)

Rose exchanged \$850 into €550

$\Rightarrow$ €550 = \$850

€1 = $\$ \frac{850}{550}$ ....(2)

from (1) and (2),

$$\$\frac{850}{550} = \$\frac{1.87}{x}$$

$$x = 1.87 \times \frac{550}{850}$$

$$= 1.21 \quad \textbf{Ans.}$$

82 (J2016/P1/Q5)

The table shows some information about the temperatures in a city.

Date	Maximum temperature	Minimum temperature
1 February	–10 °C	T °C
1 March	4 °C	–5 °C

(a) Find the difference between the maximum and minimum temperatures on 1 March. [1]

(b) The minimum temperature, T °C, on 1 February was 13 degrees lower than the minimum temperature on 1 March. Find T. [1]

Thinking Process

(a) Subtract –5 °C from 4 °C.

(b) Subtract 13 degrees from –5 °C.

Solution

(a) $4°C - (-5°C) = 9°C$ **Ans.**

(b) $-5°C - 13°C = -18°C$ **Ans.**

83 (J2016/P1/Q7)

The table shows information about some flights from Dubai to Mumbai.

Departs Dubai (local time)	03 30	16 10	21 55
Arrives Mumbai (local time)	08 10		02 30
Flight duration	3 hours 10 minutes	2 hours 55 minutes	3 hours 5 minutes

(a) Work out the time difference between Dubai and Mumbai. [1]

(b) Work out the local time in Mumbai when the 16 10 flight arrives. [1]

Thinking Process

(a) To find the time difference ✐ find the time of arrival in Dubai local time.

(b) Compute the arrival time in Dubai local time and then add the time difference.

Solution

(a) 0330 + 0310 = 0640

∴ arrival time in Dubai local time = 0640

Mumbai local time at arrival = 0810

0810 – 0640

= 0770 – 0640

= 0130

∴ time difference = 1 hour 30 minutes. **Ans.**

(b) 1610 + 0255 = 1865

= 1905

∴ arrival time in Dubai local time = 1905

local time in Mumbai at arrival = 1905 + 0130

= 2035 **Ans.**

84 (J2016/P1/Q16)

Anil has some sweets with a mass of 600 g, correct to the nearest 10 grams.

(a) Write down the lower bound of the mass of sweets. [1]

(b) Anil sells the sweets in small portions. Each portion has a mass of 25 g, correct to the nearest gram. He sells 10 portions of the sweets. Calculate the lower bound of the mass of sweets remaining. [2]

Thinking Process

(a) ✐ Divide 10 by 2. Subtract the value from 600.

(b) To find the lower bound of the mass of remaining sweets ✐ find the upper bound of the mass of sweets sold.

Solution

(a) Lower bound of the mass = 600 – 5

= 595 g. **Ans.**

(b) Upper bound of mass of one portion = 25 + 0.5

= 25.5 g

Upper bound of mass of 10 portions = 25.5 × 10

= 255 g

∴ lower bound of mass of remaining sweets

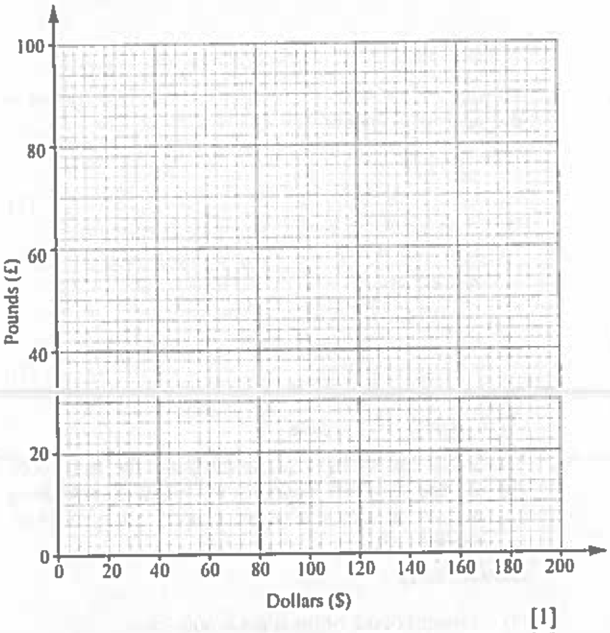
= lower bound of total mass of sweets – upper bound of mass of sweets sold

= 595 – 255 = 340 g **Ans**

85 *(J2016/P1/Q3)*

It is given that 100 dollars ($) is equivalent to 56 pounds (£).

(a) Use this information to draw a conversion graph between pounds and dollars on the grid below.



[1]

(b) Use your graph to convert $64 to pounds. [1]

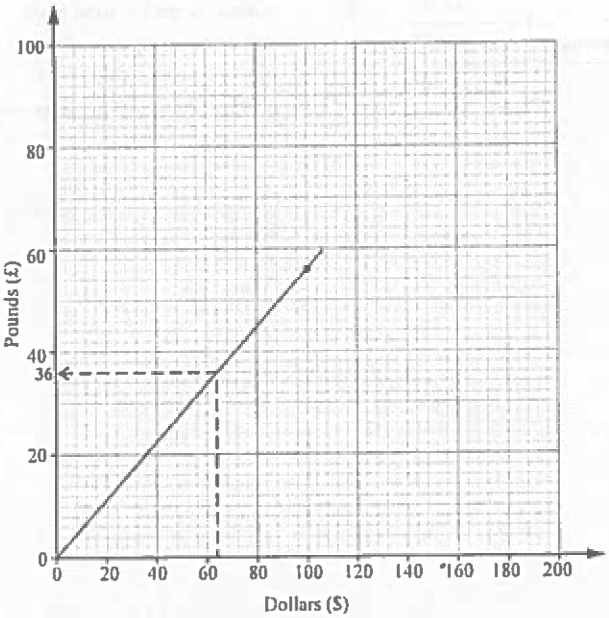
Thinking Process

(a) Plot a straight line taking two points preferably (0, 0) and (100, 56).

(b) From graph, find the value of pounds that corresponds to $64.

Solution

(a)



(b) From graph, $ 64 = £ 36 Ans.

86 *(J2016/P2/Q1)*

(a) Each year the Reds play the Blues in a baseball match.
In 2014, there were 40500 tickets sold for the match.
In 2015, the number of tickets sold was 2.4% more than in 2014.
Calculate the number of tickets sold for the match in 2015. [1]

(b) In 2015, the cost per ticket for the match was $68.25.
The cost per ticket for the match increased by 5% from 2014 to 2015.
Calculate the cost per ticket for the match in 2014. [2]

(c) Calculate the percentage increase, from 2014 to 2015, in the total money taken for the match. [3]

Thinking Process

(a) To find the total number of tickets sold ✐ find 2.4% of 40500.

(b) $68.25 represents 105% of the cost in 2014. Find 100% of the amount.

(c) To calculate the percentage increase ✐ find the increase in total money earned.

Solution

(a) 2.4% of 40500 $= \frac{2.4}{100} \times 40500$

$= 972$

∴ number of tickets sold in 2015 = 40500 + 972

= 41472 Ans.

(b) 105% of the cost in 2014 = $68.25

100% of the cost in 2014 $= \$(\frac{68.25}{105} \times 100)$

= $65 Ans.

(c) Total money earned in 2014 = 40500 × $65

= $2632500

Total money earned in 2015 = 41472 × $68.25

= $2830464

increase in total money = $2830464 − $2632500

= $197964

percentage increase $= \frac{\$197964}{2632500} \times 100$

= 7.52% Ans.

87 *(N2016/P1/Q17)*

(a) Some money is shared between Ali, Ben and Carl in the ratio 5 : 3 : 2.
Ben receives $60.
How much money is shared? [1]

(b) Express the ratio $3\frac{1}{2}$ hours : 14 minutes in the form k : 1. [2]

Thinking Process

(a) 3 parts represent \$60. Find the amount that represents 10 parts.

(b) Express $3\frac{1}{2}$ hours into minutes. Reduce the ratio into smallest integers.

Solution

(a) Sum of ratio = 5 + 3 + 2 = 10

3 parts —— \$60

10 parts —— $\$\frac{60}{3} \times 10 = \$200$

∴ total amount to be shared = \$200 **Ans.**

(b) $3\frac{1}{2}$ hours : 14 minutes

210 minutes : 14 minutes

210 : 14

15 : 1 **Ans.**

88 *(N2016/P1/Q19)*

A box has a mass of 1.7 kg, correct to the nearest 0.1 kg.

(a) Write down the lower bound for the mass of the box. [1]

(b) The box holds 100 jars.
Each jar has a mass of 140 grams, correct to the nearest 10 grams.
Calculate the lower bound of the **total** mass of the box and 100 jars.
Give your answer in kilograms. [2]

Thinking Process

(a) To find the lower bound ✐ consider the error $(0.1 \div 2 = 0.05)$ in the measurement.

(b) Subtract 5 grams $(10 \div 2 = 5)$ from 140 grams first and then find the lower bound for the total mass.

Solution

(a) Lower bound for the mass of the box

= 1.7 − 0.05 = 1.65 kg **Ans.**

(b) Lower bound of the mass of one jar = 140 − 5

= 135 grams.

Lower bound of mass of 100 jars = 135 × 100

= 13500 grams

= 13.5 kg

∴ lower bound of the total mass = 13.5 + 1.65

= 15.15 kg **Ans.**

89 *(N2016/P2 Q1)*

The basic price of the 2016 model of a car is \$21 000.
Sayeed and Rasheed each buy this model of car.

(a) (i) Sayeed pays a deposit of \$756.
Calculate the deposit Sayeed pays as a percentage of the basic price. [1]

(ii) He then pays 24 monthly payments of \$922.25.
Calculate the **total** amount that Sayeed pays as a percentage of the basic price. [2]

(b) Rasheed pays a deposit of \$381 followed by 36 equal monthly payments.
The **total** amount that he pays is 127% of the basic price of \$21 000.
Calculate Rasheed's monthly payment. [3]

(c) \$21 000 represented an increase of 5% on the basic price of the 2015 model.
Calculate the difference between the basic prices of the 2015 and 2016 models. [3]

Thinking Process

(a) (ii) Multiply \$922.25 by 24 to find the total monthly payments. Add initial deposit to it and express the answer as a percentage of \$21000.

(b) Calculate 127% of \$21000. Subtract the initial deposit of \$381 from it, then divide by 36 to find the monthly payment.

(c) ✐ \$21000 is equivalent to 105%. Hence find 100% of the amount.

Solution

(a) (i) $\frac{\$756}{\$21\,000} \times 100 = 3.6\%$ **Ans.**

(ii) Total monthly payments = \$922.25 × 24

= \$22 134

total amount paid for car = \$756 + \$22 134

= \$22890

percentage = $\frac{\$22890}{\$21\,000} \times 100 = 109\%$ **Ans.**

(b) Total amount paid for car = $\frac{127}{100} \times \$21000$

= \$26670

initial deposit = \$381

⇒ total monthly payments = \$26670 − \$381

= \$26289

∴ one month payment = $\frac{\$26289}{36}$

= \$730.25 **Ans.**

(c) 105% —— \$21000

100% —— $\$\frac{21000}{105} \times 100 = \$20000$

∴ basic price of 2015 model = \$20 000

price difference = \$21000 − \$20000

= \$1000 **Ans.**

90 (J2017/P1/Q6)

A thermometer measures temperature correct to the nearest degree.
The outside temperature is measured as −8 °C.

(a) Write down the upper bound of the outside temperature. [1]

(b) The inside temperature is measured as 10 °C. Calculate the lower bound of the difference between the outside temperature and the inside temperature. [1]

Thinking Process

(a) To find the upper bound ⁄ add 0.5° (i.e. 1÷2) to −8 °C.

(b) To find the difference h subtract the lower bound of inside temperature from the upper bound of outside temperature

Solution

(a) Upper bound of the outside temperature
$= -8° + 0.5° = -7.5°$ **Ans.**

(b) Lower bound of the difference
= lower bound of the inside temperature
− upper bound of the outside temperature
$= (10° - 0.5°) - (-7.5°)$
$= 9.5° + 7.5° = 17°$ **Ans.**

91 (J2017/P1/Q8)

(a) A film starts at 22 35 and finishes at 01 20. How long, in hours and minutes, does the film last? [1]

(b) On 1 May, Leila starts to go swimming every day.
She swims 30 lengths of the swimming pool every day. The swimming pool is 20 m long.
Work out the date when Leila has swum a total of 10 km. [2]

Thinking Process

(a) Subtract 2235 from 0120 ⁄ Express 0120 as 2480.

(b) Find the total distance she swims in one day, then divide it by 10 km.

Solution

(a) 01 20 − 22 35
= 24 80 − 22 35 = 02 45
∴ duration of film = 2 hours 45 minutes. **Ans.**

(b) Distance swum in one day $= 30 \times 20 = 600$ m

Number of days to swim 10 km $= \dfrac{10000}{600}$

$= \dfrac{50}{3} = 16.67$

∴ Leila will finish 10 km on 17 May. **Ans.**

92 (J2017/P1/Q13)

(a) Rani has \$240.
She spends $\frac{5}{8}$ of this on a new phone.
Work out the cost of the phone. [1]

(b) Anna invests \$150 in an account that pays simple interest.
She leaves the money in the account for 4 years.
At the end of 4 years she has \$162.
Work out the rate of simple interest paid per year. [2]

Thinking Process

(a) Multiply $\frac{5}{8}$ by 240.

(b) Find the simple interest. Apply $I = \frac{PRT}{100}$.

Solution

(a) Cost of the phone $= \frac{5}{8} \times \$240$
$= \$150$ **Ans.**

(b) Simple interest = \$162 − \$150 = \$12

$$I = \frac{PRT}{100}$$

$$12 = \frac{150 \times R \times 4}{100}$$

$$1200 = 600 \times R$$

$$R = 2$$

∴ rate of simple interest per year = 2% **Ans.**

93 (J2017/P2/Q1)

(a)

FLIGHTS TO SYDNEY
Cost per person: \$1199

ACCOMMODATION
Cost per adult per night: \$55
Cost per child per night: \$40

INSURANCE COVER FOR UP TO 20 DAYS
Cost per adult: \$40 and Cost per child: \$30
OR
Cost for family (2 adults and up to 4 children): \$155

A family of 2 adults and 3 children travel to Sydney for a holiday lasting 14 nights.
Calculate the **lowest total cost** of the flight, accommodation and insurance for their holiday. [3]

(b)

BONUS CARS
\$42 per day for any mileage

VALUE CARS
\$20 per day **and** \$0.50 per mile

The family hires a car for 14 days and estimates their total mileage will be 750 miles.
Which company charges less for this hire and by how much? [3]

Thinking Process

(a) To find the cost of flight ✐ multiply \$1199 by 5 people.
Calculate the accommodation cost of 2 adults for 14 nights. Calculate the accommodation cost of 3 children for 14 nights.
Compare the total cost of two insurance covers and use the lowest one. Add up all the charges to find the lowest total cost.

(b) To check which company charges less ✐ find the total cost of hiring a car for 14 days from each company.

Solution

(a) Total cost of flight for 5 persons $= \$1199 \times 5$
$= \$5995$

Accommodation cost for 2 adults $= 2(\$55 \times 14)$
$= \$1540$

Accommodation cost for 3 children $= 3(\$40 \times 14)$
$= \$1680$

Cost of insurance for option 1 $= 2(\$40) + 3(\$30)$
$= \$170$

Cost of insurance for option 2 $= \$155$

$\therefore$ Total lowest cost
$= \$5995 + \$1540 + \$1680 + \$155$
$= \$9370$ **Ans.**

(b) Cost of hiring from Bonus Cars $= \$42 \times 14$
$= \$588$

Cost of hiring from Value Cars
$= (\$20 \times 14) + (\$0.50 \times 750)$
$= \$280 + \$375 = \$655$

difference in charges $= \$655 - \$588 = \$67$

$\therefore$ Bonus Cars charges less by \$67 **Ans.**

94 *(J2017/P2 Q2)*

The table below shows the population, given to the nearest thousand, of some countries.

Country	Population in 2014	Population in 2015
Pakistan	185 133 000	188 169 000
China	1 393 784 000	1 402 007 000
South Korea	49 512 000	49 765 000
Thailand	67 223 000	67 438 000

(a) In 2015, how much larger was the population of Pakistan than the population of South Korea? [1]

(b) Which country had the smallest increase in population between 2014 and 2015? [1]

(c) Write the population of South Korea in 2014 in standard form. [1]

(d) Find the percentage increase in population of Pakistan from 2014 to 2015. [2]

(e) The population of Cambodia in 2015 was 15 677 000.
Given that the increase in population from 2014 to 2015 was 1.68%, calculate the population of Cambodia in 2014.
Give your answer correct to 3 significant figures. [3]

Thinking Process

(a) Subtract the population of South Korea from that of Pakistan

(c)✐ Recall that standard form is $A \times 10^n$, where $1 \le A < 10$.

(d) Percentage increase $= \dfrac{\text{Increase}}{\text{actual value}} \times 100$

(e) ✐ \$677000 is equivalent to $(100 + 1.68)\%$. Hence find 100% of the population.

Solution

(a) $188\,169\,000 - 49\,765\,000$
$= 138\,404\,000$ **Ans.**

(b) Thailand **Ans.**

> Population increase of Thailand
> $= 67\,438\,000 - 67\,223\,000$
> $= 215\,000$ which is the lowest as compared to other countries

(c) $49\,512\,000 = 4.95 \times 10^7$ **Ans.**

(d) Percentage increase
$= \dfrac{188\,169\,000 - 185133\,000}{185133\,000} \times 100$
$= \dfrac{3036000}{185133\,000} \times 100 = 1.64\%$ **Ans.**

(e) $101.68\% \longrightarrow 15\,677\,000$

$100\% \longrightarrow \dfrac{15\,677\,000}{101.68} \times 100 = 15\,417\,978$

$\therefore$ population of Cambodia in 2014

$= 15\,417\,978 \approx 15\,400\,000$ (3sf) **Ans.**

95 *(N2017/P1/Q5)*

The timetable for buses from A to E, calling at B, C and D, is given below.

A	08 12	08 42	and every 30 minutes until	17 12
B	08 33	09 03	and every 30 minutes until	17 33
C	08 48	09 18	and every 30 minutes until	17 48
D	09 05	09 35	and every 30 minutes until	18 05
E	09 20	09 50	and every 30 minutes until	18 20

(a) How many minutes does each journey from A to E take? [1]

(b) Sharon has an appointment at D at 3.30 p.m. What is the latest time she can catch a bus from B? [1]

Thinking Process

(a) Subtract 08 12 from 09 20 ✎ Express 09 20 as 08 80.

(b) Observe the arrival times of the bus at D. Note that to be on time for appointment, she should take the bus that arrives at D at 3.05 pm.

Solution

(a) 09 20 − 08 12

= 08 80 − 08 12 = 68 minutes **Ans.**

(b) The latest arrival time of the bus at D before appointment is at 15 05 or 3.05 pm.

$\therefore$ The latest time she can catch bus from B is at 14 33 **Ans.**

96 *(N2017/P1/Q6)*

(a)

The diagram shows a scale from 3.8 to 3.9, divided into five equal parts.

What is the value at the mark labelled P? [1]

(b)

The points X and Y lie on the line AB such that $AX : XY : YB = 3 : 2 : 4$. $AB = 18$ cm.

Find XY. [1]

Thinking Process

(a) To find the value at P ✎ find the length of one part on the scale.

(b) To find XY ✎ find the length that represents 2 units. Note that 9 units represent 18 cm.

Solution

(a) Length of one part $= \dfrac{3.9 - 3.8}{5}$

$= \dfrac{0.1}{5} = 0.02$

$\therefore \ P = 3.8 + 2(0.02)$

$= 3.8 + 0.04 = 3.84$ **Ans.**

(b) Sum of ratio $= 3 + 2 + 4 = 9$

$\therefore \ XY = \dfrac{2}{9} \times 18 = 4$ cm **Ans.**

97 *(N2017/P1/Q9)*

The area of a rectangle is given as 8 cm², correct to the nearest cm².

(a) Write down the lower bound for the area of the rectangle. [1]

(b) The width of the rectangle is given as 2 cm, correct to the nearest cm.

Calculate the lower bound for the length of the rectangle. [1]

Thinking Process

(a) To find the lower bound ✎ subtract 0.5 cm (i.e. 1 ÷ 2) from given area.

(b) To find the lower bound for the length ✎ divide the lower bound for area by the upper bound for width.

Solution

(a) Lower bound for the area $= 8 - 0.5$

$= 7.5$ cm² **Ans.**

(b) Lower bound for the length

$= \dfrac{\text{lower bound for the area}}{\text{upper bound for the width}}$

$= \dfrac{7.5}{2 + 0.5}$

$= \dfrac{7.5}{2.5} = 3$ cm **Ans.**

98 *(N2017/P1/Q17)*

(a) Find 110% of 70. [1]

(b) When new, a car was worth \$15 000.

After one year it was worth \$12 000.

Calculate the percentage reduction in its value. [2]

Thinking Process

(b) Percentage decrease $= \dfrac{\text{decrease in price}}{\text{original amount}} \times 100$

Solution

(a) $110\% \times 70$

$= \frac{110}{100} \times 70 = 77$ **Ans.**

(b) Percentage reduction $= \frac{15000 - 12000}{15000} \times 100$

$= \frac{3000}{15000} \times 100$

$= \frac{300}{15} = 20\%$ **Ans.**

99 *(N2017/P2/Q1)*

(a) Sara buys a new car.
The cash price of the car is \$4500.
She can pay for the car using option A or option B.

Option A
Pay $\frac{1}{5}$ of the cash price then 12 monthly payments of \$340

Option B
Pay 12% of the cash price then 24 monthly payments of \$195

Which option is cheaper and by how much? [4]

(b) Sara's car uses 5.2 litres of petrol for each 100 km she drives.
Petrol costs \$0.85 per litre.
Sara drives 240 km.
Calculate the cost of the petrol used for this journey.
Give your answer correct to the nearest cent. [3]

(c) Sara pays a total of \$322 for her car insurance.
This total is made up of a basic charge plus 15% sales tax.
Calculate the amount of sales tax that Sara pays. [3]

Thinking Process

(a) Option A: Multiply $\frac{1}{5}$ by \$4500 and add it to total monthly payments.
Option B: Find 12% of \$4500 and add it to total monthly payments. Find the difference between total payments to check which option is cheaper.

(b) To find the cost of the petrol & calculate how many litres were used to travel 240 km.

(c) Insurance = basic charge + 15% of basic charge.

Solution

(a) Option A.

$\frac{1}{5} \times \$4500 = \$900$

monthly payments $= 12 \times \$340 = \$4080$

total payment $= \$90 + \$4080 = \$4980$

Option B.

$\frac{12}{100} \times \$4500 = \$540$

monthly payments $= 24 \times \$195 = \$4680$

total payment $= \$540 + \$4680 = \$5220$

difference in total payments $= \$5220 - \$4980$

$= \$240$

∴ Option A is cheaper by \$240 **Ans.**

(b) 100 km —— 5.2 litres

240 km —— $\frac{5.2}{100} \times 240 = 12.48$ litres

∴ cost of the petrol $= \$0.85 \times 12.48$

$= \$10.608 \approx \$10.61$ **Ans.**

(c) Let basic charge be x.

$x + 15\%$ of $x = \$322$

$x + \frac{15}{100}x = \$322$

$\frac{115}{100}x = \$322$

$x = \$322 \times \frac{100}{115} = \$280$

∴ sales tax $= \frac{15}{100} \times \$280 = \$42$ **Ans.**

100 *(J2018/P1/Q2)*

(a) Cecil bought a camera for \$120.
After two years he sold it for \$90.
Calculate the percentage loss. [1]

(b) Some money is shared between Miriam and Nina in the ratio 2 : 3.
What percentage of the total money shared does Miriam receive? [1]

(c) Given that $a:b = 5:6$ and $b:c = 3:8$ find $a:b:c$. [1]

Thinking Process

(a) Express the loss as a percentage of the original cost.

(b) Express miriam's share as a percentage of total ratio.

Solution

(a) Percentage loss $= \dfrac{\$120 - \$90}{\$120} \times 100$

$= \dfrac{30}{120} \times 100 = 25\%$ **Ans.**

(b) Miriam share $= \dfrac{2}{2+3} \times 100$

$= \dfrac{2}{5} \times 100 = 40\%$ **Ans.**

(c) $a : b : c$

5 : 6

↘ ↑ ↘

3 : 8

$\Rightarrow a:b:c = 15:18:48$

$= 5:6:16$ **Ans.**

101 *(J2018/P1/Q6)*

A rectangle has length 64 mm and width 37 mm each correct to the nearest millimetre.

(a) Write down the lower bound for the length. [1]

(b) Calculate the lower bound for the perimeter of the rectangle. [1]

Thinking Process

(a) To find the lower bound ✐ subtract 0.5 (i.e. 1÷2) from length 64.

(b) Apply, Perimeter = 2 (*l* x *w*). ✐ subtract 0.5 from 37.

Solution

(a) Lower bound for the length $= 64 - 0.5$

$= 63.50$ mm **Ans.**

(b) Lower bound for the perimeter

$= 2(l \times w)$

$= 2(63.50 + 36.50)$

$= 2(100) = 200$ mm **Ans.**

102 *(J2018/P2/Q1)*

(a) Each week Leah works 5 days and is paid a total of $682.
Each day she works from 08 45 until 12 15 and then from 13 15 until 17 30.
Calculate Leah's hourly rate of pay. [2]

(b) Carlos buys a new bicycle.
After one year he sells it for $231.
He makes a loss of 16% on the price he paid.
Calculate the price Carlos paid for the bicycle. [2]

(c) The exchange rate between dollars ($) and euros (€) is $1 = €0.44.
Henry changes $850 to euros for his holiday.
He spends €260 when he is on holiday.
He changes the rest of the money back to dollars at the same exchange rate.
Calculate how much money in dollars he receives.
Give your answer correct to the nearest dollar. [3]

(d) Anya has $3000 to invest in a savings account for 3 years.
She can choose from these two accounts.

Account A	
Year 1	1.1% interest
Year 2	1.2% interest added to end of Year 1 total
Year 3	1.4% interest added to end of Year 2 total

Account B
Fixed rate of compound interest 1.3% per year

She chooses the account that will give her more money at the end of the 3 years.
Decide which account she chooses and find the amount she will have in her account at the end of 3 years. [4]

Thinking Process

(a) Compute the payment for one day. Find the total working hours in a day. ✐ Write 1215 as 1175.

(b) ✐ 84% represent $231. Find 100% of the amount.

(c) To get $850 worth of euros ✐ multiply $850 by 0.44. Subtract €260 from the result and convert the remaining amount back into dollars.

(d) For Account A, calculate the total amount she gets each year separately.
For Account B, apply formula of compound interest, $A = P\left(1 + \frac{r}{100}\right)^n$

Solution

(a) Leah's pay for one day $= \dfrac{\$682}{5} = \$136.40$

Working hours:

$1215 = 1175$

$1175 - 0845 = 0330$

$= 3.5$ hours

Also, $1730 - 1315 = 0415$

$= 4.25$ hours

Total working hours in a day $= 3.5 + 4.25 = 7.75$

$\therefore$ Leah's hourly rate of pay $= \dfrac{\$136.40}{7.75}$

$= \$17.60$ **Ans.**

(b) $100\% - 16\% = 84\%$

84% —— $\$231$

100% —— $\frac{\$231}{84} \times 100 = \$275$

∴ Carlos paid $275 for the bicycle **Ans.**

(c) $\$1 = €0.44$

$\$850 = €0.44 \times 850$

$= €374$

∴ Henry gets 374 euros.

Money spent on holiday = €260

Money left with Henry = €374 − €260 = €114

€0.44 —— $1

€114 —— $\frac{1}{0.44} \times 114 = \$259$

∴ Henry receives $259 **Ans.**

(d) **Account A**

Anya's investment after year 1

$= \$3000 + \left(\$3000 \times \frac{1.1}{100}\right) = \$3033$

Anya's investment after year 2

$= \$3033 + \left(\$3033 \times \frac{1.2}{100}\right) = \$3069.4$

Anya's investment after year 3

$= \$3069.4 + \left(\$3069.4 \times \frac{1.4}{100}\right) = \$3112.4$

Account B

$P = 3000, \quad n = 3, \quad r = 1.3$

Using, $A = P\left(1 + \frac{r}{100}\right)^n$

$\Rightarrow A = 3000\left(1 + \frac{1.3}{100}\right)^3 = 3118.53$

$\Rightarrow$ Anya's investment after 3 years = $3118.53

∴ Account B will give more money after 3 years. **Ans.**

103 *(N2018/P1/Q2)*

(a) 200 grams of a spice cost 85 cents.
Find the cost, in dollars, of 1 kilogram of this spice. [1]

(b) You are given that $60 : x = 3 : 2$.
Find x. [1]

Thinking Process

(a) 200 grams represents $0.85. Find 1000 gram worth of amount.

(b) Express the given ratio in fraction and solve for x.

Solution

(a) $1 \text{ kg} = 1000$ grams. 85 cents = $0.85

200 grams —— $0.85

1000 grams —— $\frac{0.85}{200} \times 1000$

$= 0.85 \times 5 = \$4.25$ **Ans.**

104 *(N2018/P2/Q1)*

(a) Kamal earned a total of $32 500 in 2017.
He paid 9% of this amount into his pension.
He paid 22% tax on the remainder of his earnings.
Calculate the amount left after paying his pension and his tax. [3]

(b) Kamal invested $1200 in a savings account paying 1.8% per year compound interest.
He left the money in the account for 5 years.
Calculate the amount of money in the account at the end of 5 years.
Give your answer correct to the nearest cent. [3]

(c) Kamal also invested some money in a different savings account for 5 years.
This account paid 2.1% per year simple interest.
At the end of 5 years there was $828.75 in the account.
Calculate the amount of money he invested in this account. [3]

(d) The exchange rate between dollars ($) and pounds (£) is $1 = £0.72.
The exchange rate between dollars and euros (€) is $1 = €1.10.
Kamal has $275.
He changes part of the $275 into pounds and receives £79.20.
He changes the remaining dollars into euros.
Calculate the amount of money he receives in euros. [3]

Thinking Process

(a) Find 9% of $32500. Subtract it from $32500. Calculate 22% of the remaining amount to find tax. Subtract the tax from the remainder amount to find the final amount left.

(b) Formula for compound interest, $A = P\left(1 + \frac{r}{100}\right)^n$

(c) P + SI = 828.75. Substitute the given values into the equation and solve for principal amount P.

(d) Divide £79.20 by 0.72 to find £79.20 worth of dollars. Subtract it from $275 and multiply remaining dollars by €1.10.

Solution

(a) Amount paid in pension = 9% of \$32500

$= \frac{9}{100} \times \$32500 = \$2925$

Remaining amount = \$32500 − \$2925

= \$29575

Amount of tax paid = 22% × \$29575

$= \frac{22}{100} \times \$29575$

= \$6506.50

Amount left with Kamal = \$29575 − \$6506.50

= \$23 068.50 **Ans.**

(b) $P = \$1200$, $r = 1.8$, $n = 5$

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow A = 1200\left(1 + \frac{1.8}{100}\right)^5 = 1200(1.018)^5 = 1311.96$$

∴ Amount of money in the account after 5 years = \$1311.96 **Ans.**

(c) Principal amount + Interest = Total amount in the account

$$\Rightarrow P + I = \$828.75$$

$$\Rightarrow P + \frac{PRT}{100} = \$828.75$$

$$\Rightarrow P + \frac{P \times 2.1 \times 5}{100} = \$828.75$$

$$\Rightarrow \frac{100P + 10.5P}{100} = \$828.75$$

$$\Rightarrow 110.5P = \$828.75 \times 100$$

$$\Rightarrow P = \frac{\$828.75 \times 100}{110.5} = \$750$$

∴ amount of money he invested = \$750 **Ans.**

(d) £0.72 = \$1

$£79.20 = \$\left(\frac{1}{0.72} \times 79.20\right) = \$110$

∴ Kamal changes \$110 into pounds.

Dollars left with Kamal = \$275 − \$110 = \$165

\$1 = € 1.10

\$165 = € (1.10 × 165) = € 181.50

∴ Kamal receives 181.50 euros. **Ans.**

105 *(N2018/P2/Q4)*

(a) Anna drives 45 km to work each day.
One day she drives the first 25 km at an average speed of 82 km/h.
She takes 36 minutes to drive the remaining distance.
Calculate her average speed, in km/h, for the whole journey. [3]

(b) Anna's journey home is 47 km, correct to the nearest kilometre.
One day her journey home takes 65 minutes, correct to the nearest 5 minutes.
Calculate the upper bound of her average speed, in km/h, for the journey home. [3]

(c) The probability that Anna arrives at work on time or early on any given day is $\frac{5}{8}$.
Calculate the probability that she is late on both Monday and Tuesday.
Give your answer as a fraction. [2]

Thinking Process

(a) Find the total time taken for the whole journey.

Apply, Average Speed = $\frac{\text{total distance}}{\text{total time}}$.

(b) To find the upper bound of the average speed add 0.5 km (i.e. 1 ÷ 2) to 47 km. Subtract 2.5 minutes (i.e. 5 ÷ 2) from 65 minutes.

(c) Find P(late) × P(late)

Solution

(a) For the first 25 km,

Time taken $= \frac{\text{distance}}{\text{speed}} = \frac{25}{82}$ hours

Total time for the journey $= \frac{25}{82} + \frac{36}{60}$

= 0.9049 hours

∴ Average Speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{45}{0.9049} = 49.73$ km/h **Ans.**

(b) Upper bound of average speed

$= \frac{\text{upper bound for the distance}}{\text{lower bound for the time}}$

$= \frac{47 + 0.5}{\frac{65 - 2.5}{60}}$

$= 47.5 \times \frac{60}{62.5} = 45.6$ km/h **Ans.**

(c) $P(\text{Anna arrives on time}) = \frac{5}{8}$

$P(\text{Anna is late}) = 1 - \frac{5}{8} = \frac{3}{8}$

∴ $P(\text{Anna is late on Monday and Tuesday})$

$= \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$ **Ans.**

Topic 2

Indices and Standard Form

1 (J2007/P1/Q12)

Evaluate

(a) 17^0, [1]

(b) $4^{\frac{5}{2}}$, [1]

(c) $(0.2)^{-2}$. [1]

Thinking Process

(b) Write 4 as 2^2

(c) Make the power of −2 positive & write 0.2 as a fraction.

Solution

(a) $17^0 = 1$ Ans.

(b) $4^{\frac{5}{2}} = \left(2^2\right)^{\frac{5}{2}} = (2)^5 = 32$ Ans.

(c) $(0.2)^{-2} = \left(\frac{2}{10}\right)^{-2} = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$ Ans.

2 (N2007/P1/Q17)

(a) It is given that $p = 4\times10^5$ and $q = 8\times10^6$. Expressing your answers in standard form, find

(i) $\frac{p}{q}$, [1]

(ii) $\sqrt[3]{q}$. [1]

(b) The numbers 225 and 540, written as the products of their prime factors, are

$$225 = 3^2\times5^2, \qquad 540 = 2^2\times3^3\times5.$$

(i) Write 2250 as the product of its prime factors. [1]

(ii) Find the smallest positive integer value of n for which $225n$ is a multiple of 540. [1]

Thinking Process

(a) (i) Divide 4 by 8 and 10^5 by 10^6 separately.

(b) (ii) To find the smallest n & Find the LCM of 225 and 540.

Solution

(a) (i) $\frac{p}{q} = \frac{4\times10^5}{8\times10^6}$

$= \frac{4}{8}\times\frac{10^5}{10^6}$

$= 0.5\times10^{-1} = 5.0\times10^{-2}$ Ans

(ii) $\sqrt[3]{q} = (q)^{\frac{1}{3}}$

$= \left(8\times10^6\right)^{\frac{1}{3}}$

$= (8)^{\frac{1}{3}}\times\left(10^6\right)^{\frac{1}{3}} = 2\times10^2$ Ans

(b) (i) $2250 = 2\times3\times3\times5\times5\times5$

$= 2\times3^2\times5^3$ Ans

(ii) $225 = 3^2\times5^2$, $540 = 2^2\times3^3\times5$

the L.C.M. of 225 and 540 is:

$2^2\times3^3\times5^2 = 2700$

smallest positive integer $= \frac{2700}{225}$

$= \frac{2^2\times3^3\times5^2}{3^2\times5^2}$

$= 2^2\times3 = 12$

$\therefore$ $n = 12$ Ans

3 (J2008/P1/Q4)

By writing each number correct to 1 significant figure, estimate the value of

$$\frac{8.62\times2.04^2}{0.285}$$ [2]

Thinking Process

Round off each number to one significant figure and simplify.

Solution

$\frac{8.62\times2.04^2}{0.285} = \frac{9\times2^2}{0.3}$

$= \frac{9\times4}{\frac{3}{10}}$

$= 36\times\frac{10}{3} = 120$ Ans.

4 (J2008/P1/Q8)

Evaluate

(a) 9^0, [1]

(b) 9^{-2}, [1]

(c) $9^{\frac{3}{2}}$. [1]

Thinking Process

(a) Use $a^0 = 1$.

(b) Use $a^{-n} = \frac{1}{a^n}$.

(c) Rewrite 9 as 3^2.

Solution

(a) $9^0 = 1$ **Ans.**

(b) $9^{-2} = \frac{1}{9^2} = \frac{1}{81}$ **Ans.**

(c) $9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}} = (3)^3 = 27$ **Ans.**

5 *(J2008/P1/Q18)*

The Earth is 1.5×10^8 kilometres from the Sun.

(a) Mercury is 5.81×10^7 kilometres from the Sun. How much nearer is the Sun to Mercury than to the Earth?
Give your answer in standard form. [2]

(b) A terametre is 10^{12} metres.
Find the distance of the Earth from the Sun in terametres. [2]

Thinking Process

(a) Understand that you need to subtract the Mercury-Sun distance from the Earth-Sun distance.

(b) divide the distance by 10^{12} ✐ change the distance into metres.

Solution

(a) Earth-Sun distance – Mercury-Sun distance

$= 1.5 \times 10^8 - 5.81 \times 10^7$

$= 15 \times 10^7 - 5.81 \times 10^7$

$= (15 - 5.81) \times 10^7 = 9.19 \times 10^7$

$\therefore$ Sun is 9.19×10^7 km nearer to Mercury than to Earth. **Ans.**

(b) Earth-Sun distance $= 1.5 \times 10^8$ kilometres

$= 1.5 \times 10^{11}$ metres

$\therefore$ distance in terametres $= \frac{1.5 \times 10^{11}}{10^{12}}$

$= 1.5 \times 10^{11} \times 10^{-12}$

$= 1.5 \times 10^{-1}$

$= 0.15$ terametres **Ans**

6 *(N2008/P1/Q7)*

It is given that $m = 2.1 \times 10^7$ and $n = 3 \times 10^4$.

Expressing your answers in standard form, find

(a) $m \div n$, [1]

(b) $n^2 + m$. [2]

Thinking Process

(a) ✐ Express m as a fraction of n.

(b) Substitute the values of m and n. Express your answer in standard form.

Solution

(a) $m \div n = \frac{m}{n}$

$= \frac{2.1 \times 10^7}{3 \times 10^4}$

$= \frac{2.1}{3} \times 10^{7-4}$

$= 0.7 \times 10^3$

$= 7 \times 10^2$ **Ans.**

(b) $n^2 + m = \left(3 \times 10^4\right)^2 + (2.1 \times 10^7)$

$= (9 \times 10^8) + (2.1 \times 10^7)$

$= 10^7(90 + 2.1)$

$= 92.1 \times 10^7$

$= 9.21 \times 10^8$ **Ans.**

7 *(J2009/P1/Q8)*

(a) Convert 0.8 kilometres into millimetres. [1]

(b) Evaluate $(6.3 \times 10^6) \div (9 \times 10^2)$, giving your answer in standard form. [2]

Thinking Process

(a) ✐ Recall, 1 km = 1000 000 mm

(b) To find $(a \times 10^m) \div (b \times 10^n)$ in standard form

✐ use $\frac{a \times 10^m}{b \times 10^n} = \left(\frac{a}{b}\right) \times 10^{m-n}$ and express $\frac{a}{b}$ in standard form to reach the final expression.

Solution

(a) 0.8 kilometres $= 0.8 \times 1000000$

$= 800000$ millimetres. **Ans.**

(b) $(6.3 \times 10^6) \div (9 \times 10^2)$

$= \frac{6.3 \times 10^6}{9 \times 10^2}$

$= \frac{6.3}{9} \times 10^4$

$= 0.7 \times 10^4 = 7.0 \times 10^3$ **Ans.**

8 *(N2009/P1/Q7)*

Tom estimated the population of five countries in 2020. The table below shows these estimates.

Country	Population
Australia	2.35×10^7
Brazil	1.95×10^9
China	1.4×10^9
Japan	1.36×10^8
United Kingdom	6.9×10^7

(a) Which country did he estimate would have a population about 20 times that of the United Kingdom? [1]

(b) How many more people did he estimate would be in Japan than in Australia?
Give your answer in Standard form. [2]

Thinking Process

(a) Multiply 20 by 6.9×10^7 and look for the country that has population nearest to your answer.
(b) Subtract the population of Australia from that of Japan.

Solution

(a) $20\times(6.9\times10^7)$

$=138\times10^7$

$=1.38\times10^9$

$\approx1.4\times10^9$ i.e. China

$\therefore$ China would have a population of about 20 times to that of the United kingdom. **Ans.**

(b) $(1.36\times10^8)-(2.35\times10^7)$

$=10^7(1.36\times10-2.35)$

$=10^7(13.6-2.35)$

$=11.25\times10^7$

$=1.125\times10^8$

$\therefore$ 1.125×10^8 more people would be in Japan than in Australia. **Ans.**

9 *(J2010/P1/Q4)*

Evaluate

(a) 9^1+9^0. [1]

(b) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$. [1]

Thinking Process

(a) evaluate 9^0 first.
(b) express 9 as base of 3.

Solution

(a) 9^1+9^0

$=9+1=10$ **Ans.**

(b) $\left(\frac{1}{9}\right)^{\frac{1}{2}}=\left[\left(\frac{1}{3}\right)^2\right]^{\frac{1}{2}}$

$=\frac{1}{3}$ **Ans.**

10 *(J2010/P2/Q3)*

The mass and diameter of the planets in the inner solar system are shown in the table.

Planet	Mass (kg)	Diameter (km)
Mercury	3.30×10^{23}	4880
Venus	4.87×10^{24}	12 100
Earth	5.97×10^{24}	12 800
Mars	6.42×10^{23}	6790

(a) List the planets in order of mass, starting with the lowest. [1]

(b) Find the radius, in kilometres, of Mars, giving your answer correct to 1 significant figure. [1]

(c) Giving your answer in standard form, find the total mass, in kilograms, of Venus and Mars. [1]

(d) [The volume of a sphere is $\frac{4}{3}\pi r^3$]

Giving your answer in standard form, find the volume, in cubic kilometres, of the Earth. [2]

Thinking Process

(b) Radius = 2 x diameter.
(d) Use the given formula to find the volume of earth in km³.

Solution

(a) Mercury, Mars, Venus, Earth. **Ans.**

(b) Radius of Mars $=\frac{6790}{2}$

$=3395\approx3000$ km (1 sf) **Ans.**

(c) Total mass of Venus and Mars

$=4.87\times10^{24}+6.42\times10^{23}$

$=5.51\times10^{24}$ kg **Ans.**

(d) Volume of earth $= \frac{4}{3} \times \pi \times \left(\frac{12800}{2}\right)^3$

$= \frac{4}{3} \times \pi \times (6400)^3$

$= 1.09821 \times 10^{12}$

$\approx 1.10 \times 10^{12}$ km^3 **Ans.**

11 *(N2010/P1/Q10)*

(a) Evaluate $5^0 - 5^{-1}$. [1]

(b) Simplify $(5x^3)^2$. [1]

(c) Simplify $\left(\frac{16}{n^{16}}\right)^{\frac{1}{2}}$. [1]

Thinking Process

(a) First evaluate 5^0 and 5^{-1} separately.

(c) Rewrite 16 as base of 4 and simplify.

Solution

(a) $5^0 - 5^{-1}$

$= 1 - \frac{1}{5}$

$= \frac{5-1}{5} = \frac{4}{5}$ **Ans.**

(b) $(5x^3)^2 = 25x^6$ **Ans.**

(c) $\left(\frac{16}{n^{16}}\right)^{\frac{1}{2}}$

$= \left(\left(\frac{4}{n^8}\right)^2\right)^{\frac{1}{2}} = \frac{4}{n^8}$ **Ans.**

12 *(J2011/P1/Q13)*

(a) The mass of one grain of rice is 0.000 02 kg. Write 0.000 02 in standard form. [1]

(b) The table shows the amount of rice grown in some countries in 2002.

	China	Brazil	India	Vietnam
Amount (tonnes)	1.2×10^8	7.6×10^6	8.0×10^7	2.1×10^7

(i) Write these amounts in order, smallest first. [1]

(ii) Calculate the difference in the amount of rice grown in Brazil and Vietnam. Give your answer in standard form. [1]

Thinking Process

(a) Recall that standard form is $A \times 10^n$, where $1 \le A < 10$.

(b) (i) To write the amounts in ascending order, Compare the powers of 10.

(ii) Subtract Brazil amount from Vietnam amount and express the answer in standard form.

Solution

(a) $0.00002 = 2 \times 10^{-5}$ **Ans.**

(b) (i) The amounts from smallest to largest are, 7.6×10^6, 2.1×10^7, 8.0×10^7, 1.2×10^8

(ii) $2.1 \times 10^7 - 7.6 \times 10^6$

$= (2.1 \times 10 - 7.6) \times 10^6$

$= (21 - 7.6) \times 10^6$

$= 13.4 \times 10^6$

$= 1.34 \times 10^7$ tonnes **Ans.**

13 *(J2011/P1/Q21)*

(a) Evaluate $\left(\frac{1}{4}\right)^{-2}$. [1]

(b) Evaluate $64^{\frac{2}{3}}$. [1]

(c) Simplify $\left(\frac{4x^2y^9}{x^4y}\right)^{\frac{1}{2}}$. [2]

Thinking Process

(a) Apply $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$

(b) Rewrite 64 as 4^3.

(c) Apply $(a^m)^n = a^{m \times n}$

Solution

(a) $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 16$ **Ans.**

(b) $64^{\frac{2}{3}} = (4^3)^{\frac{2}{3}}$

$= 4^2 = 16$ **Ans.**

(c) $\left(\frac{4x^2y^9}{x^4y}\right)^{\frac{1}{2}}$

$= \left(\frac{4y^8}{x^2}\right)^{\frac{1}{2}}$

$= \frac{4^{\frac{1}{2}} y^{8 \times \frac{1}{2}}}{x^{2 \times \frac{1}{2}}} = \frac{2y^4}{x}$ **Ans.**

14 *(N2011/P1/Q5)*

(a) Express the number 0.000 042 in standard form. [1]

(b) Calculate $(7 \times 10^{-3}) \times (3 \times 10^9)$, giving your answer in standard form. [1]

Thinking Process

(a) Standard form is in the form $A \times 10^n$ where $1 \le A < 10$

(b) Round off each number to 1 sig.fig.

Solution

(a) $0.000042 = 4.2 \times 10^{-5}$ **Ans.**

(b) $(7 \times 10^{-3}) \times (3 \times 10^9)$

$= 7 \times 3 \times 10^6$

$= 21 \times 10^6$

$= 2.1 \times 10^7$ **Ans.**

15 *(N2011/P1/Q7)*

(a) Evaluate $4^0 - 4^{-2}$. [1]

(b) Simplify $(2x^2)^3$. [1]

Thinking Process

(a) Write 4^{-2} as $\frac{1}{4^2}$. Evaluate.

Solution

(a) $4^0 - 4^{-2}$

$= 1 - \frac{1}{4^2}$

$= 1 - \frac{1}{16} = \frac{15}{16}$ **Ans.**

(b) $(2x^2)^3$

$= 2^3 x^6 = 8x^6$ **Ans.**

16 *(J2012/P1/Q18)*

(a) Find the value of

(i) $\sqrt{121}$, [1]

(ii) $\sqrt[3]{-27}$. [1]

(b) Write the following numbers in order of size, starting with the smallest.

$2^3 \quad 3^2 \quad 4^0 \quad 5^{-1}$ [1]

(c) Evaluate $16^{\frac{3}{2}}$. [1]

Thinking Process

(a) (i) Find the square root of 121.
(ii) Find the cube root of –27.

(b) Write the numbers in ordinary form first to observe the order.

(c) Rewrite 16 as 4^2.

Solution

(a) (i) $\sqrt{121}$

$= \sqrt{11^2} = 11$ **Ans.**

(ii) $\sqrt[3]{-27}$

$= (-27)^{\frac{1}{3}}$

$= \left((-3)^3\right)^{\frac{1}{3}} = -3$ **Ans.**

(b) $2^3 \quad 3^2 \quad 4^0 \quad 5^{-1}$

$= 8 \quad 9 \quad 1 \quad \frac{1}{5}$

∴ starting with the smallest,

$5^{-1} \quad 4^0 \quad 2^3 \quad 3^2$ **Ans.**

(c) $16^{\frac{3}{2}}$

$= (4^2)^{\frac{3}{2}}$

$= (4)^3 = 64$ **Ans.**

17 *(J2012/P1/Q20)*

The table below shows the populations of some countries in 2010.

Country	Population
Indonesia	2.4×10^8
Mexico	
Russia	1.4×10^8
Senegal	1.4×10^7
South Korea	4.8×10^7

(a) The population of Mexico was 111 210 000.
In the table above, complete the row for Mexico.
Give your answer in standard form, correct to two significant figures. [1]

(b) Complete the following sentences.

The population of Russia is ten times the population of ....

The population of .... is one fifth of the population of Indonesia. [2]

(c) Calculate the difference in population between South Korea and Senegal.
Give your answer in standard form. [1]

Thinking Process

(a) Express the value in standard form.

(b) Divide Russia's population by 10 and Indo-nesia's population by 5. Now look for the countries that have population nearest to your answers.

(c) Subtract by taking out common factor 10^7.

Solution

(a) Population of Mexico = 111210000

$= 1.1 \times 10^8$ (2s.f) Ans.

(b) The population of Russia is ten times the population of ......Senegal......

The population of ......South Korea...... is one fifth of the population of Indonesia.

(c) $4.8 \times 10^7 - 1.4 \times 10^7$

$= (4.8 - 1.4) \times 10^7$

$= 3.4 \times 10^7$ Ans.

18 (N2012/P1/Q11)

(a) Find the value of a when $3^a \div 3^4 = 3^2$. [1]

(b) Find the value of b when $8^b = 2$. [1]

Thinking Process

(a) Compare the powers of 3.

(b) Express 8 as 2^3.

Solution

(a) $3^a \div 3^4 = 3^2$

$3^a = 3^2 \times 3^4$

$\therefore \quad a = 2 + 4$

$= 6$ Ans.

(b) $8^b = 2$

$(2)^{3b} = (2)^1$

$\therefore \quad 3b = 1$

$b = \frac{1}{3}$ Ans.

19 (N2012/P1/Q17)

(a) Write the number 0.040 589 correct to 3 significant figures. [1]

(b) Giving your answer in standard form, evaluate $6 \times 10^{-4} + 8 \times 10^{-5}$. [1]

(c) Estimate, correct to the nearest whole number, the value of $\sqrt{97} - \sqrt{35}$.
Show clearly the approximate values you use. [1]

Thinking Process

(a) Round off the number to three significant figures.

(b) Add by taking out common factor 10^{-4}. Express the final answer in standard form. i.e, $A \times 10^n$ where $1 \le A \le 10$.

(c) Round off 97 and 35 to the nearest square number.

Solution

(a) $0.040589 \approx 0.0406$ Ans.

(b) $6 \times 10^{-4} + 8 \times 10^{-5}$

$= 10^{-4}(6 + 8 \times 10^{-1})$

$= 10^{-4}(6 + 0.8)$

$= 6.8 \times 10^{-4}$ Ans.

(c) $\sqrt{97} - \sqrt{35} \approx \sqrt{100} - \sqrt{36}$

$= 10 - 6$

$= 4$ Ans.

20 (J2013/P1/Q16)

(a) Evaluate 4^{-2}. [1]

(b) Simplify $\left(\frac{9xy^6}{x^3y^2}\right)^{\frac{1}{2}}$ [2]

Thinking Process

(a) Use $a^{-n} = \frac{1}{a^n}$

Solution

(a) $4^{-2} = \frac{1}{4^2}$

$= \frac{1}{16}$ Ans.

(b) $\left(\frac{9xy^6}{x^3y^2}\right)^{\frac{1}{2}}$

$= \left(\frac{9y^4}{x^2}\right)^{\frac{1}{2}}$

$= \left(\left(\frac{3y^2}{x}\right)^2\right)^{\frac{1}{2}} = \frac{3y^2}{x}$ Ans.

21 (J2013/P1/Q18)

The table shows information about the annual coffee production of some countries in 2010.

Country	Number of bags per year
Brazil	
Vietnam	1.85×10^7
Colombia	9.2×10^6
Indonesia	8.5×10^6

(a) In 2010, Brazil produced 48 million bags of coffee. Complete the table with the coffee production for Brazil, using standard form. [1]

(b) How many more bags of coffee were produced in Vietnam than in Colombia? [2]

(c) The mass of a bag of coffee is 60 kg.
Work out the number of kilograms of coffee produced in Indonesia.
Give your answer in standard form. [1]

Thinking Process

(a) Write 48 million in standard form.

(b) Subtract the number of bags of coffee of Vietnam from that of Colombia.

(c) Multiply 60 by 8.5 x10⁶. Re-express the answer in standard form.

Solution

(a) 48 million

$= 48000000$

$= 4.8 \times 10^7$ **Ans.**

(b) $1.85 \times 10^7 - 9.2 \times 10^6$

$= 10^6(18.5 - 9.2)$

$= 9.3 \times 10^6$ **Ans.**

(c) $60 \times (8.5 \times 10^6)$

$= 510 \times 10^6$

$= 5.1 \times 10^8$

∴ Number of kilograms of coffee produced in Indonesia $= 5.1 \times 10^8$ **Ans.**

22 *(N2013/P1/Q7)*

(a) Write the number 35 000 000 in standard form. [1]

(b) Giving your answer in standard form, evaluate

$\dfrac{4.2 \times 10^{-2}}{3 \times 10^4}$. [1]

Thinking Process

(a) Standard form $= A \times 10^n$.

(b) Use $\dfrac{a \times 10^m}{b \times 10^n} = \left(\dfrac{a}{b}\right) \times 10^{m-n}$ and express $\dfrac{a}{b}$ in standard form to reach the final expression.

Solution

(a) 35 000 000

$= 3.5 \times 10^7$ **Ans.**

(b) $\dfrac{4.2 \times 10^{-2}}{3 \times 10^4}$

$= \dfrac{4.2}{3} \times \dfrac{10^{-2}}{10^4}$

$= 1.4 \times 10^{-6}$ **Ans.**

23 *(N2013/P1/Q12)*

(a) Evaluate $\left(\dfrac{5}{3}\right)^{-2}$. [1]

(b) Simplify $\left(\dfrac{9}{t^6}\right)^{\frac{1}{2}}$. [1]

(c) Simplify $\dfrac{2x^3y}{6xy^2}$. [1]

Thinking Process

(a) $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$.

(b) Rewrite 9 as 3^2.

Solution

(a) $\left(\dfrac{5}{3}\right)^{-2}$

$= \left(\dfrac{3}{5}\right)^2 = \dfrac{9}{25}$ **Ans.**

(b) $\left(\dfrac{9}{t^6}\right)^{\frac{1}{2}}$

$= \left(\dfrac{(3)^2}{(t^3)^2}\right)^{\frac{1}{2}}$

$= \left(\left(\dfrac{3}{t^3}\right)^2\right)^{\frac{1}{2}} = \dfrac{3}{t^3}$ **Ans.**

(c) $\dfrac{2x^3y}{6xy^2}$

$= \dfrac{x^{3-1}}{3y^{2-1}} = \dfrac{x^2}{3y}$ **Ans.**

24 *(J2014/P1/Q5)*

(a) Express 0.000 085 2 in standard form. [1]

(b) Calculate $(3 \times 10^5) \div (6 \times 10^{-2})$, giving your answer in standard form. [1]

Thinking Process

(a) Recall that standard form is $A \times 10^n$, where $1 \leq A < 10$.

(b) To find $(a \times 10^m) \div (b \times 10^n)$ in standard form

use $\dfrac{a \times 10^m}{b \times 10^n} = \left(\dfrac{a}{b}\right) \times 10^{m-n}$ and express $\dfrac{a}{b}$ in standard form to reach the final expression.

Solution

(a) $0.0000852 = 8.52\times10^{-5}$ **Ans.**

(b) $(3\times10^{5})\div(6\times10^{-2})$

$=\dfrac{3\times10^{5}}{6\times10^{-2}}$

$=\dfrac{3}{6}\times10^{5-(-2)}$

$=0.5\times10^{7}$

$=5.0\times10^{6}$ **Ans.**

25 *(J2014/P1/Q19)*

(a) Evaluate

(i) $\sqrt[3]{216}$, [1]

(ii) $16^{\frac{1}{2}}-16^{0}$. [1]

(b) Simplify $\left(\dfrac{3a^2b}{12ab^4}\right)^{-2}$. [2]

Thinking Process

(a) (i) Express 216 as 6^3

(ii) Rewrite 16 as base of 4 and simplify.

(b) Apply rules of indices: $\dfrac{a^m}{a^n}=a^{m-n}$, $a^{-n}=\dfrac{1}{a^n}$

Solution

(a) (i) $\sqrt[3]{216}$

$=(216)^{\frac{1}{3}}$

$=(6\times6\times6)^{\frac{1}{3}}$

$=(6^3)^{\frac{1}{3}}=6$ **Ans.**

(ii) $16^{\frac{1}{2}}-16^{0}$

$=(4^2)^{\frac{1}{2}}-1$

$=4-1=3$ **Ans.**

(b) $\left(\dfrac{3a^2b}{12ab^4}\right)^{-2}$

$=\left(\dfrac{a}{4b^3}\right)^{-2}$

$=\left(\dfrac{4b^3}{a}\right)^{2}=\dfrac{16b^6}{a^2}$ **Ans.**

26 *(N2014/P1/Q12)*

(a) Write the number 0.000567 in standard form. [1]

(b) Gving your answer in standard form, evaluate $\dfrac{3\times10^{-5}}{5\times10^{6}}$. [2]

Thinking Process

(a) Recall that standard form is $A\times10^n$, where $1\le A<10$.

Solution

(a) $0.000567=5.67\times10^{-4}$ **Ans.**

(b) $\dfrac{3\times10^{-5}}{5\times10^{6}}$

$=\dfrac{3}{5}\times10^{-5-6}$

$=0.6\times10^{-11}$

$=6.0\times10^{-12}$ **Ans.**

27 *(N2014/P1/Q17)*

(a) Simplify $p^2(p^3-3p^{-2})$. [2]

(b) Simplify $(27x^6)^{\frac{1}{3}}$. [2]

Thinking Process

(a) Apply rules of indices: $a^m\times a^n=a^{m+n}$

(b) Express 27 as base of 3.

Solution

(a) $p^2(p^3-3p^{-2})$

$=p^{2+3}-3p^{-2+2}$

$=p^5-3$ **Ans.**

(b) $(27x^6)^{\frac{1}{3}}$

$=\left[3^3(x^2)^3\right]^{\frac{1}{3}}$

$=\left[(3x^2)^3\right]^{\frac{1}{3}}=3x^2$ **Ans.**

28 *(J2015/P1/Q9)*

(a) Write 0.000 005 21 in standard form. [1]

(b) Giving your answer in standard form, evaluate $(6\times10^7)\times(5\times10^{-3})$. [1]

Thinking Process

(a) Recall that standard form is $A\times10^n$, where $1\le A<10$

(b) Use $(a\times10^m)\times(b\times10^n)=ab\times10^{m+n}$, and further express ab in standard form to reach the final expression.

Solution

(a) $0.00000521=5.21\times10^{-6}$ **Ans.**

(b) $(6\times10^7)\times(5\times10^{-3})$

$=6\times5\times10^{7-3}$

$=30\times10^4$

$=3.0\times10^5$ **Ans.**

29 *(J2015/P1/Q16)*

(a) Evaluate

(i) 2^0+2^3. [1]

(ii) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$. [1]

(b) Simplify $(4x^2)^{-2}$. [1]

Thinking Process

(a) (i) To find the value first evaluate 2^0 and 2^3 separately.

(ii) Apply $\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$

(b) Apply $a^{-n}=\frac{1}{a^n}$

Solution

(a) (i) 2^0+2^3

$=1+8=9$ **Ans.**

(ii) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$

$=\frac{1}{9^{\frac{1}{2}}}$

$=\frac{1}{(3^2)^{\frac{1}{2}}}=\frac{1}{3}$ **Ans.**

(b) $(4x^2)^{-2}$

$=\frac{1}{(4x^2)^2}=\frac{1}{16x^4}$ **Ans.**

30 *(N2015/P1/Q10)*

(a) Express the number 0.000 004 5 in standard form. [1]

(b) $p=6\times10^8$ $q=4\times10^7$

Expressing each answer in standard form, find

(i) $p\times q$, [1]

(ii) $p-q$. [1]

Thinking Process

(a) Recall that standard form is $A\times10^n$, where $1<A<10$

(b) (i) Multiply 6 by 4 and 10^8 by 10^7 separately.

(ii) Take out common factor 10^7. Express the final answer in standard form.

Solution

(a) $0.000\,004\,5=4.5\times10^{-6}$ **Ans.**

(b) (i) $p\times q$

$=(6\times10^8)\times(4\times10^7)$

$=6\times4\times10^8\times10^7$

$=24\times10^{15}$

$=2.4\times10^{16}$ **Ans.**

(ii) $p-q$

$=6\times10^8-4\times10^7$

$=10^7(6\times10-4)$

$=10^7(60-4)$

$=56\times10^7$

$=5.6\times10^8$ **Ans.**

31 *(N2015/P1/Q11)*

(a) Evaluate $\left(\frac{3}{2}\right)^0$. [1]

(b) Evaluate $\left(\frac{3}{2}\right)^{-1}$. [1]

(c) Simplify $(9x^3)^2$. [1]

Thinking Process

(b) apply $a^{-n}=\frac{1}{a^n}=\left(\frac{1}{a}\right)^n$

(c) Apply $(a^m)^n=a^{m\times n}$

Solution

(a) $\left(\frac{3}{2}\right)^0=1$ **Ans.**

(b) $\left(\frac{3}{2}\right)^{-1}=\frac{2}{3}$ **Ans.**

(c) $(9x^3)^2$

$=9^2x^{3\times2}$

$=81x^6$ **Ans.**

32 *(J2016/P1/Q11)*

Simplify

(a) $\frac{5x^7y}{15x^3y^4}$. [1]

(b) $\left(\frac{4t^2}{v^4}\right)^{-\frac{1}{2}}$. [1]

Thinking Process

(a) Apply rules of indices: $\frac{a^m}{a^n}=a^{m-n}$, $a^{-n}=\frac{1}{a^n}$

(b) Apply $\left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^m=\frac{b^m}{a^m}$

Solution

(a) $\frac{5x^7y}{15x^3y^4}$

$=\frac{x^{7-3}}{3y^{4-1}}$

$=\frac{x^4}{3y^3}$ **Ans.**

(b) $\left(\frac{4t^2}{v^4}\right)^{-\frac{1}{2}}$

$=\left(\frac{v^4}{4t^2}\right)^{\frac{1}{2}}$

$=\frac{v^{4\times\frac{1}{2}}}{(2t)^{2\times\frac{1}{2}}}=\frac{v^2}{2t}$ **Ans.**

33 *(J2016/P1/Q22)*

The table shows the populations, correct to 2 significant figures, of some African countries in 2014.

Country	Population
Nigeria	
Sudan	3.6×10^7
Chad	1.1×10^7
Namibia	2.2×10^6

(a) In 2014, the population of Nigeria was 177 156 000.
Complete the table with the population of Nigeria using standard form, correct to 2 significant figures. [2]

(b) Complete the following.
The population of Chad was .... times the population of Namibia. [1]

(c) The population density of a country is measured as the number of people per square kilometre.
It can be found by dividing the population of the country by its area in km².
The area of Sudan is 1.86×10^6 square kilometres.
Estimate the population density of Sudan.
Give your answer correct to 1 significant figure. [2]

Thinking Process

(a) Recall that standard form is $A\times10^n$, where $1\le A<10$

(b) Divide the population of Chad by the population of Namibia

(c) Round off population and area of Sudan to one significant figure and divide.

Solution

(a) Population of Nigeria = 177 156 000

$=1.8\times10^8$ (2sf) **Ans.**

(b) $\frac{1.1\times10^7}{2.2\times10^6}$

$=\frac{1.1}{2.2}\times\frac{10^7}{10^6}$

$=\frac{11}{22}\times10=5$

∴ The population of Chad was 5 times the population of Namibia. **Ans.**

(c) $\frac{3.6\times10^7}{1.86\times10^6}$

$=\frac{4\times10^7}{2\times10^6}$

$=2\times10=20$

∴ population density of Sudan

$=20$ people/km² **Ans.**

34 *(N2016/P1/Q8)*

(a) Write the number 513 000 in standard form. [1]

(b) Expressing your answer in standard form, evaluate $(4\times10^{-5})\times(6\times10^{-4})$. [2]

Thinking Process

(a) Standard form is written in the form $A\times10^n$ where A is the number between 1 and 10.

(b) Multiply 4 by 6 and 10^{-5} by 10^{-4} separately. Re-express the answer in standard form.

Solution

(a) $513000=5.13\times10^5$ **Ans.**

(b) $(4\times10^{-5})\times(6\times10^{-4})$

$=(4\times6)\times(10^{-5}\times10^{-4})$

$=24\times10^{-9}$

$=2.4\times10\times10^{-9}=2.4\times10^{-8}$ **Ans.**

35 (N2016/P1/Q16)

(a) Evaluate $3^2 + 3^1 + 3^0$. [1]

(b) Evaluate $\left(\frac{4}{3}\right)^{-2}$. [1]

(c) Simplify $(16y^6)^{\frac{1}{2}}$. [1]

Thinking Process

(b) Use $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

(c) Express 16 as 4^2.

Solution

(a) $3^2 + 3^1 + 3^0$

$= 9 + 3 + 1 = 13$ **Ans.**

(b) $\left(\frac{4}{3}\right)^{-2}$

$= \left(\frac{3}{4}\right)^2 = \frac{9}{16}$ **Ans.**

(c) $(16y^6)^{\frac{1}{2}}$

$= (4^2 y^6)^{\frac{1}{2}}$

$= 4^{2\times\frac{1}{2}} y^{6\times\frac{1}{2}} = 4y^3$ **Ans.**

36 (N2017/P1/Q12)

$$a^x = 5$$

(a) Find a^{2x}. [1]

(b) Find a^{-x}. [1]

Thinking Process

(a) Take square on both sides.

Solution

(a) $a^x = 5$

$a^{2x} = 5^2 \Rightarrow a^{2x} = 25$ **Ans.**

(b) $a^x = 5$

$(a^x)^{-1} = (5)^{-1} \Rightarrow a^{-x} = \frac{1}{5}$ **Ans.**

37 (N2017/P1/Q16)

(a) Write the number 360 million in standard form. [1]

(b) $p = 5\times 10^9$ $q = 9\times 10^{-16}$

Expressing each answer in standard form, find

(i) $p\times q$. [1]

(ii) $\sqrt{q}$. [1]

Thinking Process

(a) Recall that standard form is in the form $A\times 10^n$ where $1 \le A < 10$.

(b) (i) Multiply 5 by 9 and 10^9 by 10^{-16} separately.

Solution

(a) 360 million = 360 000 000

$= 3.6\times 10^8$ **Ans.**

(b) (i) $p\times q = (5\times 10^9)\times(9\times 10^{-16})$

$= (5\times 9)\times(10^9\times 10^{-16})$

$= 45\times 10^{-7}$

$= 4.5\times 10^{-6}$ **Ans.**

(ii) $\sqrt{q} = \sqrt{9\times 10^{-16}}$

$= \sqrt{3^2\times(10^{-8})^2}$

$= \pm 3\times 10^{-8}$ **Ans.**

38 (J2018/P1/Q20)

$$N = 2\times 10^8$$

(a) Giving your answers in standard form, find the value of

(i) $N\times 700$. [1]

(ii) $\frac{1}{N}$. [2]

(b) Find the smallest positive integer M, given that MN is a cube number. [1]

Thinking Process

(a) Recall that standard form is $A\times 10^n$, where $1 \le A < 10$

(b) Look for a number such that the cube root of MN is a whole number.

Solution

(a) (i) $N\times 700$

$= (2\times 10^8)\times 700$

$= (2\times 10^8)\times(7\times 10^2)$

$= 14\times 10^{10} = 1.4\times 10^{11}$ **Ans.**

(ii) $\frac{1}{N} = \frac{1}{2\times 10^8}$

$= 0.5\times 10^{-8}$

$= 5\times 10^{-9}$ **Ans.**

(b) $N = 2\times 10^8$

$MN = (2\times 10^8)\times 5$

$\therefore$ $M = 5$ **Ans.**

Note that,
$MN = (2\times 10^8)\times 5$
$= 10\times 10^8 = 10^9$
which is a cube number

39 *(N2018/P1/Q9)*

$p = 8 \times 10^{-6}$ $\quad q = 2 \times 10^{11}$

Evaluate the following, giving your answers in standard form.

(a) $p \times q$ [1]

(b) $p \div q$ [1]

(c) $\sqrt[3]{p}$ [1]

Thinking Process

(a) Use $p \times q = (a \times 10^m) \times (b \times 10^n) = ab \times 10^{m+n}$ and further express ab in standard form to reach the final expression.

(b) To find $p \div q$ ✎ use $\frac{a \times 10^m}{b \times 10^n} = \left(\frac{a}{b}\right) \times 10^{m-n}$

and express $\frac{a}{b}$ in standard form.

(c) To find $\sqrt[3]{p}$ ✎ express 8×10^{-6} as $(2 \times 10^{-2})^3$.

Solution

(a) $p \times q$

$= (8 \times 10^{-6}) \times (2 \times 10^{11})$

$= 8 \times 2 \times 10^5$

$= 16 \times 10^5$

$= 1.6 \times 10^6$ **Ans.**

(b) $p \div q = \frac{p}{q}$

$= \frac{8 \times 10^{-6}}{2 \times 10^{11}}$

$= \frac{8}{2} \times \frac{10^{-6}}{10^{11}}$

$= 4 \times 10^{-6-11}$

$= 4 \times 10^{-17}$ **Ans.**

(c) $\sqrt[3]{p}$

$= \sqrt[3]{8 \times 10^{-6}}$

$= (8 \times 10^{-6})^{\frac{1}{3}}$

$= \left((2 \times 10^{-2})^3\right)^{\frac{1}{3}}$

$= 2 \times 10^{-2}$ **Ans.**

40 *(N2018/P1/Q21)*

(a) Evaluate $9^1 + 9^0$. [1]

(b) Find n, where $4^n = 2^{n-1}$. [2]

Thinking Process

(b) Express 4 as base of 2.

Solution

(a) $9^1 + 9^0$

$= 9 + 1 = 10$ **Ans.**

(b) $4^n = 2^{n-1}$

$\Rightarrow (2)^{2n} = 2^{n-1}$

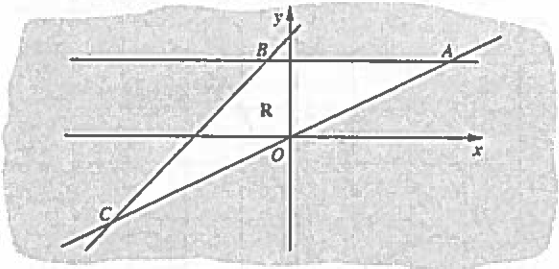
$\Rightarrow 2n = n - 1$

$\Rightarrow n = -1$ **Ans.**

Topic 3

Inequalities

1 (J2007/P1/Q10)



In the diagram, A is the point (6, 3) and C is the point $(-8, -4)$.

The equation of AB is $y = 3$ and the equation of CB is $y = x + 4$.

(a) Find the coordinates of B. [1]

(b) The unshaded region R inside triangle ABC is defined by three inequalities.

One of these is $y < x + 4$.

Write down the other two inequalities. [2]

Thinking Process

(a) To find the coordinates of B ✐ find the point of intersection of the lines AB and CB.

(b) Locate the given inequality first and then write the equations of the other two.
Convert the equations into inequalities.

Solution

(a) Equation of AB: $y = 3$ ·········(i)

equation of CB: $y = x + 4$ ·········(ii)

substituting (i) into (ii):

$3 = x + 4$

$x = -1$

also, from eq. (i), $y = 3$

∴ coordinates of B are $(-1, 3)$ Ans.

(b) Equation of AB: $y = 3$ ·········(i)

For line AC, we have points $A(6, 3)$ and $C(-8, -4)$

gradient of $AC = \frac{-4-3}{-8-6} = \frac{-7}{-14} = \frac{1}{2}$

y-intercept of $AC = 0$

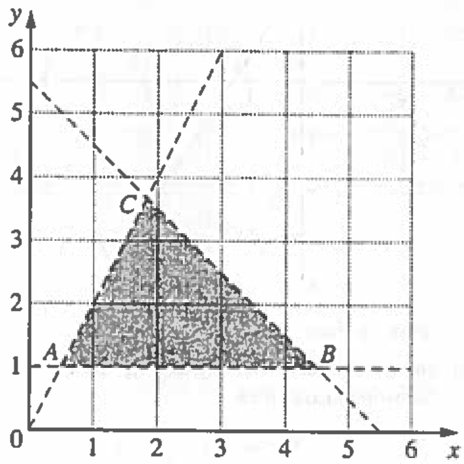
∴ equation of AC is: $y = \frac{1}{2}x$

the required inequalities are:

$y < 3$ and $y > \frac{1}{2}x$ Ans.

2 (N2007/P1/Q8)

The shaded region inside the triangle ABC is defined by three inequalities.



One of these is $x + y < 5\frac{1}{2}$.

(a) Write down the other two inequalities. [2]

(b) How many points, with integer coordinates, lie in the shaded region? [1]

Thinking Process

(a) Convert the equations into inequalities ✐ Locate the given inequality first and then write the equations of the other two.

(b) Look for integer coordinates that lie inside the shaded region.

Solution *with* TEACHER'S COMMENTS

(a) Consider two points (0, 0) and (2, 4) on the line AC

gradient $= \frac{4-0}{2-0} = 2$

y-intercept $c = 0$

∴ equation of the line AC is: $y = 2x$

and equation of the line AB is: $y = 1$

∴ the required inequalities are:

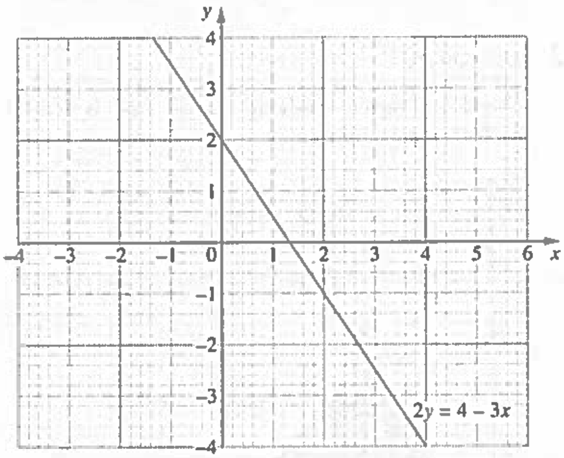
$y < 2x$ and $y > 1$ Ans

(b) Points with integer coordinates = 3 **Ans**

The three points are (2, 2), (3, 2) and (2, 3). The integer coordinates that lie on the dotted lines are not considered as they do not satisfy the respective inequalities.

3 *(J2008/P1/Q16)*

The diagram below shows the line $2y = 4 - 3x$.
On this diagram,



(a) draw the line $y = \frac{1}{2}x - 2$, [1]

(b) shade and label the region, R, defined by the following inequalities.

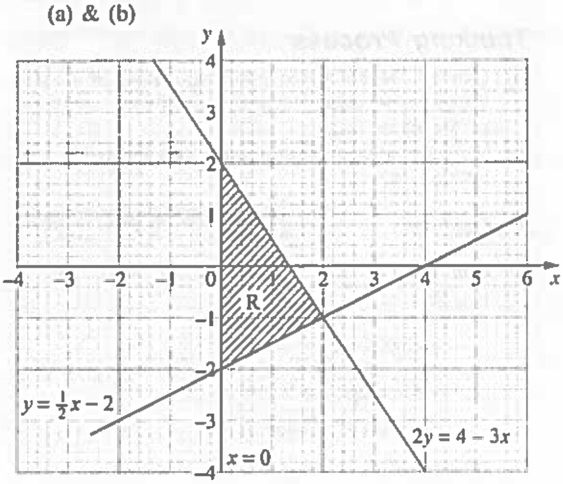
$x \geq 0 \quad 2y \leq 4 - 3x \quad y \geq \frac{1}{2}x - 2$ [2]

Thinking Process

(b) By considering the inequalities signs shade the required region.

Solution

(a) & (b)



4 *(N2008/P1/Q22)*

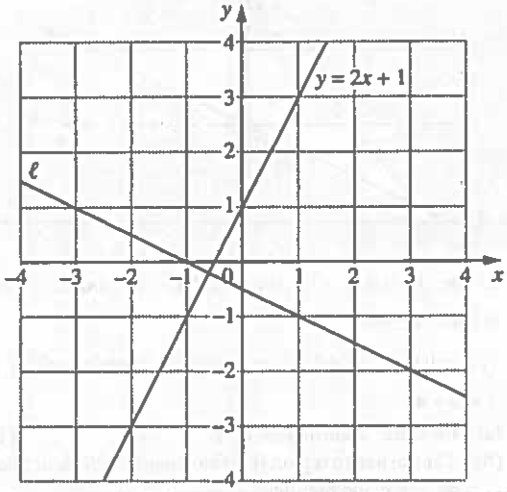
The equation of a line ℓ is $x + 2y = -1$.

(a) Write down the gradient of the line ℓ. [1]

(b) Find the equation of the line parallel to ℓ that passes through the point (0, 5). [2]

(c) The diagram in the answer space shows the line ℓ and the line $y = 2x + 1$.
On this diagram,
(i) draw the line $y = -2$,
(ii) shade and label the region, R, defined by the three inequalities

$y \geq -2 \qquad x + 2y \leq -1 \qquad y \geq 2x + 1.$



[2]

Thinking Process

(a) Re-arrange the equation in the form $y = mx + c$.
(b) Equation of straight: $y = mx + c$.
(c) (ii) Shade the region by considering the signs of each inequality.

Solution

(a) $x + 2y = -1$

$2y = -x - 1$

$y = -\frac{1}{2}x - \frac{1}{2}$

$\therefore$ gradient $= -\frac{1}{2}$ **Ans.**

(b) As the line is parallel to ℓ.

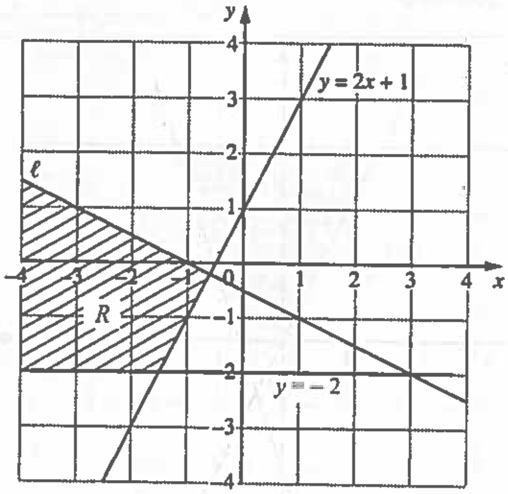
$\therefore$ equation is: $y = -\frac{1}{2}x + c$

at point (0, 5).

$5 = -\frac{1}{2}(0) + c \Rightarrow c = 5$

$\therefore$ equation of line: $y = -\frac{1}{2}x + 5$ **Ans.**

(c) (i) & (ii)



5 *(J2009/P1/Q17b)*

(b) Given that y is an integer and $-3 < 2y - 6 < 4$, list the possible values of y. [2]

Thinking Process

(b) Add 6 to all sides of inequalities, then divide by 2.

Solution

(b) $-3 < 2y - 6 < 4$

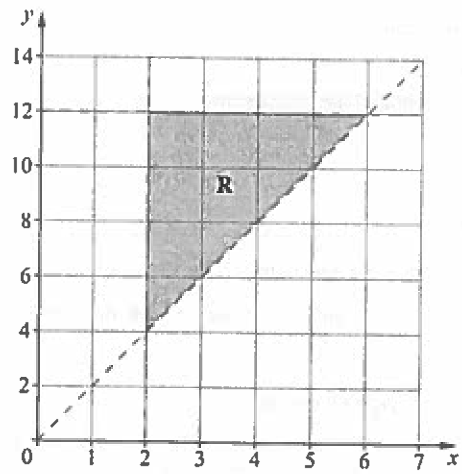
$6 - 3 < 2y < 4 + 6$

$3 < 2y < 10$

$1.5 < y < 5$

$\therefore \quad y = 2, 3, 4$ **Ans.**

6 *(J2010/P2/Q4)*



The shaded region, R, contained inside the dotted boundary lines, is defined by three inequalities.

(a) One of these inequalities is $x > 2$.
Write down the other two inequalities. [3]

(b) The points (c, d), where c and d are integers, lie in the shaded region R.
Find
(i) the maximum value of $c + d$, [1]
(ii) the value of d given that $d = 3c$. [1]

Thinking Process

(b) (i) Consider the greatest integer values of c and d that lie in the region R.
(ii) Look for the point inside region R that satisfies the given equation.

Solution *with* TEACHER's COMMENT

(a) Equation of one line: $y = 12$

For second line, taking two points (1, 2) and (2, 4),

$$\text{Gradient} = \frac{4-2}{2-1} = 2$$

y-intercept $= 0$

$\therefore$ equation is: $y = 2x$

Required inequalities are:

$y > 2x$, and $y < 12$ **Ans.**

(b) (i) Maximum value of $c + d = 5 + 11$
$= 16$ **Ans.**

> Note that the region R is inside three dotted lines. Therefore the points that lie on the boundary lines of R are not considered as they do not satisfy the respective inequalities.

(ii) The point (3, 9) in the region R satisfies the given equation $d = 3c$

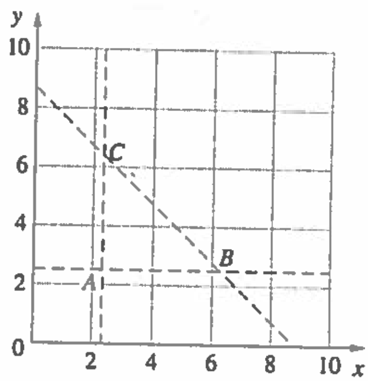
$\therefore \quad d = 9$ **Ans.**

7 *(N2010/P1/Q20)*

The three lines $3x = 7$, $2y = 5$ and $4x + 4y = 35$ intersect to form the triangle ABC, as shown in the diagram.

The region inside the triangle ABC is defined by three inequalities.

One of these is $2y > 5$.



(a) Write down the other two inequalities. [2]

(b) Find the point, with integer coordinates, that lies inside the triangle ABC and is closest to B. [1]

Thinking Process

(a) First identify the equations of the two lines and then convert them into inequalities.

(b) Look for the integer point inside triangle ABC that satisfies the given information.

Solution

(a) Line AB: $2y = 5$

Line AC: $3x = 7$

Line BC: $4x + 4y = 35$

$\therefore$ Required inequalities are:

$3x > 7$, and $4x + 4y < 35$ **Ans.**

(b) From graph, the integer point inside ΔABC and closest to B is (5, 3). **Ans.**

8 (J2011/P1/Q4)

(a) Solve $5y - 3 > 3y + 12$. [1]

(b) Write down all the integers that satisfy the inequality $-6 \le 3x < 6$. [1]

Thinking Process

(a) Arrange the inequality such that y is on one side of the inequality.

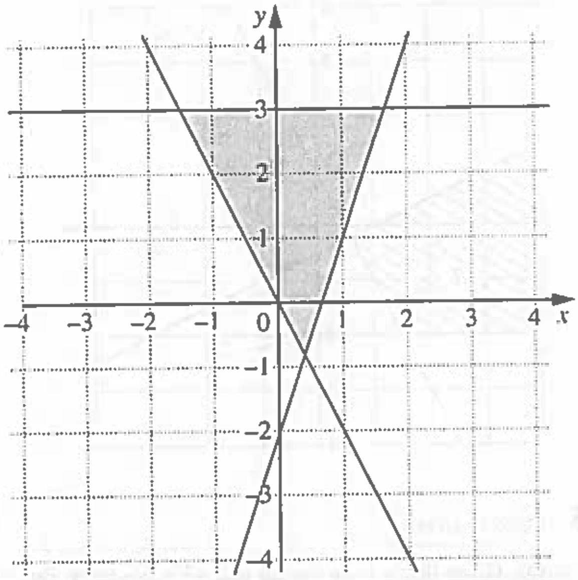
(b) Simplify the inequality and list the integers that lie within the given range.

Solution

(a)
$$5y - 3 > 3y + 12$$
$$5y - 3y > 3 + 12$$
$$2y > 15$$
$$y > \frac{15}{2} \quad \text{or} \quad y > 7.5 \quad \textbf{Ans}$$

(b)
$$-6 \le 3x < 6$$
$$-2 \le x < 2$$
$\therefore$ $x = -2, -1, 0, 1$ **Ans.**

9 (J2011/P1/Q9)



The shaded region on the diagram is represented by three inequalities.

One of these is $y \ge 3x - 2$.

Write down the other two inequalities. [2]

Thinking Process

Locate the given inequality first and then write the equations of the other two.

Solution

Equation of one line is, $y = 3$

For second line taking two points (0, 0) and (2, −4).

$$\text{gradient} = \frac{-4 - 0}{2 - 0} = -2$$

y-intercept = 0

equation is: $y = -2x$

$\therefore$ required inequalities are.

$y \le 3$ and $y \ge -2x$ **Ans.**

10 (N2011/P1/Q6)

(a) Solve the inequality $2(4 - x) < x - 10$. [1]

(b) Find the smallest integer n such that $3n > -17$. [1]

Thinking Process

(a) Arrange the inequality such that unknowns are all on one side of the inequality.

(b) Solve the inequality and find the integer which is smallest in the range formed.

Solution

(a) $2(4-x) < x-10$

$8-2x < x-10$

$-2x-x < -10-8$

$-3x < -18$

$3x > 18$

$x > 6$ **Ans.**

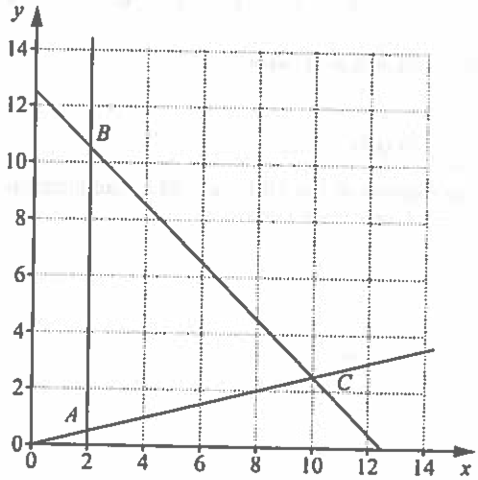
(b) $3n > -17$

$n > -\frac{17}{3}$

$n > -5.667$

$\therefore$ smallest value of $n = -5$ **Ans.**

11 *(N2011/P1/Q25)*



The diagram shows the graphs of

$x + y = 12\frac{1}{2}$,

$y = \frac{x}{4}$ and

$x = 2$.

These graphs intersect to form triangle ABC.

The region inside triangle ABC is defined by three inequalities.

One of these is $y > \frac{x}{4}$.

(a) Write down the other two inequalities. [2]

(b) $P = \{(x, y) : x \text{ and } y \text{ are integers}, (x, y) \text{ lies inside triangle } ABC\}$

$Q = \{(7, y) : y \text{ is an integer}\}$

(i) Find the member of the set P that is closest to the point C. [1]

(ii) Find $n(P \cap Q)$. [1]

Thinking Process

(a) Match the correct equation to each line. Convert the equations into inequalities.

(b) (i) Look for the integer coordinate inside triangle *ABC* and closest to point *C*.

(ii) To find $n(P \cap Q)$ List the common coordinates of sets P and Q inside the triangle *ABC*.

Solution

(a) $x > 2$ and $x + y < 12\frac{1}{2}$ **Ans.**

(b) (i) (9, 3) **Ans.**

(ii) $P \cap Q = \{(7, 2), (7, 3), (7, 4), (7, 5)\}$

$\therefore$ $n(P \cap Q) = 4$ **Ans.**

12 *(J2012/P1/Q7)*

(a) Solve $\frac{x+2}{3} \leq 2$. [1]

(b) Write down all the integers that satisfy this inequality.

$-1 \leq 4y + 3 < 11$ [1]

Thinking Process

(a) Arrange the inequality such that x is on one side on the inequality.

(b) List the integers that lie within the inequality.

Solution

(a) $\frac{x+2}{3} \leq 2$

$x + 2 \leq 6$

$x \leq 4$ **Ans.**

(b) $-1 \leq 4y + 3 < 11$

$-1-3 \leq 4y < 11-3$

$-4 \leq 4y < 8$

$-1 \leq y < 2$

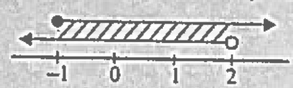
$\therefore$ $y = -1, 0, 1$ **Ans.**

Alternatively,

$-1 \leq 4y + 3$ or $4y + 3 < 11$

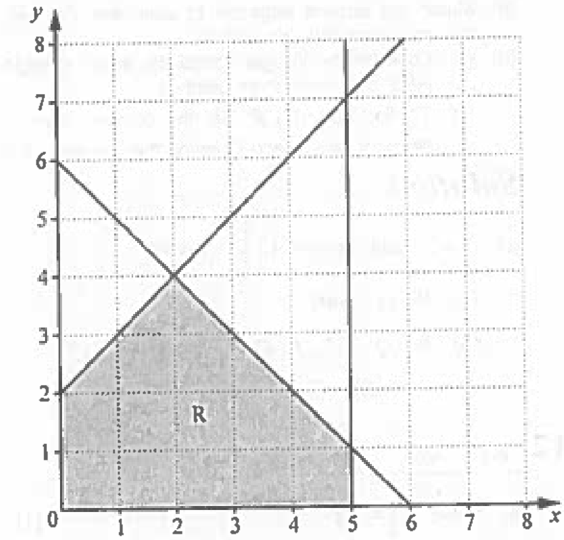
$4y \geq -1-3$ $\quad$ $4y < 8$

$y \geq -1$ $\quad$ $y < 2$



$\therefore$ $y = -1, 0, 1$ **Ans.**

13 (J2012/P2/Q1)



The diagram shows a shaded region R.

(a) Write down the name of the shaded polygon. [1]

(b) Three of the inequalities that define the region R are $x \geq 0$, $y \geq 0$ and $y \leq x + 2$.

Write down the other two inequalities that define this region. [2]

(c) On the diagram draw the line that is parallel to $y = x + 2$ and passes through the point (5, 0). [1]

(d) Find the gradient of the line that is perpendicular to $y = x + 2$. [1]

Thinking Process

(a) Note that the shaded polygon has five sides.

(b) First locate the given three inequalities and then write down the equations of the other two. Convert the equations into inequalities.

(d) Note that the given line has gradient +1.

Solution

(a) Pentagon. **Ans.**

(b) Equation of one line: $x = 5$

For second line, taking two points (6, 0) and (3, 3),

$$\text{gradient} = \frac{3-0}{3-6} = -1$$

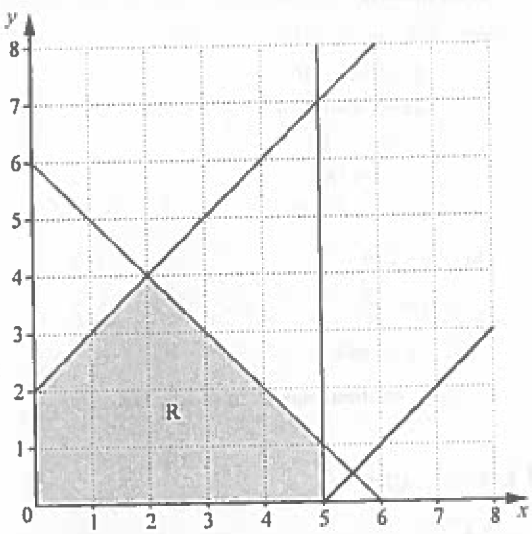
y-intercept = 6

$\therefore$ equation is: $y = -x + 6$

required inequalities are:

$x \leq 5$, and $y \leq -x + 6$ **Ans.**

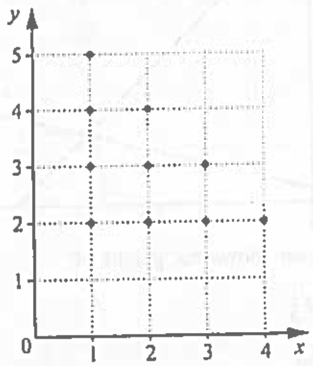
(c)



(d) Gradient = −1 **Ans.**

14 (N2012/P1/Q20)

The diagram shows 10 points, with coordinates (h, k), where h and k are integers.



(a) For these 10 points find

(i) the maximum value of $k - h$, [1]

(ii) the value of k, for the point that lies on the line $y = \frac{1}{2}x$. [1]

(b) The coordinates of the 10 points satisfy the inequalities

$h \geq a$, $k \geq b$, $h + k \leq c$.

Write down the values of a, b and c. [2]

Thinking Process

(a) (i) Look for the coordinate that gives greatest value of $k - h$.

(ii) Draw $y = \frac{1}{2}x$ on the grid and write down the y-coordinate of the point that lies on the line.

(b) To find a and b consider the smallest values of h and k respectively. To find c find the greatest value of $h + k$.

Solution with TEACHER's COMMENTS

(a) (i) Maximum value of $k - h = 5 - 1$
$= 4$ **Ans.**

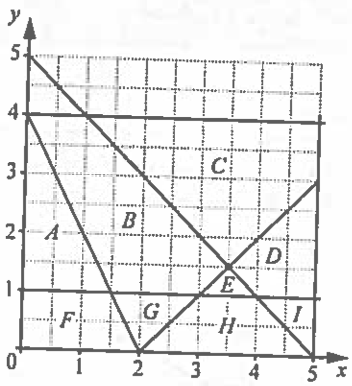
(ii) $k = 2$ **Ans.**

(b) $a = 1$ **Ans.**
$b = 2$ **Ans.**
$c = 6$ **Ans.**

> For a, note that the least value of x-coordinate is 1.
> For b, note that the least value of y-coordinate is 2.
> For c, note that the greatest value of $h + k$ is 6.

15 (J2013/P1/Q5)

The diagram shows the regions A to I.



Give the letter of the region defined by each set of inequalities.

(a) $x > 0$, $y > 0$, $y < 1$ and $y < 4 - 2x$ [1]

(b) $y > 1$, $y < x - 2$ and $y < 5 - x$ [1]

Thinking Process

First match the correct equation to each line, then identify the region by considering the signs of each inequality.

Solution

(a) F **Ans.**

(b) E **Ans.**

16 (J2013/P2 Q3 a,b)

(a) Solve $3(x - 5) = 5x - 7$. [2]

(b) (i) Solve $\frac{4y - 3}{2} \le 7$. [2]

(ii) State the integers that satisfy both $\frac{4y-3}{2} \le 7$ and $y > 2$. [1]

Thinking Process

(a) Expand and simplify for x.
(b) (ii) Use the answer of (b) (i) and list the integers that lie within the two inequalities.

Solution

(a) $3(x - 5) = 5x - 7$
$3x - 15 = 5x - 7$
$3x - 5x = -7 + 15$
$-2x = 8$
$x = -4$ **Ans.**

(b) (i) $\frac{4y-3}{2} \le 7$
$4y - 3 \le 14$
$4y \le 17$
$y \le \frac{17}{4}$
$y \le 4.25$ **Ans.**

(ii) $\frac{4y-3}{2} \le 7$ and $y > 2$.

From (b) (i), $\frac{4y-3}{2} \le 7 \Rightarrow y \le 4.25$

$\therefore$ integers that satisfy both inequalities are: 3, 4 **Ans.**

17 (N2013/P1/Q10)

Find one value of x that satisfies both $x > 4$ and $17 - 4x > 2 - x$. [2]

Thinking Process

Solve the given inequality and then write one value of x that lie within the two inequalities.

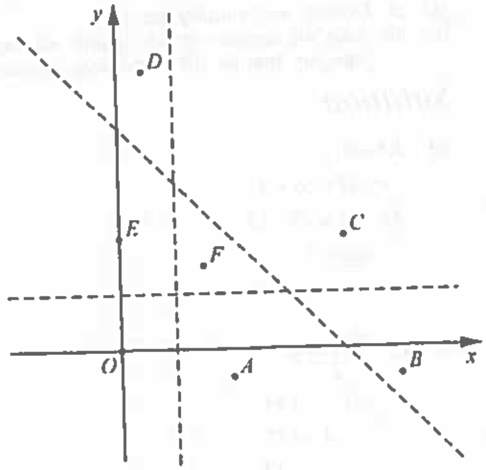
Solution

$17 - 4x > 2 - x$
$-4x + x > 2 - 17$
$-3x > -15$
$3x < 15$
$x < 5$

$\therefore$ one value of x that satisfies $x > 4$ and $x < 5$ is 4.5 **Ans.**

> The answer can be any number between 4 and 5.

18 *(N2013/P1/Q15)*



The diagram shows the three lines $x = 1$, $y = 1$ and $x + y = 4$ and the seven points O, A, B, C, D, E, and F.

(a) Which of these seven points lie in the region defined by $x + y > 4$? [1]

(b) Which one of these seven points lies in the region defined by $x < 1$, $y > 1$ and $x + y < 4$? [1]

(c) Given that O is (0, 0) and C is (4, 2), find the inequality that defines the region below the line that passes through O and C. [1]

Thinking Process

(a) & (b) ✐ Look for the points that satisfy the given conditions.

(c) First find the equation of the line OC, then change the equation into required inequality.

Solution

(a) B, C, D. **Ans.**

(b) E **Ans.**

(c) Gradient of $OC = \dfrac{2-0}{4-0} = \dfrac{1}{2}$

y-intercept of $OC = 0$

equation of OC: $y = \dfrac{1}{2}x$

∴ required inequality is: $y < \dfrac{1}{2}x$ **Ans.**

19 *(J2014/P1/Q3)*

It is given that $\dfrac{3}{4} < n < \dfrac{7}{8}$.

(a) Write down a decimal value of n that satisfies this inequality. [1]

(b) Write down a fractional value of n that satisfies this inequality. [1]

Thinking Process

(a) To find a decimal value of n ✐ Express the fractions as decimals.

(b) ✐ You may change your answer to part (a) into a fraction.

Solution with TEACHER's COMMENT

(a) $\dfrac{3}{4} < n < \dfrac{7}{8}$

$\Rightarrow 0.75 < n < 0.875$

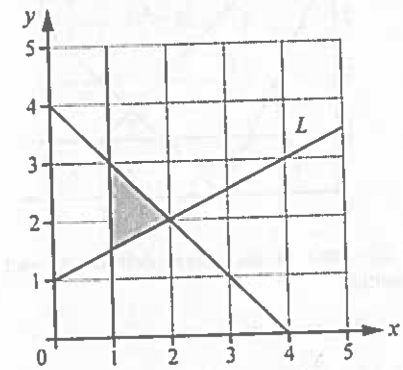
$\therefore n = 0.80$ **Ans.**

(b) $n = \dfrac{4}{5}$ **Ans.**

(a) Any answer between 0.75 and 0.875 is acceptable.

(b) Write any fraction within the given range, such as $\dfrac{5}{6}$, $\dfrac{6}{7}$, $\dfrac{13}{16}$, etc.

20 *(J2014/P1/Q15)*



(a) Find the gradient of the line L. [1]

(b) The shaded region on the diagram is defined by three inequalities.

One of these is $x + y \leq 4$.

Write down the other two inequalities. [2]

Thinking Process

(a) ✐ Apply formula, Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

(b) ✐ Locate the given inequality first and then write the equations of the other two. Convert the equations into inequalities.

Solution

(a) Taking two points (0, 1) and (4, 3) on the line,

gradient $= \dfrac{3-1}{4-0}$

$= \dfrac{2}{4} = \dfrac{1}{2}$ **Ans.**

(b) y-intercept of line $L = 1$

$\therefore$ equation of line L is: $y = \frac{1}{2}x + 1$

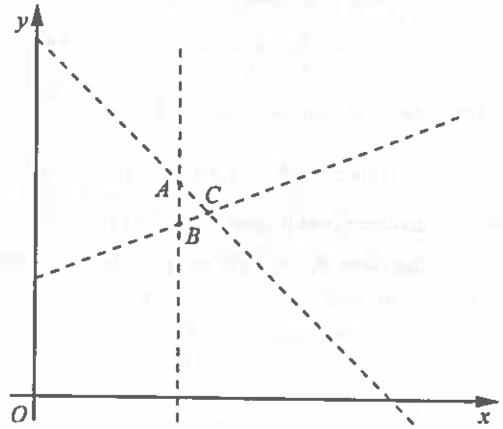
equation of second line is: $x = 1$

$\therefore$ required inequalities are:

$y \geq \frac{1}{2}x + 1$ and $x \geq 1$ **Ans.**

21 *(N2014/P1/Q23)*

The diagram shows the three lines
$x = 8$, $x + y = 21$ and $2y = 12 + x$
which intersect at the points A, B and C.



(a) Find the coordinates of B. [1]

(b) The region **inside** triangle ABC is defined by three inequalities.
One of these is $x + y < 21$.
Write down the other two inequalities. [2]

(c) Find the coordinates of the point, with integer coordinates, that is inside triangle ABC. [1]

Thinking Process

(a) First locate the equations of the two lines that cross at point B. Solve the two equations simultaneously to find coordinates of point B.

(b) First locate the given inequality and then convert the other two equations into inequalities.

(c) By inspection, find integer values of x and y that is found in the defined region

Solution

(a) The two lines that pass through point B are,

$x = 8$ and $2y = 12 + x$

substitute $x = 8$ into second equation, we have,

$2y = 12 + 8$

$2y = 20$

$y = 10$

$\therefore$ coordinates of B are $(8, 10)$ **Ans.**

(b) Required inequalities are:

$x > 8$ and $2y > 12 + x$ **Ans.**

(c) One inequality states that $x > 8$
therefore, substitute $x = 9$ in the other two inequalities,

$x + y < 21$

$\Rightarrow 9 + y < 21 \Rightarrow y < 12$

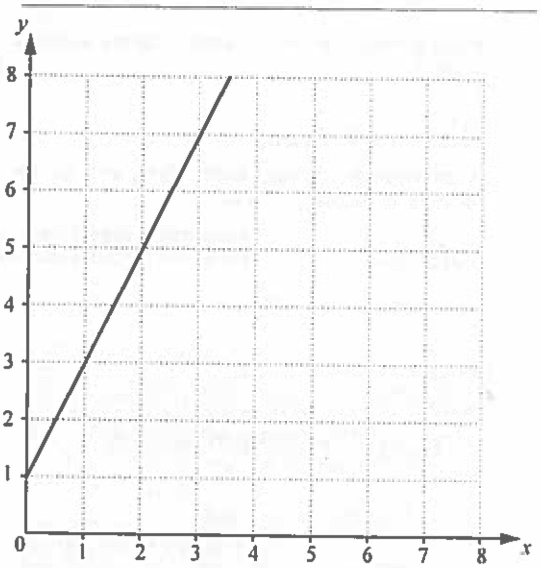
$2y > 12 + x$

$\Rightarrow 2y > 12 + 9 \Rightarrow 2y > 21 \Rightarrow y > 10.5$

from the above results we see that the integer value of y satisfied by both inequalities is 11

$\therefore$ the point inside ΔABC with integer coordinates is $(9, 11)$ **Ans.**

22 *(J2015/P1/Q11)*



The diagram shows the line $y = 2x + 1$.
The point P has coordinates (a, b) where a and b are both positive integers.
The values of a and b satisfy the inequalities
$a < 2$, $b < 7$ and $b > 2a + 1$.
Write down all the possible coordinates of P. [2]

Thinking Process

By considering the inequalities signs look for the points that satisfies the given three inequalities.

Solution

Possible coordinates of point P are:

$(1, 4)$, $(1, 5)$, $(1, 6)$ **Ans.**

23 *(J2015/P1/Q23 a)*

(a) Solve the inequalities.

$$-4 \leq 2x - 5 < 7$$ [2]

Thinking Process

(a) Add 5 to all sides of inequalities, then divide by 2.

Solution

(a)
$$-4 \le 2x - 5 < 7$$
$$5 - 4 \le 2x < 7 + 5$$
$$1 \le 2x < 12$$
$$\frac{1}{2} \le x < \frac{12}{2}$$
$$\frac{1}{2} \le x < 6 \quad \text{Ans.}$$

24 (N2015/P1/Q8)

a, b, c, d and e are five numbers, such that

$$d < a < c$$
$$a < e < c$$
$$a < b < e$$

Arrange these numbers in order, starting with the smallest. [2]

Thinking Process

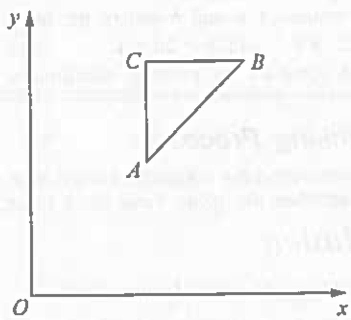
By considering the inequalities signs, arrange the numbers in increasing order.

Solution *with* TEACHER's COMMENTS

$d < a < b < e < c$ Ans.

> Note that,
> $a < b$, $a < c$, $a < e$ $\Rightarrow$ a is smaller than b, c, and e.
> $d < a$ $\Rightarrow$ d is smaller than a.
> $b < e$ and $e < c$ $\Rightarrow$ b is smaller than e and c.

25 (N2015/P1/Q18)



The sides of the triangle ABC are formed by the straight lines with equations

$$x = 3, \quad y = 6, \quad y = x + \frac{1}{2}.$$

(a) The region inside the triangle is defined by three inequalities.
Write down these three inequalities. [2]

(b) The point $(4, k)$, where k is an integer, lies inside the triangle.
Find the value of k. [1]

Thinking Process

(a) First locate the equations of the three lines that corresponds to the sides of the triangle and then convert them into inequalities.

(b) By inspection look for the integer value of y that lies inside triangle ABC.

Solution

(a) Line AB: $y = x + \frac{1}{2}$

Line AC: $x = 3$

Line CB: $y = 6$

$\therefore$ Required inequalities are:

$$y > x + \frac{1}{2}, \quad x > 3 \quad \text{and} \quad y < 6 \quad \text{Ans.}$$

(b) One inequality is: $y > x + \frac{1}{2}$

substitute $x = 4$, $y > 4 + \frac{1}{2} \Rightarrow y > 4.5$

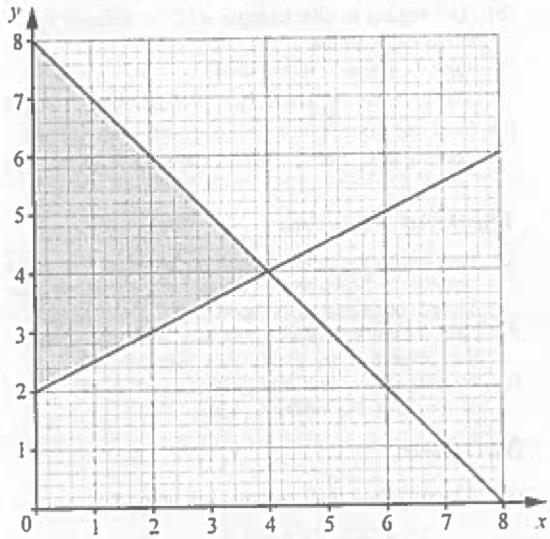
another given inequality is: $y < 6$

therefore, at $x = 4$, the integer value of y inside ΔABC is 5.

$\therefore \quad k = 5$ Ans.

26 (J2016/P1/Q15)

The diagram shows the lines $x + y = 8$ and $2y = x + 4$.



(a) The shaded region on the diagram is defined by three inequalities.
Write down these three inequalities. [2]

(b) Another region, R, is defined by the inequalities
$x + y \le 8$, $2y \le x + 4$ and $y \ge a$, where a is an integer.
This region contains 5 points with integer coordinates.
Write down the value of a. [1]

Thinking Process

(a) First match the correct equation to each line, then find the equation of the third line. Convert all three equations into inequalities.

(c) By inspection, find the third inequality such that the defined region contains 5 integer coordinates.

Solution

(a) Equation of third line: $x = 0$

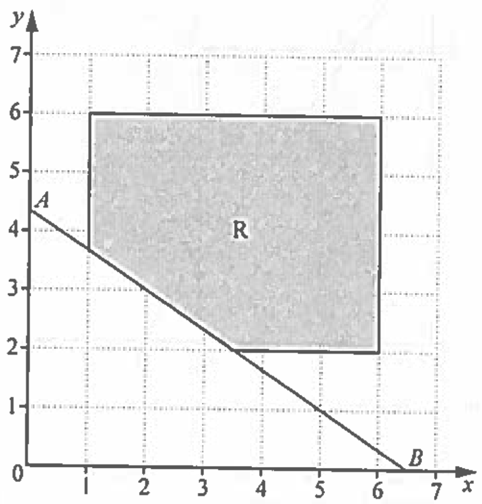
$\therefore$ required inequalities are:

$x + y \le 8,\quad 2y \ge x + 4$ and $x \ge 0$ Ans.

(b) By inspection, If the third inequality is $y \ge 3$, then the defined region contains 5 integer points.

$\therefore$ $a = 3$ Ans.

27 (N2016/P1/Q25)



In the diagram, the line $3y + 2x = 13$ meets the axes at A and B.

(a) Find the coordinates of A. [1]

(b) The shaded region R is defined by five inequalities.
Two of these are $x \le 6$ and $y \le 6$.
Write down the other three inequalities. [2]

(c) The point P is in the shaded region R.
Given that AP is as large as possible, write down the coordinates of P. [1]

Thinking Process

(a) Note that the line crosses y-axis when $x = 0$

(b) Locate the given inequality first and then write the equations of the other three. Convert the equations into inequalities.

(c) Look for the coordinate that gives greatest length of AP.

Solution

(a) $3y + 2x = 13$

At point A, $x = 0$

$\Rightarrow 3y + 2(0) = 13$

$\Rightarrow 3y = 13 \Rightarrow y = \frac{13}{3}$

$\therefore$ coordinates of A are: $(0, \frac{13}{3})$ Ans.

(b) Equations of other three lines are:

$x = 1,\quad y = 2,\quad 3y + 2x = 13$

$\therefore$ required inequalities are:

$x \ge 1,\quad y \ge 2,\quad 3y + 2x \ge 13$ Ans.

(c) From graph,
the coordinates of P are $(6, 2)$ Ans.

28 (J2017/P1/Q9)

(a) Write down all the integers that satisfy the inequality $-\frac{3}{2} \le x < 2$. [1]

(b) Complete the following inequality with a fraction.

$$\frac{3}{4} > \ldots\ldots\ldots\ldots > \frac{1}{2}$$ [1]

(c) Write down an irrational value of n that satisfies this inequality.

$$2 < n < 3$$ [1]

Thinking Process

(a) List the integers that lie within the inequality.

(b) Find the average of the two given fractions.

(c) Understand that irrational numbers are numbers that cannot be written as a simple fraction.

Solution

(a) $-\frac{3}{2} \le x < 2$

$\therefore$ the integers are $-1, 0, 1$ Ans.

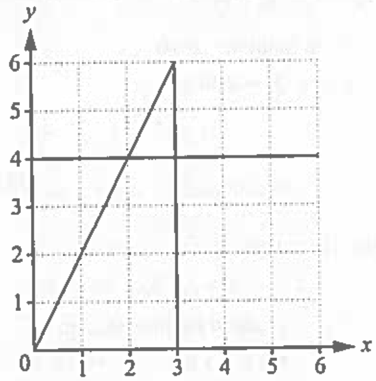
(b) $\dfrac{\frac{3}{4} + \frac{1}{2}}{2}$

$= \dfrac{\frac{5}{4}}{2} = \dfrac{5}{8}$ Ans.

(c) $n = \sqrt{5}$ Ans. Alternative solutions are $\sqrt{6}$, $\sqrt{7}$

29 *(J2017/P2/Q5 d)*

(d)



(i) Draw the graph of $x+2y=5$. [2]

(ii) Shade the region defined by these inequalities and label it R.

$x \le 3 \qquad y \le 4 \qquad y \le 2x \qquad x+2y \ge 5$

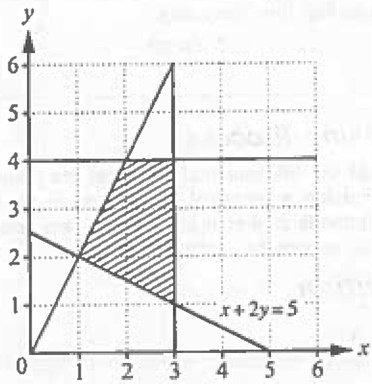
[1]

Thinking Process

(d) (ii) Shade the region by considering the signs of each inequality.

Solution

(d) (i) & (ii)



30 *(N2017/P1/Q14)*

Find the two solutions of $\frac{x}{3}-1<\frac{3x}{4}$ which are negative integers. [3]

Thinking Process

Solve the two inequality.

Solution

$\frac{x}{3}-1<\frac{3x}{4}$

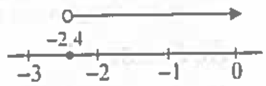
$\frac{x}{3}-\frac{3x}{4}<1$

$\frac{4x-9x}{12}<1$

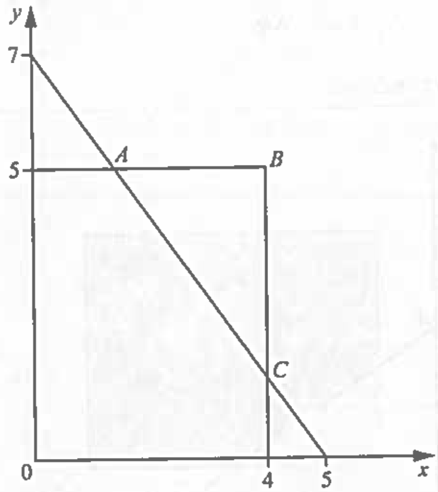
$-5x<12$

$x>-2.4$

$\therefore\ x=-2$ and -1 **Ans.**



31 *(N2017/P1/Q25)*



In the diagram, the equation of the line AC is $7x+5y=35$.

(a) Write down the three inequalities that define the region inside triangle ABC. [2]

(b) The line $y=kx$, where k is an integer, passes through triangle ABC.
Find the greatest possible value of k. [2]

Thinking Process

(a) First find the equations of the other two lines and then convert all three equations into inequalities.

(b) Greatest value of k occurs when the gradient of $y = kx$ is maximum.

Solution

(a) Equation of AB: $y=5$

Equation of BC: $x=4$

Equation of AC: $7x+5y=35$

$\therefore$ the required inequalities are:

$y<5,\quad x<4,\quad$ and $\quad 7x+5y>35$ **Ans.**

(b) The gradient k is maximum when $y = kx$ passes through A.

Equation of AC: $7x + 5y = 35$

at point A, $y = 5$

$\Rightarrow$ $7x + 5(5) = 35$ $\Rightarrow$ $7x = 10$ $\Rightarrow$ $x = \frac{10}{7}$

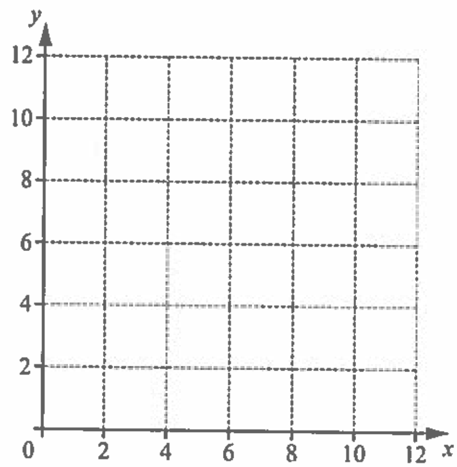
$\therefore$ coordinates of A are $(\frac{10}{7}, 5)$

substitute point A into $y = kx$,

$5 = k(\frac{10}{7})$ $\Rightarrow$ $k = \frac{35}{10} = 3.5$

given that k is an integer and $y = kx$ passes through ΔABC, therefore maximum value of $k = 3$ **Ans.**

32 *(J2018/P1/Q18)*



The region R is defined by the inequalities

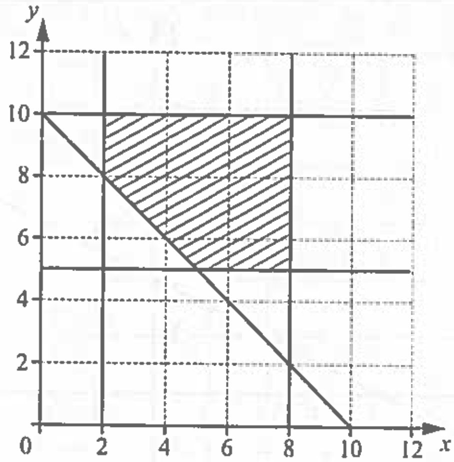
$$2 \leq x \leq 8$$
$$5 \leq y \leq 10$$
$$x + y \geq 10.$$

On the diagram, shade and label the region R. [3]

Thinking Process

Draw the lines that corresponds to the given inequalities. Then shade the region by considering the signs of each inequality.

Solution



33 *(N2018/P1/Q16)*

Find the possible values of x, given that x is an integer and $15 < 2x - 3 < 22$. [3]

Thinking Process

Solve the given inequality and list the integers that lie within the inequalities.

Solution

$15 < 2x - 3 < 22$

$\Rightarrow$ $15 + 3 < 2x < 22 + 3$

$\Rightarrow$ $18 < 2x < 25$

$\Rightarrow$ $\frac{18}{2} < x < \frac{25}{2}$

$\Rightarrow$ $9 < x < 12.5$

$\therefore$ $x = 10, 11, 12$ **Ans.**

34 *(N2018/P2/Q8a)*

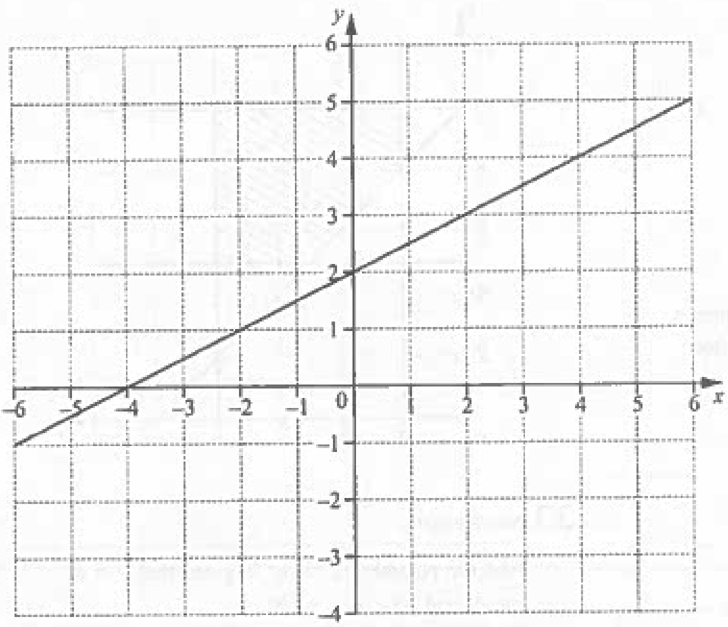
The line $2y = x + 4$ is drawn on the grid.

(i) On the grid, draw the line $x + y + 2 = 0$. [2]

(ii) The region R is represented by these three inequalities.

$$2y \geq x + 4$$
$$x + y + 2 \geq 0$$
$$y \leq 2$$

On the grid, shade and label the region R. [2]

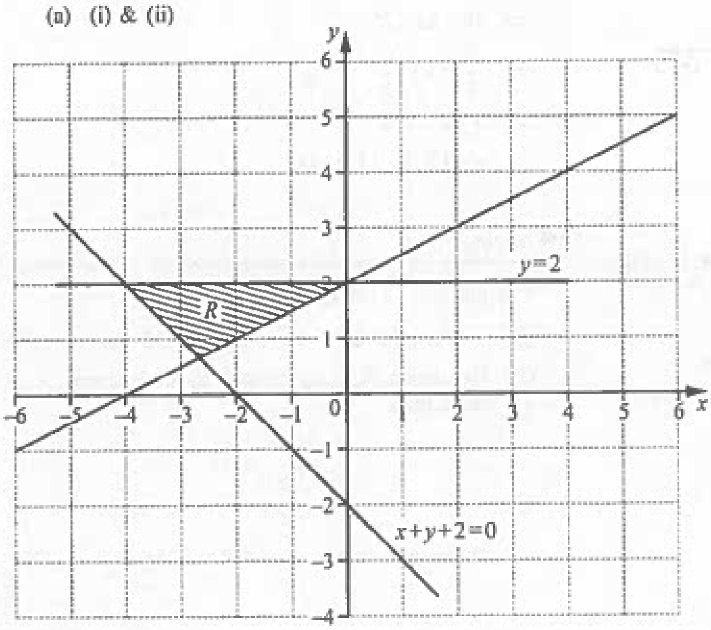


Thinking Process

(a) (ii) Draw the line y = 2. Then shade the required region by considering the inequalities signs.

Solution

(a) (i) & (ii)



Topic 4

Algebraic Expressions and Manipulations

1 (J2007/P2/Q21)

Factorise

(a) $2x^2-7x-15$. [2]

(b) $2yt-8ys-zt+4zs$. [2]

Thinking Process

(a) Factorise by sum and product method.
(b) Observe and look for the common factor in the expression.

Solution

(a) $2x^2-7x-15$

$=2x^2-10x+3x-15$

$=2x(x-5)+3(x-5)$

$=(x-5)(2x+3)$ **Ans.**

(b) $2yt-8ys-zt+4zs$

$=2y(t-4s)-z(t-4s)$

$=(t-4s)(2y-z)$ **Ans.**

2 (J2007/P2/Q3a,b)

(a) Express as a single fraction in its simplest form

$$\frac{7}{6a}-\frac{5}{9a}.$$ [2]

(b) Simplify $3b(b-1)-2(b-2)(b+2)$. [2]

Solution

(a) $\frac{7}{6a}-\frac{5}{9a}$

$=\frac{21-10}{18a}=\frac{11}{18a}$ **Ans**

(b) $3b(b-1)-2(b-2)(b+2)$

$=3b(b-1)-2(b^2-4)$

$=3b^2-3b-2b^2+8=b^2-3b+8$ **Ans**

3 (N2007/P1/Q20)

Factorise completely

(a) $15a^2+12a^3$, [1]

(b) $1-16b^2$, [1]

(c) $6cx-3cy-2dx+dy$. [2]

Thinking Process

(a) Take $3a^2$ as common.
(b) Use $a^2-b^2=(a+b)(a-b)$.
(c) By inspection look for a common factor in $6cx-3cy$ & $-2dx+dy$ respectively.

Solution

(a) $15a^2+12a^3=3a^2(5+4a)$ **Ans**

(b) $1-16b^2=(1)^2-(4b)^2$

$=(1+4b)(1-4b)$ **Ans**

(c) $6cx-3cy-2dx+dy$

$=3c(2x-y)-d(2x-y)$

$=(2x-y)(3c-d)$ **Ans**

4 (J2008/P1/Q15)

Express as a single fraction in its simplest form

$$\frac{3}{2t-1}-\frac{2}{t+2}.$$ [3]

Thinking Process

Re-express the fractions with a common denominator.

Solution

$\frac{3}{2t-1}-\frac{2}{t+2}$

$=\frac{3(t+2)-2(2t-1)}{(2t-1)(t+2)}$

$=\frac{3t+6-4t+2}{(2t-1)(t+2)}=\frac{8-t}{(2t-1)(t+2)}$ **Ans.**

5 (J2008/P1/Q19)

(a) Factorise completely

(i) $15x^2+10x$, [1]

(ii) $t^2-2t-15$. [1]

(b) Solve $4(x-0.3)=3(x-0.2)$. [2]

Thinking Process

(a) (i) Take out common factor.
(ii) Break the middle term and factorise.
(b) Expand and simplify for x.

Solution

(a) (i) $15x^2+10x=5x(3x+2)$ **Ans.**

(ii) $t^2-2t-15$

$=t^2-5t+3t-15$

$=t(t-5)+3(t-5)=(t-5)(t+3)$ **Ans.**

(b) $4(x-0.3)=3(x-0.2)$

$4x-1.2=3x-0.6$

$4x-3x=1.2-0.6$

$x=0.6$ **Ans.**

6 (J2008/P1/Q21)

(a) Solve $8 - 3t > 14 + t$. [2]

(b) Evaluate $x^2 - 6xy + 2y^2$ when $x = 2$ and $y = -3$. [2]

Thinking Process

(a) Arrange the inequality such that unknowns are all on one side of the inequality.

Solution

(a) $8 - 3t > 14 + t$

$-3t - t > 14 - 8$

$-4t > 6$

$t < \frac{6}{-4}$

$t < -\frac{3}{2} \Rightarrow t < -1.5$ **Ans.**

> When multiplying or dividing an inequality by a negative number, the inequality sign will change.

(b) $x^2 - 6xy + 2y^2$

$= (2)^2 - 6(2)(-3) + 2(-3)^2$

$= 4 + 36 + 18 = 58$ **Ans.**

7 (N2008/P1/Q21)

(a) Expand and simplify $(p - 5)(p + 4)$. [1]

(b) Factorise completely

(i) $4x^2 + 12xy + 9y^2$, [2]

(ii) $3m^2 - 48$. [2]

Thinking Process

(b) (i) Factorise the expression by grouping.
(ii) Factorise ✐ Take out common factor.

Solution

(a) $(p-5)(p+4)$

$= p^2 + 4p - 5p - 20$

$= p^2 - p - 20$ **Ans.**

(b) (i) $4x^2 + 12xy + 9y^2$

$= 4x^2 + 6xy + 6xy + 9y^2$

$= 2x(2x + 3y) + 3y(2x + 3y)$

$= (2x + 3y)(2x + 3y)$

$= (2x + 3y)^2$ **Ans.**

(ii) $3m^2 - 48$

$= 3(m^2 - 16)$

$= 3(m^2 - 4^2) = 3(m + 4)(m - 4)$ **Ans.**

8 (J2009/P1/Q4)

(a) Factorise $x^2 - y^2$. [1]

(b) Evaluate $102^2 - 98^2$. [1]

Thinking Process

(a) Recall $a^2 - b^2 = (a + b)(a - b)$
(b) ✐ Apply the formula given in part (a).

Solution

(a) $x^2 - y^2 = (x + y)(x - y)$ **Ans.**

(b) $102^2 - 98^2$

$= (102 + 98)(102 - 98)$

$= (200)(4) = 800$ **Ans.**

9 (J2009/P1/Q7)

(a) Simplify $4a^3 \times a^2$. [1]

(b) Simplify fully $3x(x + 5) - 2(x - 3)$. [2]

Thinking Process

(a) ✐ Apply the rule of indices: $a^m \times a^n = a^{m+n}$
(b) Expand and simplify.

Solution

(a) $4a^3 \times a^2$

$= 4 \times a^{3+2} = 4a^5$ **Ans.**

(b) $3x(x + 5) - 2(x - 3)$

$= 3x^2 + 15x - 2x + 6$

$= 3x^2 + 13x + 6$ **Ans.**

10 (J2009/P2/Q1 a,b)

(a) Express as a single fraction in its simplest form $\frac{2a}{3} + \frac{3}{2a}$. [1]

(b) Factorise completely $5b^2 - 10b$. [1]

Thinking Process

(a) ✐ Re-express the fractions with a common denominator.
(b) ✐ Take out the common factor.

Solution

(a) $\frac{2a}{3} + \frac{3}{2a} = \frac{4a^2 + 9}{6a}$ **Ans.**

(b) $5b^2 - 10b = 5b(b - 2)$ **Ans.**

11 *(N2009/P1/Q9)*

The force acting on an object during a collision is given by the formula

$$F = \frac{mv - mu}{t}.$$

(a) Given that $m = 4$, $v = 5$, $u = 3$ and $t = 0.01$, find the value of F. [1]

(b) Rearrange the formula to make m the subject. [2]

Thinking Process

(a) Substitute all the given values into the equation to find F.

(b) Express m in terms of F, t, u and v.

Solution

(a) $F = \frac{mv - mu}{t}$

$\Rightarrow F = \frac{(4)(5) - (4)(3)}{0.01}$

$= \frac{20 - 12}{\frac{1}{100}}$

$= 8 \times \frac{100}{1}$

$= 800$ **Ans.**

(b) $F = \frac{mv - mu}{t}$

$Ft = m(v - u)$

$\frac{Ft}{v - u} = m$

$\therefore m = \frac{Ft}{v - u}$ **Ans.**

12 *(N2009/P1/Q13)*

(a) Express $\frac{2m}{5} + \frac{m}{4}$ as a single fraction in its simplest terms. [1]

(b) Solve the inequality $5(x + 4) < 7x$. [2]

Thinking Process

(a) Find common denominator.

(b) Expand and solve for x.

Solution *with* TEACHER's COMMENTS

(a) $\frac{2m}{5} + \frac{m}{4}$

$= \frac{8m + 5m}{20}$

$= \frac{13m}{20}$ **Ans.**

(b) $5(x + 4) < 7x$

$5x + 20 < 7x$

$5x - 7x < -20$

$-2x < -20$

$x > \frac{-20}{-2}$

$x > 10$

> When multiplying or dividing an inequality by a negative number, the inequality sign will change.

13 *(N2009/P1/Q19)*

(a) Factorise completely

(i) $21a^2 - 14a$. [1]

(ii) $x^2 - 3x - 40$. [1]

(b) Given that $y = 3$ is a solution of the equation $2y^2 + ky - 27 = 0$, find the other solution. [2]

Thinking Process

(a) (i) Take out common factor.

(ii) Factorise the quadratic expression.

(b) Substitute y = 3 into given equation for k. Factorise the equation for other solution.

Solution

(a) (i) $21a^2 - 14a$

$= 7a(3a - 2)$ **Ans.**

(ii) $x^2 - 3x - 40$

$= x^2 - 8x + 5x - 40$

$= x(x - 8) + 5(x - 8)$

$= (x - 8)(x + 5)$ **Ans.**

(b) $2y^2 + ky - 27 = 0$

at $y = 3$: $2(3)^2 + k(3) - 27 = 0$

$18 + 3k - 27 = 0$

$3k = 9$

$k = 3$

substituting $k = 3$ into the equation, we have

$2y^2 + 3y - 27 = 0$

$2y^2 + 9y - 6y - 27 = 0$

$y(2y + 9) - 3(2y + 9) = 0$

$(2y + 9)(y - 3) = 0$

$2y + 9 = 0$ or $y - 3 = 0$

$y = -4.5$ or $y = 3$

$\therefore$ other solution $= -4.5$ **Ans.**

14 *(J2010/P1/Q21)*

(a) Factorise completely

(i) $3x^2 - 12x$. [1]

(ii) $x^2 - xy - 2y^2$. [1]

(b) Simplify $\dfrac{x^2+4x}{x^2-16}$. [2]

Thinking Process

(a) (i) Take out common factor.

(ii) Factorise the quadratic expression.

(b) Factorise numerator and denominator.

Solution

(a) (i) $3x^2 - 12x$

$= 3x(x-4)$ **Ans.**

(ii) $x^2 - xy - 2y^2$

$= x^2 - 2xy + xy - 2y^2$

$= x(x-2y) + y(x-2y)$

$= (x-2y)(x+y)$ **Ans.**

(b) $\dfrac{x^2+4x}{x^2-16} = \dfrac{x(x+4)}{x^2-4^2}$

$= \dfrac{x(x+4)}{(x+4)(x-4)}$

$= \dfrac{x}{x-4}$ **Ans.**

15 *(N2010/P1/Q6)*

Factorise

(a) $4t^2 - 9$. [1]

(b) $3x^2 + 5x - 2$. [1]

Thinking Process

(a) Recall $a^2 - b^2 = (a+b)(a-b)$

(b) Factorise the quadratic expression.

Solution

(a) $4t^2 - 9$

$= (2t)^2 - (3)^2$

$= (2t+3)(2t-3)$ **Ans.**

(b) $3x^2 + 5x - 2$

$= 3x^2 + 6x - x - 2$

$= 3x(x+2) - 1(x+2)$

$= (x+2)(3x-1)$ **Ans.**

16 *(N2010/P1/Q8)*

Make x the subject of the formula $y = 2x^2 + 3$. [2]

Thinking Process

Rearrange the formula and express x in terms of y.

Solution

$y = 2x^2 + 3$

$2x^2 = y - 3$

$x^2 = \dfrac{y-3}{2}$

$x = \pm\sqrt{\dfrac{y-3}{2}}$ **Ans.**

17 *(N2010/P2/Q1)*

(a) Simplify

(i) $\dfrac{x+y}{8x+8y}$ [1]

(ii) $x(3x-2) - (3x^2-5)$. [2]

(b) Solve the equation $3t - 4 = 7 + 2(t+3)$. [2]

(c) Factorise $5px - 7qx + 10py - 14qy$. [2]

(d) (i) When $x = -2$, which of the two expressions, $3x+4$ and $2-x$, has the greater value? You must show your working. [2]

(ii) Solve the inequality $3x + 4 < 2 - x$. [2]

Thinking Process

(a) (i) Take out common factor.

(ii) Expand and simplify.

(b) Remove the bracket and solve for t.

(c) Factorise by grouping.

(d) (i) Substitute the value of x into the two expressions and check.

(ii) Arrange the inequality such that unknowns are all on one side of the inequality.

Solution

(a) (i) $\dfrac{x+y}{8x+8y}$

$= \dfrac{x+y}{8(x+y)} = \dfrac{1}{8}$ **Ans.**

(ii) $x(3x-2) - (3x^2-5)$

$= 3x^2 - 2x - 3x^2 + 5$

$= 5 - 2x$ **Ans.**

(b) $3t - 4 = 7 + 2(t+3)$

$3t - 4 = 7 + 2t + 6$

$3t - 2t = 7 + 6 + 4$

$t = 17$ **Ans.**

(c) $5px - 7qx + 10py - 14qy$.

$= x(5p - 7q) + 2y(5p - 7q)$

$= (5p - 7q)(x + 2y)$ **Ans.**

(d) (i) When $x = -2$,

1st expression: $3(-2)+4 = -6+4 = -2$

2nd expression: $2-(-2) = 2+2 = 4$

$\therefore$ $2-x$ has the greater value **Ans.**

(ii) $3x+4 < 2-x$

$3x+x < 2-4$

$4x < -2$

$x < -\frac{1}{2}$ **Ans.**

18 (J2011/P1/Q15)

(a) Factorise completely $9pq-12q^2$. [1]

(b) Factorise completely $8px+4py-6x-3y$. [2]

Thinking Process

(a) Take out the common factor.

(b) Factorise by grouping.

Solution

(a) $9pq-12q^2$

$= 3q(3p-4q)$ **Ans.**

(b) $8px+4py-6x-3y$

$= 4p(2x+y)-3(2x+y)$

$= (2x+y)(4p-3)$ **Ans.**

19 (J2011/P2/Q1)

(a) Express as a single fraction in its simplest form

(i) $\frac{1}{2x}-\frac{2}{5x}$. [1]

(ii) $\frac{4}{x}+\frac{7}{x-3}$. [2]

Thinking Process

(a) (i), (ii) Re-express the fractions with a common denominator.

Solution

(a) (i) $\frac{1}{2x}-\frac{2}{5x}$

$= \frac{5-4}{10x}$

$= \frac{1}{10x}$ **Ans.**

(ii) $\frac{4}{x}+\frac{7}{x-3}$

$= \frac{4(x-3)+7(x)}{x(x-3)}$

$= \frac{4x-12+7x}{x(x-3)}$

$= \frac{11x-12}{x(x-3)}$ **Ans.**

20 (N2011/P1/Q23)

(a) Factorise $9x^2-1$. [1]

(b) Solve the equation $2y^2+29y-15=0$. [3]

Thinking Process

(a) Recall: $a^2-b^2 = (a+b)(a-b)$.

(b) Solve by factorisation.

Solution

(a) $9x^2-1$

$= (3x)^2-(1)^2$

$= (3x+1)(3x-1)$ **Ans.**

(b) $2y^2+29y-15=0$

$\Rightarrow$ $2y^2+30y-y-15=0$

$\Rightarrow$ $2y(y+15)-1(y+15)=0$

$\Rightarrow$ $(y+15)(2y-1)=0$

$\Rightarrow$ $y+15=0$ or $2y-1=0$

$\Rightarrow$ $y=-15$ or $y=\frac{1}{2}$

$\therefore$ $y=-15$ or $y=\frac{1}{2}$ **Ans.**

21 (N2011/P2/Q1)

(a) $A = h(4m+h)$

Express m in terms of A and h. [3]

(b) Factorise completely $3ax+5bx-6ay-10by$. [2]

(c) Solve the equation $\frac{5x-1}{9} = \frac{9}{5x-1}$. [3]

Thinking Process

(a) Make m the subject of formula.

(b) Factorise by grouping.

(c) Perform cross-multiplication.

Solution

(a) $A = h(4m+h)$

$A = 4mh+h^2$

$4mh = A-h^2$

$m = \frac{A-h^2}{4h}$ **Ans.**

(b) $3ax+5bx-6ay-10by$

$= 3ax-6ay+5bx-10by$

$= 3a(x-2y)+5b(x-2y)$

$= (x-2y)(3a+5b)$ **Ans.**

(c) $\frac{5x-1}{9}=\frac{9}{5x-1}$

$(5x-1)^2=81$

$5x-1=\pm 9$

$\Rightarrow\ 5x-1=9$ or $5x-1=-9$

$5x=10$ or $5x=-8$

$x=2$ or $x=-\frac{8}{5}=-1.6$

$\therefore\ x=2,$ or -1.6 **Ans.**

22 *(J2012/P1/Q11)*

$$c=\frac{b(a-b)}{a}$$

(a) Find c when $a=4$ and $b=-2$. [1]

(b) Rearrange the formula to make a the subject. [2]

Thinking Process

(a) To find c ✐ substitute the values of a and b into the equation.

(b) ✐ Express a in terms of b and c.

Solution

(a) $c=\frac{b(a-b)}{a}$

$=\frac{-2(4-(-2))}{4}$

$=\frac{-2(6)}{4}=-3$ **Ans.**

(b) $c=\frac{b(a-b)}{a}$

$ac=ab-b^2$

$ac-ab=-b^2$

$a(c-b)=-b^2$

$a=-\frac{b^2}{c-b}$

$=\frac{b^2}{b-c}$ **Ans.**

23 *(J2012/P1/Q25)*

(a) Factorise

(i) x^2+x-12. [1]

(ii) $25x^2-4y^2$. [1]

(b) Write as a single fraction $\frac{4}{3p}+\frac{1}{6p}$. [1]

(c) Solve the simultaneous equations.

$3x+5y=2$

$2x-3y=14$ [3]

Thinking Process

(a) (i) ✐ Factorise by sum and product method.

(ii) ✐ Use $a^2-b^2=(a+b)(a-b)$

(b) Re-express the fractions with a common denominator.

(c) From 1 equation, make x the subject of formula. Substitute the expression into the other equation to solve for y and subsequently the value of x.

Solution

(a) (i) x^2+x-12

$=x^2+4x-3x-12$

$=x(x+4)-3(x+4)$

$=(x+4)(x-3)$ **Ans.**

(a) (ii) $25x^2-4y^2$

$=(5x)^2-(2y)^2$

$=(5x+2y)(5x-2y)$ **Ans.**

(b) $\frac{4}{3p}+\frac{1}{6p}$

$=\frac{8+1}{6p}$

$=\frac{9}{6p}=\frac{3}{2p}$ **Ans.**

(c) $3x+5y=2$ ....(1)

$2x-3y=14$ ....(2)

from (1), $x=\frac{2-5y}{3}$ ....(3)

substitute (3) into (2)

$2\left(\frac{2-5y}{3}\right)-3y=14$

$4-10y-9y=42$

$-19y=38$

$y=-2$

substitute value of y into (3)

$x=\frac{2-5(-2)}{3}=\frac{12}{3}=4$

$\therefore\ x=4,\ y=-2$ **Ans.**

24 *(N2012/P1/Q7)*

Expand the brackets and simplify

(a) $6k-2(1-k)+3$, [1]

(b) $(2x-3)(x+4)$. [1]

Thinking Process

(a) & (b) ✐ Expand and simplify the like terms.

Solution

(a) $6k-2(1-k)+3$

$=6k-2+2k+3$

$=8k+1$ **Ans.**

(b) $(2x-3)(x+4)$

$=2x^2+8x-3x-12$

$=2x^2+5x-12$ **Ans.**

25 *(N2012/P1/Q15)*

Factorise completely

(a) $16p+4p^2$ [1]

(b) $xy+2ay+3ax+6a^2$. [2]

Thinking Process

(a) Take out common factor.

(b) By inspection look for the common factor.

Solution

(a) $16p+4p^2$

$=4p(4+p)$ **Ans.**

(b) $xy+2ay+3ax+6a^2$

$=xy+3ax+2ay++6a^2$

$=x(y+3a)+2a(y+3a)$

$=(y+3a)(x+2a)$ **Ans.**

26 *(N2012/P2/Q2 a,b)*

(a) Factorise $4x^2-1$. [1]

(b) $P=\dfrac{2Q+R}{R}$

(i) Find P when $R=Q$ [1]

(ii) Rearrange the formula to make R the subject. [3]

Thinking Process

(a) Recall $a^2-b^2=(a+b)(a-b)$

(b) (i) Substitute $R=Q$ into the equation.

(ii) Express R in terms of Q and P.

Solution

(a) $4x^2-1$

$=(2x)^2-(1)^2$

$=(2x+1)(2x-1)$ **Ans.**

(b) (i) $P=\dfrac{2Q+R}{R}$

substitute $R=Q$

$P=\dfrac{2Q+Q}{Q}$

$=\dfrac{3Q}{Q}=3$ **Ans.**

(ii) $P=\dfrac{2Q+R}{R}$

$PR=2Q+R$

$PR-R=2Q$

$R(P-1)=2Q$

$R=\dfrac{2Q}{P-1}$ **Ans.**

27 *(J2013/P1/Q24)*

(a) Expand and simplify $(t-5)(t+3)$. [1]

(b) Factorise $64x^2-9y^2$. [1]

(c) Factorise $6ab-2a-3a^2+4b$. [2]

(d) (i) Write x^2-6x+3 in the form $(x-a)^2+b$. [1]

(ii) Hence solve $x^2-6x+3=0$ leaving your answer in the form $p\pm\sqrt{q}$. [1]

Thinking Process

(b) Use the concept of differences of two perfect squares.

(c) Take out common factor.

(d) (i) Apply completing the square method.

(ii) Equate the equation found in (d) (i) to 0.

Solution

(a) $(t-5)(t+3)$

$=t^2+3t-5t-15$

$=t^2-2t-15$ **Ans.**

(b) $64x^2-9y^2$

$=(8x)^2-(3y)^2$

$=(8x+3y)(8x-3y)$ **Ans.**

(c) $6ab-2a-3a^2+4b$

$=6ab+4b-3a^2-2a$

$=2b(3a+2)-a(3a+2)$

$=(3a+2)(2b-a)$ **Ans.**

(d) (i) x^2-6x+3

$=x^2-2(x)(3)+(3)^2-(3)^2+3$

$=(x-3)^2-9+3$

$=(x-3)^2-6$ **Ans.**

(ii) $x^2-6x+3=0$

$\Rightarrow\ (x-3)^2-6=0$

$(x-3)^2=6$

$x-3=\pm\sqrt{6}$

$x=3\pm\sqrt{6}$ **Ans.**

28 *(N2013/P1/Q17)*

(a) Factorise $25t^2-4$. [1]

(b) Factorise completely $6r^2H-2r^2h$. [1]

(c) Factorise completely $8xy+4x-6y-3$. [2]

Thinking Process

(a) Factorise by using differences of two perfect squares.

(b) Take out common factor.

(c) Factorise by grouping.

Solution

(a) $25t^2 - 4$

$= (5t)^2 - (2)^2$

$= (5t+2)(5t-2)$ **Ans.**

(b) $6r^2H - 2r^2h$

$= 2r^2(3H - h)$ **Ans.**

(c) $8xy + 4x - 6y - 3$

$= 4x(2y+1) - 3(2y+1)$

$= (2y+1)(4x-3)$ **Ans.**

29 (N2013/P2/Q3)

(a) Find the value of $\dfrac{a+\sqrt{a^2+b^2}}{a^2-2ab}$ when $a = -4$ and $b = -3$.
Give your answer as a fraction. [2]

(b) Expand the brackets and simplify
$(3x^2-1)(2x+3) - x(9x-2)$. [2]

(c) (i) Factorise $9x^2 + 5x - 4$. [1]

(ii) Use your answer to part (c)(i) to solve the equation $9x^2 + 5x - 4 = 0$. [1]

(d) The sum of three consecutive integers is 84. Find these three integers. [2]

Thinking Process

(a) Substitute $a = -4$ and $b = -3$ into the expression and simplify.

(b) Expand and simplify the like terms.

(c) (i) Factorise by grouping.

(d) Let the integers be x, $x+1$, $x+2$. Form an equation and solve for x.

Solution

(a) $\dfrac{a+\sqrt{a^2+b^2}}{a^2-2ab}$

when $a = -4$ and $b = -3$, we have,

$= \dfrac{-4+\sqrt{(-4)^2+(-3)^2}}{(-4)^2-2(-4)(-3)}$

$= \dfrac{-4+\sqrt{16+9}}{16-24}$

$= \dfrac{-4+\sqrt{25}}{-8}$

$= \dfrac{-4+5}{-8}$

$= -\dfrac{1}{8}$ **Ans.**

(b) $(3x^2-1)(2x+3) - x(9x-2)$

$= (6x^3 + 9x^2 - 2x - 3) - (9x^2 - 2x)$

$= 6x^3 + 9x^2 - 2x - 3 - 9x^2 + 2x$

$= 6x^3 - 3$

$= 3(2x^3 - 1)$ **Ans.**

(c) (i) $9x^2 + 5x - 4$

$= 9x^2 + 9x - 4x - 4$

$= 9x(x+1) - 4(x+1)$

$= (x+1)(9x-4)$ **Ans.**

(ii) $9x^2 + 5x - 4 = 0$

$\Rightarrow (x+1)(9x-4) = 0$

$\Rightarrow x+1 = 0$ or $9x - 4 = 0$

$x = -1$ or $x = \dfrac{4}{9}$

$\therefore x = -1$ or $\dfrac{4}{9}$ **Ans.**

(d) Let the integers be x, $x+1$, $x+2$

$x + (x+1) + (x+2) = 84$

$3x + 3 = 84$

$3x = 81$

$x = 27$

$\therefore$ the integers are 27, 28, 29 **Ans.**

30 (J2014/P1/Q23)

(a) Expand and simplify $(2x+1)(x+4)$. [1]

(b) Write $\dfrac{3}{x} + \dfrac{4}{x+2}$ as a single fraction in its simplest form. [1]

(c) Solve $\dfrac{10}{x} = x + 3$. [3]

Thinking Process

(b) Re-express the fractions with a common denominator.

(c) Cross-multiply and form a quadratic equation. Solve by factorisation.

Solution

(a) $(2x+1)(x+4)$

$= 2x(x+4) + 1(x+4)$

$= 2x^2 + 8x + x + 4$

$= 2x^2 + 9x + 4$ **Ans.**

(b) $\dfrac{3}{x} + \dfrac{4}{x+2}$

$= \dfrac{3(x+2) + 4x}{x(x+2)}$

$= \dfrac{3x + 6 + 4x}{x^2 + 2x}$

$= \dfrac{7x+6}{x^2+2x}$ **Ans.**

(c) $\frac{10}{x} = x + 3$

$\Rightarrow 10 = x^2 + 3x$

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow x^2 - 2x + 5x - 10 = 0$

$\Rightarrow x(x-2) + 5(x-2) = 0$

$\Rightarrow (x-2)(x+5) = 0$

$\Rightarrow (x-2) = 0$ or $(x+5) = 0$

$\Rightarrow x = 2$ or $x = -5$ **Ans.**

31 *(N2014/P1/Q18)*

(a) Factorise completely $4a - 16a^2$. [1]

(b) Factorise $9b^2 - c^2$. [1]

(c) Factorise $x^2 - 5y - xy + 5x$. [2]

Thinking Process

(a) Take out common factor.
(b) Use the concept of differences between 2 perfect squares.
(c) By inspection look for the common factor.

Solution

(a) $4a - 16a^2$

$4a(1-4a)$ **Ans.**

(b) $9b^2 - c^2$

$= (3b)^2 - (c)^2 = (3b+c)(3b-c)$ **Ans.**

(c) $x^2 - 5y - xy + 5x$

$= x^2 - xy + 5x - 5y$

$= x(x-y) + 5(x-y)$

$= (x-y)(x+5)$ **Ans.**

32 *(J2015/P1/Q15)*

$c = \sqrt{8a - 3b}$

(a) Find c when $a = 3$ and $b = -4$. [1]

(b) Rearrange the formula to make b the subject. [2]

Thinking Process

(a) Substitute the values of a and b into the expression.
(b) Express b in terms of a and c.

Solution

(a) $c = \sqrt{8a - 3b}$

$= \sqrt{8(3) - 3(-4)}$

$= \sqrt{24 + 12}$

$= \sqrt{36} = \pm 6$ **Ans.**

(b) $c = \sqrt{8a - 3b}$

$\Rightarrow c^2 = 8a - 3b$

$\Rightarrow 3b = 8a - c^2 \Rightarrow b = \frac{8a - c^2}{3}$ **Ans.**

33 *(J2015/P1/Q18)*

(a) Factorise completely $p^2q - pq$. [1]

(b) (i) Factorise $5x^2 + x - 4$. [1]

(ii) Hence solve $5x^2 + x - 4 = 0$. [1]

Thinking Process

(a) Take out common factor.
(b) (i) Factorise by sum and product method.
(ii) Use the expression found in part (i) and solve for x.

Solution

(a) $p^2q - pq$

$= pq(p-1)$ **Ans.**

(b) (i) $5x^2 + x - 4$

$= 5x^2 + 5x - 4x - 4$

$= 5x(x+1) - 4(x+1)$

$= (x+1)(5x-4)$ **Ans.**

(ii) $5x^2 + x - 4 = 0$

$\Rightarrow (x+1)(5x-4) = 0$

$\Rightarrow x + 1 = 0$ or $5x - 4 = 0$

$\Rightarrow x = -1$ or $x = \frac{4}{5}$

$\therefore x = -1$ or $\frac{4}{5}$ **Ans.**

34 *(N2015/P1/Q16)*

(a) Factorise

(i) $4p^2 - 9q^2$, [1]

(ii) $2n^2 + 5n - 3$. [1]

(b) Express $\frac{3}{4x} + \frac{2}{3y}$ as a single fraction. [1]

Thinking Process

(a) (i) Recall $a^2 - b^2 = (a+b)(a-b)$.
(ii) Break the middle term and factorise.
(b) Re-express the fractions with a common denominator.

Solution

(a) (i) $4p^2 - 9q^2$

$= (2p)^2 - (3q)^2$

$= (2p+3q)(2p-3q)$ **Ans.**

(ii) $2n^2+5n-3$

$=2n^2+6n-n-3$

$=2n(n+3)-1(n+3)$

$=(n+3)(2n-1)$ **Ans.**

(b) $\frac{3}{4x}+\frac{2}{3y}$

$=\frac{3(3y)+2(4x)}{12xy}$

$=\frac{9y+8x}{12xy}$ **Ans.**

35 *(J2016/P1/Q26)*

(a) Make p the subject of the formula $t=\frac{p+3}{p-4}$. [3]

(b) Simplify fully $\frac{4x^2-1}{2x^2-9x-5}$. [3]

Thinking Process

(a) Express p in terms of t.
(b) Factorise numerator and denominator.

Solution

(a) $t=\frac{p+3}{p-4}$

$t(p-4)=p+3$

$tp-4t=p+3$

$tp-p=3+4t$

$p(t-1)=3+4t$

$p=\frac{3+4t}{t-1}$ **Ans.**

(b) $\frac{4x^2-1}{2x^2-9x-5}$

$=\frac{(2x)^2-1^2}{2x^2-10x+x-5}$

$=\frac{(2x+1)(2x-1)}{2x(x-5)+1(x-5)}$ $\quad a^2-b^2=(a+b)(a-b)$

$=\frac{(2x+1)(2x-1)}{(x-5)(2x+1)}$

$=\frac{2x-1}{x-5}$ **Ans.**

36 *(N2016/P2/Q2)*

(a) Simplify $\frac{3a^2}{10bc}\div\frac{9a}{5b^2c}$. [2]

(b) Simplify $\frac{h-k}{5h-5k}$. [2]

(c) Factorise $9m^2-4n^2$. [1]

(d) Factorise $q(p-2)+3(2-p)$. [2]

(e) (i) Find the two solutions of $5x-1=\pm 9$. [2]

(ii) The solutions of $5x-1=\pm 9$ are also the solutions of $5x^2+Bx+C=0$, where B and C are integers.
Find B and C. [2]

Thinking Process

(a) Apply rules of indices: $a^m \div a^n = a^{m-n}$
(b) Take out common factor.
(c) Use the concept of differences between 2 perfect squares.
(d) Factorise by grouping.
(e) (ii) By taking square on both sides, form a quadratic equation and find B and C by comparision.

Solution

(a) $\frac{3a^2}{10bc}\div\frac{9a}{5b^2c}$

$=\frac{3a^2}{10bc}\times\frac{5b^2c}{9a}$

$=\frac{a}{2}\times\frac{b}{3}=\frac{ab}{6}$ **Ans.**

(b) $\frac{h-k}{5h-5k}$

$=\frac{h-k}{5(h-k)}=\frac{1}{5}$ **Ans.**

(c) $9m^2-4n^2$

$=(3m)^2-(2n)^2$

$=(3m+2n)(3m-2n)$ **Ans.**

(d) $q(p-2)+3(2-p)$

$=q(p-2)-3(p-2)$

$=(p-2)(q-3)$ **Ans.**

(e) (i) $5x-1=\pm 9$

Either, $5x-1=9$ or $5x-1=-9$

$5x=10$ $\quad 5x=-8$

$x=2$ $\quad x=-\frac{8}{5}$

$\therefore\ x=2$ or $-\frac{8}{5}$ **Ans.**

(ii) $5x-1=\pm 9$

taking square on both sides.

$\Rightarrow (5x-1)^2=81$

$\Rightarrow 25x^2-10x+1=81$

$\Rightarrow 25x^2-10x-80=0$ (divide by 5)

$\Rightarrow 5x^2-2x-16=0$

comparing it with $5x^2+Bx+C=0$

$B=-2,\ C=-16$ **Ans.**

37 *(J2017/P1/Q23)*

(a) Solve $\frac{7x}{4-3x}=3$. [2]

(b) Simplify fully $\frac{4x^2-9}{2x^2-13x+15}$. [3]

Thinking Process

(a) Cross-multiply the equation and solve for x.
(b) Factorise numerator and denominator.

Solution

(a) $\frac{7x}{4-3x}=3$

$7x=3(4-3x)$

$7x=12-9x$

$16x=12 \Rightarrow x=\frac{12}{16}=\frac{3}{4}$ **Ans.**

(b) $\frac{4x^2-9}{2x^2-13x+15}$

$=\frac{(2x)^2-(3)^2}{2x^2-10x-3x+15}$

$=\frac{(2x-3)(2x+3)}{2x(x-5)-3(x-5)}$

$=\frac{(2x-3)(2x+3)}{(x-5)(2x-3)}$

$=\frac{2x+3}{x-5}$ **Ans.**

38 *(J2017/P2/Q5 a,b,c)*

(a) Express as a single fraction, as simply as possible, $\frac{1}{2x}+\frac{2}{5x}$. [1]

(b) Simplify $4(3x-2y+1)-(5x-3y+1)$. [2]

(c) Solve $3x^2-x-5=0$, giving your answers correct to 2 decimal places. [3]

Thinking Process

(a) Re-express the fractions with a common denominator.

(c) Apply formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ and solve for x.

Solution

(a) $\frac{1}{2x}+\frac{2}{5x}$

$=\frac{5+4}{10x}=\frac{9}{10x}$ **Ans.**

(b) $4(3x-2y+1)-(5x-3y+1)$

$=12x-8y+4-5x+3y-1$

$=7x-5y+3$ **Ans.**

(c) $3x^2-x-5=0$

Using quadratic formula,

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-5)}}{2(3)}$

$=\frac{1\pm\sqrt{61}}{6}$

$\Rightarrow x=\frac{1+\sqrt{61}}{6}$ or $x=\frac{1-\sqrt{61}}{6}$

$\therefore x=1.47$ or -1.14 (3 sf) **Ans.**

39 *(N2017/P1/Q22)*

(a) Factorise $9a^2-6a$. [1]

(b) Factorise $4-25t^2$. [1]

(c) Factorise $6cd-xy+2cx-3dy$. [2]

Thinking Process

(a) Take out common factor.
(b) Recall $a^2-b^2=(a+b)(a-b)$
(c) Factorise by grouping.

Solution

(a) $9a^2-6a$

$=3a(3a-2)$ **Ans.**

(b) $4-25t^2$

$=(2)^2-(5t)^2=(2+5t)(2-5t)$ **Ans.**

(c) $6cd-xy+2cx-3dy$

$=2cx+6cd-xy-3dy$

$=2c(x+3d)-y(x+3d)$

$=(x+3d)(2c-y)$ **Ans.**

40 *(J2018/P1/Q5)*

(a) Factorise $25t^2-4$. [1]

(b) Factorise $x^2-6x-3xy+18y$. [2]

Thinking Process

(a) Apply, $a^2-b^2=(a+b)(a-b)$
(b) Factorise by grouping.

Solution

(a) $25t^2-4$

$=(5t)^2-(2)^2=(5t+2)(5t-2)$ **Ans.**

(b) $x^2-6x-3xy+18y$

$=x(x-6)-3y(x-6)$

$=(x-6)(x-3y)$ **Ans.**

41 (J2018/P1/Q9)

Express each of the following as a single fraction in its simplest form.

(a) $\frac{2}{3a}+\frac{5}{2a}$ [1]

(b) $\frac{5}{2b^2}+\frac{15}{4b^3}$ [2]

Thinking Process

(a) Re-express the fractions with a common denominator.

(b) Rewrite $\frac{5}{2b^2}+\frac{15}{4b^3}$ as $\frac{5}{2b^2}\times\frac{4b^3}{15}$ and simplify.

Solution

(a) $\frac{2}{3a}+\frac{5}{2a}$

$=\frac{4+15}{6a}=\frac{19}{6a}$ **Ans.**

(b) $\frac{5}{2b^2}+\frac{15}{4b^3}$

$=\frac{5}{2b^2}\times\frac{4b^3}{15}=\frac{2b}{3}$ **Ans.**

42 (N2018/P1/Q5)

(a) Simplify $4c-3(2c-5)$. [1]

(b) Factorise $8-10y+12x-15xy$ [2]

Thinking Process

(b) Factorise by grouping.

Solution

(a) $4c-3(2c-5)$

$=4c-6c+15$

$=15-2c$ **Ans.**

(b) $8-10y+12x-15xy$

$=2(4-5y)+3x(4-5y)$

$=(4-5y)(2+3x)$ **Ans.**

43 (N2018/P2/Q3)

(a) Express as a single fraction in its simplest form

$\frac{3}{y-1}-\frac{5}{y+6}$. [3]

(b) Simplify $\frac{2v^2-5v-12}{v^2-16}$. [3]

(c) Solve $3(x^2+3)=11x$.

Show your working and give your answers correct to 3 significant figures. [4]

Thinking Process

(a) Re-express the fractions with a common denominator.

(b) Simplify by grouping.

(c) Re-arrange to form a quadratic equation apply quadratic formula and solve for x.

Solution

(a) $\frac{3}{y-1}-\frac{5}{y+6}$

$=\frac{3(y+6)-5(y-1)}{(y-1)(y+6)}$

$=\frac{3y+18-5y+5}{(y-1)(y+6)}=\frac{23-2y}{(y-1)(y+6)}$ **Ans.**

(b) $\frac{2v^2-5v-12}{v^2-16}$

$=\frac{2v^2-8v+3v-12}{v^2-(4)^2}$

$=\frac{2v(v-4)+3(v-4)}{(v-4)(v+4)}$

$=\frac{(v-4)(2v+3)}{(v-4)(v+4)}$

$=\frac{2v+3}{v+4}$ **Ans.**

(c) $3(x^2+3)=11x$

$\Rightarrow\ 3x^2+9=11x\ \Rightarrow\ 3x^2-11x+9=0$

Using quadratic formula,

$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(3)(9)}}{2(3)}$

$=\frac{11\pm\sqrt{121-108}}{6}=\frac{11\pm\sqrt{13}}{6}$

$\Rightarrow\ x=\frac{11+\sqrt{13}}{6}$ or $x=\frac{11-\sqrt{13}}{6}$

$=2.43$ $=1.23$

$\therefore\ x=2.43$ or 1.23 **Ans.**

Topic 4a

Variations

1 (N2007/P1/Q12)

(a) When an object is falling, the air resistance varies as the square of the speed.
At a certain speed, the resistance is 30 newtons.
What is the resistance at twice this speed?

(b) y is inversely proportional to x. [1]

(c) Given that $y = 6$ when $x = 4$, find the value of y when $x = 3$. [2]

Thinking Process

(a) Write down a formula involving the two quantities and a constant k.

(b) Write down an equation involving y and x and a constant k.

Solution

(a) Let r and v be the resistance and speed respectively.

$\therefore \quad r \propto v^2$

$\Rightarrow \quad r = kv^2$

when $r = 30$ newtons

$30 = kv^2$

$\Rightarrow \quad k = \dfrac{30}{v^2}$

when speed $= 2v$

$\Rightarrow \quad r = k(2v)^2$

$r = \dfrac{30}{v^2}(4v^2) = 30(4) = 120$ newtons Ans

(b) $y \propto \dfrac{1}{x}$

$\Rightarrow \quad y = \dfrac{k}{x}$

$y = 6$ when $x = 4$

$6 = \dfrac{k}{4} \quad \Rightarrow \quad k = 24$

$\therefore \quad y = \dfrac{24}{x}$

when $x = 3$

$y = \dfrac{24}{3} = 8$ Ans

2 (J2008/P1/Q10)

It is given that y is directly proportional to the square of x and that $y = 1$ when $x = \dfrac{1}{2}$.

Find

(a) the formula for y in terms of x, [2]

(b) the values of x when $y = 9$. [1]

Thinking Process

(a) To form a formula use the fact that y is proportional to x^2

(b) Substitute the value of y into the formula derived in part (a) to find x.

Solution

(a) $y \propto x^2$

$\Rightarrow \quad y = kx^2$

given that $y = 1$ when $x = \dfrac{1}{2}$

$\Rightarrow \quad 1 = k\left(\dfrac{1}{2}\right)^2$

$k = 4$

$\therefore \quad y = 4x^2$ Ans.

(b) Putting $y = 9$ in $y = 4x^2$, we have

$9 = 4x^2$

$x^2 = \dfrac{9}{4}$

$x = \pm\dfrac{3}{2} = \pm 1.5$

$\therefore \quad x = 1.5$ and -1.5 Ans.

3 (N2008/P1/Q10)

T is inversely proportional to the square of L.
Given that $T = 9$ when $L = 2$, find

(a) the formula for T in terms of L, [2]

(b) the values of L when $T = 25$. [1]

Thinking Process

(a) Write $T \propto \dfrac{1}{L^2}$. Find T in terms of L.

(b) Substitute the given value to find L.

Solution

(a) $T \propto \dfrac{1}{L^2}$

$\Rightarrow \quad T = \dfrac{k}{L^2}$

$\Rightarrow \quad 9 = \dfrac{k}{2^2}$

$\Rightarrow \quad k = 36$

$\therefore \quad T = \dfrac{36}{L^2}$ Ans.

(b) when $T = 25$,

$$25 = \frac{36}{L^2}$$
$$L^2 = \frac{36}{25}$$
$$L = \pm\frac{6}{5}$$
$$= \pm 1\tfrac{1}{5} \quad \textbf{Ans.}$$

4 *(J2009/P1/Q12)*

y is directly proportional to the square root of x.
Given that $y = 12$ when $x = 36$,
find

(a) the formula for y in terms of x, [2]

(b) the value of x when $y = 10$. [1]

Thinking Process

(a) To form a formula ✎ use the fact that y is directly proportional to $\sqrt{x}$.

(b) ✎ Substitute the value of y into the formula derived in part (a) to find x.

Solution

(a) $y \propto \sqrt{x}$

$\Rightarrow \quad y = k\sqrt{x}$

given that $y = 12$ when $x = 36$

$\Rightarrow \quad 12 = k\sqrt{36}$

$12 = k(6)$

$k = 2$

$\therefore$ required formula: $y = 2\sqrt{x}$ **Ans.**

(b) Putting $y = 10$ in $y = 2\sqrt{x}$, we have

$10 = 2\sqrt{x}$

$\sqrt{x} = 5$

squaring both sides

$x = 25$ **Ans.**

5 *(N2009/P1/Q6)*

y is inversely proportional to x.
Given that $y = 250$ when $x = 4$, find y when $x = 80$. [2]

Thinking Process

Write $y \propto \frac{1}{x}$. Find y in terms of x.

Solution

$y \propto \frac{1}{x} \quad \Rightarrow \quad y = \frac{k}{x}$

$y = 250$ when $x = 4$

$\Rightarrow \quad 250 = \frac{k}{4}$

$k = 1000$

$\therefore \quad y = \frac{1000}{x}$

when $x = 80$, $y = \frac{1000}{80}$

$= \frac{25}{2} = 12.5$ **Ans.**

6 *(J2010/P1/Q12)*

It is given that y is inversely proportional to the square of x and that $y = 48$ when $x = \frac{1}{2}$.
Find

(a) the formula for y in terms of x, [2]

(b) the values of x when $y = 3$. [1]

Thinking Process

(a) To form a formula ✎ use the fact that y is inversely proportional to x^2. Compute the constant of proportionality k by substituting the given values of x and y.

(b) ✎ Substitute the value $y = 3$ into the equation derived in part (a) to compute x.

Solution

(a) $y \propto \frac{1}{x^2}$

$\Rightarrow \quad y = \frac{k}{x^2}$, where k is a constant.

when $y = 48$, $x = \frac{1}{2}$

$$48 = \frac{k}{\left(\frac{1}{2}\right)^2}$$

$48\left(\frac{1}{4}\right) = k$

$k = 12$

$\therefore \quad y = \frac{12}{x^2}$ **Ans.**

(b) When $y = 3$,

$$3 = \frac{12}{x^2}$$
$$x^2 = \frac{12}{3}$$
$$x^2 = 4$$
$$x = \pm 2 \quad \textbf{Ans.}$$

7 *(N2010/P1/Q7)*

y is directly proportional to the square of x.
Given that $y = 50$ when $x = 5$, find the value of y when $x = 3$. [2]

Thinking Process

Write $y = kx^2$. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 3$ into the equation to compute y.

Solution

$y \propto x^2 \Rightarrow y = kx^2$

given that $y = 50$ when $x = 5$

$\Rightarrow 50 = k(5)^2$

$k = 2$

$\therefore y = 2x^2$

when $x = 3$, $y = 2(3)^2 = 18$ **Ans.**

8 *(J2011/P1/Q8)*

y is directly proportional to the square of x.
Given that $y = 2$ when $x = 4$, find y when $x = 10$. [2]

Thinking Process

$y \propto x^2 \Rightarrow y = kx^2$. Substitute given values to solve for k. Substitute $x = 10$ to find y.

Solution

$y \propto x^2 \Rightarrow y = kx^2$

$\Rightarrow 2 = k(4)^2$

$\Rightarrow k = \frac{1}{8}$

$\therefore y = \frac{1}{8}x^2$

when $x = 10$, $y = \frac{1}{8}(10)^2$

$= \frac{1}{8}(100)$

$= \frac{25}{2} = 12\frac{1}{2}$ **Ans.**

9 *(N2011/P1/Q13)*

y is inversely proportional to x.
The table shows some values of x and y.

x	3	4	q	n
y	20	p	5	m

(a) Find p. [1]
(b) Find q. [1]
(c) Express m in terms of n. [1]

Thinking Process

(a) Substitute $x = 4$ into the equation obtained.
(b) Substitute $y = 5$ into the equation obtained.
(c) Substitute $x = n$ and $y = m$ into the equation obtained in (a).

Solution

(a) $y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x}$

substitute $x = 3$, $y = 20$ into the equation.

$20 = \frac{k}{3} \Rightarrow k = 60$

$\therefore y = \frac{60}{x}$

when $x = 4$, $y = p$

$\Rightarrow p = \frac{60}{4} = 15$ **Ans.**

(b) $y = \frac{60}{x}$

when $x = q$, $y = 5$

$\Rightarrow 5 = \frac{60}{q}$

$q = \frac{60}{5} = 12$ **Ans.**

(c) $y = \frac{60}{x}$

$x = n$ and $y = m$

$\Rightarrow m = \frac{60}{n}$ **Ans.**

10 *(J2012/P1/Q5)*

y is inversely proportional to the square of x.
Given that $y = 2$ when $x = 6$, find the value of y when $x = 2$. [2]

Thinking Process

Use the fact that y is inversely proportional to x^2. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 2$ into the equation to find y.

Solution

$y \propto \frac{1}{x^2} \Rightarrow y = k(\frac{1}{x^2})$

given that $y = 2$ when $x = 6$

$\Rightarrow 2 = \frac{k}{(6)^2}$

$\Rightarrow k = 2 \times 36$

$= 72$

$\therefore y = \frac{72}{x^2}$

when $x = 2$, $y = \frac{72}{(2)^2}$

$= \frac{72}{4} = 18$ **Ans.**

11 *(N2012/P1/Q12)*

y is directly proportional to the square of x.
Given that $y = 32$ when $x = 4$, find y when $x = 3$. [2]

Thinking Process

Use the fact that y is directly proportional to x^2. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 3$ into the equation to find y.

Solution

$y \propto x^2 \Rightarrow y = kx^2$

given that $y = 32$ when $x = 4$.

$\Rightarrow 32 = k(4)^2$

$32 = 16k$

$k = 2$

$\therefore \quad y = 2x^2$

when $x = 3$, $y = 2(3)^2 = 18$ **Ans.**

12 *(J2013/P1/Q15)*

P is directly proportional to the square of Q.
When $P = 9$, $Q = 6$.

(a) Find the formula for P in terms of Q. [1]

(b) Find the values of Q when $P = 25$. [2]

Thinking Process

(a) To form a formula ∯ use the fact that P is directly proportional to Q^2. Compute the constant of proportionality k by substituting the given values of P and Q.

(b) To find the values of Q ∯ substitute $P = 25$ into the equation derived in part (a).

Solution

(a) $P \propto Q^2$

$\Rightarrow P = kQ^2$, where k is a constant.

when $P = 9$, $Q = 6$

$\Rightarrow 9 = k(6)^2$

$9 = 36k$

$k = \frac{9}{36} = \frac{1}{4}$

$\therefore \quad P = \frac{1}{4}Q^2$ **Ans.**

(b) When $P = 25$,

$25 = \frac{1}{4}Q^2$

$Q^2 = 100$

$Q = \pm 10$ **Ans.**

13 *(N2013/P1/Q6)*

y is inversely proportional to x.
Given that $y = 20$ when $x = 2$, find y when $x = 5$. [2]

Thinking Process

Write down a formula involving the two quantities and a constant k. Substitute the values of x and y to find k. Substitute $x = 5$ into the equation to find y.

Solution

$y \propto \frac{1}{x}$

$\Rightarrow \quad y = \frac{k}{x}$, where k is a constant.

given that $y = 20$ when $x = 2$.

$\Rightarrow \quad 20 = \frac{k}{2} \quad \Rightarrow \quad k = 40$

$\therefore \quad y = \frac{40}{x}$

when $x = 5$,

$y = \frac{40}{5} = 8$ **Ans.**

14 *(J2014/P1/Q7)*

The cost of a mirror is directly proportional to the square of its width.
A mirror of width 40 cm costs \$24.
Work out the cost of a mirror of width 60 cm. [2]

Thinking Process

Write down a formula involving the two quantities and a constant k. Compute the constant k by substituting the given values of width and cost. Substitute 60 cm into the formula to find cost.

Solution

$\text{cost} \propto (\text{width})^2 \Rightarrow \text{cost} = k(\text{width})^2$

given that cost = \$24 when width = 40 cm

$\Rightarrow 24 = k(40)^2$

$\Rightarrow 24 = k(1600)$

$\Rightarrow \quad k = \frac{24}{1600} \quad \Rightarrow \quad k = \frac{3}{200}$

$\therefore \quad \text{cost} = \frac{3}{200}(\text{width})^2$

when width = 60 cm, $\text{cost} = \frac{3}{200}(60)^2$

$= \frac{3}{200}(3600) = \$54$ **Ans.**

15 *(N2014/P1/Q10)*

y is inversely proportional to x.
Given that $y = 9$ when $x = 8$, find y when $x = 6$. [2]

Thinking Process

Write $y \propto \frac{1}{x}$. Find y in terms of x. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 6$ into the equation to find y.

Solution

$y \propto \frac{1}{x} \Rightarrow y = k(\frac{1}{x})$

given that $y = 9$ when $x = 8$

$\Rightarrow 9 = k(\frac{1}{8}) \Rightarrow k = 72$

$\therefore \quad y = \frac{72}{x}$

when $x = 6$, $y = \frac{72}{6}$

$= 12$ **Ans.**

16 *(J2015/P1/Q7)*

y is inversely proportional to the square of x.
Given that $y = 24$ when $x = 2$, find y when $x = 8$. [2]

Thinking Process

Use the fact that y is inversely proportional to x^2. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 8$ into the equation to find y.

Solution

$y \propto \frac{1}{x^2} \Rightarrow y = k(\frac{1}{x^2})$

given that $y = 24$ when $x = 2$

$\Rightarrow 24 = \frac{k}{(2)^2}$

$\Rightarrow k = 24 \times 4 = 96$

$\therefore \quad y = \frac{96}{x^2}$

when $x = 8$, $y = \frac{96}{(8)^2}$

$= \frac{96}{64}$

$= \frac{3}{2} = 1.5$ **Ans.**

17 *(N2015/P1/Q3)*

y varies directly as the square root of x.
Given that $y = 18$ when $x = 9$, find y when $x = 4$. [2]

Thinking Process

Write $y = k\sqrt{x}$. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 4$ into the equation to compute y.

Solution

$y \propto \sqrt{x} \Rightarrow y = k\sqrt{x}$

given that $y = 18$ when $x = 9$

$\Rightarrow 18 = k\sqrt{9}$

$18 = k(3)$

$k = 6$

$\therefore \quad y = 6\sqrt{x}$

when $x = 4$, $y = 6\sqrt{4}$

$= 6(2) = 12$ **Ans.**

18 *(J2016/P1/Q8)*

y is directly proportional to the square of x.
When $x = 10$, $y = 20$.
Find the value of y when $x = 6$. [2]

Thinking Process

Use the fact that y is directly proportional to x^2. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 6$ into the equation to find y.

Solution

$y \propto x^2 \Rightarrow y = kx^2$

given that, $x = 10$, $y = 20$

$\Rightarrow 20 = k(10)^2$

$\Rightarrow k = \frac{20}{100} = \frac{1}{5}$

$\therefore \quad y = \frac{1}{5}x^2$

when $x = 6$, $y = \frac{1}{5}(6)^2$

$= \frac{36}{5} = 7\frac{1}{5}$ **Ans.**

19 *(N2016/P1/Q11)*

y varies inversely as the square of x.

(a) When $x = 2$, $y = 9$.
Find the value of y when $x = 3$. [2]

(b) When $x = n$, $y = p$.
Write down an expression for y, in terms of p, when $x = 2n$. [1]

Thinking Process

(a) Write $y \propto \frac{1}{x^2}$. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 3$ into the equation to find y.

(b) To find an expression substitute the given values of x and y into the equation derived in part (a).

Solution

(a) $y \propto \frac{1}{x^2} \Rightarrow y = k(\frac{1}{x^2})$

given that $y = 9$ when $x = 2$

$\Rightarrow 9 = \frac{k}{(2)^2}$

$\Rightarrow k = 9 \times 4 = 36$

$\therefore y = \frac{36}{x^2}$

when $x = 3$, $y = \frac{36}{(3)^2}$

$= \frac{36}{9} = 4$ **Ans.**

(b) $y = \frac{36}{x^2}$

when $x = n$, $y = p$

$\Rightarrow p = \frac{36}{n^2} \Rightarrow n^2 = \frac{36}{p}$

when $x = 2n$, $y = \frac{36}{(2n)^2}$

$= \frac{36}{4n^2}$

$= \frac{9}{n^2}$

$= \frac{9}{\frac{36}{p}}$

$= 9 \times \frac{p}{36} = \frac{p}{4}$

$\therefore y = \frac{p}{4}$ **Ans.**

20 *(J2017 P1 Q12)*

y is inversely proportional to the square of x.
The table shows some values for x and y.

x	2	4	p
y	3	$\frac{3}{4}$	48

(a) Find the equation connecting x and y. [2]

(b) Find the value of p. [1]

Thinking Process

(a) Write $y \propto \frac{1}{x^2}$. Find y in terms of x.

(b) Substitute $y = 48$ into the equation derived in part (a) to compute p.

Solution

(a) $y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$, where k is a constant.

when $x = 2$, $y = 3$

$\Rightarrow 3 = \frac{k}{2^2} \Rightarrow k = 12$

$\therefore y = \frac{12}{x^2}$ **Ans.**

(b) $y = \frac{12}{x^2}$

when $x = p$, $y = 48$

$\therefore 48 = \frac{12}{p^2}$

$p^2 = \frac{12}{48} \Rightarrow p^2 = \frac{1}{4} \Rightarrow p = \pm\frac{1}{2}$ **Ans.**

21 *(N2017 P1 Q3)*

y is inversely proportional to x.

Given that $y = \frac{1}{6}$ when $x = 30$, find y when $x = 10$ [2]

Thinking Process

Write $y = \frac{k}{x}$. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 10$ into the equation to compute y.

Solution

$y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x}$, where k is a constant.

Given that, $y = \frac{1}{6}$ when $x = 30$.

$\Rightarrow \frac{1}{6} = \frac{k}{30} \Rightarrow k = 5$

$\therefore y = \frac{5}{x}$

when $x = 10$, $y = \frac{5}{10} = \frac{1}{2}$ **Ans.**

22 *(J2018 P1 Q4)*

y is inversely proportional to the square of x.

Given that $y = 10$ when $x = 3$, find y when $x = \frac{1}{2}$. [2]

Thinking Process

Write $y \propto \frac{1}{x^2}$. Find y in terms of x. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = \frac{1}{2}$ into the equation to find y.

Solution

$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$, where k is a constant.

when $x = 3$, $y = 10$,

$10 = \frac{k}{(3)^2} \Rightarrow k = 90$

$\therefore \quad y = \frac{90}{x^2}$

when $x = \frac{1}{2}$, $y = \frac{90}{\left(\frac{1}{2}\right)^2}$

$= 90 \times 4 = 360$ **Ans.**

23 *(N2018/P1/Q3)*

y is inversely proportional to x.

Given that $y = 30$ when $x = \frac{1}{3}$, find y when $x = 5$.

[2]

Thinking Process

Write $y \propto \frac{1}{x}$. Find y in terms of x. Compute the constant of proportionality k by substituting the given values of x and y. Substitute $x = 5$ into the equation to find y.

Solution

$y \propto \frac{1}{x} \Rightarrow y = k(\frac{1}{x})$

given that $y = 30$ when $x = \frac{1}{3}$

$\Rightarrow 30 = k(\frac{1}{\frac{1}{3}}) \Rightarrow 30 = 3k \Rightarrow k = 10$

$\therefore \quad y = \frac{10}{x}$

when $x = 5$, $y = \frac{10}{5} = 2$ **Ans.**

Topic 5

Solutions of Equations and Simultaneous Equations

1 (J2007/P1/Q22)

(a) Solve

(i) $9 - k < 7$, [1]

(ii) $\frac{5}{2t} = \frac{1}{12}$. [1]

(b) Solve the simultaneous equations

$x + y = 29$,

$4x = 95 - 2y$. [3]

Thinking Process

(a) (ii) Cross multiply and find the value of t.
(b) To solve for x and y ✐ eliminate one of the variables.

Solution

(a) (i) $9 - k < 7$

$-k < 7 - 9$

$-k < -2$

$k > 2$ Ans.

(ii) $\frac{5}{2t} = \frac{1}{12}$

$5 \times 12 = 2t$

$2t = 60$

$t = 30$ Ans.

(b) Let $x + y = 29$ ....(A)

$4x + 2y = 95$ ....(B)

(A)×4: $4x + 4y = 116$ ....(C)

(C)−(B): $4x + 4y = 116$

$4x + 2y = 95$

$-\quad -\quad -$

$2y = 21$

$\Rightarrow y = \frac{21}{2}$

$\Rightarrow y = 10\frac{1}{2}$ Ans.

Subst. into (A): $x + \frac{21}{2} = 29$

$\Rightarrow x = 29 - \frac{21}{2} \Rightarrow x = \frac{58 - 21}{2}$

$\Rightarrow x = \frac{37}{2} = 18\frac{1}{2}$ Ans.

2 (J2007/P2/Q10)

It is given that $y = \frac{3x^2 - 12}{5}$.

(a) Find y when $x = -3$. [1]

(b) Find the values of x when $y = 0$. [2]

(c) For values of x in the range $-3 \le x \le 2$, write down

(i) the largest value of y, [1]

(ii) the smallest value of y. [1]

(d) Express x in terms of y. [2]

(e) It is also given that $y = \frac{t-3}{2}$ when $x = t$.

(i) Show that t satisfies the equation $6t^2 - 5t - 9 = 0$. [1]

(ii) Solve the equation $6t^2 - 5t - 9 = 0$, giving each answer correct to two significant figures. [4]

Thinking Process

(a) Substitute the value of x in the given equation.
(b) Substitute the value of y in the given equation and solve for x.
(c) For largest and smallest values of y ✐ substitute the values of x in the given range and investigate.
(e) (i) Substitute the given values of x and y in the equation. Rearrange it to get the required quadratic equation.
(ii) To find t ✐ Apply quadratic formula.

Solution

(a) When $x = -3$

$y = \frac{3(-3)^2 - 12}{5}$

$= \frac{3(9) - 12}{5}$

$= \frac{15}{5} = 3$ Ans

(b) When $y = 0$

$0 = \frac{3x^2 - 12}{5}$

$0 \times 5 = 3x^2 - 12$

$3(x^2 - 4) = 0$

$x^2 - 4 = 0$

$(x + 2)(x - 2) = 0$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ or $x = 2$ Ans

(c) (i) Largest value of y is when $x = -3$

$y = \frac{3(-3)^2 - 12}{5} = \frac{15}{5} = 3$

∴ largest value of $y = 3$ Ans

(ii) Smallest value of y is when $x = 0$

$y = \frac{3(0)^2 - 12}{5} = -\frac{12}{5} = -2.4$

∴ smallest value of $y = -2.4$ Ans

(d) $y = \frac{3x^2 - 12}{5}$

$5y = 3x^2 - 12$

$3x^2 = 5y + 12$

$x^2 = \frac{5y+12}{3}$

$x = \pm\sqrt{\frac{5y+12}{3}}$ **Ans**

(e) (i) $y = \frac{3x^2 - 12}{5}$.

putting $y = \frac{t-3}{2}$ and $x = t$, we have

$\frac{t-3}{2} = \frac{3(t)^2 - 12}{5}$

$5(t-3) = 2(3t^2 - 12)$

$5t - 15 = 6t^2 - 24$

$6t^2 - 5t - 9 = 0$ **Shown**

(ii) $6t^2 - 5t - 9 = 0$

$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(6)(-9)}}{2(6)}$

$t = \frac{5 \pm \sqrt{25 + 216}}{12}$

$t = \frac{5 \pm \sqrt{241}}{12}$

$t = \frac{5 + \sqrt{241}}{12}$ or $\frac{5 - \sqrt{241}}{12}$

$\therefore$ $t = 1.7$ or -0.88 (2 sf) **Ans.**

3 *(N2007/P1/Q14)*

Solve the equations

(a) $\frac{24}{x-4} = 1$, [1]

(b) $12 - 2(5 - y) = 5y$. [2]

Thinking Process

(a) Make x the subject of the formula.
(b) Expand and make y the subject.

Solution

(a) $\frac{24}{x-4} = 1$

$24 = x - 4$

$x = 24 + 4 = 28$ **Ans**

(b) $12 - 2(5 - y) = 5y$

$12 - 10 + 2y = 5y$

$2 = 3y \Rightarrow y = \frac{2}{3}$ **Ans**

4 *(N2007/P1/Q16)*

Solve the simultaneous equations

$$2x - y = 16,$$
$$3x + 2y = 17.$$

[3]

Thinking Process

From 1 equation, make x the subject of formula. Substitute the expression into the other equation to solve for y and substitute the value of y back into the expression of x to find the value of x.

Solution

$2x - y = 16$ ....(i)

$3x + 2y = 17$ ....(ii)

From (i): $x = \frac{16 + y}{2}$ ....(iii)

Substitute (iii) into (ii):

$3\left(\frac{16+y}{2}\right) + 2y = 17$

$\frac{48 + 3y}{2} + 2y = 17$

$\frac{48 + 3y + 4y}{2} = 17$

$48 + 7y = 34$

$7y = -14$

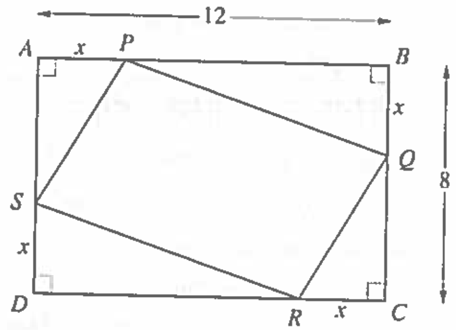
$y = -2$

Substitute $y = -2$ into (iii):

$x = \frac{16 + (-2)}{2} = \frac{14}{2} = 7$

$\therefore$ $x = 7$, $y = -2$ **Ans**

5 *(J2008/P2 Q10)*



In the diagram, $ABCD$ is a rectangle.
AB = 12 cm and BC = 8 cm.
$AP = BQ = CR = DS = x$ centimetres.

(a) Find an expression, in terms of x, for

(i) the length of QC, [1]

(ii) the area of triangle CRQ. [1]

(b) Hence show that the area, in square centimetres, of the quadrilateral $PQRS$ is $2x^2 - 20x + 96$. [3]

(c) When the area of quadrilateral $PQRS$ is 60 cm^2, form an equation in x and show that it simplifies to

$x^2 - 10x + 18 = 0.$ [1]

(d) Solve the equation $x^2 - 10x + 18 = 0$, giving each answer correct to 2 decimal places. [3]

(e) It is given that $2x^2 - 20x + 96 = 2(x-5)^2 + K$.

(i) Find the value of K. [1]

(ii) Hence write down the smallest possible area of the quadrilateral $PQRS$ and the value of x at which it occurs. [2]

Thinking Process

(a) (i) Note that $BC = 8$ cm.

(b) Subtract the area of four triangles from the rectangle $ABCD$.

(d) Apply quadratic formula.

(e) (i) Expand the RHS and find K by comparison.

(ii) Understand that when area of $PQRS$ is written in the form $2(x-5)^2 + K$, then K represents the minimum area and it occurs when $x = 5$.

Solution

(a) (i) $QC = (8 - x)$ cm. **Ans.**

(ii) Area of $\Delta CRQ = \frac{1}{2} \times RC \times QC$

$= \frac{1}{2}x(8-x)$

$= \frac{1}{2}(8x - x^2)$ cm^2 **Ans.**

(b) In ΔBQP, $PB = 12 - x$

$\therefore$ area of $\Delta BQP = \frac{1}{2}x(12-x) = \frac{1}{2}(12x - x^2)$ cm^2

Total area of four triangles

= area of ΔBQP + area of ΔDSR

+ area of ΔAPS + area of ΔCRQ

$= 2(\Delta BQP) + 2(\Delta CRQ)$

$= 2\left(\frac{1}{2}(12x - x^2)\right) + 2\left(\frac{1}{2}(8x - x^2)\right)$

$= 12x - x^2 + 8x - x^2 = (20x - 2x^2)$ cm^2

Area of rectangle $ABCD = 12 \times 8 = 96$ cm^2

$\therefore$ area of $PQRS = 96 - (20x - 2x^2)$

$= 96 - 20x + 2x^2$

$= 2x^2 - 20x + 96$ cm^2 **Shown.**

(c) Area of $PQRS = 60$ cm^2

$\therefore$ $2x^2 - 20x + 96 = 60$

$2x^2 - 20x + 96 - 60 = 0$

$2x^2 - 20x + 36 = 0$

$x^2 - 10x + 18 = 0$ **Shown.**

(d) $x^2 - 10x + 18 = 0$

applying quadratic formula

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(18)}}{2(1)}$

$= \frac{10 \pm \sqrt{100 - 72}}{2}$

$= \frac{10 \pm \sqrt{28}}{2}$

$\Rightarrow x = \frac{10 + \sqrt{28}}{2}$ or $x = \frac{10 - \sqrt{28}}{2}$

$= 7.65$ $= 2.35$

$\therefore$ $x = 7.65$ or 2.35 **Ans.**

(e) (i) $2x^2 - 20x + 96 = 2(x-5)^2 + K$

$= 2(x^2 - 10x + 25) + K$

$= 2x^2 - 20x + 50 + K$

$\therefore$ the given equation becomes

$2x^2 - 20x + 96 = 2x^2 - 20x + 50 + K$

by comparison, we have

$96 = 50 + K$

$K = 46$ **Ans.**

(ii) Area of quadrilateral $PQRS$ is

$2x^2 - 20x + 96 = 2(x-5)^2 + K$

$= 2(x-5)^2 + 46$ $(\because K = 46)$

$\therefore$ smallest possible area of $PQRS = 46$ cm^2 and it occurs when $x = 5$ cm **Ans.**

6 (J2008/P1/Q12)

Solve the simultaneous equations

$2x - 3y = 13,$

$3x + y = 3.$ [3]

Thinking Process

From 1 equation, make x the subject of formula. Substitute the expression into the other equation to solve for y and substitute the value of y back into the expression of x to find the value of x.

Solution

$2x - 3y = 13$ ·········(i)

$3x + y = 3$ ·········(ii)

From (ii): $y = 3 - 3x$ ·········(iii)

Substitute eq.(iii) into eq.(i):

$2x - 3(3 - 3x) = 13$

$2x - 9 + 9x = 13$

$11x = 22$

$x = 2$

Substitute $x=2$ into (iii):

$y=3-3(2)=3-6=-3$

$\therefore\ x=2,\ y=-3$ **Ans.**

7 *(N2008/P1/Q16)*

(a) Solve the inequality $3-2x<5$. [2]

(b) Solve the equation $3(y+2)=2(2y-7)+y$. [2]

Thinking Process

(a) Arrange the inequality such that x is on one side of the inequality.

(b) Expand the expression and solve for y.

Solution with TEACHER'S COMMENT

(a) $3-2x<5$

$-2x<5-3$

$-2x<2$

$x>\frac{2}{-2}$

$x>-1$ **Ans.**

> When multiplying or dividing an inequality by a negative number, the inequality sign will change.

(b) $3(y+2)=2(2y-7)+y$

$3y+6=4y-14+y$

$3y+6=5y-14$

$2y=20 \Rightarrow y=10$ **Ans.**

8 *(N2008/P2/Q3)*

(a) Solve the equation $\frac{2p+1}{3}=1+\frac{p-3}{2}$. [3]

(b) Simplify $\frac{2v-6}{v^2-2v-3}$. [3]

(c) The tens digit of a number is x and the units digit is y.

Hence the value of the number is $10x+y$.

For example, if $x=5$ and $y=6$, the number would be $10\times5+6=56$.

(i) When the digits x and y are reversed, the value of the number is increased by 63.
Show that $y-x=7$. [2]

(ii) The sum of the original number and the number with reversed digits is 99.

(a) Show that $x+y=9$. [1]

(b) Hence find the value of x and the value of y. [2]

Thinking Process

(c) (i) When digits are reversed, y becomes the tens digit and x becomes the unit digit. The number would then be $10y+x$.
Understand the given condition and prove.

(ii) (b) Solve the two equations simultaneously for x and y. The two equations are given in part (i) and (ii) (a).

Solution

(a) $\frac{2p+1}{3}=1+\frac{p-3}{2}$

$\frac{2p+1}{3}=\frac{2+p-3}{2}$

$2(2p+1)=3(p-1)$

$4p+2=3p-3$

$p=-5$ **Ans.**

(b) $\frac{2v-6}{v^2-2v-3}$

$=\frac{2v-6}{v^2-3v+v-3}$

$=\frac{2v-6}{v(v-3)+1(v-3)}$

$=\frac{2(v-3)}{(v-3)(v+1)}=\frac{2}{v+1}$ **Ans.**

(c) (i) Original number $=10x+y$

When the digits are reversed, the new number is $=10y+x$

according to the given condition

$(10y+x)-(10x+y)=63$

$10y+x-10x-y=63$

$9y-9x=63$

$y-x=7$ **Shown.**

(ii) (a) $(10y+x)+(10x+y)=99$

$11y+11x=99$

$11(y+x)=99$

$y+x=9$ **Shown.**

(b) $y-x=7$ ………(i)

$x+y=9$ ………(ii)

from eq. (i), $y=7+x$, put in eq.(ii)

$x+(7+x)=9$

$2x+7=9$

$2x=2 \Rightarrow x=1$ **Ans.**

putting this value in eq.(i)

$y-1=7 \Rightarrow y=8$ **Ans.**

9 *(N2008/P2/Q7)*

A light aircraft flew from Maseru to Nata and returned to Maseru.

(a) The distance from Maseru to Nata is 1080km.

(i) On the outward flight, the average speed of the aircraft was x kilometres per hour.
Write down an expression, in terms of x, for the time taken in hours. [1]

(ii) On the return flight, the average speed was 30 km/h greater than the average speed on the outward flight. Write down an expression, in terms of x, for the time taken, in hours, on the return flight. [1]

(b) The time taken on the return flight was half an hour less than the time taken on the outward flight. Form an equation in x and show that it reduces to $x^2+30x-64800=0$. [3]

(c) Solve the equation $x^2+30x-64800=0$. [4]

(d) Calculate

(i) the time taken, in hours, on the outward flight. [1]

(ii) the average speed for the whole flight from Maseru to Nata and back to Maseru. [2]

Thinking Process

(a) (i) & (ii) ✎ speed $=\frac{\text{distance}}{\text{time}}$.

(b) Use the expressions of time found in part (a) and form up an equation.

(c) Apply quadratic formula and solve for x.

(d) (i) Substitute the value of x in the answer to part (a) (i).

Solution

(a) (i) Average speed $= x$ km/h

$\therefore$ Time $=\frac{\text{distance}}{\text{speed}}=\frac{1080}{x}$ hrs **Ans.**

(ii) Average speed $=(x+30)$ km/h

$\therefore$ time $=\frac{1080}{x+30}$ hrs **Ans.**

(b) According to the given information

$$\frac{1080}{x}-\frac{1080}{x+30}=\frac{1}{2}$$

$$1080\left(\frac{1}{x}-\frac{1}{x+30}\right)=\frac{1}{2}$$

$$1080\left(\frac{x+30-x}{x(x+30)}\right)=\frac{1}{2}$$

$$1080\left(\frac{30}{x^2+30x}\right)=\frac{1}{2}$$

$$\frac{32400}{x^2+30x}=\frac{1}{2}$$

$$64800=x^2+30x$$

$$x^2+30x-64800=0 \quad \textbf{Shown.}$$

(c) $x^2+30x-64800=0$

by quadratic formula,

$$x=\frac{-30\pm\sqrt{(30)^2-4(1)(-64800)}}{2(1)}$$

$$=\frac{-30\pm\sqrt{260100}}{2}$$

$$=\frac{-30\pm 510}{2}$$

$$=\frac{-30+510}{2} \text{ or } x=\frac{-30-510}{2}$$

$$=240 \text{ or } =-270$$

$\therefore x=240$ or -270 **Ans.**

(d) (i) Using answer to part (a) (i) and part (c),

Time taken on the outward flight

$$=\frac{1080}{240}=4.5 \text{ hours} \quad \textbf{Ans.}$$

(ii) Time taken on the return flight $=\frac{1080}{240+30}$

$= 4.0$ hours

total distance covered $= 2(1080) = 2160$ km

total time taken $= 4.5 + 4 = 8.5$ hours

$\therefore$ Average speed $=\frac{\text{total distance}}{\text{total time taken}}$

$$=\frac{2160}{8.5}=254.12$$

≈ 254 km/h **Ans.**

10 *(J2009/P1/Q17a)*

(a) Solve $\frac{3x-2}{5}=\frac{x}{3}$. [2]

Thinking Process

(a) Cross multiply and solve for x.

Solution

(a) $$\frac{3x-2}{5}=\frac{x}{3}$$

$$3(3x-2)=5x$$

$$9x-6=5x$$

$$4x=6$$

$$x=\frac{6}{4}=\frac{3}{2}=1\frac{1}{2} \quad \textbf{Ans.}$$

11 *(J2009/P2/Q1d)*

(d) Solve the equation $3x^2+11x-7=0$, giving each answer correct to 2 decimal places. [4]

Thinking Process

(d) Solve the equation by using quadratic formula.

Solution

(d) $3x^2+11x-7=0$

Applying quadratic formula

$$x=\frac{-11\pm\sqrt{11^2-4(3)(-7)}}{2(3)}$$

$$=\frac{-11\pm\sqrt{121+84}}{6}$$

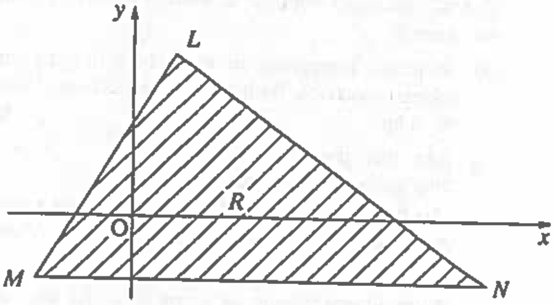
$$=\frac{-11\pm\sqrt{205}}{6}$$

$$=\frac{-11+\sqrt{205}}{6} \text{ or } =\frac{-11-\sqrt{205}}{6}$$

$\therefore x=0.55$ or -4.22 (2dp) **Ans.**

12 (N2009/P1/Q24)

The diagram below shows triangle LMN.



The equations of the lines LM and LN are $2y = 3x + 5$ and $x + 4y = 24$ respectively.

(a) Solve the simultaneous equations

$$x + 4y = 24,$$
$$2y = 3x + 5.$$

Hence write down the coordinates of L. [3]

(b) M is $(-3, -2)$ and MN is parallel to the x-axis.
The shaded region, R, inside triangle LMN, is defined by three inequalities.
One of these is $2y \leqslant 3x + 5$.
Write down the other two inequalities. [2]

Thinking Process

(a) Make y the subject of formula from equation $2y = 3x + 5$. Substitute y into the other equation to find x and the value of y subsequently.

(b) First locate the given inequality and then write the equations of the other two. Convert the equations into inequalities.

Solution

(a) $x + 4y = 24$ ....(1)

$2y = 3x + 5$ ....(2)

From eq. (2): $y = \dfrac{3x+5}{2}$

substitute value of y into eq. (1),

$$x + 4\left(\frac{3x+5}{2}\right) = 24$$
$$x + 6x + 10 = 24$$
$$7x = 14$$
$$x = 2$$

substitute $x = 2$ into eq. (2)

$$2y = 3(2) + 5$$
$$2y = 11 \Rightarrow y = 5.5$$

∴ coordinates of L: $(2, 5.5)$ **Ans.**

(b) Equation of LN: $x + 4y = 24$

Equation of MN: $y = -2$

∴ required inequalities are:

$x + 4y < 24$ and $y > -2$ **Ans.**

13 (N2009/P2/Q1)

Solve the equations

(a) $2^y = 8$, [1]

(b) $3p + 4 = 8 - 2(p - 3)$, [2]

(c) $\dfrac{18}{q} - \dfrac{16}{q+2} = 1$. [3]

(d) $5x^2 + x - 7 = 0$, giving each solution correct to 2 decimal places. [4]

Thinking Process

(a) Write 8 in index form.

(b) Expand and solve for p.

(c) Make a common denominator on the left hand side and solve.

(d) Apply quadratic formula.

Solution

(a)
$$2^y = 8$$
$$2^y = 2^3$$

∴ $y = 3$ **Ans.**

(b)
$$3p + 4 = 8 - 2(p - 3)$$
$$3p + 4 = 8 - 2p + 6$$
$$3p + 2p = 8 + 6 - 4$$
$$5p = 10$$

$p = 2$ **Ans.**

(c)
$$\frac{18}{q} - \frac{16}{q+2} = 1$$
$$\frac{18(q+2) - 16q}{q(q+2)} = 1$$
$$18q + 36 - 16q = q(q+2)$$
$$2q + 36 = q^2 + 2q$$
$$q^2 = 36$$

$q = \pm 6$ **Ans.**

(d) $5x^2 + x - 7 = 0$

Applying quadratic formula,

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(5)(-7)}}{2(5)}$$
$$= \frac{-1 \pm \sqrt{141}}{10}$$

$x = \dfrac{-1 + \sqrt{141}}{10}$ or $x = \dfrac{-1 - \sqrt{141}}{10}$

$= 1.0874$ or $x = -1.2874$

∴ $x = 1.09$ or -1.29 (to 2 dp) **Ans.**

14 *(J2010/P1/Q6)*

(a) Solve $\frac{3}{x-1} = 2$. [1]

(b) Given that $p = 2t - r$, express t in terms of p and r. [1]

Thinking Process

(a) Make x the subject.
(b) Make t the subject of the formula.

Solution

(a) $\frac{3}{x-1} = 2$

$3 = 2x - 2$

$2x = 5$

$x = \frac{5}{2} = 2.5$ **Ans.**

(b) $p = 2t - r$

$2t = p + r$

$t = \frac{p+r}{2}$ **Ans.**

15 *(J2010/P1/Q13)*

Solve the simultaneous equations.

$$3x + 2y = 7$$
$$x - 3y = 17$$ [3]

Thinking Process

From 2nd equation, make x the subject of formula. Substitute the expression into the 1st equation to solve for y. Substitute the value of y back into the expression of x to find the value of x.

Solution

$3x + 2y = 7$ ....(1)

$x - 3y = 17$ ....(2)

From (2): $x = 17 + 3y$ ....(3)

Substitute (3) into (1):

$3(17 + 3y) + 2y = 7$

$51 + 9y + 2y = 7$

$11y = -44$

$y = -4$

Substitute $y = -4$ into (3):

$x = 17 + 3(-4)$

$= 17 - 12 = 5$

$\therefore$ $x = 5$, $y = -4$ **Ans.**

16 *(J2010/P2/Q8)*

Ahmed throws a ball to John.
The ball travels 10 metres at an average speed of x metres per second.

(a) Write an expression, in terms of x, for the time taken, in seconds, for the ball to travel from Ahmed to John. [1]

(b) John then throws the ball to Pierre.
The ball travels 15 metres.
The ball's average speed is 0.5 metres per second greater than the ball's average speed from Ahmed to John.
Write an expression, in terms of x, for the time taken, in seconds, for the ball to travel from John to Pierre. [1]

(c) The time taken between John catching the ball and then throwing it to Pierre is 2 seconds.
The total time taken for the ball to travel from Ahmed to Pierre is 7 seconds.
Write down an equation in x, and show that it simplifies to
$$2x^2 - 9x - 2 = 0.$$ [3]

(d) Solve the equation $2x^2 - 9x - 2 = 0$, giving each answer correct to 2 decimal places. [4]

(e) (i) Find the average speed, in metres per second, of the ball as it travels from John to Pierre. [1]

(ii) How much longer does it take for the ball to travel from John to Pierre than from Ahmed to John?
Give your answer in seconds. [2]

Thinking Process

(a) Apply, time = $\frac{\text{distance}}{\text{speed}}$

(b) Apply, time = $\frac{\text{distance}}{\text{speed}}$. Note that this time average speed is 0.5 m/s greater than x.

(c) To form an equation note that the total of the two times taken found in (a) and (b) is equal to 5 seconds.

(d) Solve by formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(e) (ii) Substitute the value of x into your answers to parts (a) and (b) and find their difference in seconds.

Solution

(a) Time taken $= \frac{10}{x}$ seconds. **Ans.**

(b) Time taken $= \frac{15}{x + 0.5}$ seconds. **Ans.**

(c) Total time during which the ball was moving is 5 seconds.

$\Rightarrow \quad \dfrac{10}{x}+\dfrac{15}{x+0.5}=5$

$\dfrac{10(x+0.5)+15x}{x(x+0.5)}=5$

$10x+5+15x=5x(x+0.5)$

$25x+5=5x^2+2.5x$

$5x^2-22.5x-5=0$

multiply by 2

$10x^2-45x-10=0$

divide by 5

$2x^2-9x-2=0$ **Shown.**

(d) $2x^2-9x-2=0$

By formula,

$x=\dfrac{-(-9)\pm\sqrt{(-9)^2-4(2)(-2)}}{2(2)}$

$=\dfrac{9\pm\sqrt{97}}{4}$

$\Rightarrow \quad x=\dfrac{9+\sqrt{97}}{4}$ or $x=\dfrac{9-\sqrt{97}}{4}$

$\Rightarrow \quad x=4.71$ or $x=-0.21$ (2 dp) **Ans.**

(e) (i) Average speed $=x+0.5$

$=4.71+0.5$

$=5.21$ m/s **Ans.**

(ii) Time taken for the ball to travel from Ahmed to John $=\dfrac{10}{4.71}=2.12$ seconds.

Time taken for the ball to travel from John to Pierre $=\dfrac{15}{4.71+0.5}=2.88$ seconds.

$2.88-2.12=0.76$

$\therefore$ the ball takes 0.76 seconds more to travel from John to Pierre than from Ahmed to John. **Ans.**

17 *(N2010/P1/Q14)*

Solve the simultaneous equations.

$$3y=2x$$
$$x+2y=21$$

[3]

Thinking Process

Substitute the value of y from 1st equation into the 2nd equation and subsequently solve for values of x and y.

Solution

$3y=2x \Rightarrow y=\dfrac{2}{3}x$ ………(1)

$x+2y=21$ ………(2)

Substitute (1) into (2):

$x+2\left(\dfrac{2}{3}x\right)=21$

$x+\dfrac{4}{3}x=21$

$\dfrac{3x+4x}{3}=21$

$7x=63$

$x=9$

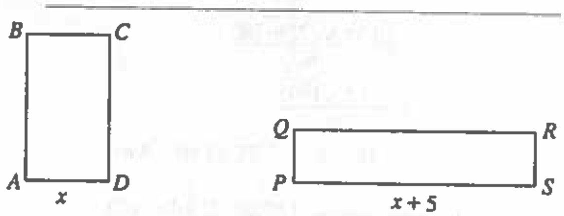
Substitute $x=9$ into (1):

$y=\dfrac{2}{3}(9)$

$=6$

$\therefore \quad x=9, \quad y=6$ **Ans.**

18 *(N2010/P2/Q7)*



$ABCD$ and $PQRS$ are rectangles.
Each rectangle has an area of 13 cm².
$AD=x$ centimetres and $PS=(x+5)$ centimetres.

(a) Find, in terms of x, an expression for

(i) AB, [1]

(ii) PQ. [1]

(b) Given that AB is 3 cm greater than PQ, form an equation in x and show that it simplifies to

$$3x^2+15x-65=0.$$ [3]

(c) Solve the equation $3x^2+15x-65=0$, giving each answer correct to 2 decimal places. [4]

(d) (i) Show that the perimeter of $ABCD$ is 14.9 cm, correct to 3 significant figures. [1]

(ii) Find the difference between the perimeters of the two rectangles. [2]

Thinking Process

(a) (i) & (ii) Apply Area = length × width.

(b) The difference between (a) (i) and (ii) = 3. Simplify the equation obtained.

(c) Solve by quadratic formula.

(d) (i) Substitute the value of x into the expressions of AB and AD and find the perimeter.

(ii) Find the perimeter of $PQRS$ and calculate the difference.

Solution

(a) (i) $AB = \frac{13}{x}$ cm **Ans.**

(ii) $PQ = \frac{13}{x+5}$ cm **Ans.**

(b) $\frac{13}{x} - \frac{13}{x+5} = 3$

$\Rightarrow 13\left(\frac{1}{x} - \frac{1}{x+5}\right) = 3$

$\Rightarrow 13\left(\frac{x+5-x}{x(x+5)}\right) = 3$

$\Rightarrow 65 = 3x(x+5)$

$\Rightarrow 65 = 3x^2 + 15x$

$\Rightarrow 3x^2 + 15x - 65 = 0$ **Shown.**

(c) $3x^2 + 15x - 65 = 0$

Using quadratic formula,

$$x = \frac{-15 \pm \sqrt{15^2 - 4(3)(-65)}}{2(3)}$$

$$= \frac{-15 \pm \sqrt{225 + 780}}{6}$$

$$= \frac{-15 \pm \sqrt{1005}}{6}$$

$\Rightarrow x = 2.78$ or -7.78 (3 sf) **Ans.**

(d) (i) Perimeter of $ABCD = 2(AB + AD)$

$= 2(\frac{13}{2.78} + 2.78)$

$= 2(7.4563)$

$= 14.9$ cm (3 sf) **Ans.**

(ii) Perimeter of $PQRS = 2(PQ + PS)$

$= 2\left(\frac{13}{2.78+5} + (2.78+5)\right)$

$= 2(1.671 + 7.78)$

$= 18.9$ cm (3 sf)

Difference between $ABCD$ and $PQRS$

$= 18.9 - 14.9 = 4$ cm **Ans.**

19 *(J2011/P1/Q25)*

(a) Solve $10 - 3(2x-1) = 3x + 1$. [2]

(b) Solve the simultaneous equations.

$4x + 3y = 11$

$2x - 5y = 25$ [3]

Thinking Process

(a) Rearrange x on one side of equation.

(b) Multiply $2x - 5y = 25$ by 2, eliminate x, then find y.

Solution

(a) $10 - 3(2x-1) = 3x + 1$

$10 - 6x + 3 = 3x + 1$

$10 + 3 - 1 = 3x + 6x$

$12 = 9x$

$x = \frac{12}{9}$

$= \frac{4}{3} = 1\frac{1}{3}$ **Ans.**

(b) Let $4x + 3y = 11$ ....(A)

$2x - 5y = 25$ ....(B)

(B)×2: $4x - 10y = 50$ ....(C)

(A) − (C): $4x + 3y = 11$

$4x - 10y = 50$

$-\quad +\quad -$

$13y = -39$

$\Rightarrow y = -\frac{39}{13} = -3$

subst. y into (A): $4x + 3(-3) = 11$

$4x - 9 = 11$

$4x = 20$

$x = 5$

$\therefore x = 5,\ y = -3$ **Ans.**

20 *(N2011/P1/Q19)*

Solve the simultaneous equations.

$2x + 3y = 0$

$x + 4y = -15$ [3]

Thinking Process

Substitute $x = -4y - 15$ from 2nd equation into first equation and subsequently solve for values of x and y.

Solution

$2x + 3y = 0$ ....(1)

$x + 4y = -15 \Rightarrow x = -15 - 4y$............(2)

substitute (2) into (1)

$2(-15 - 4y) + 3y = 0$

$-30 - 8y + 3y = 0$

$-5y = 30$

$y = -6$

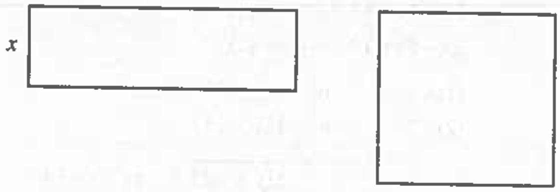
substitute $y = -6$ into (2)

$x = -15 - 4(-6)$

$= -15 + 24 = 9$

$\therefore x = 9,\ y = -6$ **Ans.**

21 *(N2011/P2/Q10)*



A piece of wire, 28 cm in length, is cut into two parts. One part is used to make a rectangle and the other a square.
The length of the rectangle is three times its width.
The width of the rectangle is x centimetres.

(a) (i) Write down an expression, in terms of x, for the length of the rectangle. [1]

(ii) Find, and simplify, an expression, in terms of x, for the length of a side of the square. [2]

(b) It is given that the area of the rectangle is equal to the area of the square.

(i) Form an equation in x and show that it reduces to $x^2 - 28x + 49 = 0$. [2]

(ii) Solve the equation $x^2 - 28x + 49 = 0$, giving each solution correct to 3 significant figures. [4]

(iii) Which solution represents the width of the rectangle?
Give a reason for your answer. [2]

(iv) Calculate the area of the square. [1]

Thinking Process

(a) (ii) To find the expression find the perimeter of the rectangle and subtract it from 28 cm to find the perimeter of square.

(b) (i) Use given information to form an equation.

(ii) To solve the equation apply quadratic formula.

(iii) Identify which value is suitable for the width of the rectangle. Give a reason.

Solution

(a) (i) Length of the rectangle $= 3x$ cm **Ans.**

(ii) Perimeter of the rectangle $= 2(l + b)$

$= 2(3x + x) = 8x$ cm

length of wire used to make the square

$= (28 - 8x)$ cm

$\therefore$ length of one side of the square

$= \dfrac{28 - 8x}{4} = (7 - 2x)$ cm **Ans.**

(b) (i) Area of rectangle = Area of square

$\Rightarrow (x)(3x) = (7 - 2x)^2$

$\Rightarrow 3x^2 = 49 - 28x + 4x^2$

$\Rightarrow x^2 - 28x + 49 = 0$ **Shown.**

(ii) $x^2 - 28x + 49 = 0$

by formula.

$x = \dfrac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(49)}}{2}$

$= \dfrac{28 \pm \sqrt{588}}{2}$

$\Rightarrow x = \dfrac{28 + \sqrt{588}}{2}$ or $x = \dfrac{28 - \sqrt{588}}{2}$

$\Rightarrow x = 26.124$ or $x = 1.876$

$\therefore x = 26.1$ or 1.88 (3 sf) **Ans.**

(iii) Width of the rectangle is 1.88 cm, because the perimeter of the rectangle with this width is well below 28 cm, which is the total length of wire.

(iv) Area of the square $= (7 - 2x)^2$

$= (7 - 2(1.876))^2$

$= (3.248)^2$

$= 10.55$

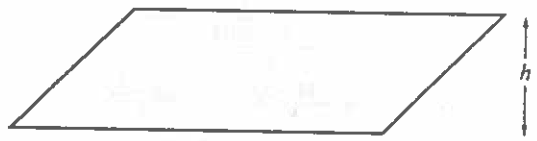
$\approx 10.6 \text{ cm}^2$ **Ans.**

22 *(J2012/P2/Q2)*

(a) Solve the equation $\dfrac{7x + 1}{4} - \dfrac{x}{2} = 1$. [2]

(b) Solve the equation $y^2 - 81 = 0$. [1]

(c)



The length of the base of a parallelogram is 6 cm more than its perpendicular height, h cm.
The area of this parallelogram is 33.25 cm^2

(i) Show that h satisfies the equation
$4h^2 + 24h - 133 = 0$. [2]

(ii) Solve the equation $4h^2 + 24h - 133 = 0$. [3]

(iii) Find the length of the base of the parallelogram. [1]

Thinking Process

(a) Find common denominator. Cross multiply and solve for x.

(b) Recall $a^2 - b^2 = (a + b)(a - b)$.

(c) (i) Apply: Area of parallelogram = base × height.

(ii) Apply quadratic formula and solve for h.

(iii) Use the value of h found in (ii) to find the base length.

Solution

(a) $\frac{7x+1}{4} - \frac{x}{2} = 1$

$\frac{7x+1-2x}{4} = 1$

$5x+1 = 4 \Rightarrow x = \frac{3}{5}$ **Ans.**

(b) $y^2 - 81 = 0$

$\Rightarrow y^2 - 9^2 = 0$

$\Rightarrow (y+9)(y-9) = 0$

$\therefore y = -9$ or $y = 9$ **Ans.**

(c) (i) Perpendicular height $= h$ cm,

$\therefore$ length of base $= (6+h)$ cm

area of parallelogram = base × height

$\Rightarrow 33.25 = (6+h)(h)$

$\Rightarrow \frac{3325}{100} = 6h + h^2$

$\Rightarrow \frac{133}{4} = 6h + h^2$

$\Rightarrow 133 = 24h + 4h^2$

$\Rightarrow 4h^2 + 24h - 133 = 0$ **(Shown).**

(ii) $4h^2 + 24h - 133 = 0$

by quadratic formula,

$h = \frac{-24 \pm \sqrt{24^2 - 4(4)(-133)}}{2(4)}$

$= \frac{-24 \pm \sqrt{576 + 2128}}{8}$

$= \frac{-24 \pm \sqrt{2704}}{8}$

$= \frac{-24+52}{8}$ or $\frac{-24-52}{8}$

$\Rightarrow h = 3.5$ or -9.5 **Ans.**

(iii) length of base $= (6+h)$ cm

$= 6 + 3.5$

$= 9.5$ cm **Ans.**

23 (N2012/P2/Q2 c)

(c) Solve the simultaneous equations.

$3x + 4y = 17$

$2x - 5y = 19$ [3]

Thinking Process

(c) Eliminate one of the variables and solve for x and y.

Solution

(c) $3x + 4y = 17$ ....(1)

$2x - 5y = 19$ ....(2)

$(1) \times 2:$ $6x + 8y = 34$

$(2) \times 3:$ $6x - 15y = 57$

$- \quad + \quad -$

$23y = -23 \Rightarrow y = -1$

subst. $y = -1$ into (1).

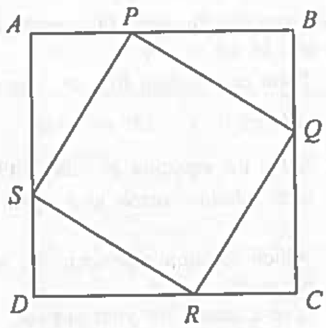
$3x + 4(-1) = 17$

$3x = 21$

$x = 7$

$\therefore x = 7, y = -1$ **Ans.**

24 (N2012/P2/Q10)



$ABCD$ is a square.

$AP = BQ = CR = DS$.

(a) Giving reasons, show that triangles PAS and QBP are congruent. [3]

(b) The length of a side of the square $ABCD$ is 40 cm and $AP = x$ cm.

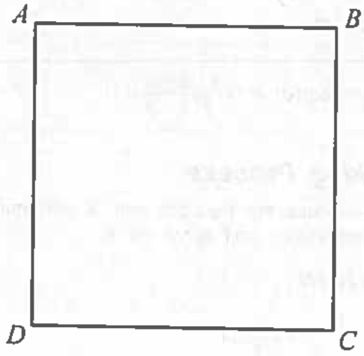
(i) Write down an expression for PB in terms of x. [1]

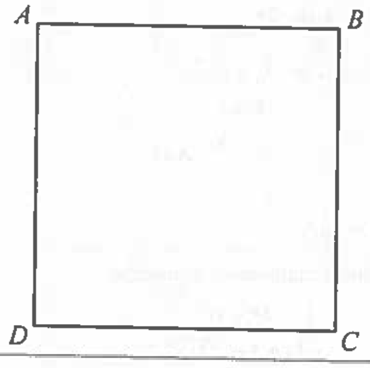
(ii) Show that the area, y cm², of PQRS is given by $y = 1600 - 80x + 2x^2$ [2]

(c) (i) When $y = 1100$, show that $x^2 - 40x + 250 = 0$. [1]

(ii) Solve the equation $x^2 - 40x + 250 = 0$. Give each answer correct to 1 decimal place. [3]

(d) Two outlines of $ABCD$ are drawn to scale in the answer space below. The scale is 1 : 10. Draw accurately the quadrilateral $PQRS$ corresponding to each value of x found above.





[2]

Thinking Process

(a) Prove: $AP = BQ$, $AS = BP$ and $\angle PAS = \angle QBP$.

(b) (ii) Use pythagoras theorem to find PQ. Apply Area = length × width.

(c) (i) ✐ Substitute $y = 1100$ in the equation given in (b)(ii).

(ii) ✐ Solve by quadratic formula.

(d) To construct the quadrilateral $PQRS$ ✐ change the values of x according to the given scale.

Solution

(a) $AB = AD$ (given)

$DS = AP$ (given)

$\therefore \quad PB = SA$

also, $AP = BQ$ (given)

$P\hat{A}S = Q\hat{B}P = 90^\circ$ (ABCD is a square)

$\therefore \quad \Delta PAS \equiv \Delta QBP$ (SAS) Shown.

(b) (i) $PB = (40 - x)$ cm **Ans.**

(ii) In ΔPBQ, using pythagoras theorem.

$PQ^2 = PB^2 + BQ^2$

$= (40 - x)^2 + x^2$

area of $PQRS$:

$y = PQ \times PS$

$= PQ^2$

$= (40 - x)^2 + x^2$

$= 1600 - 80x + x^2 + x^2$

$= 1600 - 80x + 2x^2$ **Shown.**

$\Delta PAS \equiv \Delta QBP$
$\therefore PS = PQ$

(c) (i) $y = 1600 - 80x + 2x^2$

when $y = 1100$,

$\Rightarrow 1100 = 1600 - 80x + 2x^2$

$\Rightarrow 2x^2 - 80x + 500 = 0$

$\Rightarrow x^2 - 40x + 250 = 0$ **Shown.**

(ii) $x^2 - 40x + 250 = 0$

using quadratic formula.

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(1)(250)}}{2(1)}$$

$$= \frac{40 \pm \sqrt{1600 - 1000}}{2}$$

$$= \frac{40 \pm \sqrt{600}}{2}$$

$\Rightarrow x = \frac{40 + \sqrt{600}}{2}$ or $x = \frac{40 - \sqrt{600}}{2}$

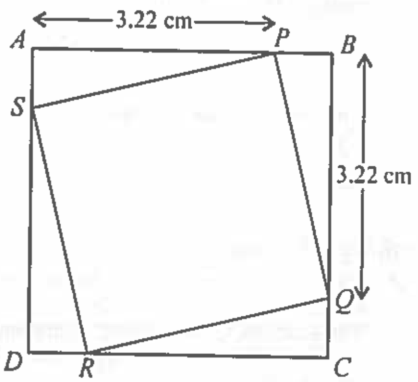
$\Rightarrow x = 32.25$ or $x = 7.75$

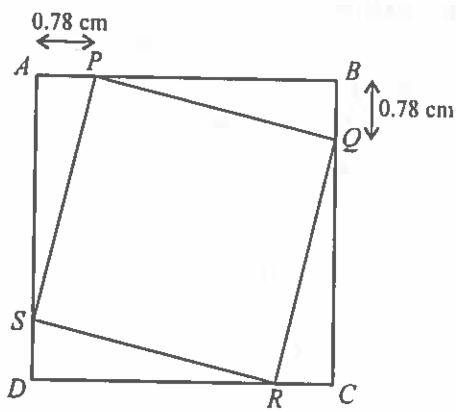
$\therefore \quad x = 32.2$ or 7.8 (1dp) **Ans.**

(d) Given scale:

10	:	1
32.2	:	3.22
7.8	:	0.78

$\therefore \quad AP = 3.22$ cm and 0.78 cm respectively.





25 *(J2013/P1/Q20)*

(a) Solve $\frac{3x}{4}+\frac{2x-1}{2}=3$. [2]

(b) Write as a single fraction in its simplest form
$\frac{5}{x+4}+\frac{2}{x-1}$. [2]

Thinking Process

(a) Re-express the fractions with a common denominator.

(b) Find common denominator. Solve for x.

Solution

(a)
$$\frac{3x}{4}+\frac{2x-1}{2}=3$$
$$\frac{3x+2(2x-1)}{4}=3$$
$$\frac{3x+4x-2}{4}=3$$
$$7x-2=12$$
$$7x=14 \Rightarrow x=2 \quad \textbf{Ans.}$$

(b)
$$\frac{5}{x+4}+\frac{2}{x-1}$$
$$=\frac{5(x-1)+2(x+4)}{(x+4)(x-1)}$$
$$=\frac{5x-5+2x+8}{(x+4)(x-1)}$$
$$=\frac{7x+3}{(x+4)(x-1)} \quad \textbf{Ans.}$$

26 *(J2013/P2/Q3 c)*

(c) Solve the simultaneous equations.
$$2x-y=6$$
$$4x+3y=-3$$
[3]

Thinking Process

(c) From 1st equation, make y the subject of formula. Substitute the expression into the 2nd equation to solve for x. Substitute the value of x back into the expression of y to find the value of y.

Solution

(c) $2x-y=6$ ....(1)

$4x+3y=-3$ ....(2)

From (1): $y=2x-6$ ....(3)

Substitute (3) into (2):
$$4x+3(2x-6)=-3$$
$$4x+6x-18=-3$$
$$10x=15 \Rightarrow x=1.5$$

Substitute $x=1.5$ into (3):
$$y=2(1.5)-6$$
$$=3-6=-3$$

$\therefore \quad x=1.5, \quad y=-3$ **Ans.**

27 *(N2013/P1/Q8)*

Solve the equation $\frac{3x+1}{2}-\frac{x}{3}=1$. [2]

Thinking Process

Re-express the fractions with a common denominator and solve for x.

Solution

$$\frac{3x+1}{2}-\frac{x}{3}=1$$
$$\frac{3(3x+1)-2x}{6}=1$$
$$9x+3-2x=6$$
$$7x=3$$
$$x=\frac{3}{7} \quad \textbf{Ans.}$$

28 *(N2013/P1/Q13)*

Solve the simultaneous equations.
$$4x-3y=14$$
$$2x+y=-3$$
[3]

Thinking Process

Substitute the value of y from 2nd equation into the 1st equation and subsequently solve for values of x and y.

Solution

$4x-3y=14$ ....(1)

$2x+y=-3$ ....(2)

From (2): $y=-2x-3$ ....(3)

substitute (3) into (1):
$$4x-3(-2x-3)=14$$
$$4x+6x+9=14$$
$$10x=5$$
$$x=\frac{1}{2}$$

Substitute $x=\frac{1}{2}$ into (3).
$$y=-2(\tfrac{1}{2})-3$$
$$=-1-3=-4$$

$\therefore \quad x=\frac{1}{2}, \quad y=-4$ **Ans.**

29 *(N2013/P2/Q11)*

(a) Express as a single fraction, in its simplest form, $\frac{7}{p+2}-\frac{4}{2p-3}$. [3]

(b) The distance between London and York is 320 km.
A train takes x hours to travel between London and York.

(i) Write down an expression, in terms of x, for the average speed of the train. [1]

(ii) A car takes $2\frac{1}{2}$ hours longer than a train to travel between London and York. The average speed of the train is 80 km/h greater than the average speed of the car. Form an equation in x and show that it simplifies to $2x^2+5x-20=0$. [3]

(iii) Solve the equation $2x^2+5x-20=0$, giving your answers correct to 2 decimal places. [3]

(iv) Hence find the average speed of the car correct to the nearest km/h. [2]

Thinking Process

(a) Re-express the fractions with a common denominator.

(b) (i) Speed $=\dfrac{\text{distance}}{\text{time}}$.

(ii) Apply speed $=\dfrac{\text{distance}}{\text{time}}$ to find the speed of the car. Use the two expressions of speed to form up an equation.

(iii) Apply quadratic formula and solve for x.

(iv) Substitute the value of x found in (iii) into the expression for speed of the car.

Solution

(a) $\dfrac{7}{p+2}-\dfrac{4}{2p-3}$

$=\dfrac{7(2p-3)-4(p+2)}{(p+2)(2p-3)}$

$=\dfrac{14p-21-4p-8}{(p+2)(2p-3)}$

$=\dfrac{10p-29}{(p+2)(2p-3)}$ **Ans.**

(b) (i) Average speed $=\dfrac{320}{x}$ km/h **Ans.**

(ii) Average speed of the car

$=\dfrac{320}{x+\frac{5}{2}}$

$=\dfrac{320}{\frac{2x+5}{2}}=(\dfrac{640}{2x+5})$ km/h.

according to given information,

$\dfrac{320}{x}-\dfrac{640}{2x+5}=80$

$320\left(\dfrac{1}{x}-\dfrac{2}{2x+5}\right)=80$

$\dfrac{2x+5-2x}{x(2x+5)}=\dfrac{80}{320}$

$\dfrac{5}{x(2x+5)}=\dfrac{1}{4}$

$20=x(2x+5)$

$2x^2+5x-20=0$ **Shown.**

(iii) $2x^2+5x-20=0$

applying quadratic formula,

$x=\dfrac{-5\pm\sqrt{(5)^2-4(2)(-20)}}{2(2)}$

$=\dfrac{-5\pm\sqrt{25+160}}{4}$

$=\dfrac{-5\pm\sqrt{185}}{4}$

$\Rightarrow x=\dfrac{-5+\sqrt{185}}{4}$ or $x=\dfrac{-5-\sqrt{185}}{4}$

$\therefore x=2.15$ or -4.65 **Ans.**

(iv) Average speed of the car $=\dfrac{640}{2x+5}$

$=\dfrac{640}{2(2.15)+5}$

$=68.817$

≈ 69 km/h **Ans.**

30 *(J2014/P1/Q25)*

In quadrilateral $ABCD$

angle $A=(2y+x)°$
angle $B=(3y+x)°$
angle $C=(2y+10)°$
angle $D=(3x+5)°$

(a) By finding the sum of the angles in the quadrilateral, show that $7y+5x=345$. [1]

(b) Given that angle $A=90°$ then $2y+x=90$. Solve the simultaneous equations to find x and y.

$7y+5x=345$
$2y+x=90$ [3]

(c) Find the size of the smallest angle in the quadrilateral. [1]

Thinking Process

(a) To find x apply, sum of angles in a quadrilateral = 360°.

(b) Substitute the value of x from 2nd equation into the 1st equation and solve for y. Substitute the value of y back into the expression of x and solve for x.

Solution

(a) Sum of angles in a quadrilateral = 360°

$\Rightarrow (2y+x)+(3y+x)+(2y+10)+(3x+5)=360$

$\Rightarrow 2y+x+3y+x+2y+10+3x+5=360$

$\Rightarrow 7y+5x+15=360$

$\Rightarrow 7y+5x=345$ **Shown.**

(b) $7y+5x=345$ ....(1)

$2y+x=90$ ....(2)

From (2): $x=90-2y$ ....(3)

substitute (3) into (1):

$$7y+5(90-2y)=345$$
$$7y+450-10y=345$$
$$-3y=-105$$
$$y=\frac{105}{3}=35$$

Substitute $y=35$ into (3).

$$x=90-2(35)$$
$$=90-70=20$$

$\therefore\ x=20,\quad y=35$ **Ans.**

(c) Smallest angle = angle D

$$=(3x+5)^\circ$$
$$=(3(20)+5)^\circ$$
$$=65^\circ$$ **Ans.**

31 (J2014/P2/Q2)

(a) $f=\dfrac{6c^2-d}{4}$

(i) Find f when $c=8$ and $d=-4$. [1]

(ii) Express c in terms of d and f. [2]

(b) Solve $17-5x\le 2x+3$. [2]

(c) Factorise $9-25x^2$. [1]

(d) Factorise completely $8px+6qy-3qx-16py$. [2]

(e) Solve $5x^2+6x-13=0$.
Give your answers correct to two decimal places. [4]

Thinking Process

(a) (i) Substitute the given values into the equation to find f.

(ii) Make c the subject of the formula.

(b) Arrange the inequality such that the unknowns are all on one side of the inequality.

(c) Apply, $a^2-b^2=(a+b)(a-b)$.

(d) Factorise by grouping.

(e) Apply quadratic formula and solve for x.

Solution

(a) (i) $f=\dfrac{6c^2-d}{4}$

$$=\frac{6(8)^2-(-4)}{4}$$
$$=\frac{6(64)+4}{4}$$
$$=\frac{388}{4}=97$$ **Ans.**

(ii) $f=\dfrac{6c^2-d}{4}$

$$4f=6c^2-d$$
$$6c^2=4f+d$$
$$c^2=\frac{4f+d}{6}$$
$$c=\pm\sqrt{\frac{4f+d}{6}}$$ **Ans.**

(b) $17-5x\le 2x+3$

$$-5x-2x\le 3-17$$
$$-7x\le -14$$
$$x\ge\frac{-14}{-7}$$
$$x\ge 2$$ **Ans.**

(c) $9-25x^2$

$$=(3)^2-(5x)^2$$
$$=(3+5x)(3-5x)$$ **Ans.**

(d) $8px+6qy-3qx-16py$

$$=8px-16py-3qx+6qy$$
$$=8p(x-2y)-3q(x-2y)$$
$$=(x-2y)(8p-3q)$$ **Ans.**

(e) $5x^2+6x-13=0$

by quadratic formula,

$$x=\frac{-6\pm\sqrt{(6)^2-4(5)(-13)}}{2(5)}$$
$$=\frac{-6\pm\sqrt{36+260}}{10}$$
$$=\frac{-6\pm\sqrt{296}}{10}$$
$$\Rightarrow x=\frac{-6+\sqrt{296}}{10}\quad\text{or}\quad x=\frac{-6-\sqrt{296}}{10}$$
$$=1.1205\qquad\qquad =-2.320$$

$\therefore\ x=1.12$ or -2.32 (2dp) **Ans.**

32 (N2014/P2/Q8)

(a) $T=2\pi\sqrt{\dfrac{h}{g}}$

(i) Find T when $h=125$ and $g=981$. [1]

(ii) Make h the subject of the formula. [3]

(b) Solve the equation $45-(p+3)=2p$. [2]

(c) Solve the equation $\dfrac{2x-3}{4}+\dfrac{5-x}{3}=0$. [3]

(d) Solve the equation $3y^2+11y+4=0$.
Give your answers correct to 2 decimal places. [3]

Thinking Process

(a) (i) To find T substitute the values of h and g into the given equation.

(ii) Express h in terms of T and g.

(c) Add the 2 fractions by finding a common denominator. Solve for x.

(d) Solve by quadratic formula.

Solution

(a) (i) $T = 2\pi\sqrt{\frac{h}{g}}$

when $h = 125$, $g = 981$, $T = 2\pi\sqrt{\frac{125}{981}}$

$= 2 \times \pi \times 0.357$

$= 2.24$ **Ans.**

(ii) $T = 2\pi\sqrt{\frac{h}{g}}$

$\Rightarrow \frac{T}{2\pi} = \sqrt{\frac{h}{g}}$

squaring both sides

$\Rightarrow \frac{T^2}{4\pi^2} = \frac{h}{g} \Rightarrow h = \frac{T^2 g}{4\pi^2}$ **Ans.**

(b) $45 - (p+3) = 2p$

$45 - p - 3 = 2p$

$42 = 3p$

$p = 14$ **Ans.**

(c) $\frac{2x-3}{4} + \frac{5-x}{3} = 0$

$\frac{3(2x-3) + 4(5-x)}{12} = 0$

$6x - 9 + 20 - 4x = 0$

$2x + 11 = 0$

$2x = -11$

$x = -5.5$ **Ans.**

(d) $3y^2 + 11y + 4 = 0$

using quadratic formula,

$y = \frac{-11 \pm \sqrt{(11)^2 - 4(3)(4)}}{2(3)}$

$= \frac{-11 \pm \sqrt{121 - 48}}{6}$

$= \frac{-11 \pm \sqrt{73}}{6}$

$\Rightarrow y = \frac{-11 + \sqrt{73}}{6}$ or $y = \frac{-11 - \sqrt{73}}{6}$

$= -0.4093$ $= -3.2573$

$\therefore$ $y = -0.41$ or -3.26 **Ans.**

33 (J2015/P1/Q23b)

Solve the simultaneous equations.

$3x + 4y = 3$

$2x - y = 13$ [3]

Thinking Process

From 2nd equation, make y the subject of formula. Substitute the expression into the 1st equation to solve for x. Substitute the value of x back into the expression of y to find the value of y.

Solution

$3x + 4y = 3$ ....(1)

$2x - y = 13$ ....(2)

From (2): $y = 2x - 13$ ....(3)

Substitute (3) into (1):

$3x + 4(2x - 13) = 3$

$3x + 8x - 52 = 3$

$11x = 55$

$x = 5$

Substitute $x = 5$ into (3):

$y = 2(5) - 13$

$= 10 - 13 = -3$

$\therefore$ $x = 5$, $y = -3$ **Ans.**

34 (J2015/P2/Q9)

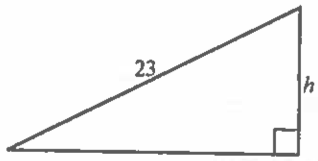
(a) Factorise completely

(i) $4x^3 - 10xy$. [1]

(ii) $9a^2 - b^2$. [1]

(b) Solve $\frac{7}{3-2m} = 4$. [2]

(c)



A right-angled triangle has a base that is 7 cm longer than its height, h cm.

The hypotenuse of the triangle is 23 cm.

(i) Show that h satisfies the equation

$h^2 + 7h - 240 = 0$. [2]

(ii) Write down an expression, in terms of h, for the area of the triangle. [1]

(iii) Hence state the exact area of the triangle. [1]

(iv) Solve $h^2 + 7h - 240 = 0$, giving your answers correct to 1 decimal place. [3]

(v) Calculate the perimeter of the triangle. [1]

Thinking Process

(a) (i) Take out common factor.

(ii) Recall $a^2 - b^2 = (a+b)(a-b)$

(b) Perform cross-multiplication.

(c) (i) Find the length of base of the triangle and then apply pythagoras theorem.

(iii) Substitute the value of $h^2 + 7h$ from part (c) (i) into the answer of (c) (ii).

(iv) Solve by quadratic formula.

Solution

(a) (i) $4x^3 - 10xy$

$= 2x(2x^2 - 5y)$ **Ans.**

(ii) $9a^2 - b^2$

$= (3a)^2 - b^2$

$= (3a-b)(3a+b)$ **Ans.**

(b) $\frac{7}{3-2m} = 4$

$7 = 4(3-2m)$

$7 = 12 - 8m$

$8m = 5$

$m = \frac{5}{8}$ **Ans.**

(c) (i) Length of base of triangle

$= (7+h)$ cm

Applying pythagoras theorem.

$(7+h)^2 + h^2 = (23)^2$

$(49 + 14h + h^2) + h^2 = 529$

$2h^2 + 14h - 480 = 0$

$2(h^2 + 7h - 240) = 0$

$h^2 + 7h - 240 = 0$ **Shown.**

(ii) Area of triangle

$= \frac{1}{2}(7+h)(h)$

$= \frac{1}{2}(7h + h^2)$ cm^2

(iii) From part (c) (i).

$h^2 + 7h - 240 = 0$

$\Rightarrow h^2 + 7h = 240$

area of triangle $= \frac{1}{2}(7h + h^2)$

$= \frac{1}{2}(240)$

$= 120$ cm^2 **Ans.**

(iv) $h^2 + 7h - 240 = 0$

using quadratic formula,

$$h = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-240)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{49 + 960}}{2}$$

$$= \frac{-7 \pm \sqrt{1009}}{2}$$

$\Rightarrow h = \frac{-7 + \sqrt{1009}}{2}$ or $h = \frac{-7 - \sqrt{1009}}{2}$

$= 12.38$ $\quad = -19.38$

$\therefore h = 12.4$ or -19.4 **Ans.**

(v) Perimeter of the triangle $= h + h + 7 + 23$

$= 2h + 30$

$= 2(12.38) + 30$

$= 54.76$

≈ 54.8 cm **Ans.**

35 *(N2015/P2/Q6)*

(a) (i) Solve the equation

$$\left(x + \frac{7}{2}\right) = \pm \frac{\sqrt{5}}{2}.$$

Give both answers correct to 2 decimal places. [2]

(ii) The solutions of $\left(x + \frac{7}{2}\right) = \pm\frac{\sqrt{5}}{2}$ are also the solutions of $x^2 + Bx + C = 0$, where B and C are integers. Find B and C. [3]

(b) Solve the inequality $7 - 3x > 13$. [2]

(c) Factorise $6x - 3yt + 18y - xt$. [2]

(d) Solve these simultaneous equations.

$3a + 4b = -13$

$5a + 6b = -11$ [4]

Thinking Process

(a) (ii) Form a quadratic equation by taking square on both sides. Compare it with given equation to find B and C.

(b) Arrange the inequality such that x is on one side of the inequality.

(c) Factorise by grouping.

(d) Eliminate one of the variables and solve for a and b.

Solution

(a) (i) $\left(x + \frac{7}{2}\right) = \pm \frac{\sqrt{5}}{2}$

$\Rightarrow x + \frac{7}{2} = \frac{\sqrt{5}}{2}$ or $x + \frac{7}{2} = -\frac{\sqrt{5}}{2}$

$x = \frac{\sqrt{5}}{2} - \frac{7}{2}$ or $x = -\frac{\sqrt{5}}{2} - \frac{7}{2}$

$x = -2.382$ or $x = -4.618$

$\therefore x = -2.38$ or -4.62 **Ans.**

(ii) $\left(x+\frac{7}{2}\right)=\pm\frac{\sqrt{5}}{2}$

$\left(x+\frac{7}{2}\right)^2=\left(\pm\frac{\sqrt{5}}{2}\right)^2$

$x^2+7x+\frac{49}{4}=\frac{5}{4}$

$x^2+7x+11=0$

comparing it with x^2+Bx+C

$B=7, \quad C=11$ **Ans.**

(b) $7-3x>13$

$-3x>13-7$

$-3x>6$

$x<\frac{6}{-3}$

$x<-2$ **Ans.**

(c) $6x-3yt+18y-xt$

$=6x-xt-3yt+18y$

$=x(6-t)-3y(t-6)$

$=x(6-t)+3y(6-t)$

$=(6-t)(x+3y)$ **Ans.**

(d) $3a+4b=-13$ ....(1)

$5a+6b=-11$ ....(2)

$(1)\times 3: \quad 9a+12b=-39$

$(2)\times 2: \quad 10a+12b=-22$

$\quad - \quad - \quad +$

$-a \quad =-17 \Rightarrow a=17$

substitute $a=17$ into (1).

$3(17)+4b=-13$

$51+4b=-13$

$4b=-64$

$b=-16$

$\therefore \; a=17, \; b=-16$ **Ans.**

36 *(J2016/P1/Q10)*

Solve the simultaneous equations.

$6x+y=1$

$4x-y=4$ [2]

Thinking Process

Eliminate one of the variables and solve for x and y.

Solution

$6x+y=1$ ....(1)

$4x-y=4$ ....(2)

Adding both equations.

$6x+y=1$

$4x-y=4$

$10x \quad =5 \Rightarrow x=\frac{1}{2}$

substitute $x=\frac{1}{2}$ into (1).

$\Rightarrow 6(\frac{1}{2})+y=1$

$\Rightarrow 3+y=1 \Rightarrow y=-2$

$\therefore \; x=\frac{1}{2}, \; y=-2$ **Ans.**

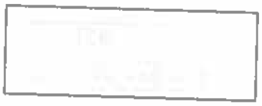
37 *(J2016 P2 Q5)*

(a) Factorise fully $8x^2y-12x^5$. [1]

(b) Solve $4x-2(x+5)=3$. [2]

(c) Solve $7-5y<20$. [2]

(d) A rectangle has length $2x$ cm, perimeter 18 cm and area 10 cm^2.



$2x$

(i) Show that $2x^2-9x+5=0$. [2]

(ii) Solve $2x^2-9x+5=0$, giving your answers correct to 2 decimal places. [3]

(iii) Find the difference between the length and the width of the rectangle. [1]

Thinking Process

(a) Take out common factor.

(c) Arrange the inequality such that y is on one side of the inequality.

(d) (i) Find an expression for the width of the rectangle. Then use the given information to form the required equation.

(ii) Solve by quadratic formula.

(iii) Substitute the value of x found in (ii) into the expressions of length and width and find the difference.

Solution

(a) $8x^2y-12x^5$

$=4x^2(2y-3x^3)$ **Ans.**

(b) $4x-2(x+5)=3$

$4x-2x-10=3$

$2x=13$

$x=\frac{13}{2}=6.5$ **Ans.**

(c) $7-5y<20$

$-5y<20-7$

$-5y<13$

$y>-\frac{13}{5}$

$y>-2.6$ **Ans.**

(d) (i) Let the width of the rectangle be w,

given that, perimeter = 18 cm

$\Rightarrow 2(2x)+2w=18$

$2w=18-4x$

$w=\frac{2(9-2x)}{2}=9-2x$ cm

given that, area = 10 cm^2

$\Rightarrow 2x\times w=10$

$2x\times(9-2x)=10$

$x(9-2x)=5$

$9x-2x^2=5$

$2x^2-9x+5=0$ **Shown.**

(ii) $2x^2-9x+5=0$

using quadratic formula,

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(5)}}{2(2)}$$

$$=\frac{9\pm\sqrt{81-40}}{4}$$

$$=\frac{9\pm\sqrt{41}}{4}$$

$\Rightarrow x=\frac{9+\sqrt{41}}{4}$ or $x=\frac{9-\sqrt{41}}{4}$

$=3.85$ $=0.65$

$\therefore\ x=3.85$ or 0.65 **Ans.**

(iii) Taking $x=3.85$,

Length $=2x$

$=2(3.85)=7.7$ cm

width $=9-2x$

$=9-2(3.85)=1.3$ cm

difference $=7.7-1.3=6.4$ cm **Ans.**

38 *(N2016/P1/Q15)*

Solve the simultaneous equations.

$3x+\ \ y=\ \ 9$

$2x+3y=-8$ [3]

Thinking Process

Substitute the value of y from 1st equation into the 2nd equation and subsequently solve for values of x and y.

Solution

$3x+y=9$ .... (1)

$2x+3y=-8$ .... (2)

From (1): $y=9-3x$ .... (3)

substitute (3) into (2):

$2x+3(9-3x)=-8$

$2x+27-9x=-8$

$-7x=-35$

$x=5$

Substitute $x=5$ into (3).

$y=9-3(5)$

$=9-15=-6$

$\therefore\ x=5,\ y=-6$ **Ans.**

39 *(N2016/P1/Q20)*

Solve the equation $\frac{2x-1}{4}+\frac{x-2}{3}=2.$ [3]

Thinking Process

Add the 2 fractions by finding common denominator and solve for x.

Solution

$$\frac{2x-1}{4}+\frac{x-2}{3}=2$$

$$\frac{3(2x-1)+4(x-2)}{12}=2$$

$6x-3+4x-8=24$

$10x=35$

$x=3.5$ **Ans.**

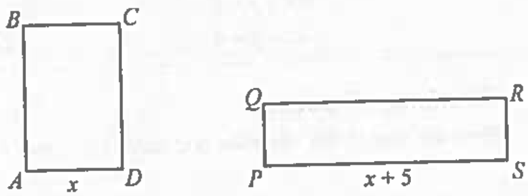
40 *(N2016/P2/Q7)*

(a) $x=\sqrt{a^2+b^2}$

(i) Calculate x when $a=-0.73$ and $b=1.84$. [1]

(ii) Express b in terms of x and a. [2]

(b)



$ABCD$ and $PQRS$ are rectangles.

$AD=x$ cm and $PS=(x+5)$ cm.

Each rectangle has an area of 17 cm^2.

(i) Write down an expression for PQ in terms of x. [1]

(ii) AB is 3 cm longer than PQ.

Form an equation in x and show that it simplifies to $3x^2+15x-85=0$. [3]

(iii) Solve the equation $3x^2+15x-85=0$.
Give your solutions correct to 3 significant figures. [3]

(iv) Find the perimeter of the rectangle $PQRS$. [2]

Thinking Process

(a) (i) Substitute the given values into the equation.
(ii) Make b the subject of the formula.

(b) (i) To find PQ apply, Area = length × width.
(ii) Apply, Area = length × width and simplify the equation obtained.
(iii) Apply quadratic formula and solve for x.
(iv) Substitute the value of x into the expressions of PQ and PS and find the perimeter.

Solution

(a) (i) $x=\sqrt{a^2+b^2}$

$\Rightarrow x=\sqrt{(-0.73)^2+(1.84)^2}$

$\Rightarrow x=\sqrt{3.9185}$

$\Rightarrow x=1.98$ **Ans.**

(ii) $x=\sqrt{a^2+b^2}$

$\Rightarrow x^2=a^2+b^2$

$\Rightarrow b^2=x^2-a^2$

$\Rightarrow b=\sqrt{(x+a)(x-a)}$ **Ans.**

(b) (i) Area of $PQRS = PS\times PQ$

$17=(x+5)\times PQ$

$PQ=\dfrac{17}{x+5}$ cm **Ans.**

(ii) $AB=(\dfrac{17}{x+5}+3)$ cm

Area of $ABCD = AD\times AB$

$17=x\times(\dfrac{17}{x+5}+3)$

$17=\dfrac{17x}{x+5}+3x$

$17=\dfrac{17x+3x(x+5)}{x+5}$

$17(x+5)=17x+3x^2+15x$

$17x+85=3x^2+32x$

$3x^2+15x-85=0$ **Shown.**

(iii) $3x^2+15x-85=0$

by quadratic formula,

$x=\dfrac{-15\pm\sqrt{(15)^2-4(3)(-85)}}{2(3)}$

$=\dfrac{-15\pm\sqrt{1245}}{6}$

$\Rightarrow x=\dfrac{-15+\sqrt{1245}}{6}$ or $x=\dfrac{-15-\sqrt{1245}}{6}$

$=3.38$ $\quad$ $=-8.38$

$\therefore x=3.38$ or -8.38 **Ans.**

(iv) $PQ=\dfrac{17}{3.38+5}=\dfrac{17}{8.38}$ cm

$PS=3.38+5=8.38$ cm

perimeter of $PQRS=2(PS+PQ)$

$=2\left(8.38+\dfrac{17}{8.38}\right)$

$=2(10.409)$

$=20.82$ cm **Ans.**

41 *(J2017/P1/Q11)*

Solve the simultaneous equations.

$5x-2y=16$

$3x+4y=7$ [3]

Thinking Process

Multiply the first equation by 2, eliminate y and find x. Substitute the value of x back into the equation to find the value of y.

Solution

$5x-2y=16$ .... (1)

$3x+4y=7$ .... (2)

eq. (1)×2: $\quad 10x-4y=32$

$\qquad 3x+4y=7$

$13x+0=39 \Rightarrow 13x=39$

$x=3$

substitute $x=3$ into eq. (2),

$3(3)+4y=7$

$4y=-2$

$y=-\dfrac{1}{2}$

$\because x=3,\ y=-\dfrac{1}{2}$ **Ans.**

42 *(N2017/P2/Q5)*

(a) Express as a single fraction in its simplest form $\dfrac{4}{x-2}-\dfrac{5}{x+1}$. [2]

(b) Solve $2x(x+1)=3(4-x)$. [3]

(c) Anil and Yasmin buy some pens and notebooks from the same shop.
Anil buys 3 pens and 2 notebooks for \$4.80.
Yasmin buys 5 pens and 4 notebooks for \$9.00.

(i) Form a pair of simultaneous equations to represent this information. [1]

(ii) Solve the simultaneous equations to find the cost of a pen and the cost of a notebook. [3]

Thinking Process

(a) Re-express the fractions with a common denominator.
(b) Solve by factorisation.
(c) (ii) Solve by either elimination method or substitution method.

Solution

(a) $\dfrac{4}{x-2}-\dfrac{5}{x+1}$

$=\dfrac{4(x+1)-5(x-2)}{(x-2)(x+1)}$

$=\dfrac{4x+4-5x+10}{(x-2)(x+1)}$

$=\dfrac{14-x}{(x-2)(x+1)}$ **Ans.**

(b) $2x(x+1)=3(4-x)$

$2x^2+2x=12-3x$

$2x^2+5x-12=0$

$2x^2-3x+8x-12=0$

$x(2x-3)+4(2x-3)=0$

$(2x-3)(x+4)=0$

$\Rightarrow\ 2x-3=0$ or $x+4=0$

$x=\dfrac{3}{2}$ $\quad x=-4$

$\therefore\ x=\dfrac{3}{2}$ or -4 **Ans.**

(c) (i) Let p be the cost of a pen and n be the cost of a notebook.

$\therefore$ according to given information,

$3p+2n=4.8$

$5p+4n=9$ **Ans.**

(ii) $3p+2n=4.8$ .... (1)

$5p+4n=9$ .... (2)

eq.(1)×2: $6p+4n=9.6$ .... (3)

eq.(3) − eq.(2): $6p-5p=9.6-9$

$p=0.6$

substitute $p=0.6$ into eq.(1)

$3(0.6)+2n=4.8$

$1.8+2n=4.8$

$2n=3$

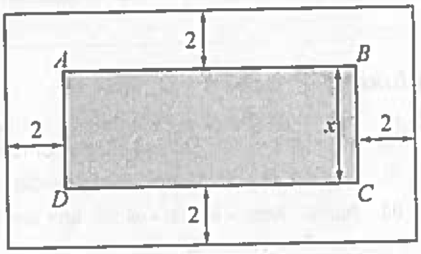
$n=1.5$

$\therefore$ cost of pen = \$0.6,

cost of notebook = \$1.5 **Ans.**

43 *(N2017/P2/Q10)*

A rectangular picture, $ABCD$, is placed inside a rectangular frame.



The length, AB, of the picture is three times its height, x cm.
The width of the frame is 2 cm.

(a) The total area of the picture and the frame is 476 cm².
Form an equation in x and show that it simplifies to $3x^2+16x-460=0$. [4]

(b) Solve the equation $3x^2+16x-460=0$. [3]

(c) Find the height and length of the frame. [2]

(d) The frame is made from wood.
The wood is 5 mm thick.
The mass of 1 cm³ of the wood is 0.7 g.
Calculate the mass of wood used in the frame. [3]

Thinking Process

(a) Find the height and length of the frame in terms of x. Apply, Area = length × width
(b) Apply quadratic formula and solve for x.
(c) Use the value of x found in (b) to find the height and length of the frame.
(d) To calculate the mass ∮ calculate the volume of wood used in the frame.

Solution

(a) Given that, $AB=3x$ cm, $BC=x$ cm

$\therefore$ length of frame $=3x+4$

width of frame $=x+4$

Total area $=l\times w$

$\Rightarrow\ 476=(3x+4)(x+4)$

$\Rightarrow\ 476=3x^2+12x+4x+16$

$\Rightarrow\ 3x^2+16x-460=0$ **Shown.**

(b) $3x^2+16x-460=0$

applying quadratic formula,

$$x=\frac{-16\pm\sqrt{(16)^2-4(3)(-460)}}{2(3)}$$

$$=\frac{-16\pm\sqrt{256+5520}}{6}$$

$$=\frac{-16\pm\sqrt{5776}}{6}=\frac{-16\pm 76}{6}$$

$\Rightarrow\ x=\frac{-16+76}{6}$ or $x=\frac{-16-76}{6}$

$\therefore\ x=10$ or -15.33 **Ans.**

(c) Length of frame $=3x+4$

$=3(10)+4=34$ cm **Ans.**

width of frame $=x+4$

$=10+4=14$ cm **Ans.**

(d) Area of picture $=3x\times x$

$=3(10)\times 10=300\ \text{cm}^2$

Area of frame = total area of picture and frame − area of picture

$=476-300=176\ \text{cm}^2$

$\therefore$ Volume of wood used $=176\times 0.5=88\ \text{cm}^3$

Note that: 5 mm = 0.5 cm

mass of 1 cm^3 of wood = 0.7 g.

mass of 88 cm^3 of wood $=0.7\times 88=61.6$ g

$\therefore$ mass of wood used = 61.6 g **Ans.**

44 (J2018/P1/Q8)

Solve $\frac{4}{x-11}=\frac{1}{3x}$. [2]

Thinking Process

Cross multiply and solve for x.

Solution

$$\frac{4}{x-11}=\frac{1}{3x}$$

$12x=x-11$

$11x=-11\ \Rightarrow\ x=-1$ **Ans.**

45 (J2018/P2/Q3)

(a) Solve $3(x+10)=12-7x$. [2]

(b) Solve the simultaneous equations.
Show your working.

$4x-3y=28$

$6x+y=9$ [3]

(c) Simplify $\frac{v^2-8v}{2v^2-13v-24}$. [3]

Thinking Process

(a) Expand and solve for x.

(b) Substitute the value of y from 2nd equation into the 1st equation and solve for x. Substitute the value of x back into the expression of y and solve for y.

(c) Factorise numerator and denominator.

Solution

(a) $3(x+10)=12-7x$

$3x+30=12-7x$

$10x=-18$

$x=-1.8$ **Ans.**

(b) $4x-3y=28$ ....(1)

$6x+y=9$ ....(2)

From (2): $y=9-6x$ ....(3)

substitute (3) into (1):

$4x-3(9-6x)=28$

$4x-27+18x=28$

$22x=55\ \Rightarrow\ x=\frac{55}{22}=2.5$

Substitute $x=2.5$ into (3).

$y=9-6(2.5)=-6$

$\therefore\ x=2.5,\ y=-6$ **Ans.**

(c) $\frac{v^2-8v}{2v^2-13v-24}$

$=\frac{v(v-8)}{2v^2-16v+3v-24}$

$=\frac{v(v-8)}{2v(v-8)+3(v-8)}$

$=\frac{v(v-8)}{(v-8)(2v+3)}=\frac{v}{2v+3}$ **Ans.**

46 (J2018/P2/Q9)

(a) On Monday, Ravi goes on a 20 km run.

(i) His average speed for the first 12 km is x km/h.
Write down an expression, in terms of x, for the time taken for the first 12 km.
Give your answer in minutes. [1]

(ii) His average speed for the final 8 km of the run is 1.5 km/h slower than for the first 12 km.
Write an expression, in terms of x, for the time taken for the final 8 km of the run.
Give your answer in minutes. [1]

(iii) Ravi takes 110 minutes to complete the full 20 km.
Form an equation in x and show that it simplifies to $22x^2-273x+216=0$. [4]

(iv) Solve the equation $22x^2-273x+216=0$.
Show your working and give each answer correct to 2 decimal places. [3]

(b) On Friday, Ravi ran the whole 20 km at the same average speed that he ran the final 8 km on Monday.
Calculate the time Ravi took to run 20 km on Friday.
Give your answer in hours and minutes, correct to the nearest minute. [3]

Thinking Process

(a) (i) Apply, time = $\frac{\text{distance}}{\text{speed}}$

(ii) Apply, time = $\frac{\text{distance}}{\text{speed}}$. Note that this time average speed is 1.5 km/h slower than x.

(iii) To form an equation add up the two times and equate to 110.

(iv) Apply quadratic formula and solve for x.

(b) Substitute the value of x into your answer to part (a) (ii) and Calculate the time.

Solution

(a) (i) Time taken $= \frac{12}{x}$ hours

$= \frac{12}{x} \times 60 = \frac{720}{x}$ minutes **Ans.**

(ii) Time taken $= \frac{8}{x-1.5}$ hours

$= \frac{8}{x-1.5} \times 60$

$= \frac{480}{x-1.5}$ minutes **Ans.**

(iii) Total time to complete the run = 110 minutes

$\Rightarrow \frac{720}{x} + \frac{480}{x-1.5} = 110$

$\Rightarrow \frac{720(x-1.5)+480x}{x(x-1.5)} = 110$

$\Rightarrow 720(x-1.5)+480x = 110x(x-1.5)$

$\Rightarrow 720x - 1080 + 480x = 110x^2 - 165x$

$\Rightarrow 110x^2 - 1365x + 1080 = 0$ (divide by 5)

$\Rightarrow 22x^2 - 273x + 216 = 0$ **Shown.**

(iv) $22x^2 - 273x + 216 = 0$

Using quadratic formula,

$$x = \frac{-(-273) \pm \sqrt{(-273)^2 - 4(22)(216)}}{2(22)}$$

$$= \frac{273 \pm \sqrt{55521}}{44}$$

$\Rightarrow x = \frac{273 + 235.63}{44}$ or $x = \frac{273 - 235.63}{44}$

$= 11.56$ $= 0.85$

$\therefore x = 11.56$ or 0.85 **Ans.**

(b) Average speed $= x - 1.5$

$= 11.56 - 1.5 = 10.06$ km/h

Time taken $= \frac{20}{10.06}$

$= 1.99$ hours

$= 1$ hour 59 minutes **Ans.**

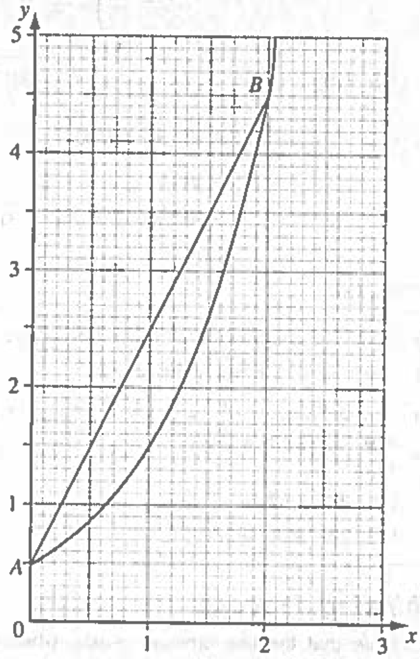
Topic 6

Co-ordinate Geometry

1 (J2007/P1/Q13)

(a) Given that $2y = 3^x$, find x when $y = 40\frac{1}{2}$. [1]

(b) The points, $A(0, \frac{1}{2})$ and $B(2, 4\frac{1}{2})$, lie on the curve as shown in the diagram.



(i) Calculate the gradient of the straight line AB. [1]

(ii) Using the diagram, estimate the value of x at which the gradient of the curve is equal to the gradient of the straight line AB. [1]

Thinking Process

(a) Substitute the given value of y and solve the equation.

(b) (i) Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$.

(ii) Draw a tangent to the curve, parallel to line AB and look for the required value of x.

Solution

(a) $2y = 3^x$

substituting $y = 40\frac{1}{2}$, we have

$2\left(40\frac{1}{2}\right) = 3^x \Rightarrow 2\left(\frac{81}{2}\right) = 3^x$

$\Rightarrow 3^4 = 3^x$

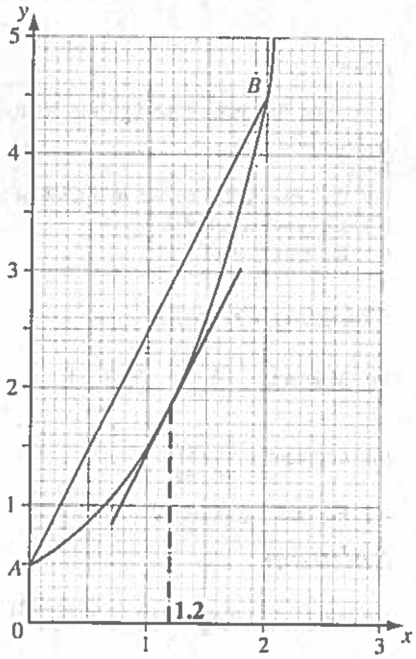
$\therefore x = 4$ **Ans.**

(b) (i) We have $A(0, \frac{1}{2})$ and $B(2, 4\frac{1}{2})$

$\therefore$ gradient of $AB = \frac{\frac{9}{2} - \frac{1}{2}}{2 - 0}$

$= \frac{\frac{8}{2}}{2} = \frac{4}{2} = 2$ **Ans.**

(ii)



By drawing a tangent parallel to line AB, we see that the gradient of the curve is equal to the gradient of line AB when $x = 1.2$ **Ans.**

2 (N2007/P2/Q11a)

(a) P is the point (2, 9) and Q is the point (4, 6). Find

(i) the length of PQ, [2]

(ii) the equation of the line PQ. [2]

Thinking Process

(a) (i) Length or distance between two points: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

(ii) Equation of a straight line: $y = mx + C$

Solution

(a) (i) Length of $PQ = \sqrt{(4-2)^2+(6-9)^2}$

$= \sqrt{(2)^2+(-3)^2}$

$= \sqrt{4+9}$

$= \sqrt{13} = 3.61$ units Ans

(ii) Gradient $PQ = \frac{6-9}{4-2} = -\frac{3}{2}$

equation of line through P and Q is

$y = -\frac{3}{2}x + c$

at $P(2, 9)$

$9 = -\frac{3}{2}(2) + c \Rightarrow c = 12$

$\therefore\ y = -\frac{3}{2}x + 12$ Ans

3 *(J2008/P1/Q17)*

A straight line passes through the points P (1, 2) and Q (5, −14).
Find
(a) the coordinates of the midpoint of PQ, [1]
(b) the gradient of PQ, [1]
(c) the equation of PQ. [2]

Thinking Process

(a) Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

(b) Gradient $= \frac{y_1-y_2}{x_1-x_2}$.

(c) Equation: $y = mx + c$.

Solution

(a) Midpoint of $PQ = \left(\frac{1+5}{2}, \frac{2+(-14)}{2}\right)$

$= \left(\frac{6}{2}, \frac{-12}{2}\right) = (3, -6)$ Ans.

(b) Gradient of $PQ = \frac{2-(-14)}{1-5} = \frac{16}{-4} = -4$ Ans.

(c) Equation of PQ: $y = mx + c$

$\Rightarrow\ y = -4x + c$

for c, put $P(1, 2)$

$\Rightarrow\ 2 = -4(1) + c$

$c = 2 + 4 = 6$

$\therefore\ y = -4x + 6$ Ans.

4 *(J2009/P2/Q1c)*

(c) The points P and Q are (4, 7) and (8, −3) respectively.
Find
(i) the midpoint of PQ, [1]
(ii) the length of PQ. [2]

Thinking Process

(c) (i) Mid-point formula, $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(ii) Distance formula: $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Solution

(c) (i) midpoint of $PQ = \left(\frac{4+8}{2}, \frac{7-3}{2}\right)$

$= \left(\frac{12}{2}, \frac{4}{2}\right) = (6, 2)$ Ans.

(ii) length of $PQ = \sqrt{(4-8)^2+(7-(-3))^2}$

$= \sqrt{(-4)^2+(10)^2}$

$= \sqrt{116}$

$= 10.77 \approx 10.8$ units (3sf) Ans.

5 *(N2009/P1/Q14)*

(a) Find the coordinates of the point where the line $2y = 3x + 15$ crosses the y-axis. [1]

(b) The coordinates of the points P and Q are (−1, 10) and (3, 4) respectively.
Find
(i) the gradient of PQ, [1]
(ii) the midpoint of PQ. [1]

Thinking Process

(a) Note that the line crosses y-axis, when $x = 0$.

(b) (i) Apply formula, Gradient $= \frac{y_2-y_1}{x_2-x_1}$.

(ii) Midpoint, $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Solution

(a) $2y = 3x + 15$

The line meets y-axis when $x = 0$,

$\therefore\ 2y = 3(0) + 15 = 15$

$y = \frac{15}{2} = 7\frac{1}{2}$

$\therefore$ coordinates are: $(0, 7\frac{1}{2})$ Ans.

(b) (i) Gradient of $PQ = \frac{4-10}{3-(-1)}$

$= \frac{-6}{4}$

$= -\frac{3}{2} = -1.5$ **Ans.**

(ii) Midpoint of $PQ = \left(\frac{-1+3}{2}, \frac{10+4}{2}\right)$

$= (1, 7)$ **Ans.**

6 *(J2010/P1/Q14)*

A straight line passes through the points $P(-8, 10)$ and $Q(4, 1)$.

Find

(a) the coordinates of the midpoint of PQ, [1]

(b) the equation of PQ. [2]

Thinking Process

(a) Mid-point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(b) To find equation use $y = mx + c$, where m is the gradient and c is the y-intercept.

Solution

(a) Midpoint of $PQ = \left(\frac{-8+4}{2}, \frac{10+1}{2}\right)$

$= (-2, 5.5)$ **Ans.**

(b) Gradient of $PQ = \frac{10-1}{-8-4} = \frac{9}{-12} = -\frac{3}{4}$

Equation of PQ is:

$y = -\frac{3}{4}x + c$

using $Q(4, 1)$

$1 = -\frac{3}{4}(4) + c \Rightarrow c = 4$

$\therefore\ y = -\frac{3}{4}x + 4$ **Ans.**

7 *(J2011/P1/Q24)*

P is the point $(-2, 1)$ and Q is the point $(3, 7)$.

(a) M is the midpoint of PQ. Find the coordinates of M. [1]

(b) Find the gradient of the line PQ. [1]

(c) The line with equation $2y + 3x + k = 0$ passes through the point P.

(i) Find k. [2]

(ii) Find the gradient of this line. [1]

Thinking Process

(a) Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

(b) Gradient $= \frac{y_2-y_1}{x_2-x_1}$

(c) (i) Substitute point P into the equation to solve for k.

(ii) Rearrange the equation in the form $y = mx + c$.

Solution

(a) Mid-point, $M = \left(\frac{-2+3}{2}, \frac{1+7}{2}\right)$

$= \left(\frac{1}{2}, 4\right)$ **Ans.**

(b) Gradient of $PQ = \frac{7-1}{3-(-2)}$

$= \frac{6}{5} = 1\frac{1}{5}$ **Ans.**

(c) (i) $2y + 3x + k = 0$

the line passes through point $P(-2, 1)$

$\Rightarrow 2(1) + 3(-2) + k = 0$

$2 - 6 + k = 0$

$k = 4$ **Ans.**

(ii) $2y + 3x + k = 0$

$2y = -3x - k$

$y = -\frac{3}{2}x - \frac{k}{2}$

$\therefore$ gradient, $m = -\frac{3}{2}$ **Ans.**

8 *(N2011/P2/Q6)*

The point A is $(0, 7)$, and the point B is $(6, 9)$.

(a) Express $\vec{AB}$ as a column vector. [1]

(b) Find the gradient of AB. [1]

(c) The equation of the line AB is $x + Py + Q = 0$. Find P and Q. [2]

(d) The point C is $(12, 2)$.

(i) Given that C is the midpoint of BM, find the coordinates of M. [1]

(ii) Calculate AC. [1]

(iii) The point D lies on the line AB. The line CD is parallel to the y-axis.

(a) Find the coordinates of D. [2]

(b) Express $\vec{AD}$ in terms of $\vec{AB}$. [1]

Thinking Process

(a) $\vec{AB} = \vec{OB} - \vec{OA}$

(b) Gradient between two points $= \frac{y_2-y_1}{x_2-x_1}$

(c) Find the equation of the line AB. Compare it with the given equation to identify P and Q.

(d) (i) To find M apply the mid-point formula.

(ii) Apply length formula: $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

(iii) To find D solve the equations of AB and CD simultaneously.

Solution

(a) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \begin{pmatrix}6\\9\end{pmatrix} - \begin{pmatrix}0\\7\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$ **Ans.**

(b) Gradient of $AB = \dfrac{9-7}{6-0}$

$= \dfrac{2}{6} = \dfrac{1}{3}$ **Ans.**

(c) Equation of AB: $y = \frac{1}{3}x + c$

at $A(0, 7)$, $7 = \frac{1}{3}(0) + c \Rightarrow c = 7$

$\therefore\ y = \frac{1}{3}x + 7$

$\Rightarrow 3y = x + 21$

$\Rightarrow x - 3y + 21 = 0$

comparing it with $x + Py + Q = 0$

$P = -3,\ Q = 21$ **Ans.**

(d) (i) Let coordinates of M be (x, y)

$\left(\dfrac{6+x}{2}, \dfrac{9+y}{2}\right) = (12, 2)$

$\Rightarrow \dfrac{6+x}{2} = 12, \quad \dfrac{9+y}{2} = 2$

$\Rightarrow 6 + x = 24, \quad 9 + y = 4$

$\Rightarrow x = 18, \quad y = -5$

$\therefore\ M$ is $(18, -5)$ **Ans.**

(ii) $AC = \sqrt{(12-0)^2 + (2-7)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= 13$ units **Ans.**

(iii) (a) Equation of AB: $y = \frac{1}{3}x + 7$

equation of CD: $x = 12$ ($\because CD \parallel y$-axis)

lines AB and CD meet at D,

$\Rightarrow y = \frac{1}{3}(12) + 7 = 11$

$\therefore$ coordinates of D: $(12, 11)$ **Ans.**

(b) $\vec{AD} = \vec{OD} - \vec{OA}$

$= \begin{pmatrix}12\\11\end{pmatrix} - \begin{pmatrix}0\\7\end{pmatrix}$

$= \begin{pmatrix}12\\4\end{pmatrix}$

$= 2\begin{pmatrix}6\\2\end{pmatrix} = 2\vec{AB}$

$\therefore\ \vec{AD} = 2\vec{AB}$ **Ans.**

9 (J2012/P1/Q14)

A is the point $(0, 4)$ and B is the point $(-6, 1)$.

(a) M is the midpoint of the line AB.
Find the coordinates of M. [1]

(b) Find the equation of the line AB. [2]

Thinking Process

(a) ∅ Mid-point $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

(b) To find equation ∅ use $y = mx + c$, where m is the gradient and c is the y-intercept.

Solution

(a) $M = \left(\dfrac{0-6}{2}, \dfrac{4+1}{2}\right) = (-3, 2.5)$ **Ans.**

(b) Gradient of $AB = \dfrac{1-4}{-6-0} = \dfrac{-3}{-6} = \dfrac{1}{2}$

$\Rightarrow$ equation of AB is: $y = \frac{1}{2}x + c$

using point $A(0, 4)$

$4 = \frac{1}{2}(0) + c \Rightarrow c = 4$

$\therefore\ y = \frac{1}{2}x + 4$ **Ans.**

10 (J2013/P1/Q4)

A line has equation $3y = 2 - x$.

(a) Find the gradient of the line. [1]

(b) The line passes through the point $(5, k)$.
Find the value of k. [1]

Thinking Process

(a) Rearrange the equation in the form $y = mx + c$

(b) To find k ∅ substitute $(5, k)$ into the equation of line.

Solution

(a) $3y = 2 - x$

$y = \dfrac{2}{3} - \dfrac{x}{3}$

$= -\dfrac{1}{3}x + \dfrac{2}{3}$

$\therefore$ gradient of the line $= -\dfrac{1}{3}$ **Ans.**

(b) $3y = 2 - x$

substitute $(5, k)$ into the equation.

$3k = 2 - 5$

$3k = -3$

$k = -1$ **Ans.**

11 *(N2013/P1/Q19)*

P is (–1, 3) and Q is (5, –1).

(a) Find the coordinates of the midpoint of PQ. [1]

(b) Find the gradient of the line PQ. [1]

(c) Given that the length of $PQ = 2\sqrt{n}$ units, where n is an integer, find the value of n. [2]

Thinking Process

(a) Mid-point $= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

(b) Gradient $= \dfrac{y_1-y_2}{x_1-x_2}$.

(c) Length $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Solution

(a) Midpoint of $PQ = \left(\dfrac{-1+5}{2}, \dfrac{3-1}{2}\right)$

$= (2, 1)$ **Ans.**

(b) Gradient of $PQ = \dfrac{-1-3}{5-(-1)}$

$= \dfrac{-4}{6} = -\dfrac{2}{3}$ **Ans.**

(c) Length of $PQ = \sqrt{(-1-5)^2+(3-(-1))^2}$

$2\sqrt{n} = \sqrt{(-6)^2+(4)^2}$

$2\sqrt{n} = \sqrt{52}$

taking square of both sides,

$4n = 52$

$n = 13$ **Ans.**

12 *(N2014/P1/Q25)*

P is (–4, 4) and Q is (3, –2).
M is the midpoint of PQ.

(a) Find the coordinates of M. [1]

(b) Find the gradient of the line PQ. [1]

(c) Q is the midpoint of the line PQR.

(i) Find the coordinates of R. [2]

(ii) Write down the value of $\dfrac{PM}{MR}$. [1]

Thinking Process

(a) Mid-point $= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

(b) Gradient $= \dfrac{y_1-y_2}{x_1-x_2}$

(c) (i) Let R be (x, y). Equate the midpoint of PQ to the midpoint of QR.

(ii) Apply length formula to find the ratio.

Solution

(a) $M = \left(\dfrac{-4+3}{2}, \dfrac{4-2}{2}\right)$

$= (-\dfrac{1}{2}, 1)$ **Ans.**

(b) Gradient of $PQ = \dfrac{-2-4}{3-(-4)}$

$= -\dfrac{6}{7}$ **Ans.**

(c) (i) Let the coordinates of R be (x, y)

midpoint of $PR = \left(\dfrac{-4+x}{2}, \dfrac{4+y}{2}\right)$

since Q is the midpoint of line PR,

$\Rightarrow \left(\dfrac{-4+x}{2}, \dfrac{4+y}{2}\right) = (3, -2)$

$\Rightarrow \dfrac{-4+x}{2} = 3 \quad , \quad \dfrac{4+y}{2} = -2$

$-4+x = 6 \quad , \quad 4+y = -4$

$x = 10 \quad , \quad y = -8$

$\therefore$ R is $(10, -8)$ **Ans.**

(ii) Using vector method,

$\vec{PM} = \vec{OM} - \vec{OP}$

$= \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix} - \begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} -\frac{7}{2} \\ -3 \end{pmatrix}$

$\vec{MR} = \vec{OR} - \vec{OM}$

$= \begin{pmatrix} 10 \\ -8 \end{pmatrix} - \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$

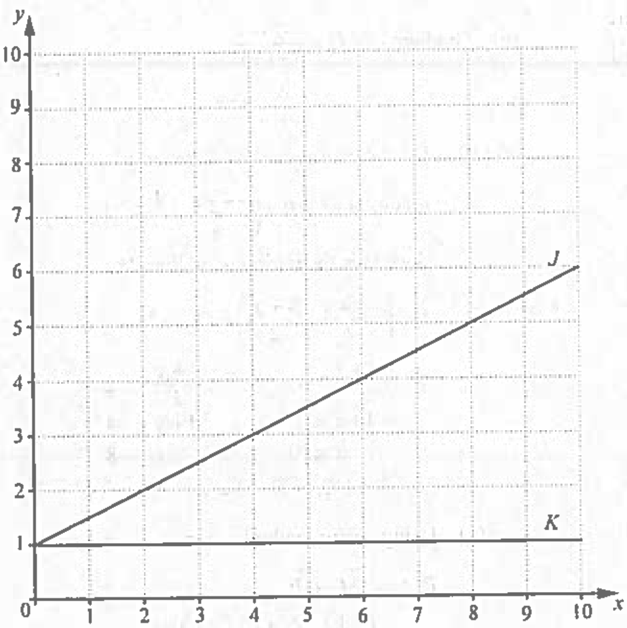
$= \begin{pmatrix} \frac{21}{2} \\ -9 \end{pmatrix} = 3\begin{pmatrix} \frac{7}{2} \\ -3 \end{pmatrix} = 3\vec{PM}$

$\therefore$ $3PM = MR \Rightarrow \dfrac{PM}{MR} = \dfrac{1}{3}$ **Ans.**

13 (J2015/P2/Q1)

(a) Simplify $\frac{4x-1}{3}+\frac{3x+5}{2}$. [2]

(b)



(i) Find the gradient of line J. [1]

(ii) Write down the equation of line K. [1]

(iii) Draw a line, L, through (6, 1) such that the area enclosed between J, K and L is 6 cm². [1]

(iv) Find the equation of line L. [2]

(v) The line N is perpendicular to line J at (2, 2).
Find the coordinates of the point where line N crosses the y-axis. [2]

Thinking Process

(a) Add the two fractions by finding a common denominator.

(b) (i) Gradient of a line = $\frac{\text{vertical change}}{\text{horizontal change}}$.

(ii) ✐ Note that line K is parallel to x-axis.

(iii) To locate the line L ✐ find the height of the triangle enclosed by lines J, K, and L by using the given area.

(iv) To find the equation ✐ use $y = mx + c$, where m is the gradient and c is the y-intercept.

(v) Find the equation of line N. Put $x = 0$ into the line to find the y-intercept.

Solution

(a) $\frac{4x-1}{3}+\frac{3x+5}{2}$

$=\frac{2(4x-1)+3(3x+5)}{6}$

$=\frac{8x-2+9x+15}{6}$

$=\frac{17x+13}{6}$ **Ans.**

(b) (i) Gradient line $J = \frac{5}{10}$

$= \frac{1}{2}$ **Ans.**

(ii) Equation of line K: $y = 1$ **Ans.**

(iii) Area of $\Delta JKL = 6$ cm²

$\Rightarrow \frac{1}{2} \times 6 \times h = 6$

$3h = 6$

$h = 2$

$\therefore$ height of $\Delta JKL = 2$ cm.

For line L Refer to figure on the next page.

(iv) Taking two points (4, 3) and (6, 1) on the line L.

gradient, $m = \frac{1-3}{6-4}$

$= \frac{-2}{2} = -1$

$\therefore$ equation of line L is: $y = -x + c$

using point (6, 1),

$1 = -6 + c \Rightarrow c = 7$

$\therefore$ $y = -x + 7$ **Ans.**

(v) Line N is perpendicular to line J

$\Rightarrow$ gradient of $N \times$ gradient of $J = -1$

gradient of $N \times \frac{1}{2} = -1$

gradient of $N = -2$

equation of line N is: $y = -2x + c$

using point (2, 2).

$2 = -2(2) + c \Rightarrow c = 6$

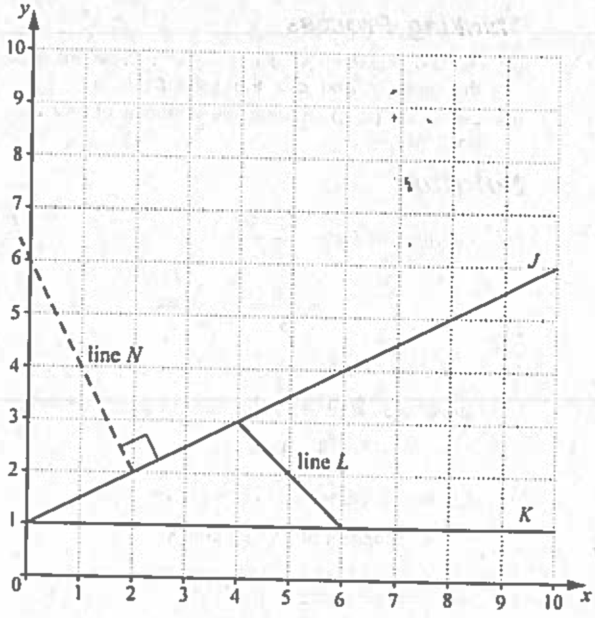
$\therefore$ equation is: $y = -2x + 6$

line N crosses the y-axis when $x = 0$

$\Rightarrow y = -2(0) + 6 = 6$

$\therefore$ required coordinates are: (0, 6) **Ans.**

Alternative Method:
Place a set square along line J and draw a perpendicular line at point (2, 2). The perpendicular line crosses the y-axis at (0, 6).



14 (N2015/P1/Q22)

P is the point $(1, -3)$ and Q is the point $(7, 2)$.

(a) Find the coordinates of the midpoint of PQ. [1]

(b) Find the gradient of the line PQ. [1]

(c) The line, L, with equation $2x - 5y = k$, passes through the point Q.

(i) Find the value of k. [1]

(ii) The line $x + Ay = 3$ is parallel to L. Find the value of A. [1]

Thinking Process

(a) Mid-point $= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

(b) Gradient $= \dfrac{y_1-y_2}{x_1-x_2}$

(c) (i) Substitute point Q into the equation of line L and solve for k.

(ii) Parallel lines have the same gradient. Rearrange the equations of both lines in the form $y = mx + c$, and equate their gradients to find A.

Solution

(a) Midpoint of $PQ = \left(\dfrac{1+7}{2}, \dfrac{-3+2}{2}\right)$

$= \left(4, -\dfrac{1}{2}\right)$ **Ans.**

(b) Gradient of $PQ = \dfrac{2-(-3)}{7-1} = \dfrac{5}{6}$ **Ans.**

(c) (i) $2x - 5y = k$

the line passes through point $Q(7, 2)$

$\Rightarrow 2(7) - 5(2) = k$

$14 - 10 = k$

$k = 4$ **Ans.**

(ii) $x + Ay = 3$

$Ay = -x + 3$

$y = \dfrac{-1}{A}x + \dfrac{3}{A}$

$\therefore$ gradient $= \dfrac{-1}{A}$

Line L: $2x - 5y = k$

$5y = 2x - k$

$y = \dfrac{2}{5}x - \dfrac{k}{5}$

$\therefore$ gradient $= \dfrac{2}{5}$

since the two lines are parallel,

$\therefore \dfrac{2}{5} = \dfrac{-1}{A}$

$\Rightarrow 2A = -5 \Rightarrow A = -\dfrac{5}{2}$ **Ans.**

15 (J2016/P1/Q14)

The coordinates of the midpoint of the line AB are $(1, 2)$.
The length of the line AB is 10 units.

(a) If the gradient of AB is 0, find the coordinates of A and B. [1]

(b) If the gradient of AB is $\dfrac{3}{4}$, find the coordinates of A and B. [2]

Thinking Process

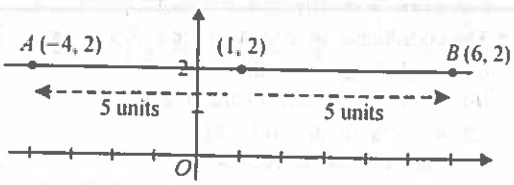
(a) Draw a quick sketch to find the coordinates of A and B. Note that line AB is parallel to x-axis.

(b) Draw the line AB using the given gradient. To find coordinates of A, point $(1, 2)$ moves 3 units down and 4 units to the right. To find coordinates of B, point $(1, 2)$ moves 4 units to the right and 3 units up.

Solution

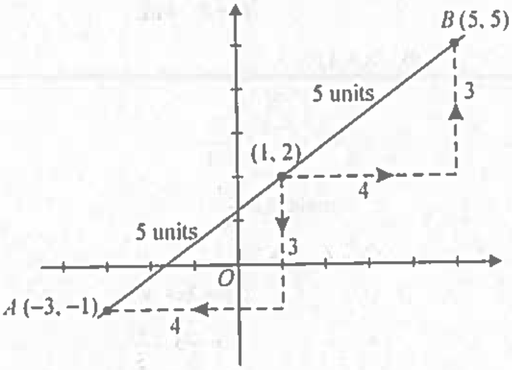
(a) The gradient of line AB is 0, therefore line AB is parallel to x-axis and meets y-axis at $y = 2$

$\therefore$ by drawing a quick sketch, coordinates of A and B are: $A(-4, 2)$, $B(6, 2)$ **Ans.**



(b) The gradient of line $AB = \frac{3}{4}$

$\therefore$ by drawing a quick sketch, coordinates of A and B are: $A(-3, -1)$, $B(5, 5)$ **Ans.**



16 (J2017/P1/Q16)

A is the point (0, 3), B is the point (1, 5) and C is the point (p, −1).

(a) Find the equation of the line AB. [2]

(b) The gradient of the line BC is $-\frac{3}{4}$.
Find the value of p. [2]

Thinking Process

(a) To find equation ✎ use $y = mx + c$, where m is the gradient and c is the y-intercept.

(b) Equate gradient of BC to $-\frac{3}{4}$.

Solution

(a) Gradient of $AB = \frac{5-3}{1-0} = 2$

Equation of AB is: $y = 2x + c$

using $A(0, 3)$

$3 = 2(0) + c \Rightarrow c = 3$

$\therefore$ equation of AB: $y = 2x + 3$ **Ans.**

(b) Gradient of $BC = \frac{-1-5}{p-1} = \frac{-6}{p-1}$

$\Rightarrow -\frac{3}{4} = -\frac{6}{p-1}$

$3p - 3 = 24$

$3p = 27$

$p = 9$ **Ans.**

17 (N2017/P1/Q20)

The coordinates of P and M are (−3, 10) and (0, 4).

(a) Find the gradient of the line PM. [1]

(b) Find the equation of the line PM. [1]

(c) M is the midpoint of PQ.
Find the coordinates of Q. [2]

Thinking Process

(b) To find equation ✎ use $y = mx + c$, where m is the gradient and c is the y-intercept.

(c) Let Q be (x, y). Equate the midpoint of PQ to point M.

Solution

(a) Gradient of $PM = \frac{4-10}{0-(-3)}$

$= \frac{-6}{3} = -2$ **Ans.**

(b) Equation of PM is: $y = -2x + c$

substitute $M(0, 4)$, $4 = -2(0) + c \Rightarrow c = 4$

$\therefore y = -2x + 4$ **Ans.**

(c) Let the coordinates of Q be (x, y)

midpoint of PQ = point M

$\Rightarrow \left(\frac{-3+x}{2}, \frac{10+y}{2}\right) = (0, 4)$

$\Rightarrow \frac{-3+x}{2} = 0$, $\frac{10+y}{2} = 4$

$-3 + x = 0$, $10 + y = 8$

$x = 3$, $y = -2$

$\therefore$ coordinates of Q are (3, −2) **Ans.**

18 (J2018/P2/Q10)

A is the point (−4, −1), B is the point (2, 2) and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$.

(a) Find the coordinates of the midpoint of AB. [1]

(b) Find the gradient of AB. [1]

(c) Show that BC is perpendicular to AB. [2]

(d) $ABCD$ is a rectangle.
Find the coordinates of point D. [2]

(e) Calculate the perimeter of rectangle $ABCD$. [4]

Thinking Process

(a) Mid-point $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(b) Gradient $= \frac{y_1 - y_2}{x_1 - x_2}$

(c) ✎ Find gradient of BC. Show that (gradient of AB) × (gradient of BC) = −1.

(d) Note that $\overrightarrow{AD} = \overrightarrow{BC}$. Find $\overrightarrow{OD}$ using vector subtraction.

(e) Perimeter of a rectangle = 2 (length + width).

Solution

(a) Midpoint of $AB = \left(\frac{-4+2}{2}, \frac{-1+2}{2}\right)$

$= (-1, \frac{1}{2})$ **Ans.**

(b) Gradient of $AB = \frac{2-(-1)}{2-(-4)}$

$= \frac{3}{6} = \frac{1}{2}$ **Ans.**

(c) $\vec{BC} = \begin{pmatrix} 4 \\ -8 \end{pmatrix} \Rightarrow$ gradient of $BC = \frac{-8}{4} = -2$

Gradient of $AB \times$ Gradient of $BC = \frac{1}{2} \times -2$

$= -1$

$\therefore$ BC is perpendicular to AB **Shown.**

(d) $ABCD$ is a rectangle

$\Rightarrow \vec{AD} = \vec{BC}$

$\Rightarrow \vec{OD} - \vec{OA} = \vec{BC}$

$\Rightarrow \vec{OD} = \vec{OA} + \vec{BC}$

$= \begin{pmatrix} -4 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ -8 \end{pmatrix} = \begin{pmatrix} 0 \\ -9 \end{pmatrix}$

$\therefore$ coordinates of D are $(0, -9)$ **Ans.**

(e) Length of $AB = \sqrt{(2+4)^2 + (2+1)^2}$

$= \sqrt{36+9} = \sqrt{45}$

Length of $BC = \sqrt{(4)^2 + (-8)^2}$

$= \sqrt{16+64} = \sqrt{80}$

Perimeter of $ABCD = 2(\sqrt{45} + \sqrt{80})$

$= 31.30$ **Ans.**

19 *(N2018/P1/Q20)*

P is the point $(-3, 4)$, Q is the point $(5, 1)$.

(a) M is the midpoint of PQ.
Find the coordinates of M. [1]

(b) Find the gradient of PQ. [1]

(c) R is the point $(-6, 0)$, O is the point $(0, 0)$.
Which of the points, R or P, is closer to O?
Show your working. [2]

Thinking Process

(a) Mid-point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(b) Gradient $= \frac{y_1-y_2}{x_1-x_2}$

(c) To check which point is closer to O ∴ find the lengths of OR and OP.

Solution

(a) Midpoint of PQ, $M = \left(\frac{-3+5}{2}, \frac{4+1}{2}\right)$

$= (1, \frac{5}{2})$ **Ans.**

(b) Gradient of $PQ = \frac{1-4}{5-(-3)}$

$= -\frac{3}{8}$ **Ans.**

(c) Length of $OR = \sqrt{(-6-0)^2 + (0-0)^2}$

$= \sqrt{36} = 6$

Length of $OP = \sqrt{(-3-0)^2 + (4-0)^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = 5$

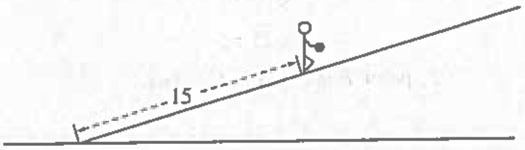
$\therefore$ point P is closer to O **Ans.**

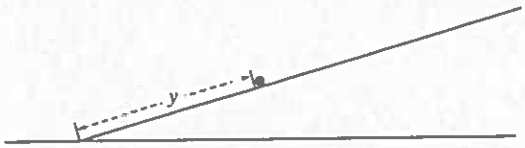
Topic 7

Graphs of Functions and Graphical Solutions

1 (J2007/P2/Q8)

Answer the whole of this question on a sheet of graph paper.





Adam stood on a slope, 15 m from the bottom. He rolled a heavy ball directly up the slope.
After t seconds the ball was y metres from the bottom of the slope.
The table below gives some values of t and the corresponding values of y.

t	0	1	2	2.5	3	3.5	4	4.5	5	5.5
y	15	22	25	25	24	22	19	15	10	4

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $0 \le y \le 30$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Extend the curve to find the value of t when the ball reached the bottom of the slope. [1]

(c) (i) By drawing a tangent, find the gradient of the curve when $t = 3.5$. [2]

(ii) State briefly what this gradient represents. [1]

(d) Immediately after he rolled the ball, Adam ran down the slope at a constant speed of 1.5 m/s.

(i) Write down the distance of Adam from the bottom of the slope when

(a) $t = 0$,

(b) $t = 4$. [2]

(ii) On the same axes, draw the graph that represents the distance of Adam from the bottom of the slope for $0 \le t \le 6$. [2]

(iii) Hence find the distance of Adam from the bottom of the slope when the ball passed him. [1]

Thinking Process

(c) (ii) y represents the distance and t represents time. Therefore gradient is the rate of change of distance i.e. speed.

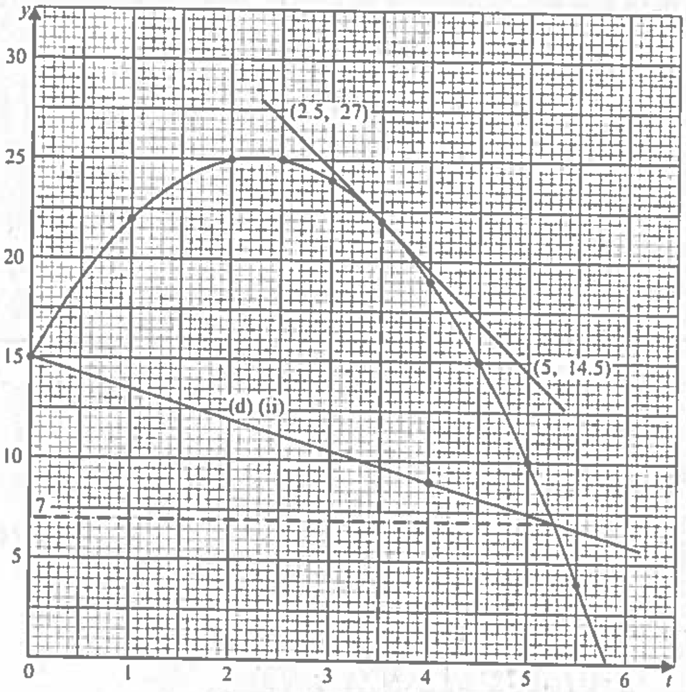
(d) (i) (a) Note that when $t = 0$, adam is 15 m from the bottom.

(ii) Using the information from part (d) (i), draw a straight line on the same graph.

(iii) Note the distance where the two graphs meet.

Solution

(a)



(b) From graph we see that the ball reaches the bottom of the slope when time t = 5.8 sec. **Ans**

(c) (i) gradient $= \dfrac{14.5 - 27}{5 - 2.5}$

$= -\dfrac{12.5}{2.5} = -5$ **Ans**

(ii) The gradient represents the speed of the ball at 3.5 sec.

(d) (i) (a) When $t = 0$, Adam is 15m from the bottom.

$\therefore$ distance from bottom = 15 m **Ans**

(b) distance = speed × time = $1.5 \times 4 = 6$ m

Adam is 6m below from its starting point.

$\therefore$ distance from bottom $= 15 - 6 = 9$ m **Ans**

(ii) refer to graph

(iii) required distance = 7 m **Ans**

2 *(N2007/P2/Q8)*

Answer the whole of this question on a sheet of graph paper.

A stone was thrown from the top of a vertical cliff. Its position during the flight is represented by the equation $y = 24 + 10x - x^2$, where y metres is the height of the stone above the sea and x metres is the horizontal distance from the cliff.

(a) Solve the equation $0 = 24 + 10x - x^2$. [2]

(b) The table shows some values of x and the corresponding values of y.

x	0	2	4	6	8	10
y	24	40	48	48	40	24

(i) Using a scale of 1 cm to represent 1 metre, draw a horizontal x-axis for $0 \le x \le 14$.
Using a scale of 2 cm to represent 10 metres, draw a vertical y-axis for $0 \le y \le 50$.
On your axes, plot the points from the table and join them with a smooth curve. [3]

(ii) Use your answer to part (a) to complete the graph which represents the flight of the stone. [1]

(iii) Find the height of the stone above the sea when its horizontal distance from the cliff was 7m. [1]

(iv) Use your graph to find how far the stone travelled horizontally while it was 6m or more above the top of the cliff. [2]

(c) It is given that $24 + 10x - x^2 = p - (x - 5)^2$.

(i) Find the value of p. [1]

(ii) **Hence** find

(a) the greatest height of the stone above the sea, [1]

(b) the horizontal distance from the cliff when the stone was at its greatest height. [1]

Thinking Process

(b) (ii) Part (a) gives the values of x when $y = 0$. Therefore extend your curve to meet x-axis at a point you have calculated in part (a).

(iv) From graph, find the two values of x when the stone was 6 m or more above the top of cliff. Note that the height of the cliff is 24 metres. Therefore the stone is 24 + 6 = 30 m above the sea level.

(c) (i) Solve the equation for p.

Solution

(a) $0 = 24 + 10x - x^2$

$\Rightarrow \quad x^2 - 10x - 24 = 0$

$x^2 + 2x - 12x - 24 = 0$

$x(x + 2) - 12(x + 2) = 0$

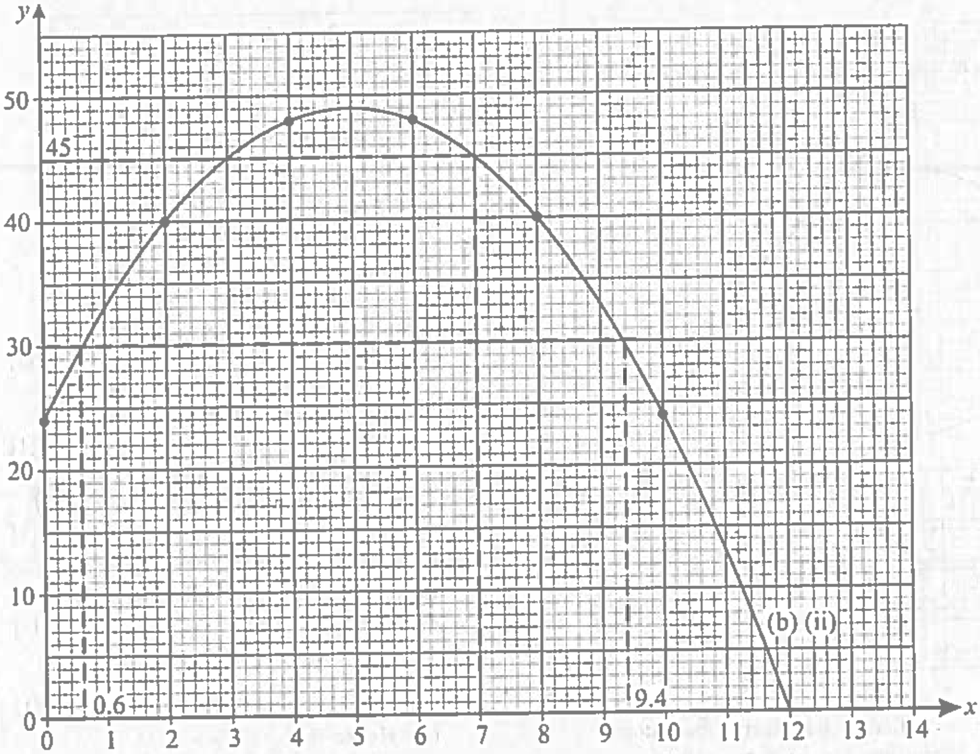
$(x + 2)(x - 12) = 0$

$(x + 2) = 0$ or $(x - 12) = 0$

$x = -2$ or $x = 12$

$\therefore \quad x = -2$, or 12 **Ans**

(b) (i) & (ii) Refer to graph



(iii) From graph, the height of stone when horizontal distance was 7m is 45 m Ans.

(iv) The height of the cliff is 24 m. When the stone was 6 m above the top of the cliff, then the total height above the sea level is $24+6=30$ m

$\therefore$ from graph, the distance travelled by the stone when it was 30 m above the sea level or 6 m above the top of the cliff is: $9.4-0.6=8.8$ m Ans.

(c) (i) $24+10x-x^2=p-(x-5)^2$

$24+10x-x^2=p-(x^2-10x+25)$

$24+10x-x^2=p-x^2+10x-25$

$p=24+25$

$p=49$ Ans

(ii) (a) Greatest height reached = 49 m Ans

(b) Req. horizontal distance = 5 m Ans

3 *(J2008/P2 Q8)*

Answer the whole of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y, correct to one decimal place, for

$$y=\frac{4}{5}\times 2^x.$$

x	−2	−1	0	1	2	2.5	3	3.5	4
y	p	0.4	0.8	1.6	3.2	4.5	6.4	9.1	12.8

(a) Calculate p. [1]

(b) Using a scale of 2cm to represent 1 unit, draw a horizontal x-axis for $-2 \le x \le 4$.
Using a scale of 2cm to represent 2 units, draw a vertical y-axis for $0 \le y \le 14$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) As x decreases, what value does y approach? [1]

(d) By drawing a tangent, find the gradient of the curve at the point (3, 6.4). [2]

(e) (i) On the axes used in part (b), draw the graph of $y = 8 - 2x$. [2]

(ii) Write down the coordinates of the point where the line intersects the curve. [1]

(iii) The x coordinate of thispoint of intersection satisfies the equation $2^x = Ax + B$. Find the value of A and the value of B. [2]

Thinking Process

(c) Observe the shape of the curve and make a conclusion.

(d) Draw tangent at $x = 3$. Take two points on the tangent and find the gradient.

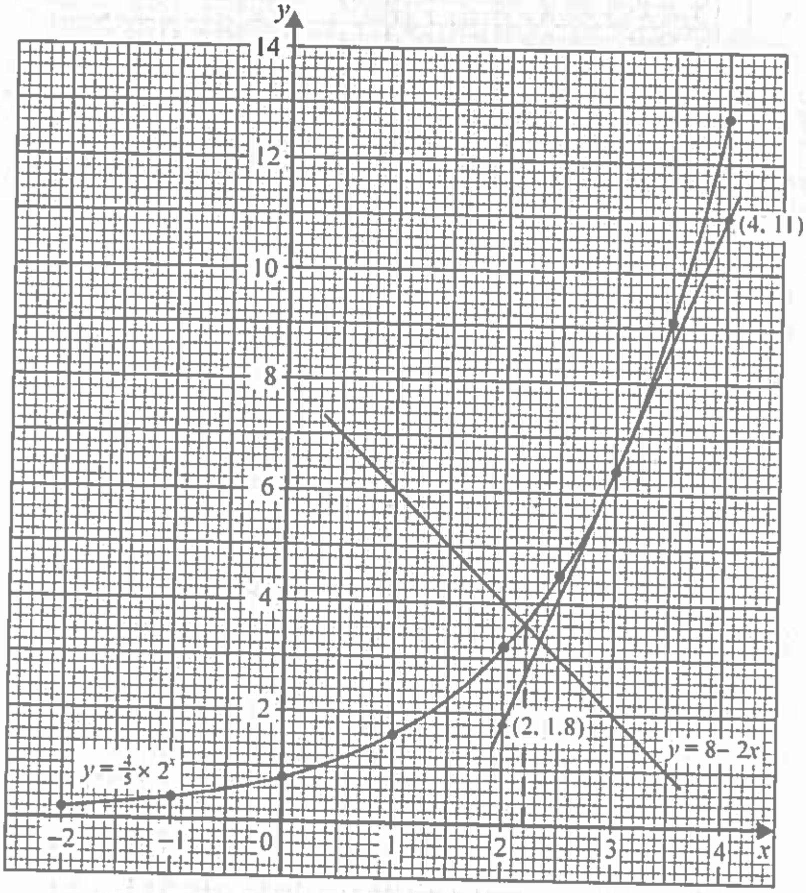
(e) (iii) Substitute eq. of line into eq. of curve. Rearrange the equation in the form $2^x = Ax + B$.

Solution

(a) $y = \frac{4}{5} \times 2^{-2} = \frac{4}{5} \times \frac{1}{2^2} = 0.2$,

$\therefore p = 0.2$ **Ans.**

(b)



(c) From graph, we see that as x decreases, the y values of the curve approaches to 0. **Ans.**

(d) Gradient at $x = 3$ is

$\frac{11 - 1.8}{4 - 2} = \frac{9.2}{2} = 4.6$ **Ans.**

(e) (i) Refer to graph

(ii) (2.2, 3.6) **Ans.**

(iii) Eq.of curve: $y = \frac{4}{5} \times 2^x$,

Eq.of line: $y = 8 - 2x$

$\therefore \frac{4}{5} \times 2^x = 8 - 2x$

$2^x = \frac{5}{4}(8 - 2x)$

$2^x = 10 - \frac{5}{2}x$

or $2^x = -\frac{5}{2}x + 10$

comparing it with $2^x = Ax + B$, we have

$A = -\frac{5}{2} = -2.5$, and $B = 10$ **Ans.**

4 (N2008/P1/Q17)

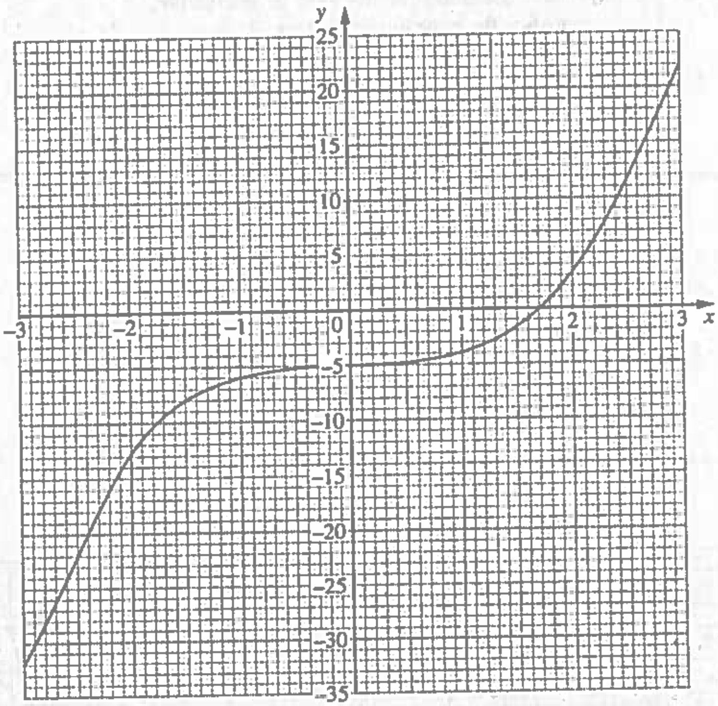
The curve $y = x^3 - 5$ is shown on the axes below.

(a) Use the graph to find an approximate value of $\sqrt[3]{5}$. [1]

(b) (i) On the axes above, draw the graph of $y = 15 - 5x$. [1]

(ii) Write down the coordinates of the point where the graphs cross. [1]

(iii) The x coordinate of the point where the graphs cross is a solution of the equation $x^3 = a + bx$. Find the value of a and the value of b. [1]



Thinking Process

(a) Note that when $y = 0$, $x = \sqrt[3]{5}$.

(b) (iii) Substitute equation of curve into equation of line. Rearrange the resulting equation in the form $x^3 = a + bx$. Find the values of a and b by comparison.

Solution

(a) $y = x^3 - 5$

when $y = 0$,

$0 = x^3 - 5$

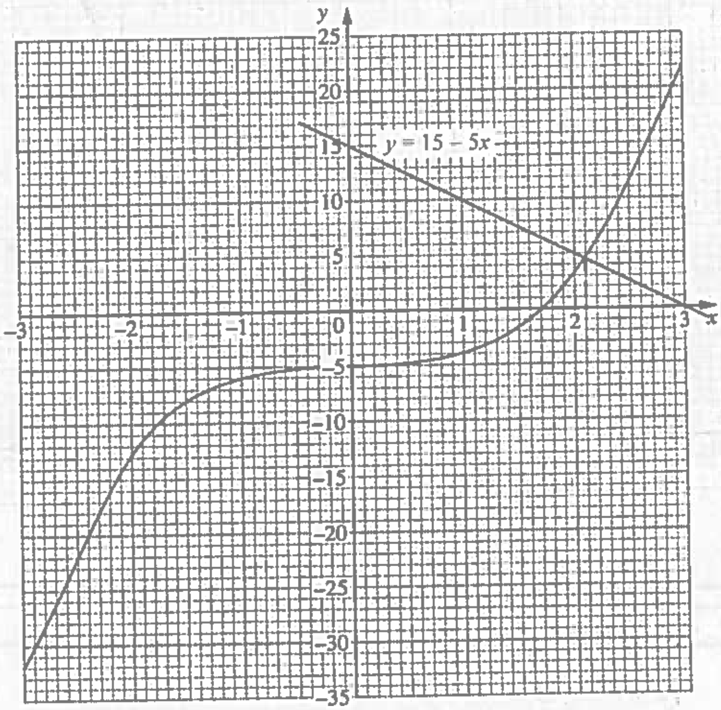
$x^3 = 5$

$x = \sqrt[3]{5}$

from graph we see that, when $y = 0$, $x = 1.7$

$\therefore$ approximate value of $\sqrt[3]{5}$ is 1.7 **Ans.**

(b) (i) Refer to graph.



(ii) (2.1, 4.5) **Ans.**

(iii) Eq. of curve: $y = x^3 - 5$

eq. of line: $y = 15 - 5x$

$\therefore \quad x^3 - 5 = 15 - 5x$

$x^3 = 20 - 5x$

comparing it with $x^3 = a + bx$, we have,

$a = 20$ and

$b = -5$ **Ans.**

5 *(N2008/P2/Q10)*

Answer the whole of this question on a sheet of graph paper.

The number of bacteria in a colony doubles every half hour.
The colony starts with 50 bacteria.
The table below shows the number of bacteria in the colony after t hours.

Time (t hours)	0	0.5	1	1.5	2	2.5	3	3.5
Number of bacteria (y)	50	100	200	400	800	1600	3200	6400

(a) Using a scale of 4 cm to represent 1 hour, draw a horizontal t-axis for $0 \le t \le 4$.
Using a scale of 2 cm to represent 1000 bacteria, draw a vertical y-axis for $0 \le y \le 7000$.
On your axes, plot the points given in the table, and join them with a smooth curve. [3]

(b) Use your graph to find the number of bacteria in the colony when $t = 2.75$. [1]

(c) (i) By drawing a tangent, find the gradient of the graph when $t = 2.5$. [2]
(ii) State briefly what this gradient represents. [1]

(d) The number of bacteria in another colony is given by the equation.
$$y = 6500 - 1000t.$$
(i) On the same axes, draw a graph to represent the number of bacteria in this colony. [2]
(ii) Find the value of t when the number of bacteria in each colony is the same. [1]

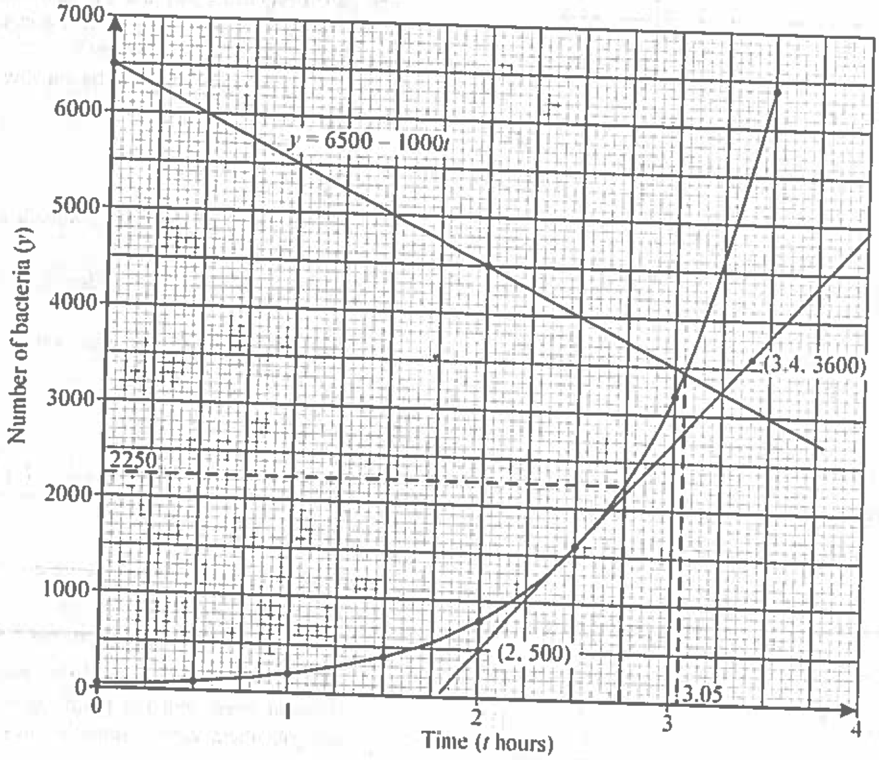
(e) Given that the equation of the first graph is $y = ka^t$, find the value of
(i) k, [1]
(ii) a, [1]

Thinking Process

(a) Plot the points given in the table.
(c) (ii) x-axis represents time and y-axis represents number of bacteria. Therefore the gradient represents rate of increase.
(d) (ii) From graph, find the time at the point where the line meets the curve.
(e) Substitute the first two coordinates from the table into the given equation.

Solution

(a)



(b) When $t = 2.75$, no. of bacteria, $y = 2250$ **Ans.**

(c) (i) Gradient $= \dfrac{3600 - 500}{3.4 - 2}$

$= 2214.3 \approx 2210$ (3sf) **Ans.**

(ii) The gradient represents the rate at which the number of bacteria is increasing.

(d) (i) Refer to graph.

(ii) From graph, $t = 3.05$ hrs. **Ans.**

(e) (i) $y = ka^t$

taking (0, 50) from the table

$50 = ka^0 \Rightarrow k = 50$ **Ans.**

(ii) Taking another point (0.5, 100) from the table and the value of k, we have

$100 = 50a^{0.5}$

$a^{0.5} = 2$

squaring both sides

$(a^{\frac{1}{2}})^2 = (2)^2$

$a = 4$ **Ans.**

6 *(J2009/P1/Q13)*

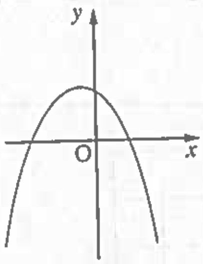


Figure 1

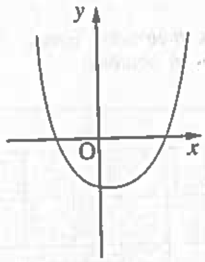


Figure 2

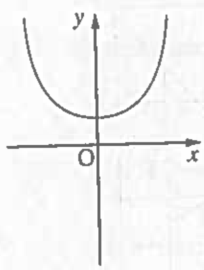


Figure 3

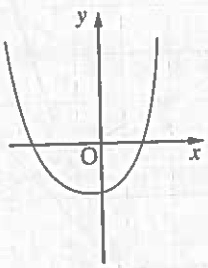


Figure 4

Which of the figures shown above could be the graph of

(a) $y = x^2 + 2$, [1]

(b) $y = (x - 2)(x + 1)$, [1]

(c) $y = 2 - x - x^2$. [1]

Thinking Process

(a) Observe that it is a quadratic graph which is symmetrical about the y-axis.

(b) Observe that it is a quadratic graph which cuts the x-axis at $x = 2$ and $x = -1$.

(c) Observe that x^2 is negative. Therefore it is a quadratic graph opening downwards.

Solution

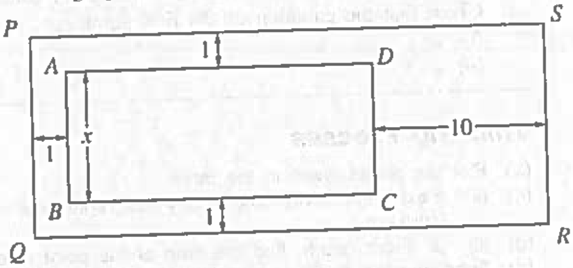
(a) $y = x^2 + 2 \rightarrow$ Figure 3 **Ans.**

(b) $y = (x - 2)(x + 1) \rightarrow$ Figure 2 **Ans.**

(c) $y = 2 - x - x^2 \rightarrow$ Figure 1 **Ans.**

7 *(J2009 P2/Q8)*

Answer THE WHOLE of this question on a sheet of graph paper.



The diagram represents a rectangular pond, $ABCD$, surrounded by a paved region.
The paved region has widths 1 m and 10 m as shown.
The pond and paved region form a rectangle $PQRS$.
The area of the pond is 168 m².

(a) Taking the length of AB to be x metres, write down expressions, in terms of x, for
(i) PQ,
(ii) BC,
(iii) QR. [2]

(b) Hence show that the area, y square metres, of the paved region, is given by
$$y = 22 + 11x + \frac{336}{x}.$$ [2]

(c) The table below shows some values of x and the corresponding values of y.

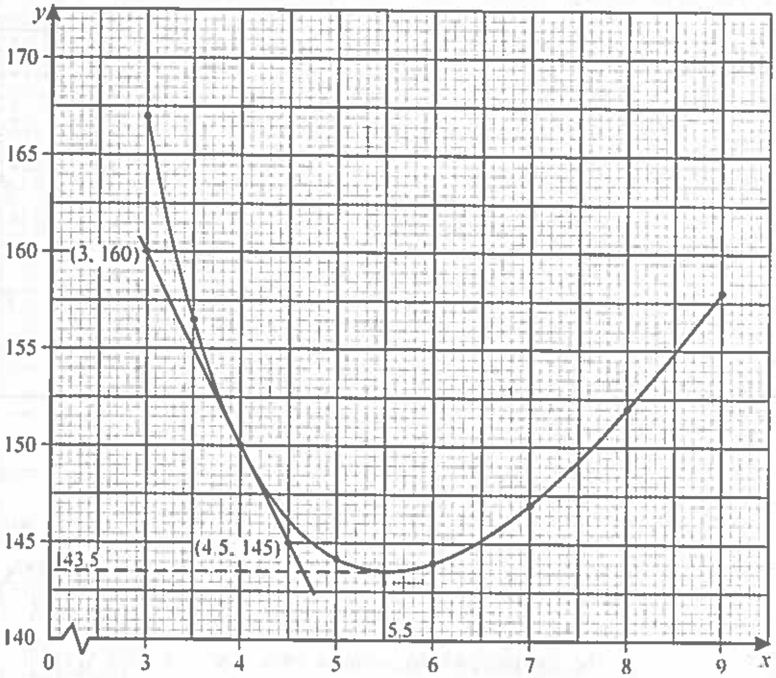
x	3	3.5	4	5	6	7	8	9
y	167	156.5	150	144.2	144	147	152	p

Calculate p. [1]

(d) Using a scale of 2 cm to represent 1 metre, draw a horizontal x-axis for $3 \le x \le 9$.
Using a scale of 2 cm to represent 5 square metres, draw a vertical y-axis for $140 \le y \le 170$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(e) By drawing a tangent, find the gradient of the curve at (4, 150). [2]

(f) Use your graph to find
   (i) the smallest area of the paved region, [1]
   (ii) the length of PQ when the area of the paved region is smallest. [1]



Thinking Process

(a) (i) Note that paved region is of 1 m width.
   (ii) To find BC Apply the formula area = length × width .
   (iii) Add 11m to BC.

(b) Subtract area of $ABCD$ from area of $PQRS$.

(f) (i) From graph, find the value of y that corresponds to the least value of the curve.
   (ii) From graph, find x when y is minimum and use it to find PQ.

Solution

(a) (i) $PQ = (x+2)$ m Ans.

(ii) Area of pond $ABCD = 168$ m^2

$\therefore\ x \times BC = 168$

$BC = \frac{168}{x}$ m Ans.

(iii) $QR = BC + 11$

$= \left(\frac{168}{x} + 11\right)$ m Ans.

(b) Area of paved region

= area of $PQRS$ − area of $ABCD$

$\Rightarrow\ y = PQ \times QR - 168$

$= (x+2)\left(\frac{168}{x} + 11\right) - 168$

$= 168 + 11x + \frac{336}{x} + 22 - 168$

$= 22 + 11x + \frac{336}{x}$ Shown.

(c) $p = 22 + 11(9) + \frac{336}{9}$

$= 22 + 99 + 37.333 = 158.3$ Ans.

(d) Refer to graph.

(e) Taking two points (4.5, 145) & (3, 160) on the tangent,

Gradient $= \frac{160-145}{3-4.5} = -10$ Ans.

(f) (i) From graph. Smallest area = 143.5 m^2 Ans.

(ii) The area of paved region is least at $x = 5.5$

$\therefore\ PQ = x + 2$

$= 5.5 + 2 = 7.5$ m Ans.

8 *(N2009/P2 Q8)*

Answer THE WHOLE of this question on a sheet of graph paper.

(a) The variables x and y are connected by the equation

$$y = 4x^3 - 18x^2 + 20x.$$

The table below shows some values of x and the corresponding values of y.

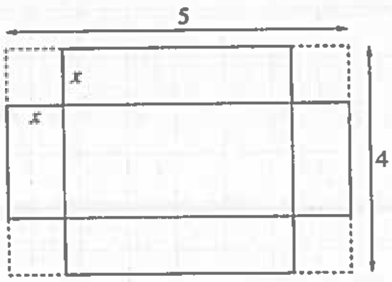
x	0	0.5	1	1.5	2	2.5	3	3.5
y	0	6	6	3	0	0	6	p

(i) Calculate the value of p. [1]

(ii) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 4$.
Using a scale of 1 cm to represent 2 units, draw a vertical y-axis for $-4 \le y \le 24$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(iii) Using your graph, find the values of x when $y = 4$. [2]

(b) A rectangular card is 5 cm long and 4 cm wide. As shown in the diagram, a square of side x centimetres is cut off from each corner.



The card is then folded to make an open box of height x centimetres.

(i) Write down expressions, in terms of x, for the length and width of the box. [1]

(ii) Show that the volume, V cubic centimetres, of the box is given by the equation.

$V = 4x^3 - 18x^2 + 20x$. [2]

(iii) Which value of x found in (a)(iii) cannot be the height of a box with a volume of 4 cm^3? [1]

(iv) Using the graph drawn in part (a)(ii), find

(a) the greatest possible volume of a box made from this card, [1]

(b) the height of the box with the greatest volume. [1]

Thinking Process

(a) (i) Substitute $x = 3.5$ into y.

(ii) Plot the points from the table using the given scales.

(iii) Draw a line at $y = 4$, and find the corresponding values of x.

(b) (i) Subtract $2x$ from the length and width of the rectangular card.

(ii) Express the volume of the box in terms of x.

(iii) Substitute the values of x into length and width of the box and inspect.

(iv) (a) & (b) Find the greatest volume from the graph and the corresponding value of x.

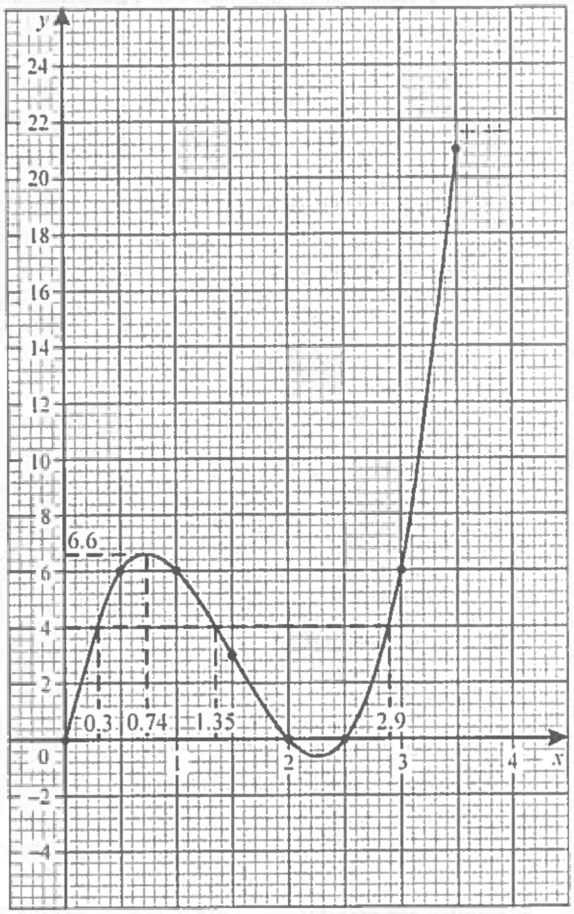
Solution *with* TEACHER's COMMENTS

(a) (i) $p = 4(3.5)^3 - 18(3.5)^2 + 20(3.5)$

$= 171.5 - 220.5 + 70$

$= 21$ **Ans.**

(ii)



(iii) From the graph, when $y = 4$,

$x = 0.3$, 1.35 and 2.9 **Ans.**

(b) (i) Length $= (5 - 2x)$ cm **Ans.**

Width $= (4 - 2x)$ cm **Ans.**

(ii) Volume $= l \times w \times h$

$\Rightarrow V = (5 - 2x)(4 - 2x)(x)$

$= x(20 - 18x + 4x^2)$

$= 20x - 18x^2 + 4x^3$

$= 4x^3 - 18x^2 + 20x$ **Shown.**

(iii) $x = 2.9$ cannot be the height. **Ans.**

When $x = 2.9$,
Length of the box $= 5 - 2(2.9) = -0.8$
Width of the box $= 4 - 2(2.9) = -1.8$
Note that both answers are negative.
Therefore 2.9 cm cannot be the height.

(iv) (a) From graph, the greatest volume of the box $= 6.6\text{ cm}^3$ **Ans.**

(b) Height of the box, $x = 0.74$ cm **Ans.**

9 *(J2010/P2/Q6)*

Answer THE WHOLE of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y for

$$y = \frac{2^x}{4}.$$

x	−1	0	1	2	3	4	5
y	m	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	n

(a) Calculate the values of m and n. [2]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-1 \le x \le 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \le y \le 8$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equations

(i) $\frac{2^x}{4} = 3$. [1]

(ii) $2^x = 6$. [1]

(d) The equation $y = \frac{2^x}{4}$ can be written in the form $y = 2^t$.

(i) Find an expression for t in terms of x. [1]

(ii) Hence, find the equation of the line that can be drawn on your graph to evaluate y when $t = -\frac{3}{4}$. [1]

Thinking Process

(a) To find m & n ✎ substitute $x = -1$ & $x = 5$ into the equation respectively.

(c) (i) ✎ Draw a dotted line at $y = 3$ and find the corresponding x-values.

(ii) ✎ Draw a dotted line at $y = 1.5$ and find the corresponding x-values.

(d) (i) Equate the two given equations, simplify and make t the subject.

(ii) Substitute the value of t into the expression found in (d)(i) to find x. Substitute this value of x into the equation of curve and simplify.

Solution

(a) $m = \frac{2^{-1}}{4} = \frac{1}{8} = 0.125$ **Ans.**

$n = \frac{2^5}{4} = \frac{32}{4} = 8$ **Ans.**

(b) Refer to graph below.

(c) (i) $\frac{2^x}{4} = 3 \Rightarrow y = 3$

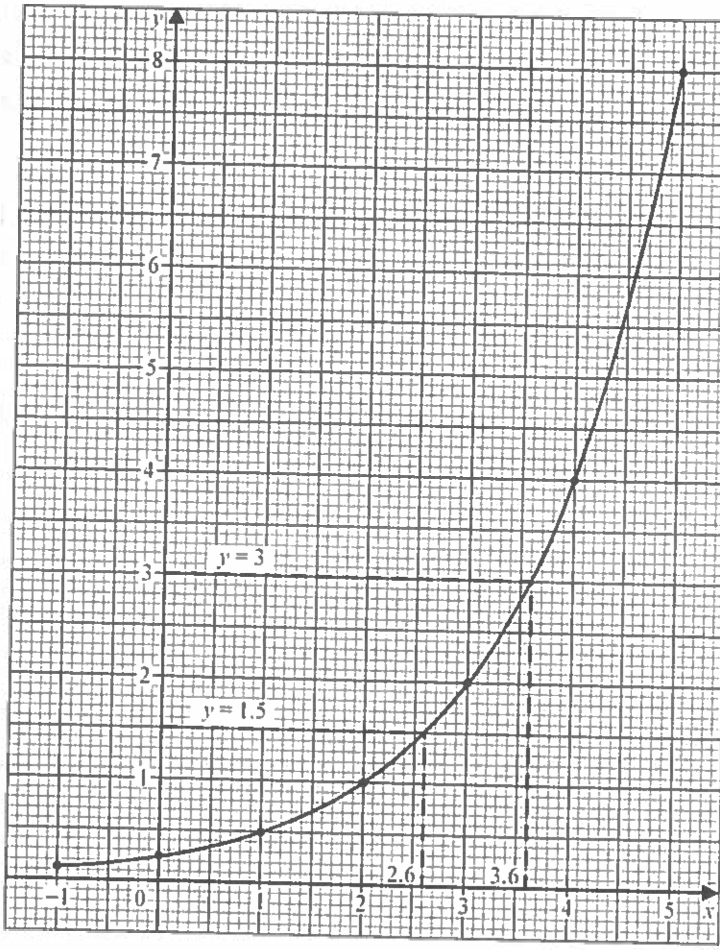
From graph,
when $y = 3$, $x = 3.6$ **Ans.**

(ii) $2^x = 6$

$\frac{2^x}{4} = \frac{6}{4}$

$\Rightarrow y = \frac{6}{4} = 1.5$

From graph,
when $y = 1.5$, $x = 2.6$ **Ans.**



(d) (i) Equating $y = 2^t$ and $y = \frac{2^x}{4}$,

$$2^t = \frac{2^x}{4}$$

$$2^t = \frac{2^x}{2^2}$$

$$2^t = 2^x \times 2^{-2}$$

$\therefore \quad t = x - 2$ **Ans.**

(ii) when $t = -\frac{3}{4}$,

$$-\frac{3}{4} = x - 2$$

$$x = 2 - \frac{3}{4} = \frac{5}{4}$$

Substitute this value of x into equation of curve,

$$y = \frac{2^{\frac{5}{4}}}{4}$$

$\Rightarrow y = 2^{\frac{5}{4}} \times 2^{-2} \quad \Rightarrow \quad y = 2^{-\frac{3}{4}}$ **Ans.**

10 *(N2010 P2/Q8)*

Answer THE WHOLE of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x^3}{10} - \frac{x}{2}.$$

The table below shows some corresponding values of x and y.

x	0	1	2	3	4	4.5
y	0	−0.4	−0.2	1.2	4.4	p

(a) Calculate p. [1]

(b) Using a scale of 2 cm to 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 5$ and a vertical y-axis for $-1 \le y \le 7$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equation $\frac{x^3}{10} - \frac{x}{2} = 0.3$ for values of x in the range $0 \le x \le 5$. [1]

(d) (i) Draw the chord joining the two points (0, 0) and (3, 1.2) and calculate its gradient. [1]

(ii) Draw a tangent at the point where the gradient of the curve is equal to the gradient of the chord. [1]

(e) (i) On the same axes, draw the graph of the straight line $y = -x + 6$. [2]

(ii) Write down the x coordinate of the point where the line crosses the curve. [1]

(iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$. Find A and B. [2]

Thinking Process

(a) Substitute $x = 4.5$ into y.
(b) Plot the points given in the table.
(c) Draw a dotted line at $y = 0.3$ and find the corresponding x-values.
(d) (i) Draw a line that joins the two given points. Use the given points to find the gradient.
(ii) Draw a tangent to the curve at a point such that the tangent is parallel to the line drawn in (d) (i).
(e) (iii) Equate and subsequently simplify the equations of the curve and the straight line.

Solution

(a) $p = \frac{4.5^3}{10} - \frac{4.5}{2}$
$= 9.1125 - 2.25$
$= 6.863 \approx 6.9$ **Ans.**

(b) Refer to graph next page.

(c) $\frac{x^3}{10} - \frac{x}{2} = 0.3 \Rightarrow y = 0.3$
from graph,
when $y = 0.3$, $x = 2.5$ **Ans.**

(d) (i) Refer to graph for the chord.

Gradient $= \frac{1.2 - 0}{3 - 0} = 0.4$ **Ans.**

(ii) Refer to graph.

(e) (i) Refer to graph.

(ii) $x = 3.5$ **Ans.**

(iii) Equation of curve: $y = \frac{x^3}{10} - \frac{x}{2}$

Equation of line: $y = -x + 6$

Equating both equations,

$$\frac{x^3}{10} - \frac{x}{2} = -x + 6$$

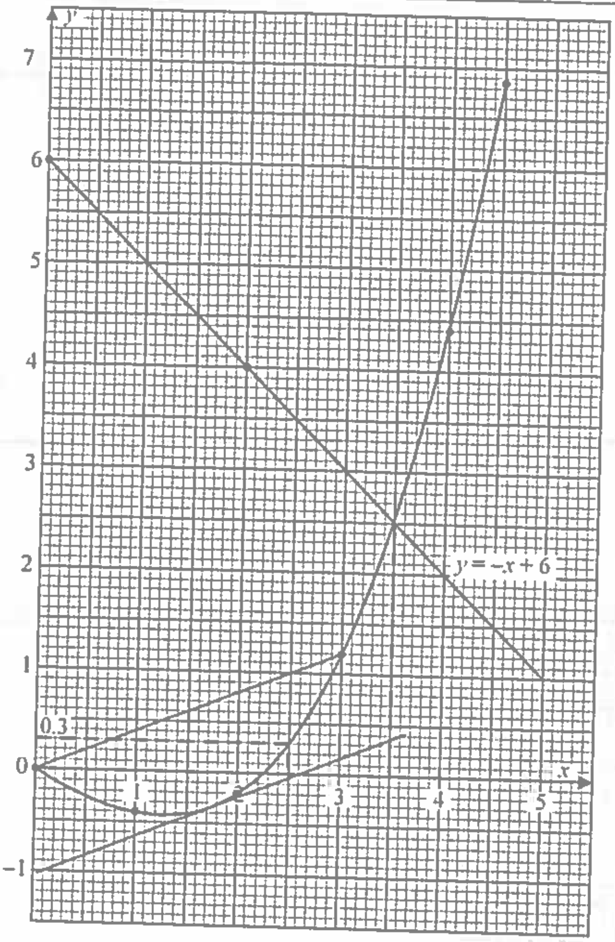
$$\Rightarrow \frac{2x^3 - 10x}{20} = -x + 6$$

$$\Rightarrow 2x^3 - 10x = -20x + 120$$

$$\Rightarrow 2x^3 + 10x - 120 = 0$$

$$\Rightarrow x^3 + 5x - 60 = 0$$

$\therefore \quad A = 5, \quad B = -60$ **Ans.**



11 *(J2011/P2/Q9)*

The table below shows some of the values of x and the corresponding values of y for

$$y = (2x - 3)(x + 2).$$

x	−3	−2	−1	0	1	2	3
y	9	0			−3	4	15

(a) Complete the table. [1]

(b) On the axes below, plot the points from the table and join them with a smooth curve. [2]

(c) Use your graph to

(i) solve the equation $(2x-3)(x+2) = 2$, [1]

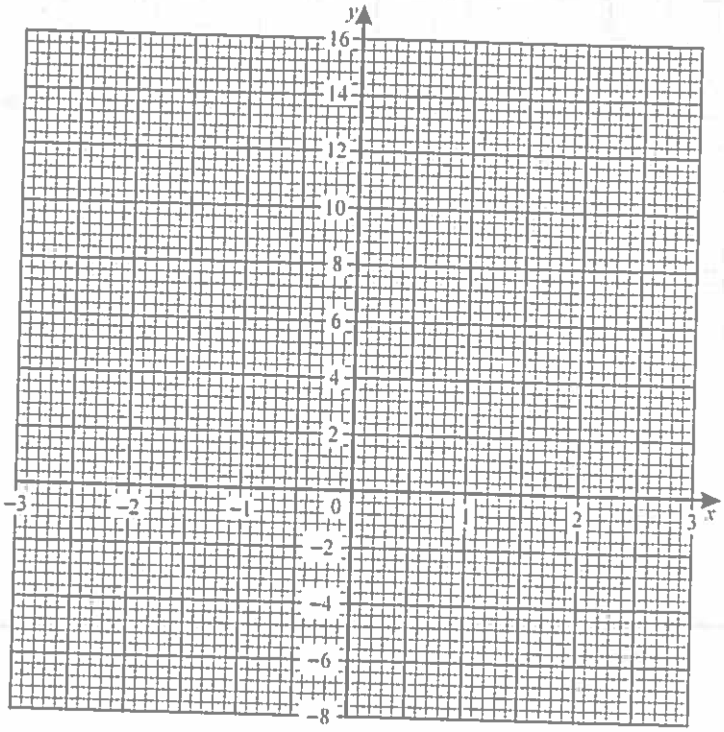
(ii) find the minimum value of y, [1]

(iii) find the gradient of the curve at (2 , 4). [2]

(d) (i) Show that the x-coordinates of the points where $y = (2x-3)(x+2)$ and $y = 1 - 2x$ would intersect are the solutions of the equation

$$2x^2 + 3x - 7 = 0.$$ [1]

(ii) Solve algebraically the equation $2x^2 + 3x - 7 = 0$, giving each answer correct to 2 decimal places. [4]



Thinking Process

(a) To complete the table ✐ Substitute $x = 0$ and $x = -1$ into the given equation.

(c) (i) Draw the line $y = 2$ and find the corresonding values of x.

(ii) From graph, find the least value of y.

(iii) Draw a tangent to the curve at $x = 2$ and find the gradient of the tangent.

(d) (i) Solve $y = (2x-3)(x+2)$ and $y = 1-2x$ simultaneously.

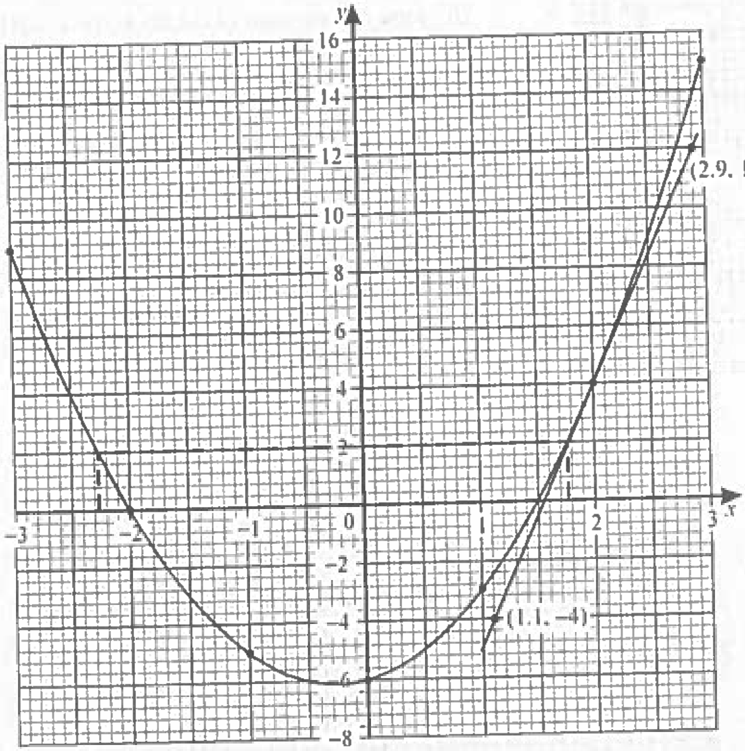
(ii) Solve the equation using quadratic formula.

Solution

(a) When $x=-1$, $y=(-2-3)(-1+2)=-5$

when $x=0$, $y=(0-3)(0+2)=-6$

(b)



(c) (i) $(2x-3)(x+2)=2$

$\Rightarrow y=2$

$\therefore$ from graph,

$x=-2.27$ or 1.77 **Ans.**

(ii) From graph, minimum value of $y=-6.1$ **Ans.**

(iii) Taking two points $(1.1, -4)$ and $(2.9, 12)$ on the tangent.

$$\text{gradient} = \frac{12-(-4)}{2.9-1.1}$$

$$= \frac{16}{1.8} = 8.89 \quad \textbf{Ans.}$$

(d) (i) $y=(2x-3)(x+2)$ ....(1)

$y=1-2x$ ....(2)

substitute (2) into (1).

$$1-2x=(2x-3)(x+2)$$

$$\Rightarrow 1-2x=2x^2+x-6$$

$$\Rightarrow 2x^2+3x-7=0$$

which is as per the given equation. **Shown.**

(ii) $2x^2+3x-7=0$

by quadratic formula.

$$x=\frac{-3\pm\sqrt{3^2-4(2)(-7)}}{2(2)}$$

$$=\frac{-3\pm\sqrt{9+56}}{4}$$

$$=\frac{-3\pm\sqrt{65}}{4}$$

$$=\frac{-3+\sqrt{65}}{4} \text{ or } \frac{-3-\sqrt{65}}{4}$$

$$\Rightarrow x=1.27 \text{ or } -2.77 \quad \textbf{Ans.}$$

12 *(N2011/P2/Q8)*

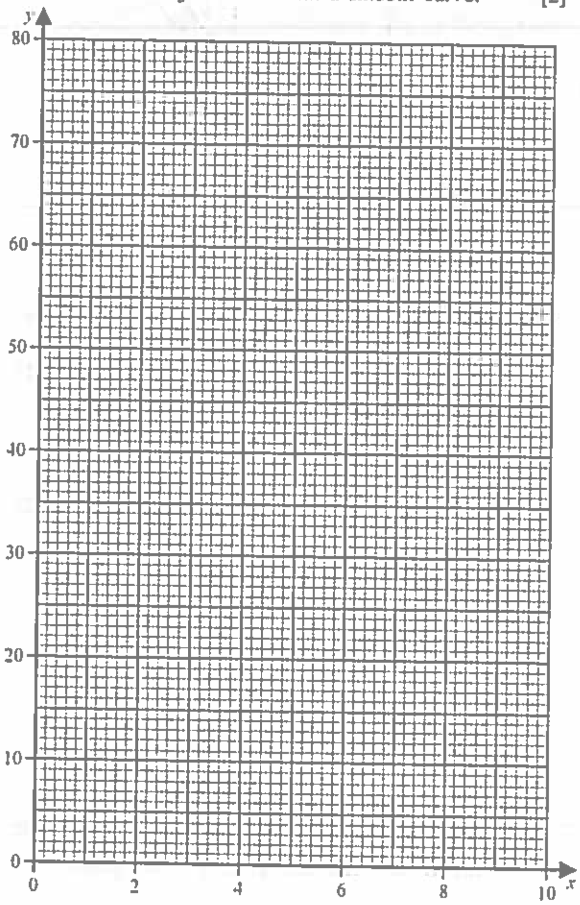
Two companies, A and B, were started 10 years ago. Initial investments of \$25 or multiples of \$25 could be made when Company A started business.

(a) The table shows the value of an initial investment of \$25 at the end of each of the next 10 years.

Number of years (x)	0	1	2	3	4	5	6	7	8	9	10
Value in dollars (y)	25	28	31	35	39	44	49	55	62	69	78

(i) Calculate the value of an initial investment of \$500 after 8 years. [1]

(ii) On the grid, plot the points given in the table and join them with a smooth curve. [2]



(iii) Using your graph, find x when the value of an initial investment of \$100 had increased to \$168. [1]

(b) An initial investment of \$25 was made when company B started business.
The value, y dollars, after x years, is given by the equation $y = 3.75x + 25$.

(i) Calculate the value of an initial investment of \$500 after 8 years. [1]

(ii) On the grid, draw the graph of $y = 3.75x + 25$. [2]

(c) Using your graphs, find the value of x when an initial investment of \$25 had increased to the same value in each company. [1]

(d) (i) By drawing a tangent to the graph representing an investment in company A, find the rate of increase of this investment when $x = 7$. [2]

(ii) State the rate of increase of an investment in company B. [1]

(iii) By drawing another tangent to the graph representing an investment in company A, find the value of x when the rates of increase of investments in each company were the same. [1]

Thinking Process

(a) (i) From table, \$25 increase to \$62 after 8 years. Using ratio concepts, find the increased value of \$500.

(iii) \$100 is increased to \$168. Using ratio, find the increased value of \$25. Use graph to find the corresponding x-value.

(b) (i) Using equation, find the increased value of \$25 after 8 years. Then use the ratio concept to find the increased value of \$500.

(c) Find the x-value where the two graphs meet.

(d) (i) Draw a tangent to the curve at $x = 7$ and find its gradient.

(ii) Identify the gradient of equation for company B.

(iii) To find a value of x 🖉 draw a tangent on company A curve such that it is parallel to the graph of company B.

Solution *with* TEACHER's COMMENTS

(a) (i) From table,

value of \$25 after 8 years = \$62

$\therefore$ value of \$500 after 8 years $= \dfrac{62}{25} \times 500$

$= \$1240$ **Ans.**

(ii) Refer to graph on next page.

(iii) \$100 —— \$168

\$25 —— $\$\dfrac{168}{100} \times 25 = \$42$

from graph,

when $y = 42$, $x = 4.6$ years **Ans.**

(b) (i) $y = 3.75x + 25$

when $x = 8$ years

$y = 3.75(8) + 25 = \$55$

$\therefore$ \$25 are increased to \$55 after 8 years.

\$25 —— \$55

\$500 —— $\$\dfrac{55}{25} \times 500 = \$1100$

$\therefore$ \$500 are increased to \$1100 after 8 years **Ans.**

(ii) Refer to graph on next page.

(c) From graph, $x = 5$ years. **Ans.**

(d) (i) Taking two points (5.6, 46) and (8.8, 66) on the tangent.

$$\text{gradient} = \frac{66-46}{8.8-5.6}$$
$$= \frac{20}{3.2} = 6.25$$

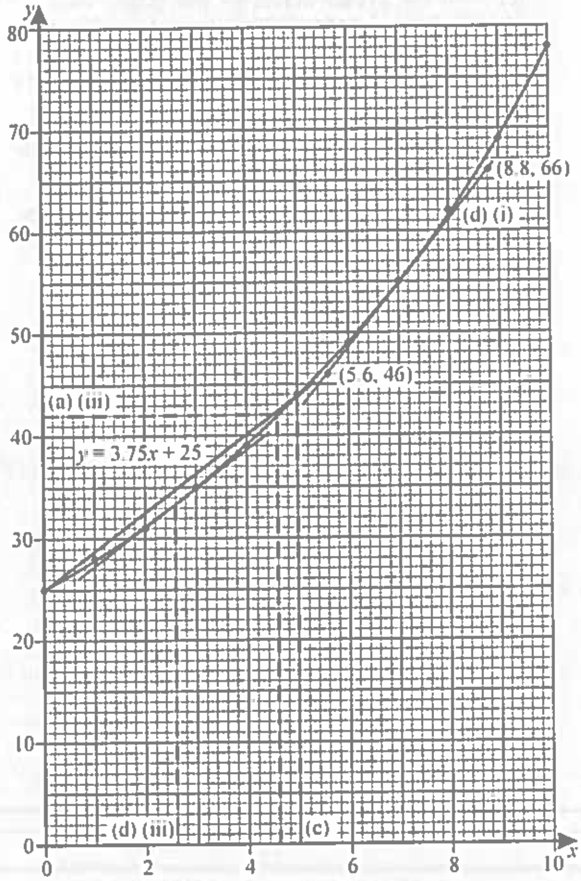
$\therefore$ rate of increase = \$6.25 per year **Ans.**

(ii) rate of increase = 3.75 **Ans.**

For company B,
$y = 3.75x + 25$
$\therefore$ gradient (or rate of increase) = 3.75

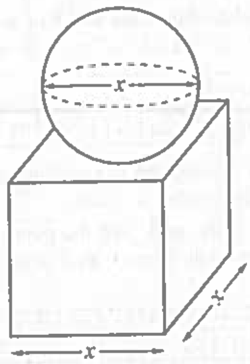
(iii) From graph, $x = 2.6$ **Ans.**

For rates of increase (or gradient) for both companies to be same, draw a tangent to the curve of company A at a point such that it is parallel to the line of company B. Write down the x-value of the point where this tangent meets the curve.



13 *(J2012/P2/Q12)*

[The volume of a sphere $= \frac{4}{3}\pi r^3$]



A solid consists of a sphere on top of a square-based cuboid.

The **diameter** of the sphere is x cm.

The base of the cuboid has sides of length x cm.

The sum of the height of the cuboid and one of the sides of the base is 8 cm.

(a) By considering the height of the cuboid, explain why it is not possible for this sphere to have a radius of 5 cm. [1]

(b) By taking the value of π as 3, show that the approximate volume, y cm³, of the solid is given by

$$y = 8x^2 - \frac{x^3}{2}.$$ [2]

(c) The table below shows some values of x and the corresponding values of y for

$$y = 8x^2 - \frac{x^3}{2}.$$

x	1	2	3	4	5	6	7
y	7.5	28		96	137.5	180	220.5

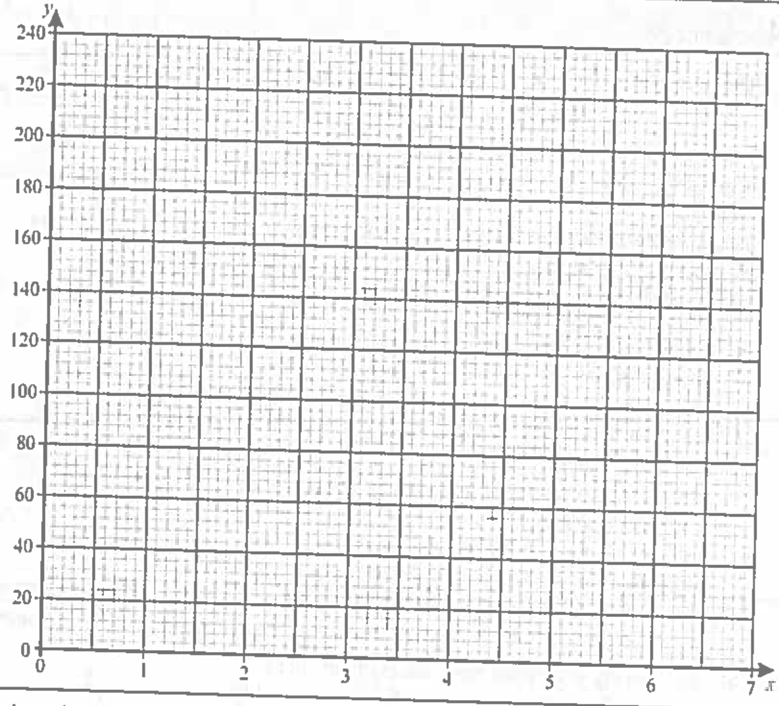
(i) Complete the table. [1]

(ii) On the grid next page, plot the graph of

$$y = 8x^2 - \frac{x^3}{2} \text{ for } 1 \le x \le 7.$$ [3]

(iii) Use your graph to find the **height of the cuboid** when the volume of the solid is 120 cm³ [2]

(d) A cylinder has radius 3 cm and length x cm. By drawing a suitable graph on the grid, estimate the value of x when the solid and the cylinder have the same volume.
Take the value of π as 3. [3]



Thinking Process

(a) Note that sum of height and one side of cuboid is 8 cm.

(b) Volume of solid = volume of sphere + volume of cuboid.

(c) (i) To complete the table substitute $x=3$ into the equation.

(iii) From graph, find x when volume is 120 cm^3 and use the given information to find height of cuboid.

(d) Compute volume of the cylinder using the information. Draw a straight line on the same graph.

Solution

(a) Let h be the cuboid height. It is given that, $h+x=8$ cm.

When diameter of sphere, $x=10$ cm, then, $h+10=8 \Rightarrow h=-2$ cm.

The height of the cuboid cannot be -2 cm, therefore the sphere cannot have a radius of 5 cm. .

(b) Let h be the cuboid height.

$\therefore \quad h+x=8 \Rightarrow h=(8-x)$ cm

Volume of solid,

$y=$ vol. of cuboid + vol. of sphere

$=(x)(x)(8-x)+\frac{4}{3}(3)(\frac{x}{2})^3$

$=8x^2-x^3+4(\frac{x^3}{8})$

$=8x^2-x^3+\frac{x^3}{2}$

$=8x^2-\frac{x^3}{2}$ **(Shown).**

(c) (i) When $x=3$,

$y=8(3)^2-\frac{3^3}{2}$

$=58.5$ **Ans.**

(ii) Refer to graph.

(iii) From graph,

when $y=120$, $x=4.6$

$\therefore$ height of cuboid,

$h=8-4.6$

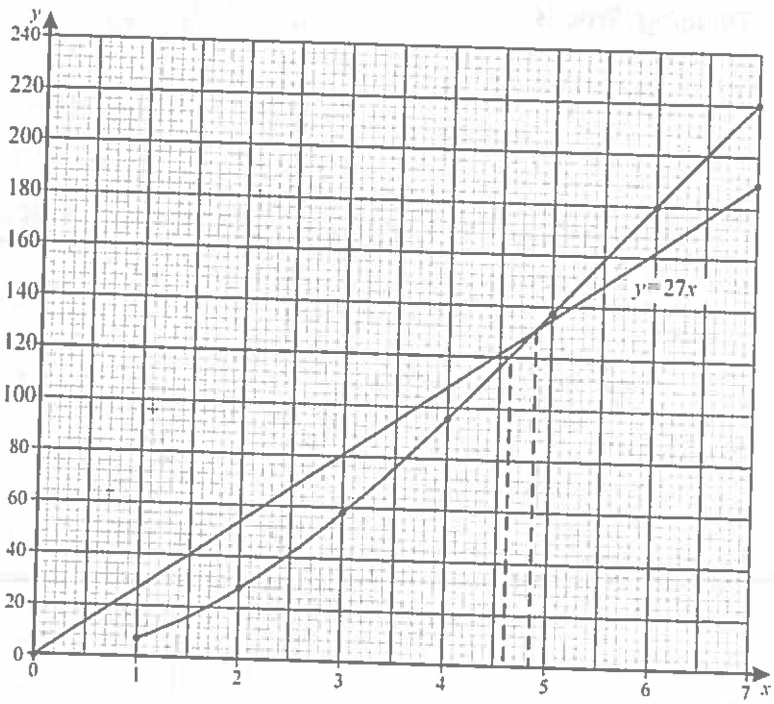
$=3.4$ cm. **Ans.**

(d) Vol. of cylinder $=\pi r^2 h$

$\Rightarrow \quad y=(3)(3)^2(x)=27x$

refer to graph for the line.

From graph, cylinder and solid have same volume at $x=4.85$ **Ans.**



14 *(N2012/P2/Q8)*

The variables x and y are connected by the equation
$$y = 1 + 2x^2 - x^3.$$
The table below shows some values of x, and the corresponding values of y, correct to 1 decimal place where appropriate.

x	−1	−0.5	0	0.5	1	1.5	2	2.5
y	4	1.6	1	1.4	2	2.1	1	p

(a) Calculate p.
Give your answer correct to 1 decimal place. [1]

(b) On the graph paper, using a scale of 2 cm to represent 1 unit on both axes,
draw a horizontal x-axis for $-2 \le x \le 3$, and draw a vertical y-axis for $-3 \le y \le 5$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find all the solutions of $1 + 2x^2 - x^3 = 2$. [2]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = -0.5$. [2]

(e) By drawing an appropriate straight line on the grid, solve the equation $1 + 2x^2 - x^3 = x$. [2]

(f) Find the range of values of k such that $1 + 2x^2 \; x^3 \; k$ has 3 solutions. [2]

Thinking Process

(a) Substitute $x = 2.5$ into y.
(b) Plot the points from the table using the given scale.
(c) Draw $y = 2$. Find intersection points where line meets the curve.
(d) Draw a tangent at $x = -0.5$. Take 2 points on the tangent and calculate the gradient.
(e) Draw $y = x$. Find intersection points where line meets the curve.
(f) Find the greatest and least values of y at which the line $y = k$ intersects the curve at three points.

Solution

with **TEACHER's COMMENTS**

(a) $p = 1 + 2(2.5)^2 - (2.5)^3$
$= 1 + 12.5 - 15.625$
$= -2.125$
≈ -2.13 **Ans.**

(b) Refer to graph.

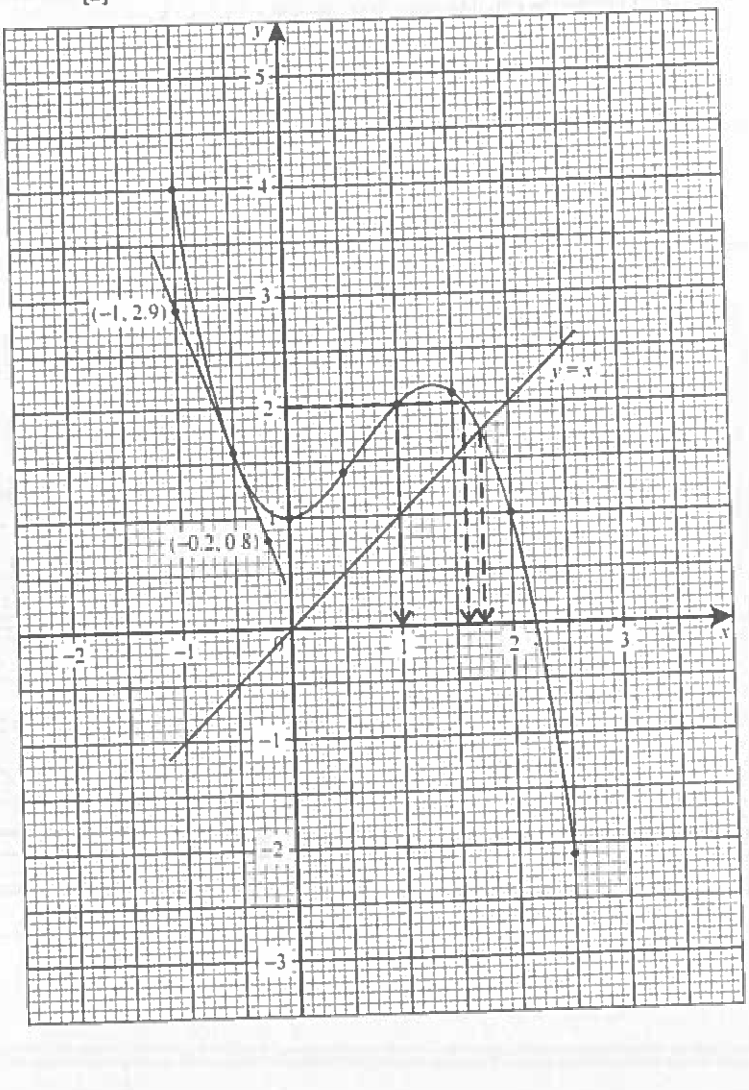
(c) $1 + 2x^2 - x^3 = 2$
$\Rightarrow \; y = 2$
$\therefore$ from graph, $x = 1$ and 1.6 **Ans.**

(d) Taking two points $(-0.2, 0.8)$ and $(-1, 2.9)$ on the tangent.

$\text{gradient} = \dfrac{2.9 - 0.8}{-1 - (-0.2)}$
$= \dfrac{2.1}{-0.8}$
$= -2.63$ **Ans.**

(e) $1 + 2x^2 - x^3 = x$
$\Rightarrow \; y = x$
$\therefore$ from graph, $x = 1.75$ **Ans.**

(f) $1 + 2x^2 - x^3 = k$
$\Rightarrow \; y = k$
$\therefore$ from graph, $y = k$ has 3 solutions in the range: $1 < k < 2.17$ **Ans.**



15 *(J2013/P2 Q11)*

(a) The table shows some values of x and the corresponding values of y for $y = 2x^3 - 3x^2 + 5$.

x	−1.5	−1	−0.5	0	0.5	1	1.5	2
y		0	4	5	4.5	4	5	9

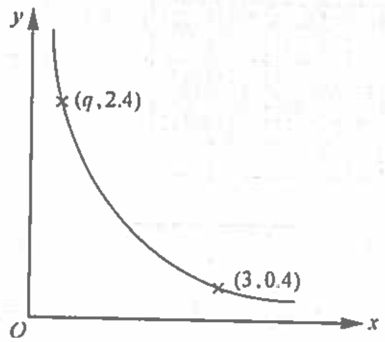
(i) Complete the table. [1]

(ii) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $-1.5 \le x \le 2$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-10 \le y \le 10$.
Draw the graph of
$y = 2x^3 - 3x^2 + 5$ for $-1.5 \le x \le 2$. [3]

(iii) Use your graph to estimate the gradient of the curve when $x = 1.5$. [2]

(iv) By drawing a suitable line on your graph, find the solution of the equation
$2x^3 - 3x^2 + 4 = 0$ [2]

(b)



The graph shows a sketch of the curve $y = \frac{p}{x}$.
Two points on the curve are (3, 0.4) and (q, 2.4).

(i) Find p and q. [2]

(ii) Calculate the gradient of the straight line joining the points (3, 0.4) and (q, 2.4). [2]

Thinking Process

(a) (i) Substitute $x = -1.5$ into y.
(iii) Draw a tangent at $x = 1.5$. Take two points on the tangent and find the gradient.
(iv) Draw a line $y = 1$ and find the intersection point where $y = 1$ meets the curve.

(b) (i) To find p ∯ substitute (3, 0.4) into the equation.
To find q ∯ substitute p & (q, 2.4) into the equation.
(ii) Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

Solution

(a) (i) When $x = -1.5$

$y = 2(-1.5)^3 - 3(-1.5)^2 + 5$
$= -6.75 - 6.75 + 5$
$= -8.5$ **Ans.**

(ii) Refer to graph on next page.

(iii) Taking two points (0.8, 2) and (1.85, 6.5) on the tangent.

gradient $= \frac{6.5 - 2}{1.85 - 0.8}$
$= \frac{4.5}{1.05}$
$= 4.2857 \approx 4.29$ (3sf) **Ans.**

(iv) $2x^3 - 3x^2 + 4 = 0$
$\Rightarrow 2x^3 - 3x^2 + 4 + 1 = 1$
$\Rightarrow 2x^3 - 3x^2 + 5 = 1$
$\Rightarrow y = 1$
$\therefore$ from graph, at $y = 1$, $x = -0.92$ **Ans.**

(b) (i) $y = \frac{p}{x}$

substitute (3, 0.4) into equation.

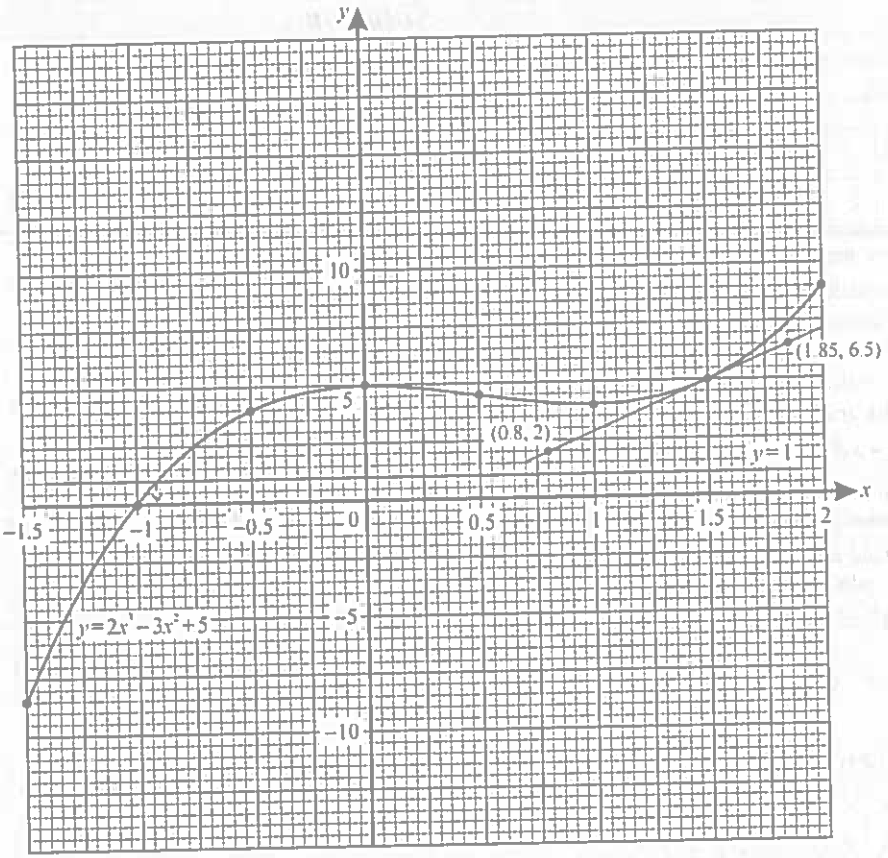
$0.4 = \frac{p}{3} \Rightarrow p = 1.2$ **Ans.**

substitute (q, 2.4) and the value of p into the equation.

$2.4 = \frac{1.2}{q}$

$q = \frac{1.2}{2.4} = 0.5$ **Ans.**

(ii) Gradient $= \frac{2.4 - 0.4}{q - 3}$
$= \frac{2}{0.5 - 3}$
$= \frac{2}{-2.5} = -0.8$ **Ans.**



16 *(N2013/P2/Q9)*

The variables x and y are connected by the equation

$y = x + \frac{1}{x}$.

The table below shows some values of x and the corresponding values of y.
The values of y are correct to 2 decimal places where appropriate.

x	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	4.25	2.5	2.08	2	2.05	2.17	2.32	2.5

(a) On the graph paper, Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $-2 \le x \le 2$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-5 \le y \le 5$.
On your axes, plot the points given in the table and join them with a smooth curve. [2]

(b) By drawing a tangent, estimate the gradient of the curve when $x = 0.75$. [2]

(c) Let $f(x) = x + \frac{1}{x}$.

(i) Given that $f(a) = b$, find $f(-a)$ in terms of b. [1]

(ii) Hence, or otherwise, complete the table below for $y = x + \frac{1}{x}$.

x	−2	−1.75	−1.5	−1.25	−1	−0.75	−0.5	−0.25
y					−2			

[1]

(iii) On the same axes, draw the graph of $y = x + \frac{1}{x}$ for $-2 \le x \le -0.25$. [1]

(iv) Write down an estimate for the gradient of the curve when $x = -0.75$. [1]

(d) (i) On the same axes, draw the graph of the straight line $y = 4 - x$. [1]

(ii) Write down the x-coordinate of each of the points where the graphs of $y = 4 - x$ and $y = x + \frac{1}{x}$ intersect. [1]

(iii) Find the equation for which these x values are the solutions. Give your equation in the form $Ax^2 + Bx + C = 0$. [2]

Thinking Process

(b) Draw tangent at $x = 3$. Take two points on the tangent and find the gradient.

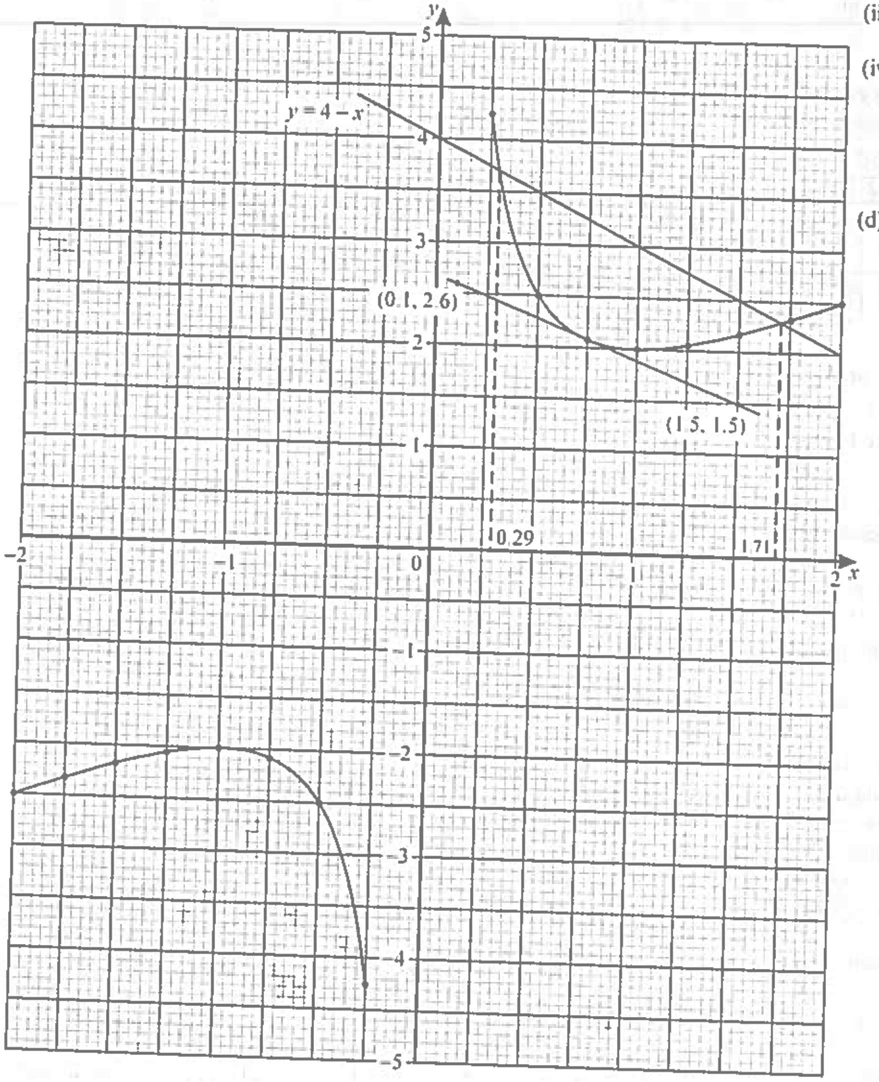
(c) (i) Equate f(a) to b and form an expression of b in terms of a. Use this expression to find f(−a) in terms of b.

(iv) To find an estimate for the gradient use the answer of part (b).

(d) (iii) Substitute $y = 4 - x$ into $y = x + \frac{1}{x}$.

Solution

(a)



(b) Taking two points (1.5, 1.5) and (0.1, 2.6) on the tangent, gradient $= \frac{2.6-1.5}{0.1-1.5}$

$= \frac{1.1}{-1.4} = -0.786$ **Ans.**

(c) (i) $f(a) = a + \frac{1}{a} \Rightarrow b = a + \frac{1}{a}$

$f(-a) = -a - \frac{1}{a}$

$= -(a + \frac{1}{a}) = -b$

$\therefore$ $f(-a) = -b$ **Ans.**

(ii)

x	−2	−1.75	−1.5	−1.25	−1	−0.75	−0.5	−0.25
y	−2.5	−2.32	−2.17	−2.05	−2	−2.08	−2.5	−4.25

(iii) Refer to graph.

(iv) The tangent at $x = -0.75$ would be parallel to the tangent drawn in part (b).

$\therefore$ gradient $= -0.786$ **Ans.**

(d) (i) Refer to graph.

(ii) $x = 1.29$ and 1.71 **Ans.**

(iii) Equation of curve: $y = x + \frac{1}{x}$

Equation of line: $y = 4 - x$

$\therefore$ $x + \frac{1}{x} = 4 - x$

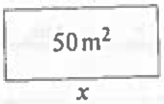
$\Rightarrow$ $\frac{x^2+1}{x} = 4 - x$

$\Rightarrow$ $x^2 + 1 = 4x - x^2$

$\Rightarrow$ $2x^2 - 4x + 1 = 0$ **Ans.**

17 (J2014/P2/Q10)

Adil wants to fence off some land as an enclosure for his chickens.
The enclosure will be a rectangle with an area of 50 m².



(a) The enclosure is x m long.
Show that the total length of fencing, L m, required for the enclosure is given by
$$L = 2x + \frac{100}{x}.$$ [2]

(b) The table below shows some values of x and the corresponding values of L, correct to one decimal place where appropriate, for
$$L = 2x + \frac{100}{x}.$$

x	2	4	6	8	10	12	14	16	18	20
L	54	33	28.7	28.5	30	32.3	35.1	38.3		

Complete the table. [2]

(c) On the grid opposite draw a horizontal x-axis for $0 \le x \le 20$ using a scale of 1 cm to represent 2 m and a vertical L-axis for $0 \le L \le 60$ using a scale of 2 cm to represent 10 m.
On the grid, plot the points given in the table and join them with a smooth curve. [3]

(d) Adil only has 40 m of fencing.
Use your graph to find the range of values of x that he can choose. [2]

(e) (i) Find the minimum length of fencing Adil could use for the enclosure. [1]

(ii) Find the length and width of the enclosure using this minimum length of fencing. Give your answers correct to the nearest metre. [1]

(f) Suggest a suitable length and width for an enclosure of area 100 m², that uses the minimum possible length of fencing. [1]

Thinking Process

(a) Apply the formula area = $l \times w$ and express the width in terms of x. Then use the formula, perimeter = $2(l + w)$ to find L.

(b) Substitute the values of x into the equation to find the missing values of L.

(d) Find the corresponding values of x when $L = 40$ m.

(e) (i) From graph, read the minimum value of curve.

(ii) To find width ✎ find the value of x when the curve is at its minimum value. Substitute it into the equation of L to find length.

(f) To find a suitable length and width ✎ observe the answers to part (e) (ii).

Solution with TEACHER'S COMMENTS

(a) Let w be the width of the rectangle

∴ Area of the rectangle $= x \times w$

$\Rightarrow \quad 50 = xw$

$\Rightarrow \quad w = \frac{50}{x}$

Total length of fencing.

$L = 2x + 2w$

$\Rightarrow L = 2x + 2\left(\frac{50}{x}\right)$

$\Rightarrow L = 2x + \frac{100}{x}$ **Shown.**

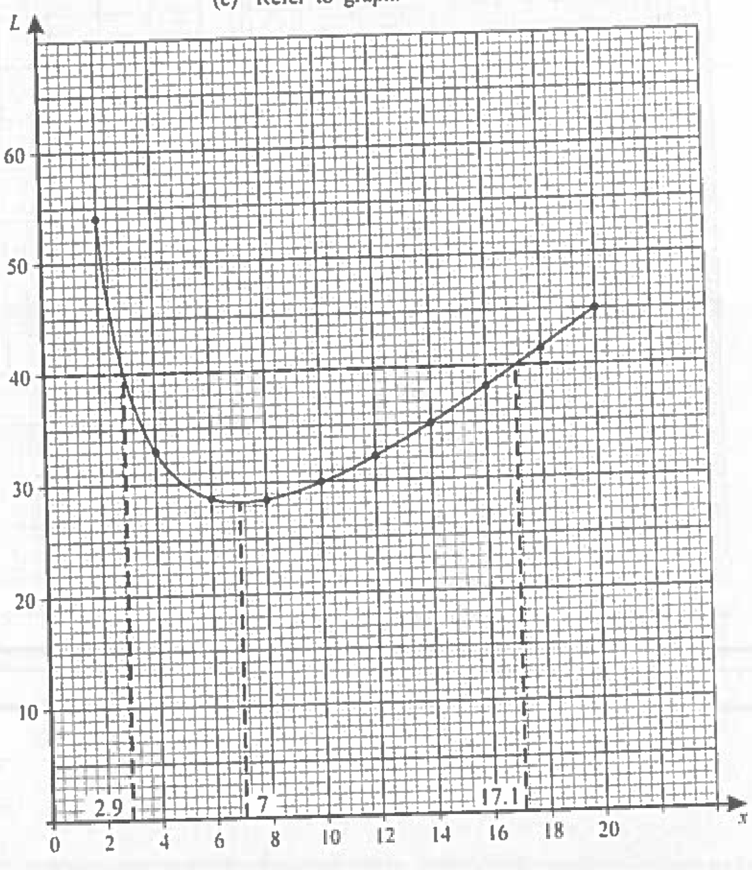
(b) When $x = 18$ $\quad L = 2(18) + \frac{100}{18}$

$= 41.56$ **Ans.**

When $x = 20$ $\quad L = 2(20) + \frac{100}{20}$

$= 45$ **Ans.**

(c) Refer to graph.



(d) From graph, when $L = 40$ m,

$2.9 \le x \le 17.1$ **Ans.**

(e) (i) From graph,

minimum length = 28.2 m. **Ans.**

(ii) From graph, when $L = 28.2$ m

length, $x = 7$ m **Ans.**

width, $w = \frac{50}{x}$

$= \frac{50}{7} = 7.14 \approx 7$ m **Ans.**

(f) Length = 10 m

Width = 10 m **Ans.**

Note that the minimum perimeter for a given area occurs when the rectangle is a square. Therefore the length and width of the enclosure would be the square root of the given area of 100 m².

18 *(N2014/P2/Q10)*

The table below is for $y = x^2 - 4x - 1$.

x	−2	−1	0	1	2	3	4	5	6
y		4	−1	−4	−5	−4	−1	4	

(a) Complete the table. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \le x \le 6$.
Using a scale of 2 cm to 5 units, draw a vertical y-axis for $-10 \le y \le 15$.
Plot the points from the table and join them with a smooth curve. [3]

(c) By drawing a tangent, estimate the gradient of the curve at $x = 3$. [2]

(d) (i) Find the least value of y. [1]

(ii) $y \le 4$ for $a \le x \le b$.
Find the least possible value of a and the greatest possible value of b. [2]

(e) Use your graph to solve the equation
$x^2 - 4x + 2 = 0$.
Show your working to explain how you used your graph. [3]

Solution

(a) When $x = -2$, $y = (-2)^2 - 4(-2) - 1 = 11$ **Ans.**

When $x = 6$, $y = (6)^2 - 4(6) - 1 = 11$ **Ans.**

(b) Refer to graph below.

(c) Taking two points (1.2, −7.5) and (5.3, 0.5) on the tangent, gradient $= \frac{0.5-(-7.5)}{5.3-1.2}$

$= \frac{0.5+7.5}{5.3-1.2}$

$= \frac{8}{4.1} = 1.95$ **Ans.**

(d) (i) Least value of $y = -5$ **Ans.**

(ii) From graph, $y \le 4$ for $-1 \le x \le 5$

$\therefore$ $a = -1$, $b = 5$ **Ans.**

(e) $x^2 - 4x + 2 = 0$

$x^2 - 4x = -2$

$x^2 - 4x - 1 = -2 - 1$

$\Rightarrow$ $y = -3$

$\therefore$ from graph, when $y = -3$,

$x = 0.6$ or 3.4 **Ans.**

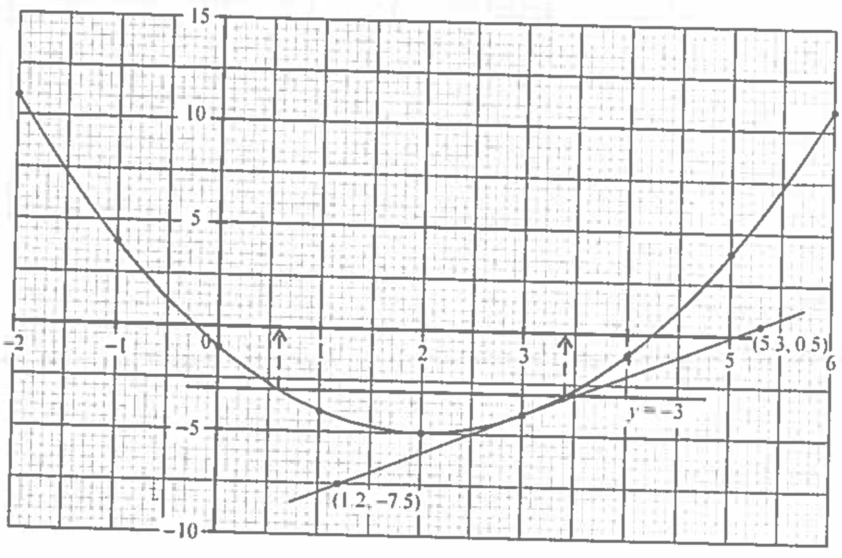
Thinking Process

(c) Draw a tangent at $x = 3$. Take two points on the tangent and find the gradient.

(d) (i) From graph, find the value of y that corresponds to the least value of the curve.

(ii) From graph, or from table, find the values of x when $y = 4$.

(e) Re-write the equation in the form $y = x^2 - 4x - 1$. Draw a straight line on the graph and find the values of x where the line intersects the curve.



19 *(J2015 P2 Q7 b)*

(i) Complete the table of values for $y=6+x-x^2$.
Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-3 \le x \le 4$.
Using a scale of 1 cm to 1 unit, draw a vertical y-axis for $-7 \le y \le 7$.
Hence draw the graph of $y=6+x-x^2$. [3]

x	−3	−2	−1	0	1	2	3	4
y	−6	0		6	6		0	−6

(ii) Use your graph to estimate the maximum value of $6+x-x^2$. [1]

(iii) By drawing the line $x+y=4$, find the approximate solutions to the equation $2+2x-x^2=0$. [2]

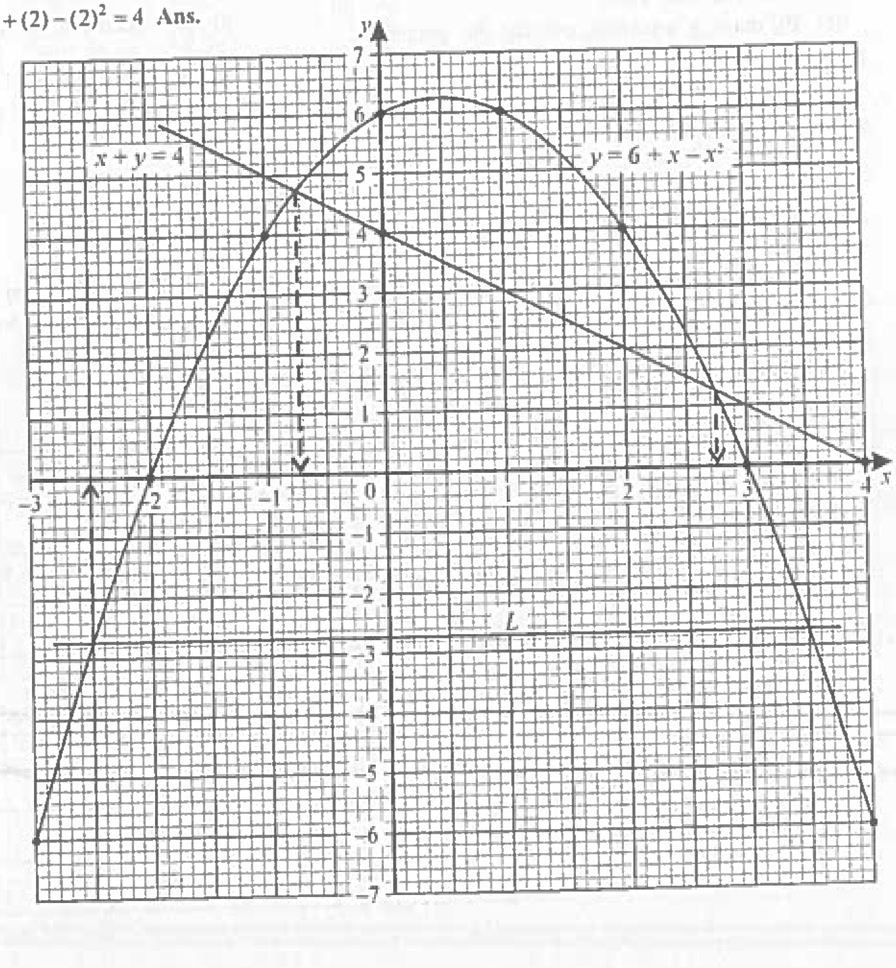
(iv) The equation $x-x^2=k$ has a solution $x=3.5$.
By drawing a suitable line on the grid, find the other solution. Label your line with the letter L. [2]

Thinking Process

(i) Substitute $x=-1$ and $x=2$ into y. Using the given scale, plot the points given in table
(ii) From graph, read the maximum value of y.
(iii) From graph, find the values of x where the given line intersects the curve.
(iv) To draw line L ∅ re-write the equation in the form $6+x-x^2=y$.

Solution

(i) When $x=-1$, $y=6+(-1)-(-1)^2=4$ Ans.
When $x=2$, $y=6+(2)-(2)^2=4$ Ans.



(ii) Maximum value = 6.25 **Ans.**

(iii) $2+2x-x^2=0$

$\Rightarrow\ 2+2x-x^2-x+4=-x+4$

$\Rightarrow\ 6+x-x^2=-x+4$

$\Rightarrow\ y=-x+4$

$\Rightarrow\ y+x=4$

$\therefore$ from graph, the line and the curve intersect at,

$x=-0.73$ or 2.75 **Ans.**

(iv) $x-x^2=k$

$\Rightarrow\ 6+x-x^2=k+6$

$\Rightarrow\ y=k+6$

which is a straight line parallel to x-axis.

$\therefore$ the graph of the line L is a straight line parallel to x-axis and passing through the curve where $x=3.5$

$\therefore$ from graph, the other solution is:

$x=-2.5$ **Ans.**

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20 *(N2015/P2/Q9)*

The distance, d metres, of a moving object from an observer after t minutes is given by

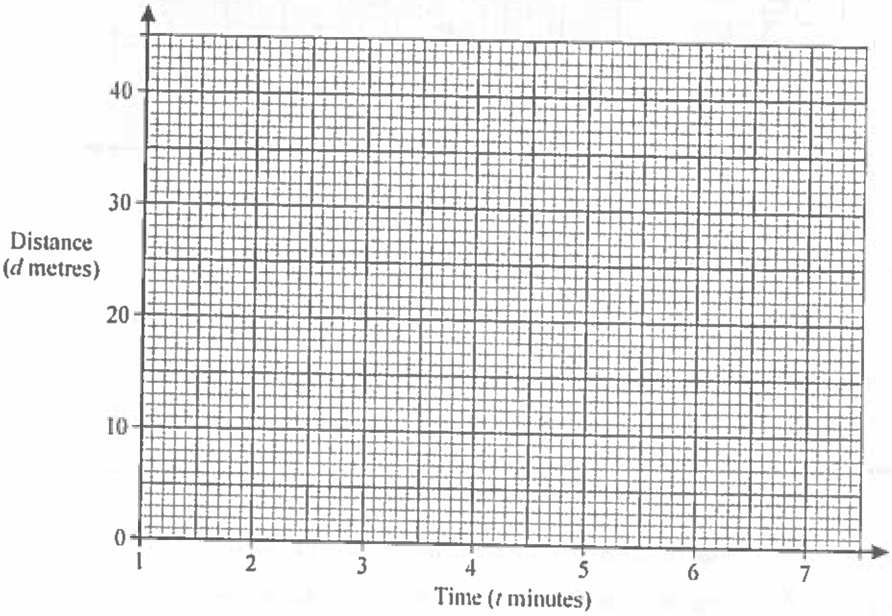
$$d=t^2+\frac{48}{t}-20\,.$$

(a) Some values of t and d are given in the table.
The values of d are given to the nearest whole number where appropriate.

t	1	1.5	2	2.5	3	3.5	4	4.5	5	6	7
d	29	14	8	5	5	6	8	11	15	24	

Complete the table. [1]

(b) On the grid, plot the points given in the table and join them with a smooth curve.



[2]

(c) (i) By drawing a tangent, calculate the gradient of the curve when $t=4$. [2]

(ii) Explain what this gradient represents. [1]

(d) For how long is the object less than 10 metres from the observer? [2]

(e) (i) Using your graph, write down the two values of t when the object is 12 metres from the observer.
For each value of t, state whether the object is moving towards or away from the observer.

When t = ..., the object is moving .... the observer.

When t = ..., the object is moving .... the observer. [2]

(ii) Write down the equation that gives the values of t when the object is 12 metres from the observer. [1]

(iii) This equation is equivalent to $t^3+At+48=0$. Find A. [1]

Thinking Process

(c) (i) Draw a tangent at $x = 4$. Take two points on the tangent and find the gradient.

(ii) Gradient is the rate of change of distance i.e. speed.

(d) To find duration of time ∯ find the values of t when $d = 10$.

(e) (ii) Substitute d = 12 into the equation of curve and simplify.

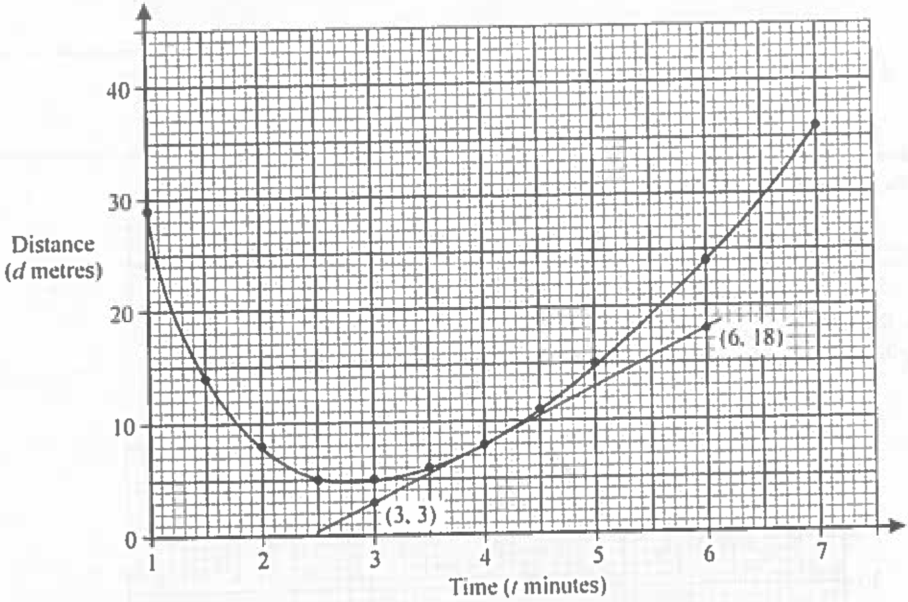
(iii) Rearrange the equation of part (ii) to form a cubic equation. Compare with the given equation to find A.

Solution

(a) When $t = 7$, $d = (7)^2 + \frac{48}{7} - 20$

$= 35.86 \approx 36$ **Ans.**

(b)



(c) (i) Taking two points (3, 3) and (6, 18) on the tangent, gradient $= \frac{18-3}{6-3}$

$= \frac{15}{3} = 5$ **Ans.**

(ii) The gradient represents the speed of the object at 4 minutes.

(d) When $d = 10$ meters, $t = 1.8$ and 4.4 minutes.

∴ duration of time $= 4.4 - 1.8$

$= 2.6$ minutes. **Ans.**

(e) (i) When t = ...1.63......, the object is moving ......towards...... the observer.

When t = ...4.65......, the object is moving ......away...... the observer.

(ii) $d = t^2 + \frac{48}{t} - 20$

when $d = 12$ m,

$12 = t^2 + \frac{48}{t} - 20 \Rightarrow t^2 + \frac{48}{t} - 32 = 0$ **Ans.**

(iii) $t^2 + \frac{48}{t} - 32 = 0$

$\Rightarrow t^3 + 48 - 32t = 0 \Rightarrow t^3 - 32t + 48 = 0$

comparing it with $t^3 + At + 48 = 0$

$A = -32$ **Ans.**

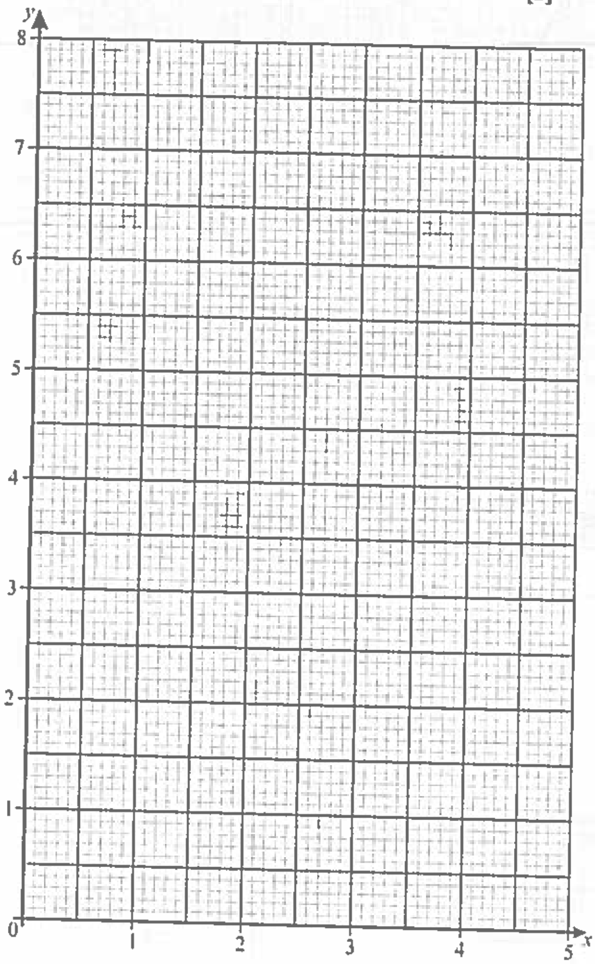
21 *(J2016/P2/Q8)*

The table below shows some values of x and the corresponding values of y for $y = \frac{1}{4} \times 2^x$.

x	0	1	2	3	4	5
y	$\frac{1}{4}$		1	2	4	8

(a) Complete the table. [1]

(b) On the grid below, draw the graph of $y = \frac{1}{4} \times 2^x$. [2]



(c) By drawing a suitable line, find the gradient of your graph where $x = 4$. [2]

(d) (i) Show that the line $2x + y = 6$, together with the graph of $y = \frac{1}{4} \times 2^x$, can be used to solve the equation $2^x + 8x - 24 = 0$. [1]

(ii) Hence solve $2^x + 8x - 24 = 0$. [2]

(e) The points P and Q are $(2, 3)$ and $(5, 4)$ respectively.

(i) Find the gradient of PQ. [1]

(ii) On the grid, draw the line l, parallel to PQ, that touches the curve $y = \frac{1}{4} \times 2^x$. [1]

(iii) Write down the equation of l. [2]

Thinking Process

(a) Substitute $x = 1$ into y.

(c) Draw a tangent at $x = 4$. Take two points on the tangent and find the gradient.

(d) (i) Substitute equation of the line into the equation of curve and rearrange the equation in the required form.

(ii) Draw the line $2x + y = 6$ on the graph and find the values of x where the line intersects the curve.

(e) (ii) Draw a line with the same gradient as PQ at a point on the curve such that the line is tangent to the curve.

(iii) To find equation l, use $y = mx + c$, where m is the gradient and c is the y-intercept.

Solution

(a) When $x = 1$,

$$y = \frac{1}{4} \times 2^1 = \frac{1}{2} \quad \textbf{Ans.}$$

(b) Refer to graph on the next page.

(c) Taking two points $(3.3, 2)$ and $(4.6, 5.7)$ on the tangent.

$$\text{gradient} = \frac{5.7 - 2}{4.6 - 3.3}$$

$$= \frac{3.7}{1.3} = 2.85 \text{ (3sf)} \quad \textbf{Ans.}$$

(d) (i) Equation of curve: $y = \frac{1}{4} \times 2^x$

Equation of line: $2x + y = 6$

substitute eq. of line into eq. of curve.

$$2x + (\frac{1}{4} \times 2^x) = 6$$

$$\frac{1}{4} \times 2^x = 6 - 2x$$

$$2^x = 24 - 8x$$

$$2^x + 8x - 24 = 0 \quad \textbf{Shown.}$$

(ii) The solution of $2^x + 8x - 24 = 0$ is the x-coordinate of the point where the line meets the curve.

$\therefore$ From graph, $x = 2.35$ **Ans.**

(e) (i) Gradient of $PQ = \frac{4-3}{5-2} = \frac{1}{3}$ Ans.

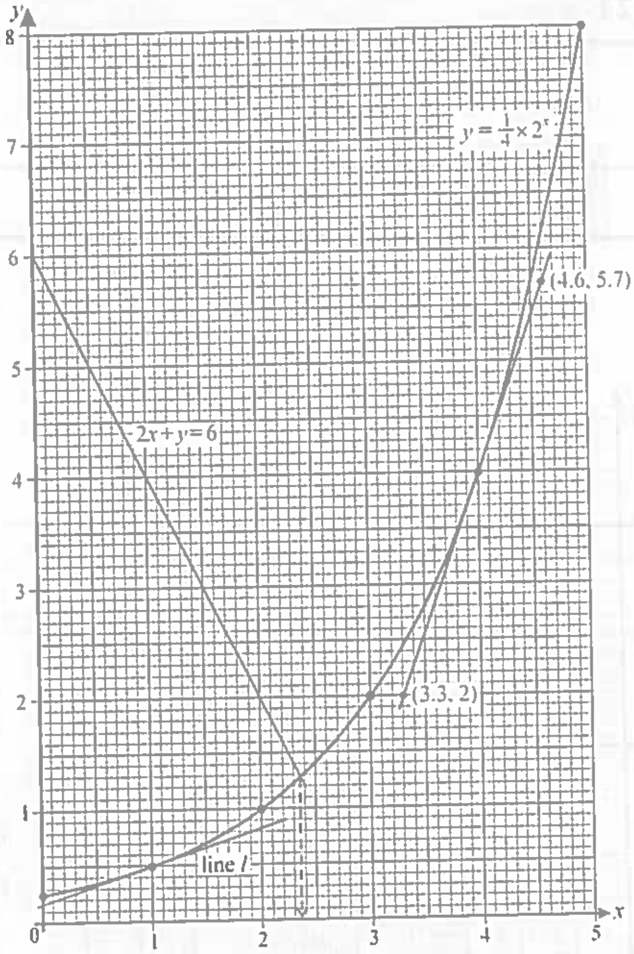
(ii) Refer to graph.

(iii) Line l is parallel to PQ,

$\therefore$ gradient of $l = \frac{1}{3}$

from graph, y-intercept of $l = 0.17$

$\therefore$ equation of l: $y = \frac{1}{3}x + 0.17$ Ans.



22 (N2016/P2/Q3)

(a) Complete the table of values for $y = \frac{x}{20}(x^2 - 10)$.

x	0	1	2	3	4	5
y	0	−0.45	−0.6	−0.15	1.2	

[1]

(b) Using a scale of 2 cm to 1 unit on both axes, draw the graph of

$y = \frac{x}{20}(x^2 - 10)$ for $0 \leq x \leq 5$. [2]

(c) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2.5$. [2]

(d) Use your graph to solve the equation

$\frac{x}{20}(x^2 - 10) = 0$ for $0 \leq x \leq 5$. [2]

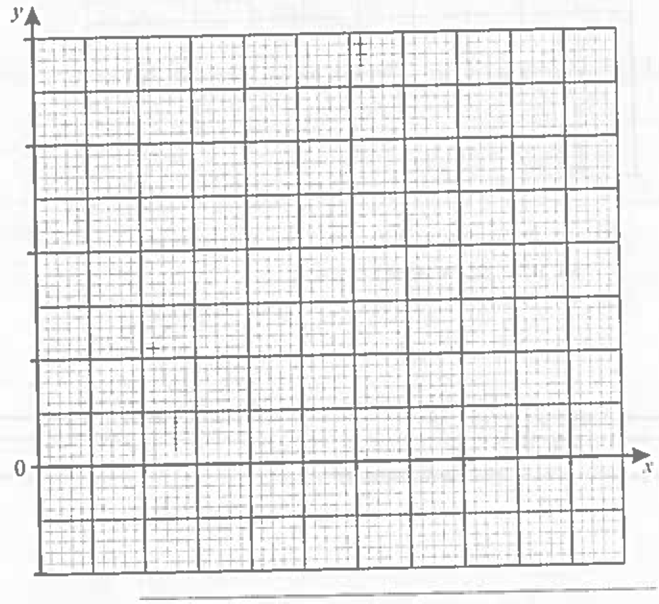
(e) The graph of $y = \frac{x}{20}(x^2 - 10)$, together with the graph of a straight line L, can be used to solve the equation

$x^3 + 10x - 80 = 0$ for $0 \leq x \leq 5$.

(i) Find the equation of line L. [2]

(ii) Draw the graph of line L on the grid. [1]

(iii) Hence solve the equation $x^3 + 10x - 80 = 0$ for $0 \leq x \leq 5$. [1]



Thinking Process

(a) Substitute $x = 5$ into the equation to find the missing value of y.

(c) Draw a tangent at $x = 2.5$. Take two points on the tangent and find the gradient.

(d) find the value of x where the curve intersects the x-axis.

(e) (i) To find the equation of line L re-arrange $x^3 + 10x - 80 = 0$ to form the equation of the curve.

(iii) Find the value of x where the curve intersects the line L.

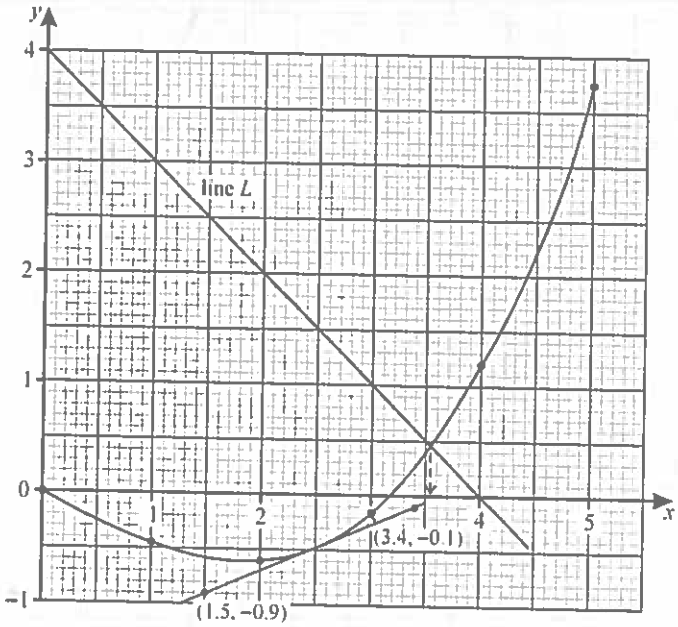
Solution

(a) When $x = 5$

$$y = \frac{5}{20}(5^2 - 10)$$

$$= \frac{1}{4}(15) = 3.75 \quad \textbf{Ans.}$$

(b)



(c) Taking two points (1.5, −0.9) and (3.4, −0.1) on the tangent,

$$\text{gradient} = \frac{-0.1 - (-0.9)}{3.4 - 1.5}$$

$$= \frac{0.8}{1.9} = 0.421 \quad \textbf{Ans.}$$

(d) $\frac{x}{20}(x^2 - 10) = 0$

$\Rightarrow \; y = 0$

$\therefore$ from graph, at $y = 0$, $x = 3.17$ **Ans.**

(e) (i) $x^3 + 10x - 80 = 0$

$$x^3 = 80 - 10x$$

$$x^3 - 10x = 80 - 10x - 10x$$

$$x(x^2 - 10) = 80 - 20x$$

$$x(x^2 - 10) = 20(4 - x)$$

$$\frac{x}{20}(x^2 - 10) = 4 - x$$

$\Rightarrow \; y = 4 - x$

$\therefore$ equation of line L is:

$y = 4 - x$ **Ans.**

(ii) Refer to graph.

(iii) From graph, $x = 3.55$ **Ans.**

23 *(J2017/P2 Q9)*

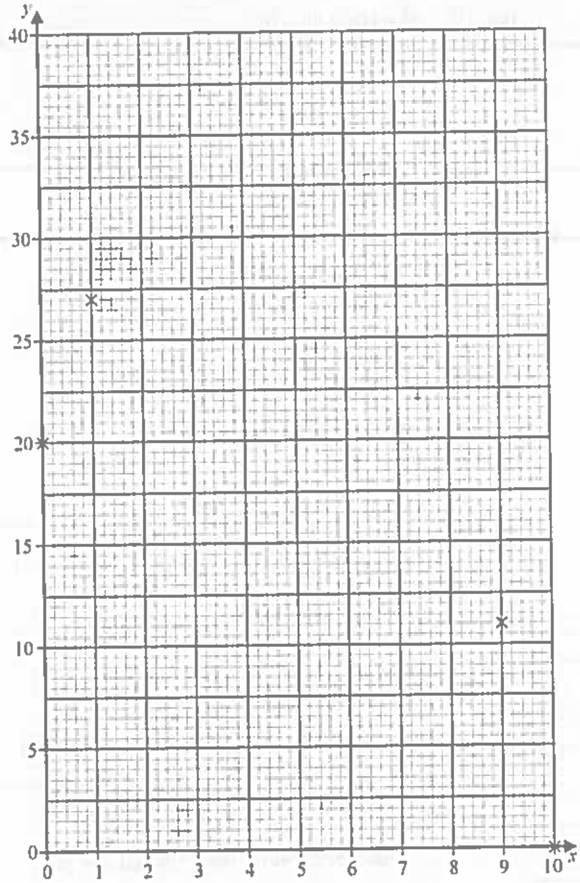
A random number, x, is generated, where x is any real number.

(a) Manuel adds 2 to x. He subtracts x from 10.
Manuel then multiplies these two results to give his number, y.
Show that $y = 20 + 8x - x^2$. [2]

(b) On the grid next page, draw the graph of $y = 20 + 8x - x^2$ for $0 \le x \le 10$.
Four points have been plotted for you. [4]

(c) On the same grid, draw a suitable line to find the value of Manuel's number, y, when it is the same as the random number, x. [2]

(d) Jolene multiplies the random number, x, by 5 and then adds 2 to give her number, z.
Calculate the possible values of x when Manuel's number, y, and Jolene's number, z, are the same. [4]



Thinking Process

(a) Use the given information to form the number y.
(b) Draw a table using values of x from $x = 1$ to $x = 10$. Find the corresponding values of y. Plot the points from the table and draw the graph.
(c) Draw $y = x$. Find the point of intersection of the line and the curve.
(d) Form Jolene's equation. Equate it to Manuel's equation.

Solution

(a) $y = (x+2)\times(10-x)$

$= 10x - x^2 + 20 - 2x$

$= 20 + 8x - x^2$ **Shown..**

(b)

x	1	2	3	4	5	6	7	8	9	10
y	27	32	35	36	35	32	27	20	11	0

Refer to graph.

(c) When Manuel's number, y is same as x.

$\Rightarrow$ $y = x$

$\therefore$ from graph, $y = 9.2$ **Ans.**

(d) Jolene's number : $z = 5x + 2$

given that Manuel's number, y, and Jolene's number, z, are same

$\Rightarrow$ $20 + 8x - x^2 = 5x + 2$

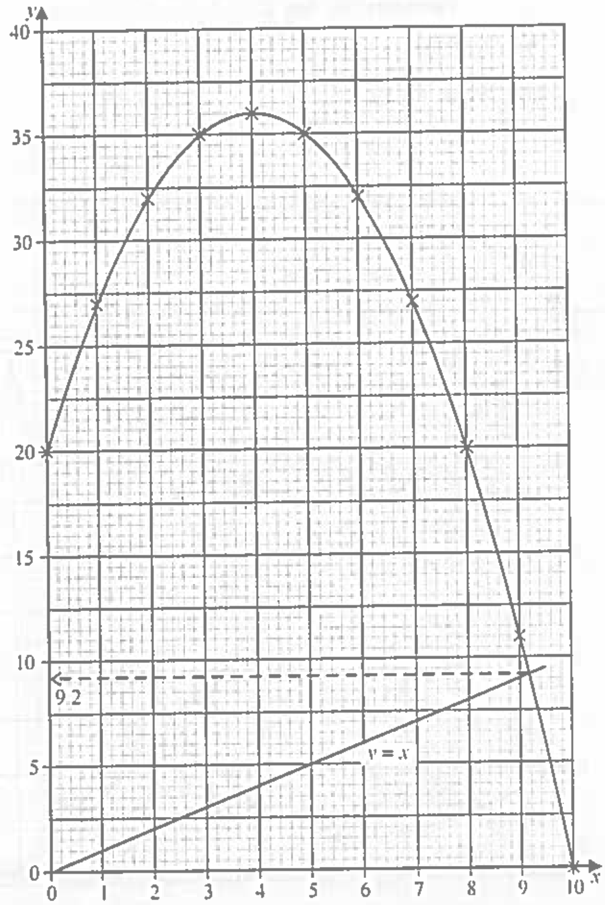
$18 + 3x - x^2 = 0$

$x^2 - 3x - 18 = 0$

$x^2 + 3x - 6x - 18 = 0$

$x(x+3) - 6(x+3) = 0 \Rightarrow (x+3)(x-6) = 0$

$\therefore$ $x = -3$, or 6 **Ans.**



24 *(N2017 P2 Q7)*

(a) The variables x and y are connected by the equation $y = 3 + x - \dfrac{x^2}{2}$.

Some corresponding values of x and y are given in the table below.

x	−3	−2	−1	0	1	2	3	4	5
y		−1	1.5	3	3.5	3	1.5	−1	

(i) Complete the table. [1]

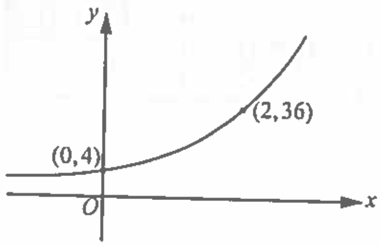
(ii) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-3 \le x \le 5$.

Using a scale of 1 cm to 1 unit, draw a vertical y-axis for $-5 \le y \le 5$.
On the grid next page, draw the graph of
$y = 3 + x - \frac{x^2}{2}$ for $-3 \le x \le 5$. [3]

(iii) By drawing a tangent, estimate the gradient of the curve at (3, 1.5). [2]

(iv) The points of intersection of the graph of $y = 3 + x - \frac{x^2}{2}$ and the line $y = k$ are the solutions of the equation $10 + 2x - x^2 = 0$.
(a) Find the value of k. [1]
(b) By drawing the line $y = k$ on your graph, find the solutions of the equation $10 + 2x - x^2 = 0$. [2]

(b) This is a sketch of the graph of $y = pa^x$, where $a > 0$.
The graph passes through the points (0, 4) and (2, 36).



(i) Write down the value of p. [1]
(ii) Find the value of a. [1]
(iii) The graph passes through the point (4, q). Find the value of q. [1]

Thinking Process

(a) (ii) Plot the points from the table using the given scale
(iii) Draw a tangent at $x = 3$. Take 2 points on the tangent and calculate the gradient.
(iv) (a) To find the value of k re-arrange $10 + 2x - x^2 = 0$ in the form of the equation of the curve.
(b) Draw the line and find the values of x where the line meets the curve.

(b) (i) Substitute (0, 4) into y.
(ii) Substitute (2, 36) and the value of p found in (i) into y
(iii) To find q h substitute the values of p, a and (4, q) into y.

Solution

(a) (i) When $x = -3$, $y = 3 + (-3) - \frac{(-3)^2}{2}$
$= -4.5$ **Ans.**

When $x = 5$, $y = 3 + (5) - \frac{(5)^2}{2}$
$= 8 - 12.5 = -4.5$ **Ans.**

(ii) Refer to graph on next page.

(iii) Taking two points (1.5, 4.4) and (4.5, −1.4) on the tangent.
Gradient $= \frac{-1.4 - 4.4}{4.5 - 1.5} = \frac{-5.8}{3} = -1.93$ **Ans.**

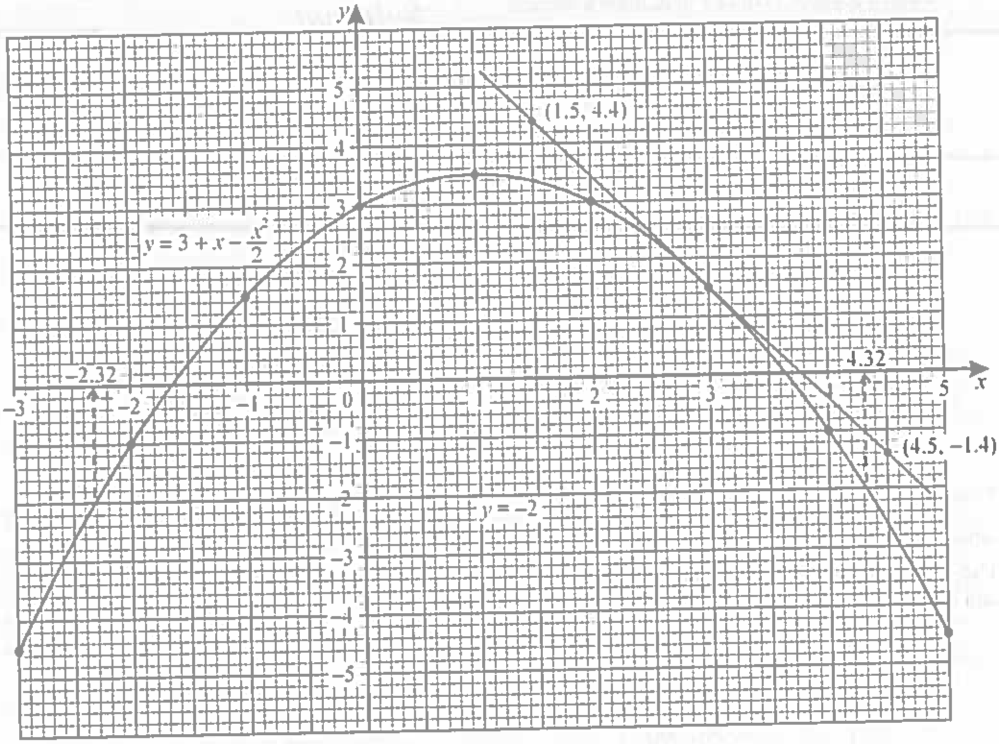
(iv) (a) $10 + 2x - x^2 = 0$
$\Rightarrow 2x - x^2 = -10$
$\Rightarrow 2(x - \frac{x^2}{2}) = -10$
$\Rightarrow x - \frac{x^2}{2} = -5$
$\Rightarrow 3 + x - \frac{x^2}{2} = -5 + 3$
$\Rightarrow y = -2$
$\therefore k = -2$ **Ans.**

(b) From graph, at $y = -2$,
$x = -2.32$ or 4.32 **Ans.**

(b) (i) $y = pa^x$
substitute (0, 4) into the equation,
$4 = pa^0 \Rightarrow p = 4$ **Ans.**

(ii) $y = pa^x$
substitute (2, 36) and the value of p.
$36 = 4a^2$
$\Rightarrow a^2 = 9 \Rightarrow a = \pm 3$
given that $a > 0$, $\therefore a = 3$ **Ans.**

(iii) The equation now becomes, $y = 4(3)^x$
Substitute (4, q) into the equation,
$q = 4(3)^4 \Rightarrow q = 324$ **Ans.**



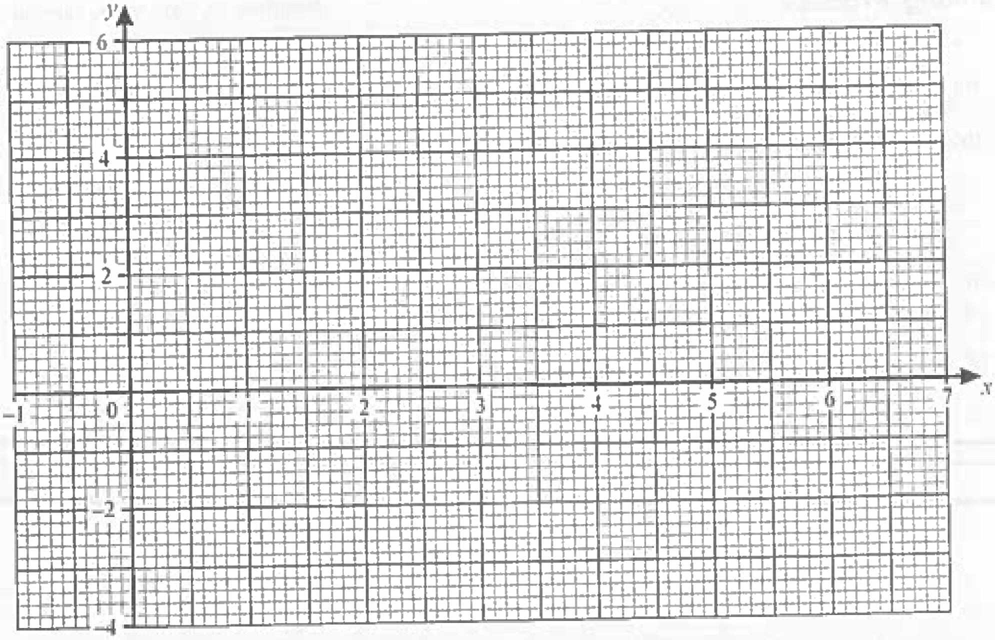
25 *(J2018/P2/Q6)*

(a) Complete the table for $y = \frac{x^2}{2} - 3x + 2$.

x	−1	0	1	2	3	4	5	6	7
y		2	−0.5	−2	−2.5	−2	−0.5	2	

[1]

(b) Draw the graph of $y = \frac{x^2}{2} - 3x + 2$ for $-1 \le x \le 7$.



[3]

(c) By drawing a tangent, estimate the gradient of the curve at $x = 1.5$. [2]

(d) Complete these inequalities to describe the range of values of x where $y \geq 0$.

$x \leq$ .... $x \geq$ ....

[2]

(e) (i) On the same grid, draw the line $4y + 3x = 12$. [2]

(ii) The x-coordinates of the points of intersection of this line and the curve are the solutions of the equation $2x^2 + Ax + B = 0$. Find the value of A and the value of B. [2]

Thinking Process

(a) Substitute $x = -1$ and $x = 7$ into the equation to obtain the missing values respectively.
(c) Draw a tangent at $x = 1.5$. Take 2 points on the tangent and calculate the gradient.
(d) To describe the range of find the intersection points where the curve meets the x-axis.
(e) (ii) Substitute the equation of curve into equation of line. Rearrange the resulting equation in the form $2x^2 + Ax + B = 0$. Find the values of A and B by comparison.

Solution

(a) When $x = -1$

$y = \frac{(-1)^2}{2} - 3(-1) + 2$

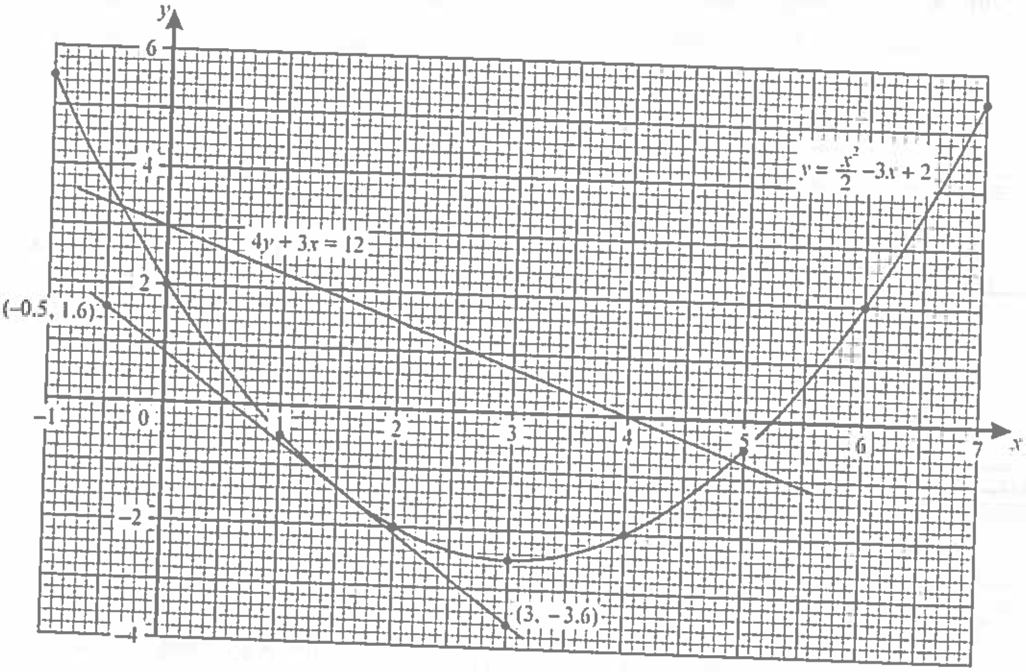
$= \frac{1}{2} + 5 = 5.5$ **Ans.**

When $x = 7$

$y = \frac{(7)^2}{2} - 3(7) + 2$

$= \frac{49}{2} - 21 + 2 = 5.5$ **Ans.**

(b)



(c) Taking two points $(-0.5, 1.6)$ and $(3, -3.6)$ on the tangent.

gradient $= \frac{-3.6 - 1.6}{3 - (-0.5)}$

$= \frac{-5.2}{3.5} = -1.49$ (3sf) **Ans.**

(d) $x \leq 0.78$, $x \geq 5.22$ **Ans.**

(e) (i) Refer to graph.

(ii) Equation of curve: $y = \frac{x^2}{2} - 3x + 2$

Equation of line: $4y + 3x = 12$

Substitute curve into line.

$4\left(\frac{x^2}{2} - 3x + 2\right) + 3x = 12$

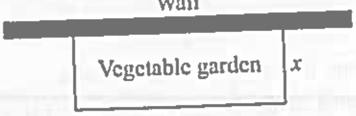
$\Rightarrow 2x^2 - 12x + 8 + 3x = 12$

$\Rightarrow 2x^2 - 9x - 4 = 0$

$\therefore A = -9, \quad B = -4$ **Ans.**

26 (N2018/P2 Q6)

Zara fences off a piece of land next to a wall to make a vegetable garden.



The garden is a rectangle with the wall as one side of the rectangle.
The area of the garden is 18 square metres.
The width of the garden is x metres.

(a) The total length of fencing required for the garden is y metres.

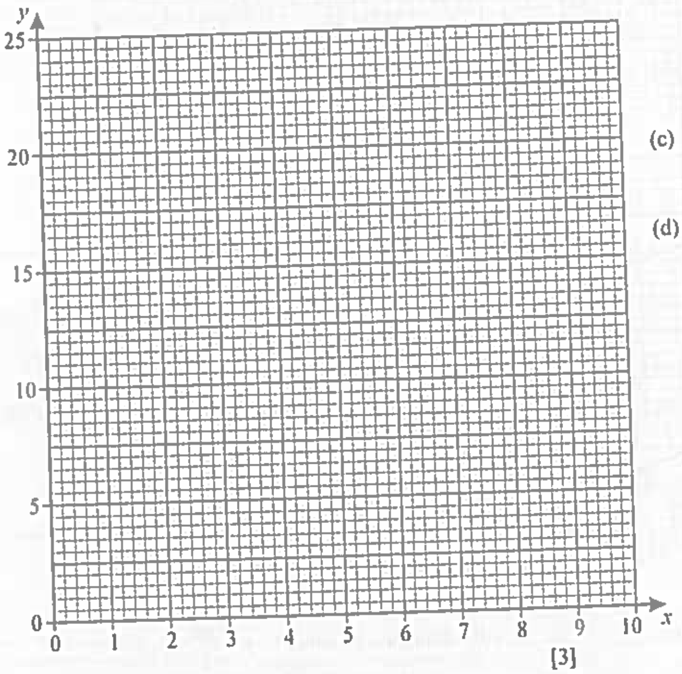
Show that $y = 2x + \frac{18}{x}$. [1]

(b) (i) Complete the table for $y = 2x + \frac{18}{x}$.

x	1	2	3	4	5	6	7	8	9
y			12	12.5	13.6	15	16.6	18.3	

[2]

(ii) On the grid, draw the graph of $y = 2x + \frac{18}{x}$ for $1 \le x \le 9$.



[3]

(c) Use your graph to find the two possible widths of the garden if 14 metres of fencing is used. [2]

(d) The fencing costs $20 per metre.

(i) Find the minimum amount it will cost Zara to build the fence. [2]

(ii) Zara wants to spend no more than $350 on the fence.
Find the greatest possible width of the garden Zara can make. [2]

Thinking Process

(a) Apply the formula area $= l \times w$ and express the length in terms of x. Then add up all three sides

(c) Find the two corresponding values of x when $y = 14$ m

(d) (i) From graph, read the value of x when y is minimum. Substitute it into the equation of y to find length of fencing. Multiply by $20 to find the cost.

(ii) One metre costs $20. Find how many metres costs $350.

Solution

(a) Area of garden = length × width

$$18 = \text{length} \times x \Rightarrow \text{length} = \frac{18}{x} \text{ m.}$$

Fencing is required on three sides of the garden.

∴ total length of fencing is.

$$y = x + x + \frac{18}{x} \Rightarrow y = 2x + \frac{18}{x} \quad \textbf{Shown.}$$

(b) (i) When $x = 1$, $y = 2(1) + \frac{18}{1} = 20$

When $x = 2$, $y = 2(2) + \frac{18}{2} = 13$

When $x = 9$, $y = 2(9) + \frac{18}{9} = 20$

(ii) Refer to graph on next page.

(c) From graph, when $y = 14$ m,

width $x = 1.65$ m or $x = 5.3$ m **Ans.**

(d) (i) From graph, least value of $y = 12$ m

So, minimum length required for fencing = 12 m

∴ Cost to build the fence $= 12 \times \$20$
$= \$240$ **Ans.**

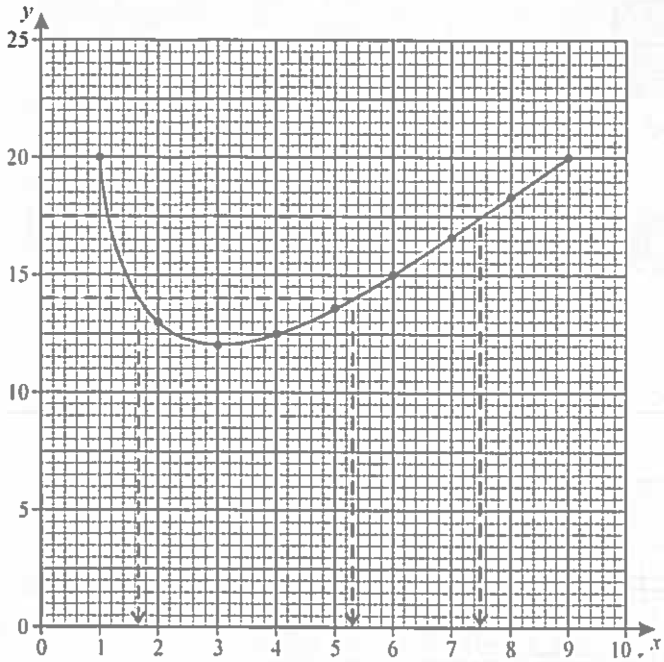
(ii) $20 —— 1 m

$350 —— $\frac{1}{20} \times 350 = 17.5$ m

∴ 17.5 metres of fencing costs $350

From graph, when $y = 17.5$,
$x = 7.5$

∴ greatest possible width = 7.5 m **Ans.**

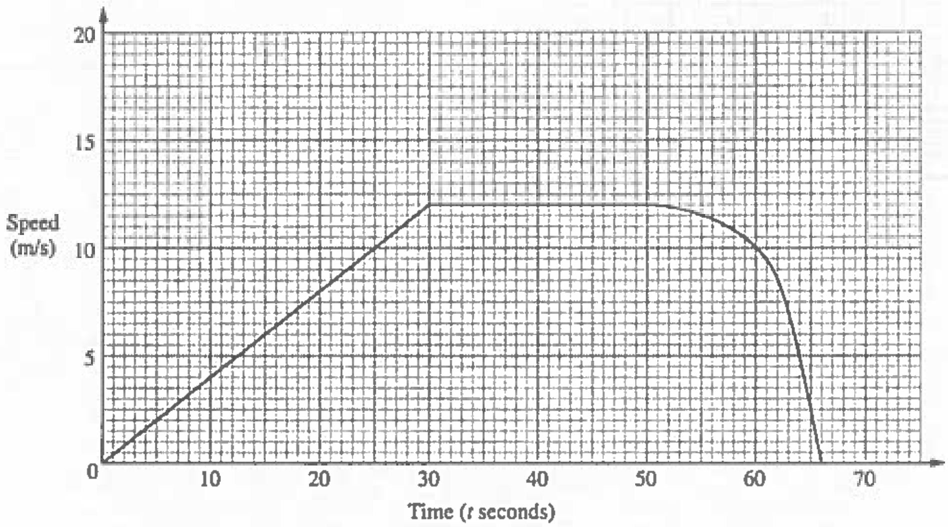

y
25
20
15
10
5
0
0 1 2 3 4 5 6 7 8 9 10
x

Topic 8

Graphs in Practical Situations and Travel Graphs

1 *(N2007/P1/Q22)*

The diagram is the speed-time graph of a cyclist's journey.



(a) Calculate the time taken to travel the first 300 metres. [2]

(b) By drawing a tangent, find the retardation of the cyclist when $t = 55$. [2]

Thinking Process

(a) By inspection look for the time whose area under graph (distance) is 300 m.

(b) Find the gradient of the tangent at $t = 55$.

Solution

(a) Distance travelled in first 30 seconds

$= \frac{1}{2} \times 30 \times 12 = 180$ m

remaining distance $= 300 - 180 = 120$ m

let 120 m be covered in t seconds.

$\therefore$ distance = area under graph

$120 = (t - 30)(12)$

$10 = t - 30$

$t = 40$ seconds

$\therefore$ time taken to travel 300 m

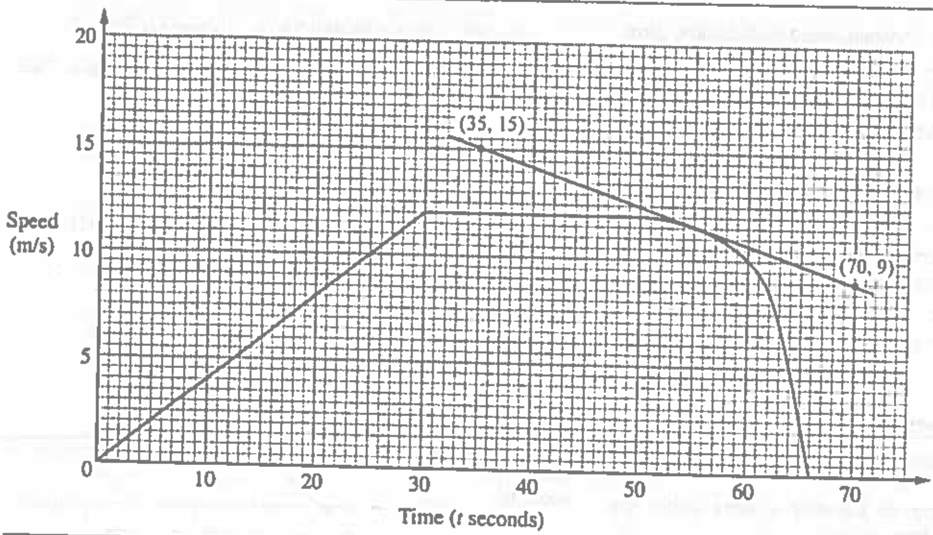
$= 40$ seconds **Ans**

> Note that area under graph from 30 seconds to t seconds is a rectangle.

(b) From graph (*next page*)

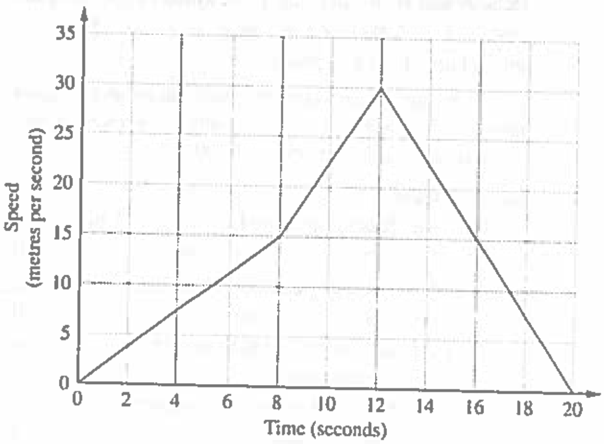
gradient of the tangent $= \frac{15 - 9}{35 - 70} = -\frac{6}{35}$

$\therefore$ retardation $= \frac{6}{35}$ m/s^2 **Ans**



2 *(J2008/P1/Q13)*

The diagram is the speed-time graph of the first 20 seconds of a motorcyclist's journey.



(a) Calculate the motorcyclist's retardation during the final 8 seconds. [1]

(b) Calculate the distance travelled in the 20 seconds. [2]

Thinking Process

(a) To find retardation ⇨ find the gradient of the line during the last 8 seconds.

(b) To find the distance ⇨ find the area under the graph from $t = 0$ to $t = 20$ seconds.

Solution with TEACHER's COMMENTS

(a) Retardation $= \dfrac{30-0}{8} = 3.75\text{ m/s}^2$ **Ans.**

Since retardation of the motorcyclist is constant
$\therefore$ retardation $= \dfrac{\text{change in speed}}{\text{time taken for the retardation to occur}}$

(b) Total distance

$= \left(\frac{1}{2}\times 8\times 15\right)+\left(\frac{1}{2}\times 4(15+30)\right)+\left(\frac{1}{2}\times 8\times 30\right)$

$= (4\times 15)+2(45)+(4\times 30)$

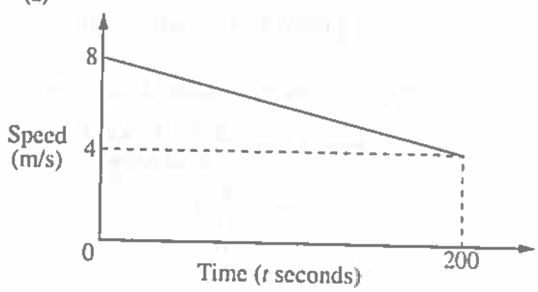
$= 60+90+120$

$= 270\text{ m}$ **Ans.**

Note that total distance = area of Δ from $t = 0$ to $t = 8$
+ area of trapezium from $t = 8$ to $t = 12$
+ area of Δ from $t = 12$ to $t = 20$

3 *(J2009/P2/Q5)*

(a)



Ali was on a training run.
The diagram is the speed-time graph of part of his run.
At $t = 0$, his speed was 8 m/s.
His speed decreased at a constant rate until it was 4 m/s at $t = 200$.

(i) Calculate

(a) his retardation during the 200 s. [1]

(b) the distance he ran during the 200 s, [2]

(c) his speed at $t = 150$. [1]

(ii) Ben ran at a constant speed in the same direction as Ali.
At $t = 0$, Ali and Ben were level.
They ran the same distance in the next 150 seconds.
Calculate Ben's speed. [2]

(b) Chris ran 200 m, correct to the nearest 10 metres.
He took 25 s, correct to the nearest second.
Find lower bounds for
(i) the distance run, [1]
(ii) his average speed. [3]

Thinking Process

(a) (i) (a) Retardation = gradient of speed-time graph.
(b) Distance travelled = area under the speed-time curve.
(c) To find speed when $t = 150$ s use

$$\text{acceleration} = \frac{\text{change in speed}}{\text{time taken}}$$

(ii) Find the distance travelled by Ali and Ben in 150 seconds. Observe that both travelled same distance in 150 seconds.

(b) (i) To find lower bound for the distance subtract $5\,(10 \div 2 = 5)$ from the given distance.
(ii) To find lower bound for the average speed subtract 5m from 200m and add 0.5s to 25s.

Solution

(a) (i) (a) Retardation $= \frac{8-4}{200} = 0.02\text{ m/s}^2$ Ans.

(b) Distance covered in 200s
= area of trapezium
$= \frac{1}{2}(200)(8+4) = 1200$ m Ans.

(c) Let x be the speed at 150 seconds,

$$\text{Retardation} = \frac{\text{change in speed}}{\text{time taken}}$$

$$0.02 = \frac{8-x}{150}$$
$$3 = 8 - x$$
$$x = 5$$

∴ speed at 150s = 5 m/s Ans.

(ii) Distance covered by Ali in 150s
$= \frac{1}{2}(150)(5+8) = 975$ m

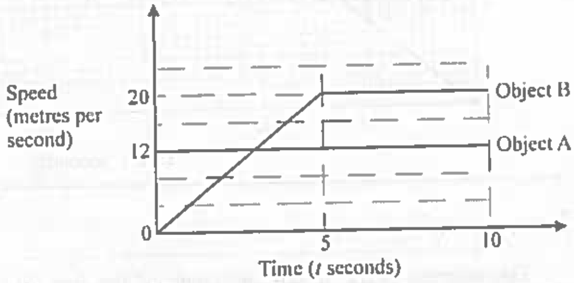
Let speed of Ben $= v$ m/s
Distance covered by Ben in 150s $= 150v$ m
since both ran the same distance.
$\Rightarrow 150v = 975$
$v = 6.5$

∴ speed of Ben = 6.5 m/s Ans.

(b) (i) Lower bound for the distance $= 200 - 5$
$= 195$ m Ans.

(ii) L. bound for av. speed $= \frac{200-5}{25+0.5}$
$= \frac{195}{25.5}$
$= 7.65$ m/s (3sf) Ans.

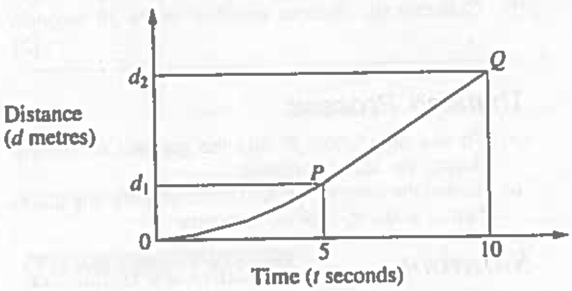
4 *(N2009/P2 Q11)*



The diagram shows the speed-time graphs of two objects, A and B, for the first 10 seconds of their motion.
Object A travelled at a constant speed of 12 m/s throughout the 10 seconds.
Object B started from rest, and accelerated at a constant rate, attaining a speed of 20 m/s after 5 seconds. It then travelled at a constant speed of 20 m/s.

(a) Calculate
(i) the distance travelled by object B during the first 5 seconds of its motion, [1]
(ii) the average speed of object B for the first 10 seconds of its motion, [2]
(iii) the value of t when both objects were travelling at the same speed, [2]
(iv) the value of t when both objects had travelled the same distance. [2]

(b) The diagram below shows the distance-time graph for object B.



In the diagram, OP is a curve and PQ is a straight line.
(i) State the values of d_1 and d_2. [1]
(ii) What does the gradient of the straight line PQ represent? [1]

(iii) Write down the gradient of the tangent to the curve at $t = 2.5$. [1]

(c) After 10 seconds, both objects slowed down at the same constant rate.
Object A came to rest after a further 9 seconds. After How many seconds from the start of its motion did object B come to rest? [2]

Thinking Process

(a) (i) Calculate the area under the curve of object B for the first 5 seconds.
(ii) Compute the area under the curve of object B for the first 10 seconds.
(iii) Find the intercept of the two speed curves. The time at which the interception occurs is the answer to the question.
(iv) Let x be the time at which both objects have travelled the same distance and solve $x (x \geq 5)$ by equating the distance travelled by objects A and B.

(b) (i) d_1 is the distance travelled in 5 seconds by object B. d_2 is the total distance covered by object B in 10 seconds.
(ii) Gradient of PQ is the constant speed of the object B.
(iii) Write down the speed of B at $t = 2.5$ s.

(c) Find the deceleration of object A. Use it to compute the time taken by object B to come to rest.

Solution

(a) (i) Distance travelled by object B for the first 5 seconds $= \frac{1}{2}(5)(20) = 50$ m **Ans.**

(ii) Distance travelled by object B for the first 10 seconds $= \frac{1}{2}(5)(20) + (5)(20)$
$= 50 + 100 = 150$ m

∴ average speed of object B for the first 10 seconds $= \frac{150}{10} = 15$ m/s **Ans.**

(iii) Let t be the time at which object B is at the same speed as object A. Using concept of similar triangles, we have

$$\frac{t}{5} = \frac{12}{20}$$

$$t = \frac{12}{20} \times 5 = 3 \text{ s.}$$

∴ both objects had same speed at $t = 3$ seconds. **Ans.**

(iv) If t is the time at which objects A and B have travelled the same distance, we have,

distance travelled by object $A = 12t$

distance travelled by object B

$$= \frac{1}{2}(20)(t + (t - 5))$$

$$= 10(2t - 5) = 20t - 50$$

equating the distance travelled by both objects.

$$20t - 50 = 12t$$

$$8t = 50$$

$$t = 6.25 \text{ s}$$

∴ both objects had covered same distance at $t = 6.25$ seconds. **Ans.**

(b) (i) $d_1 = 50$ m **Ans.**

$d_2 = 150$ m **Ans.**

> d_1 and d_2 are the distances travelled by B in 5 and 10 seconds. In part (a) (i) & (ii), these distances have already been calculated.

(ii) Gradient of PQ represents the constant speed of the object B from 5 to 10 seconds. **Ans.**

(iii) From speed-time graph,
when, $t = 2.5$, speed of $B = 10$ m/s.

∴ gradient of distance-time graph at $t = 2.5$ seconds, is 10 m/s. **Ans.**

(c) Deceleration of object $A = \frac{12 - 0}{9} = \frac{4}{3}$ m/s^2

Let object B comes to rest at t seconds. As both objects slowed down at the same constant rate.

∴ deceleration of object $B = \frac{4}{3}$ m/s^2

$$\Rightarrow \quad \frac{4}{3} = \frac{20 - 0}{t - 10}$$

$$4(t - 10) = 60$$

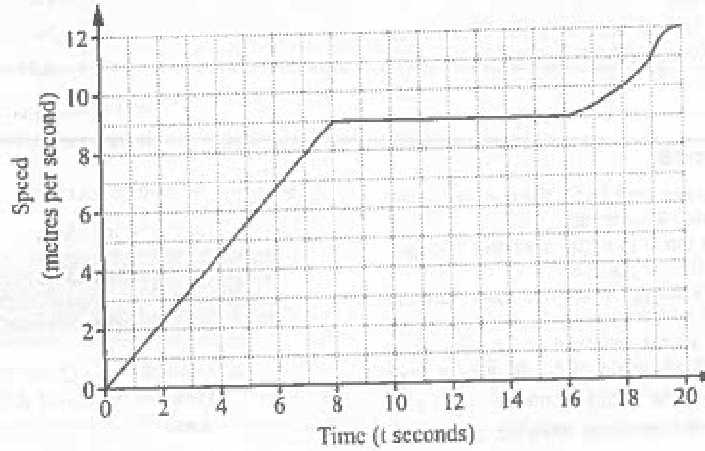
$$t - 10 = 15$$

$$t = 25 \text{ s.}$$

∴ B comes to rest at $t = 25$ seconds. **Ans.**

5 *(J2010/P1/Q25)*

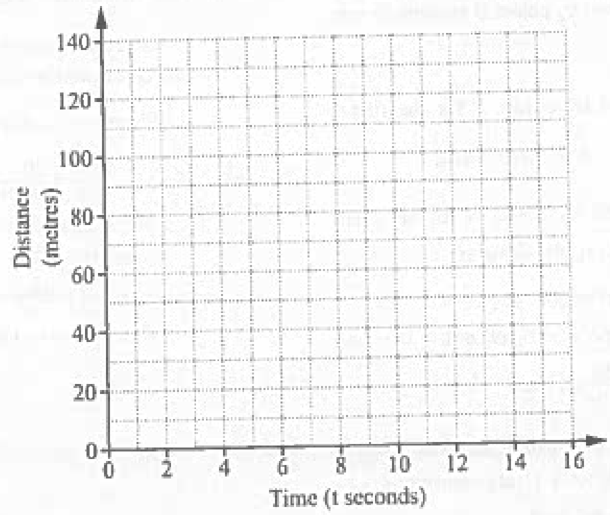
The diagram is the speed-time graph for the first 20 seconds of a cyclist's journey.



(a) Calculate the distance travelled in the first 16 seconds. [1]

(b) By drawing a tangent, find the acceleration of the cyclist when $t = 18$. [2]

(c) On the grid in the answer space, sketch the distance-time graph for the first 16 seconds of the cyclist's journey.



[2]

Thinking Process

(a) Find the area under the graph over the time-frame specified.
(b) Draw a tangent at t = 18. Take two points on the tangent and find the gradient.
(c) To sketch the distance-time graph consider the distances travelled in the first 8 seconds and the last 8 seconds separately.

Solution

(a) Distance covered in the first 16 second

$= \text{area of trapezium}$

$= \frac{1}{2}(9)(16+8)$

$= 108 \text{ m}$. **Ans.**

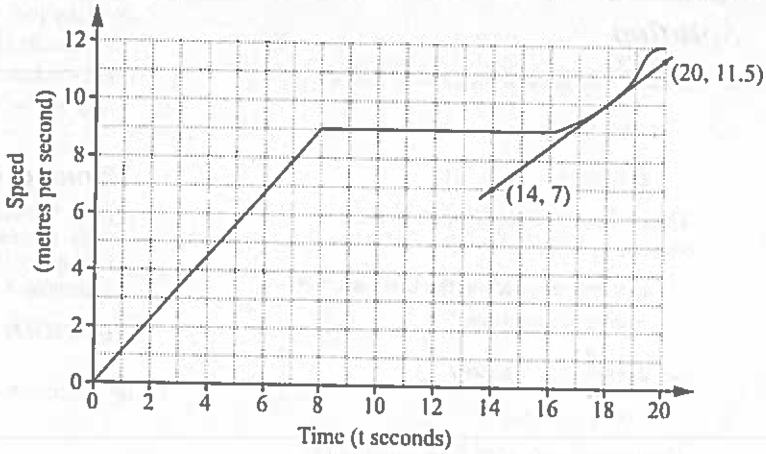
(b) Acceleration

$= \text{gradient of the tangent.}$

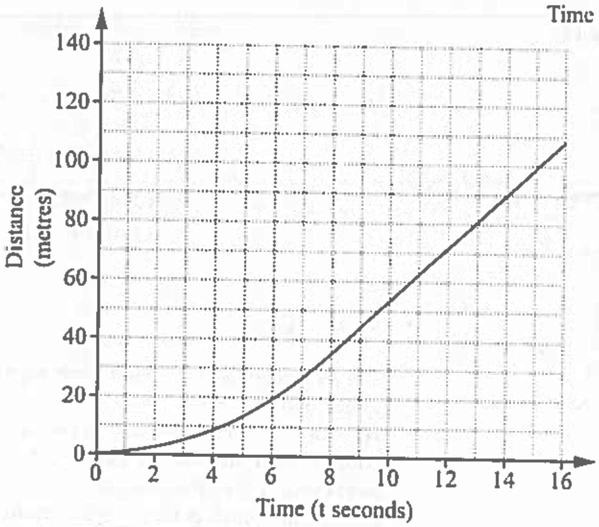
$= \frac{11.5-7}{20-14}$

$= \frac{4.5}{6}$

$= 0.75 \text{ m/s}^2$ **Ans.**



(c)



Note that
Distance covered in first 8 seconds
$= \frac{1}{2} \times 8 \times 9 = 36$ m.
Distance covered in the last 8 seconds
$= 8 \times 9 = 72$ m.

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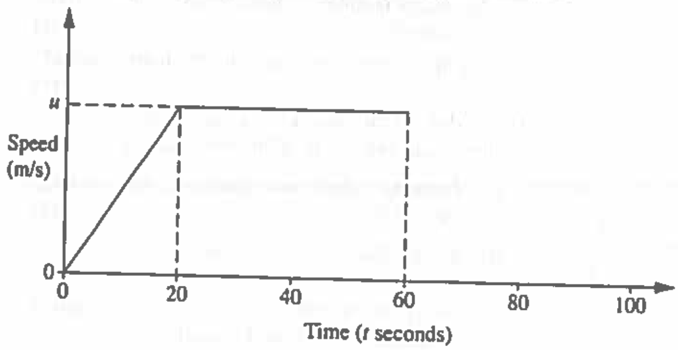
6 *(N2010/P1/Q25)*



The diagram is the speed-time graph of part of the journey of a car.
From $t = 0$ to $t = 20$ the car moves with a constant acceleration.
From $t = 20$ to $t = 60$ the car moves with a constant speed of u metres per second.

(a) When $t = 20$ the car has travelled D metres from the start.
Calculate the value of t when the car has travelled $2D$ metres from the start. [2]

(b) At $t = 60$, the car slows down with a constant deceleration.
This deceleration is half of the acceleration between $t = 0$ and $t = 20$.
During this period of deceleration, calculate the value of t when the car has a speed of $\frac{u}{4}$ metres per second. [2]

Thinking Process

(a) Find distance D in terms of u by taking area under the graph for the first 20 seconds. Find the second distance $2D$ for the first t seconds. Equate both distances for t.

(b) Use deceleration $= \frac{\text{change in speed}}{\text{time taken}}$ and solve for t according to the given information.

Solution

(a) Distance travelled in the first 20 seconds,

$=\frac{1}{2}(20)(u)=10u$

$\therefore \quad D=(10u)$ m ·········(1)

The car has travelled $2D$ metres at time t seconds from the start.

$\therefore$ Distance travelled in the first t seconds = area of trapezium

$\Rightarrow \quad 2D=\frac{1}{2}(u)(t+t-20)$

$2D=(u)(t-10)$

substituting value of D from equation (1),

$2(10u)=(u)(t-10)$

$20=t-10$

$t=30$ seconds **Ans.**

(b) Acceleration during 0 to 20 seconds $=\frac{u}{20}$ m/s^2

Given that at time t seconds, the speed of the car decreases from u m/s to $\frac{u}{4}$ m/s.

$\therefore$ deceleration at t seconds $=\frac{u-\frac{u}{4}}{t-60}$ m/s^2

given that this deceleration is half of acceleration.

$\Rightarrow \quad \frac{1}{2}\left(\frac{u}{20}\right)=\frac{u-\frac{u}{4}}{t-60}$

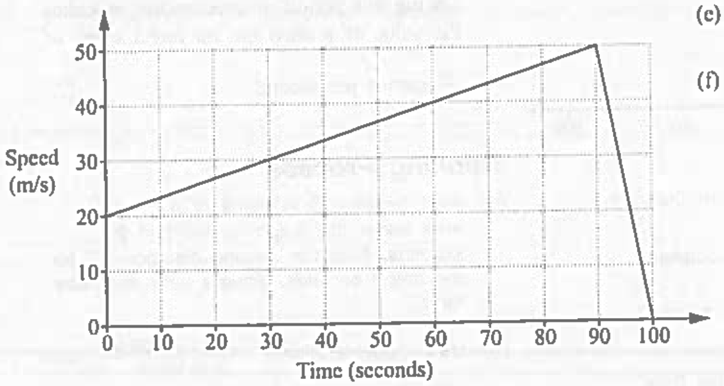
$\frac{u}{40}=\frac{4u-u}{4(t-60)}$

$4u(t-60)=3u(40)$

$t-60=30$

$t=90$ seconds **Ans.**

7 *(N2011/P1/Q21)*



The diagram is the speed-time graph of the last 100 seconds of a train's journey.

(a) Calculate the train's retardation during the last 10 seconds of the journey. [1]

(b) Calculate the distance travelled in the 100 seconds. [2]

Thinking Process

(a) To find retardation ⇒ divide the decrease in speed by the time taken.

(b) Distance travelled in 100 seconds = area of trapezium + area of triangle.

Solution

(a) Retardation $=\frac{50}{100-90}$

$=\frac{50}{10}=5$ m/s^2 **Ans.**

(b) Distance travelled $=\frac{1}{2}(90)(20+50)+\frac{1}{2}(10)(50)$

$=(45)(70)+\frac{1}{2}(500)$

$=3150+250$

$=3400$ m **Ans.**

8 *(J2011/P2 Q5)*

The distance-time graph (*on next page*) shows Ravi's cycle journey.
He sets out from home and cycles to a park.
After a short stop at the park, he then continues his journey to a shopping centre.
He stops for lunch at the shopping centre before cycling home.

(a) At what time does Ravi arrive at the park? [1]

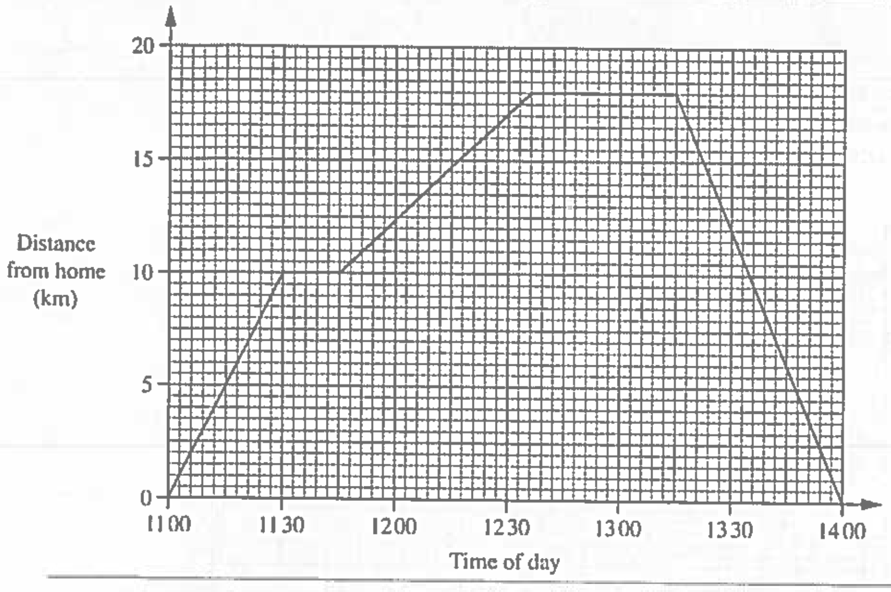
(b) How many minutes does Ravi spend at the shopping centre? [1]

(c) How far is the park from the shopping centre? [1]

(d) At what speed does Ravi cycle home? Give your answer in kilometres per hour. [1]

(e) Between which two places did Ravi cycle slowest? [1]

(f) Salim, Ravi's brother, sets out from home at 1115.
He cycles directly to the shopping centre at a constant speed of 15 km/h.
Who arrives at the shopping centre first?
How many minutes later does his brother arrive? [2]



Thinking Process

(a) Read the time from the graph.
(b) Read from the graph, the values of time where the line is horizontal.
(c) Use graph to find the required distance.
(d) Speed = distance divided by time.
(e) Ravi is slowest where the gradient of graph is least, i.e. between 1145 to 1236.
(f) On the graph, draw a straight line, starting at 1115, to show salim's journey from home to shopping centre.

Solution

(a) From graph, the time when Ravi arrives at the park = 1130 **Ans.**

(b) From graph, the time spent at the shopping centre = 1315 − 1236
= 1275 − 1236
= 39 min **Ans.**

(c) From graph, distance between park and the shopping centre = 18 − 10 = 8 km **Ans.**

(d) Time to travel from shopping centre to home = 45 minutes = 0.75 hours.

$\therefore$ Speed $= \frac{\text{distance}}{\text{time}}$

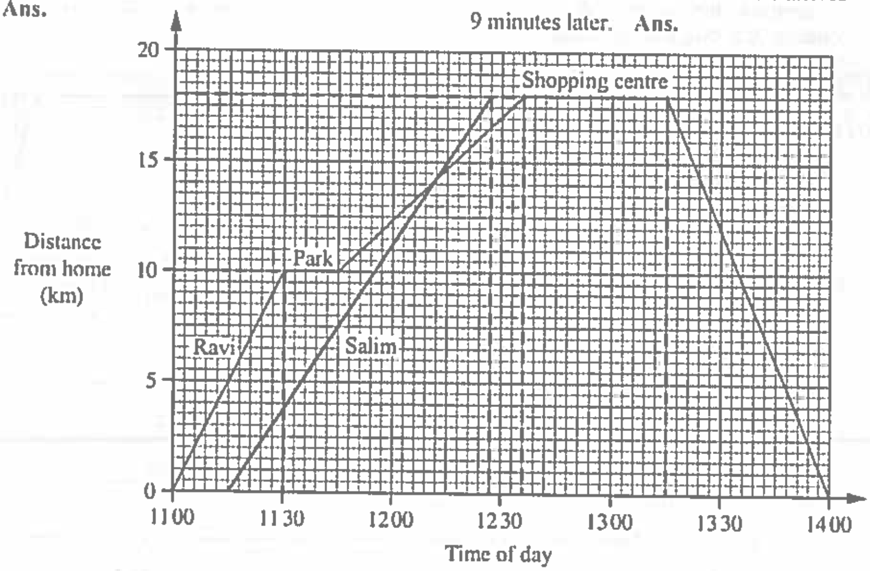
$= \frac{18}{0.75} = 24$ km/h **Ans.**

(e) Ravi was slowest between Park and Shopping centre. **Ans.**

Note that the graph has least gradient between park and shopping centre. Therefore the speed is slowest between these two points.

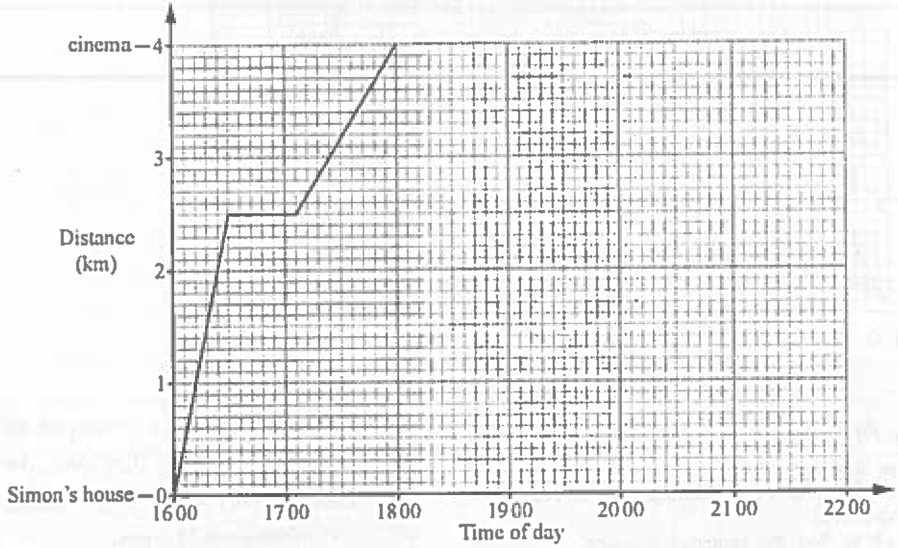
(f) From graph, Salim arrives shopping centre at 1227
Ravi arrives shopping centre at 1236

$\therefore$ Salim arrives first and his brother arrives 9 minutes later. **Ans.**



9 *(J2012/P2/Q3)*

Simon walks from his house to Juan's house.
He stays there for a short while before they walk together to the cinema.
The graph represents the journey from Simon's house to the cinema.



(a) For how many minutes does Simon stay at Juan's house? [1]

(b) At what speed does Simon walk to Juan's house? [1]

(c) Simon has a 15% discount voucher for his cinema ticket but Juan pays the full price. Simon pays \$4.42 for his ticket. How much does Juan pay? [2]

(d) They stay at the cinema for 2 hours before they each walk home at 3 km/h. Complete the graph to show this information. [2]

(e) At what time do they arrive at Juan's house? [1]

Thinking Process

(a) Read from graph.

(b) Find the gradient of the line from 1600 to 1630.

(c) 85% represent \$4.42. Find 100%.

(d) To complete the graph consider the distance travelled in one hour.

(e) Read from graph.

Solution

(a) 36 minutes.

(b) Distance travelled

= 2.5 km

time taken

= 30 minutes = 0.5 hour

$\therefore$ speed $= \frac{2.5}{0.5}$

= 5 km/h **Ans.**

(c) 85% —— \$4.42

100% —— \$ $\frac{4.42}{85} \times 100$

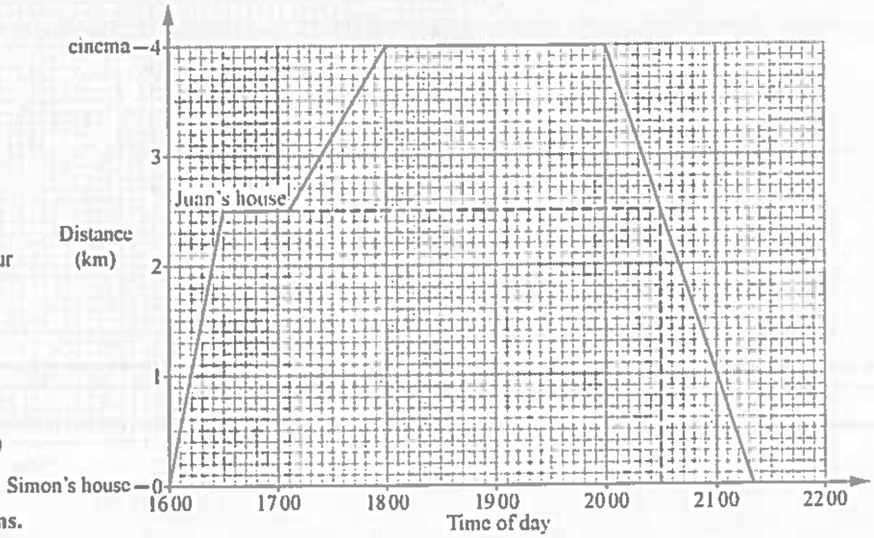
—— \$5.20

$\therefore$ Juan pays \$5.20 **Ans.**

(d) Refer to graph.

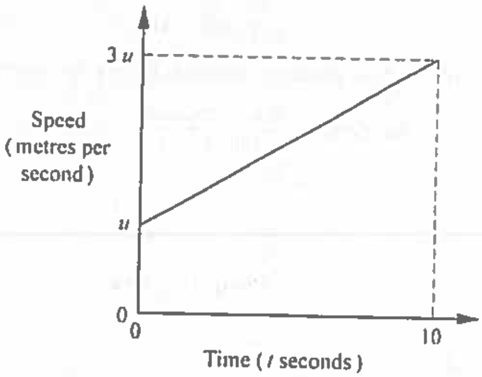
(e) From graph,

they arrive at Juan's house at 2030 **Ans.**



10 *(N2012/P1/Q25)*

The diagram is the speed-time graph of a cyclist. The cyclist accelerates uniformly from a speed of u metres per second to a speed of $3u$ metres per second in a time of 10 seconds.

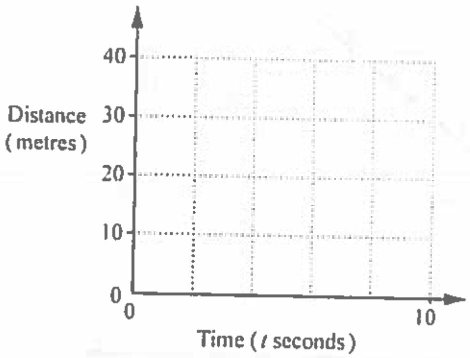


(a) Find an expression, in terms of u, for the acceleration. [1]

(b) The distance travelled by the cyclist from $t = 0$ to $t = 10$ is 40 m.

(i) Find the value of u. [2]

(ii) On the grid below, sketch the distance-time graph of the cyclist.



[2]

Thinking Process

(a) acceleration = $\dfrac{\text{change in speed}}{\text{time taken}}$.

(b) (i) Distance under a speed-time graph is the area under the graph.

(ii) From $t = 0$ to $t = 10$, the speed is increasing, hence it is represented by a curve with increasing gradient.

Solution

(a) Acceleration $= \dfrac{3u - u}{10}$

$= \dfrac{2u}{10}$

$= \dfrac{u}{5}$ m/s² **Ans.**

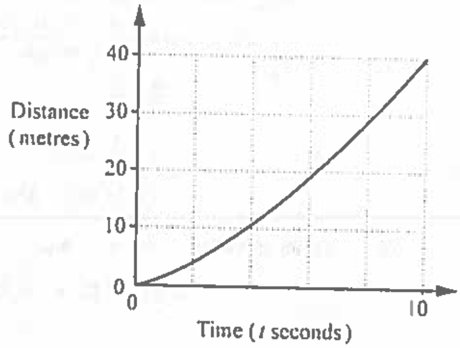
(b) (i) Distance travelled = area of trapezium

$40 = \dfrac{1}{2}(10)(u + 3u)$

$40 = 20u$

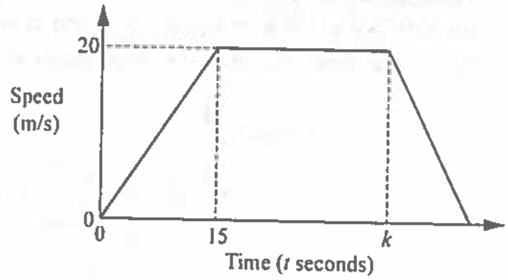
$u = 2$ **Ans.**

(ii)



11 *(N2013/P1/Q25)*

The diagram is the speed-time graph of a car's journey.



(a) Find the speed when $t = 12$. [1]

(b) Find the distance travelled by the car from $t = 0$ to $t = 15$. [1]

(c) The distance travelled by the car from $t = 0$ to $t = k$ is 750 m.
Find k. [2]

(d) The retardation of the car is 2 m/s². Find the number of seconds it takes to slow down and stop. [1]

Thinking Process

(a) To find the speed when $t = 12$ find acceleration for the 1st 15 seconds.

(b) Find the area under graph over the interval from $t = 0$ to $t = 15$.

(c) Find the area under the graph from $t = 0$ to $t = 15$ and equate it to 750 m.

(d) Use retardation = $\dfrac{\text{change in speed}}{\text{time taken}}$.

Solution

(a) Acceleration from 0 to 15 seconds $= \frac{20}{15}$

$= \frac{4}{3}$ m/s^2

let x be the speed at 12 seconds.

$$\text{acceleration} = \frac{\text{change in speed}}{\text{time taken}}$$

$$\frac{4}{3} = \frac{x}{12}$$

$$x = \frac{4}{3} \times 12$$

$x = 16$ m/s Ans.

(b) Distance travelled = area of triangle

$= \frac{1}{2}(15)(20) = 150$ m Ans.

(c) Distance travelled = area of trapezium

$$750 = \frac{1}{2}(20)(k - 15 + k)$$

$$750 = 10(2k - 15)$$

$$75 = 2k - 15$$

$$2k = 90$$

$k = 45$ Ans.

(d) Let the car takes t seconds to slow down and stop

$$\text{retardation} = \frac{\text{change in speed}}{\text{time taken}}$$

$$2 = \frac{20}{t}$$

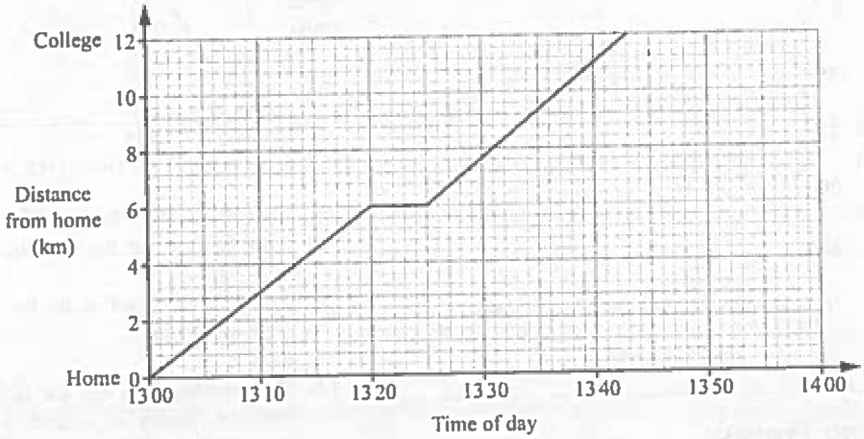
$$t = \frac{20}{2}$$

$t = 10$ seconds. Ans.

12 (J2013/P1/Q22)

Varun leaves home at 13 00 and cycles 12 km to college.
The distance-time graph below shows Varun's journey.
His sister Kiran leaves college at 13 10 and cycles home on the same road at a constant speed of 16 km/h.

(a) On the same grid, draw the distance-time graph for Kiran's journey.

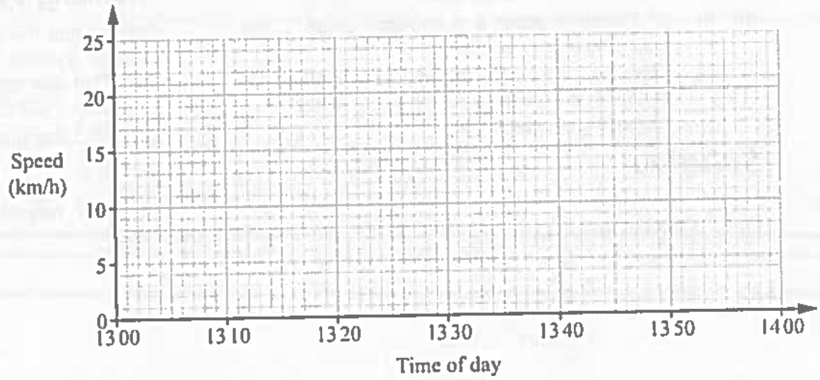


[2]

(b) How far was Kiran from home when she passed Varun? [1]

(c) Find Varun's speed for the first 20 minutes of his journey.
Give your answer in kilometres per hour. [1]

(d) On the grid, draw the speed-time graph for Varun's journey. [2]



Thinking Process

(a) Draw a straight line starting at 1310 from college to home. Note that Kiran travels 16 kilometres in one hour

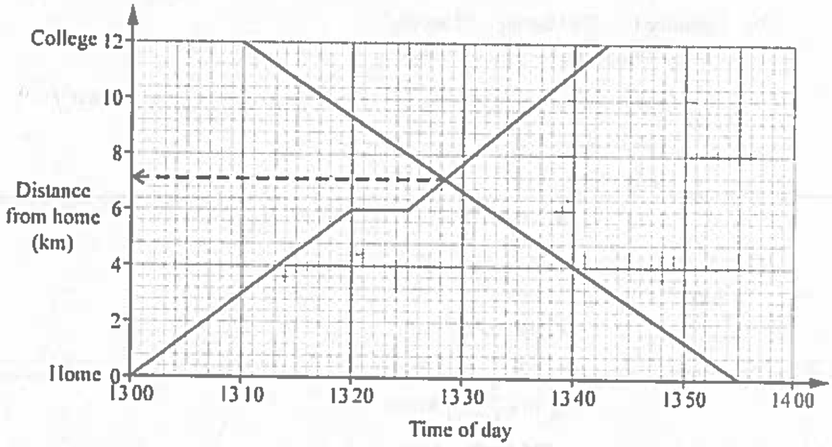
(b) ✎ Read the value from the graph where the two lines meet.

(c) Apply speed = $\frac{\text{distance}}{\text{time}}$.

(d) From 1300 to 1320, use answer from part (c). From 1320 to 1325, and from 1325 to 1343 use gradient of distance-time graph.

Solution

(a) Refer to graph.



(b) From graph, distance of Kiran from home = 7.1 km. **Ans.**

(c) 20 min = $\frac{20}{60} = \frac{1}{3}$ hours

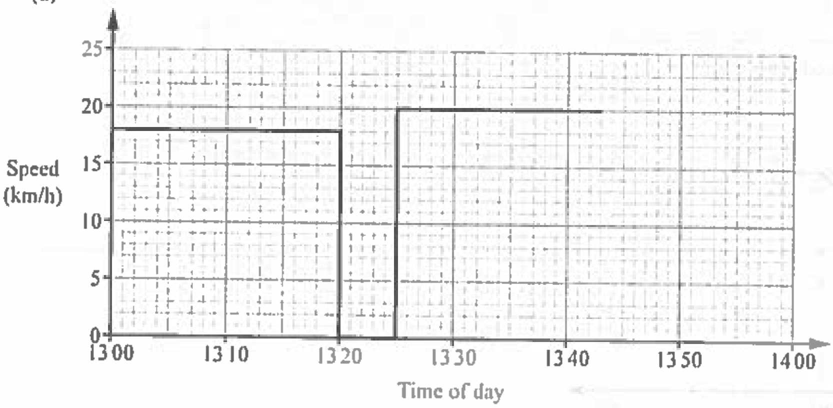
∴ Varun's speed

$= \frac{6}{\frac{1}{3}}$

$= 6 \times \frac{3}{1}$

= 18 km/h **Ans.**

(d)



From 1300 to 1320, speed = 18 km/h.

From 1320 to 1325, Varun has not covered any distance. So speed = 0 km/h.

From 1325 to 1343. time = 18 min = 0.3 h

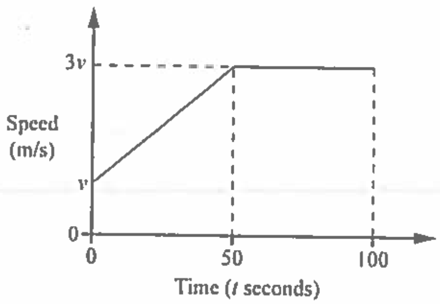
∴ speed = $\frac{6}{0.3}$ = 20 km/h.

13 *(J2014 P1 Q20)*

The diagram shows the speed-time graph for 100 seconds of a car's journey.

The car accelerates uniformly from a speed of v m/s to a speed of $3v$ m/s in 50 seconds.

It then continues at a constant speed.



(a) Find, in terms of v, the acceleration of the car in the first 50 seconds. [1]

(b) The car travels 2500 metres during the 100 seconds. Find v. [2]

(c) Find the speed of the car, in kilometres per hour, when $t = 75$. [1]

Thinking Process

(a) Acceleration = gradient of speed-time graph. Calculate gradient of line from $t = 0$ to $t = 50$.

(b) Total distance travelled in a speed-time graph = area under speed-time graph.

(c) To find the speed in km/h ✎ first find the speed in m/s. Note that 1 km = 1000 m, and 1 hour = 3600 seconds.

Solution

(a) Acceleration in first 50 seconds $= \frac{3v - v}{50}$

$= \frac{2v}{50}$

$= \frac{v}{25}$ m/s² **Ans.**

(b) Distance travelled during 100 second

= area of trapezium + area of rectangle

$\Rightarrow \quad 2500 = \frac{1}{2}(50)(v + 3v) + (50 \times 3v)$

$2500 = 100v + 150v$

$2500 = 250v$

$v = 10$ m/s **Ans.**

(c) When $t = 75$ seconds,

speed of the car $= 3v$ m/s

$= 3(10)$ m/s

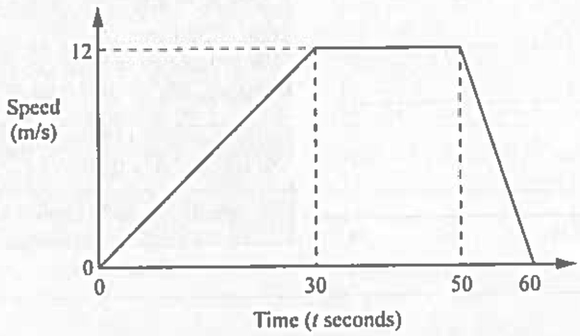
$= 30$ m/s

$= \left(30 \times \frac{3600}{1000}\right)$ km/h

$= 108$ km/h **Ans.**

14 *(N2014/P1/Q22)*

The diagram shows the speed-time graph of a cyclist's journey.



(a) Find the retardation. [1]

(b) Find the speed when $t = 9$. [1]

(c) Find the distance travelled by the cyclist from $t = 0$ to $t = 60$. [2]

Thinking Process

(a) To find retardation ✐ calculate the gradient of the line from $t = 50$ to $t = 60$.

(b) To calculate the speed when $t = 9$ ✐ calculate the gradient of the graph for the 1st 30 seconds.

(c) To find the distance ✐ find the area under the graph from $t = 0$ to $t = 60$.

Solution

(a) Retardation $= \frac{12}{60 - 50}$

$= \frac{12}{10} = 1.2$ m/s² **Ans.**

(b) Acceleration from 0 to 30 seconds $= \frac{12}{30}$

$= \frac{2}{5}$ m/s²

when $t = 9$, speed $= \frac{2}{5} \times 9$

$= \frac{18}{5} = 3.6$ m/s **Ans.**

(c) Distance travelled = area of trapezium

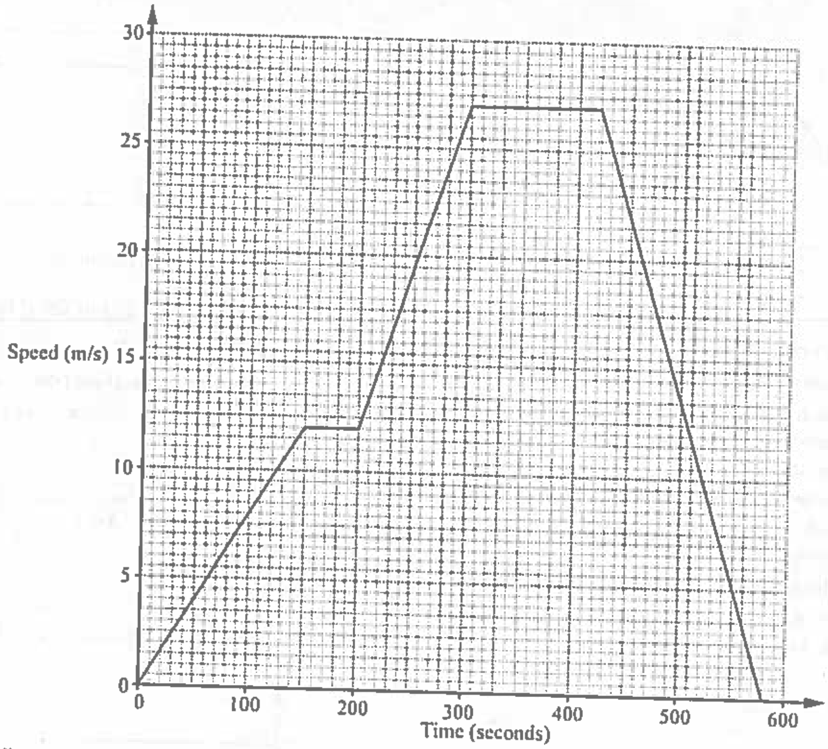
$= \frac{1}{2}(12)(20 + 60)$

$= 6(80)$

$= 480$ m **Ans.**

15 (J2015/P2 Q2)

The diagram is a speed-time graph of a train's journey between two stations.



(a) What was the maximum speed of the train? [1]

(b) Circle the statement that describes the train's motion 350 seconds after it left the first station.

Accelerating   Decelerating   Constant speed   Stopped at a station [1]

(c) Calculate the acceleration of the train during the first 150 seconds of its journey. [1]

(d) What was the speed of the train 20 seconds before it completed its journey? [1]

(e) How far did the train travel during the first 200 seconds? [2]

(f) Calculate the average speed of the train in kilometres per hour during the first 200 seconds. [2]

Thinking Process

(b) Note that from 300 to 420 seconds, the speed of the train is constant.

(c) Acceleration = $\frac{\text{change in speed}}{\text{time taken}}$

(d) Read the required speed from graph.

(e) Distance travelled in a speed-time graph = area under the speed-time graph.

(f) Distance travelled during the first 200 seconds divided by 200 seconds gives the average speed.

Solution

(a) Maximum speed = 27 m/s **Ans.**

(b) Constant speed

(c) Acceleration = $\frac{\text{change in speed}}{\text{time taken}}$

$= \frac{12}{150} = 0.08 \text{ m/s}^2$ **Ans.**

(d) From graph, the time 20 seconds before the train completes its journey, is 560 seconds.

∴ at 560 seconds, speed of train is 3.5 m/s **Ans.**

(e) Distance travelled during first 200 second

= area of trapezium

$= \frac{1}{2}(12)(200+50)$

= 1500 m **Ans.**

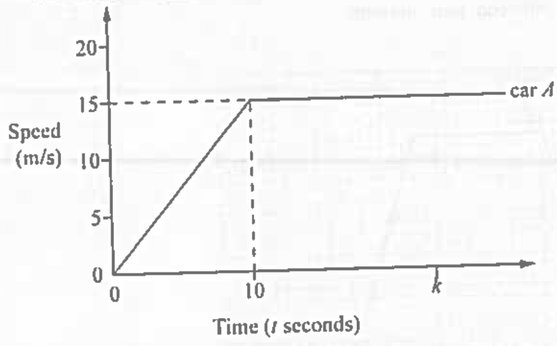
(f) Average speed = $\frac{\text{total distance travelled}}{\text{time taken}}$

$= \frac{1500}{200}$ m/s

$= (\frac{1500}{200} \times \frac{3600}{1000})$ km/h

= 27 km/h **Ans.**

16 *(N2015 P1 Q25)*



The diagram shows the speed-time graph of car A.

(a) Find the acceleration of car A when $t = 7$. [1]

(b) Find an expression, in terms of k, for the distance moved by car A between $t = 0$ and $t = k$, where $k > 10$.
Give your answer in its simplest form. [2]

(c) Car B travels at a constant speed of 12 m/s in the same direction as car A.

(i) On the diagram, sketch the speed-time graph of car B. [1]

(ii) When $t = 0$, car B passes car A.
When $t = k$, car A overtakes car B.
Find the value of k. [1]

Thinking Process

(a) To find acceleration, calculate the gradient of the line from $t = 0$ to $t = 10$.

(b) To find the distance, find the area under the graph from $t = 0$ to $t = k$.

(c) (ii) Note that when car A overtakes car B, then both cars have travelled the same distance from $t = 0$ to $t = k$.

Solution

(a) When $t = 7$,

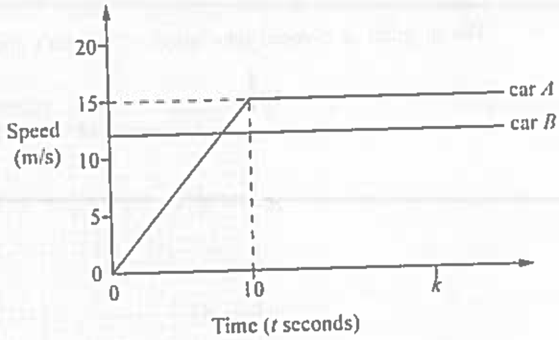
$$\text{Acceleration} = \frac{15}{10}$$
$$= 1.5 \text{ m/s}^2 \quad \textbf{Ans.}$$

The acceleration is constant from $t = 0$ to $t = 10$.

(b) Distance travelled = area of trapezium

$$= \frac{1}{2}(15)(k + k - 10)$$
$$= \frac{1}{2}(15)(2k - 10)$$
$$= (15)(k - 5)$$
$$= (15k - 75) \text{ m} \quad \textbf{Ans.}$$

(c) (i)



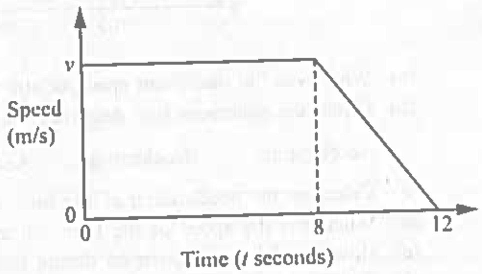
(ii) Distance travelled by car B during 0 to k seconds = $12k$ m

When car A overtakes car B, both cars have travelled same distance from $t = 0$ to $t = k$.

$$\therefore \quad 12k = 15k - 75$$
$$3k = 75$$
$$k = 25 \quad \textbf{Ans.}$$

17 *(J2016 P1 Q18)*

The diagram is the speed-time graph for part of a car's journey.



The retardation of the car between $t = 8$ and $t = 12$ is 4 m/s².

(a) Find v. [1]

(b) Find the total distance travelled by the car in the 12 seconds. [2]

Thinking Process

(a) Apply retardation = $\dfrac{\text{change in speed}}{\text{time interval}}$.

(b) Total distance travelled = area under speed-time graph.

Solution

(a)
$$\text{Retardation} = \frac{\text{change in speed}}{\text{time taken}}$$
$$4 = \frac{v - 0}{12 - 8}$$
$$4 = \frac{v}{4}$$
$$v = 16 \text{ m/s} \quad \textbf{Ans.}$$

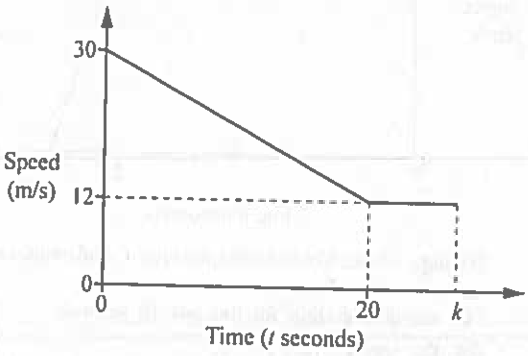
(b) Total distance travelled = area of trapezium

$= \frac{1}{2}(v)(8+12)$

$= \frac{1}{2}(16)(20)$

$= 160$ m **Ans.**

18 (N2016 P1 Q27)



The diagram shows the speed-time graph of a car which slows down from 30 m/s to 12 m/s in 20 seconds, and then continues at a speed of 12 m/s.

(a) Find the retardation when $t = 10$. [1]

(b) Find the distance travelled by the car between $t = 0$ and $t = 20$. [2]

(c) The distance travelled by the car between $t = 20$ and $t = k$ is 60 m. Find the value of k. [2]

Thinking Process

(a) To find retardation ✎ calculate the gradient of the line from $t = 0$ to $t = 20$.

(b) ✎ Find the area under the graph from $t = 0$ to $t = 20$.

(c) To find the value of k ✎ consider the area under the graph from $t = 20$ to $t = k$ seconds.

Solution

(a) When $t = 10$,

retardation $= \frac{30-12}{20}$

$= \frac{18}{20} = 0.9$ m/s^2 **Ans.**

(b) Distance travelled = area of trapezium

$= \frac{1}{2}(20)(12+30)$

$= (10)(42)$

$= 420$ m **Ans.**

(c) Distance travelled = area of rectangle

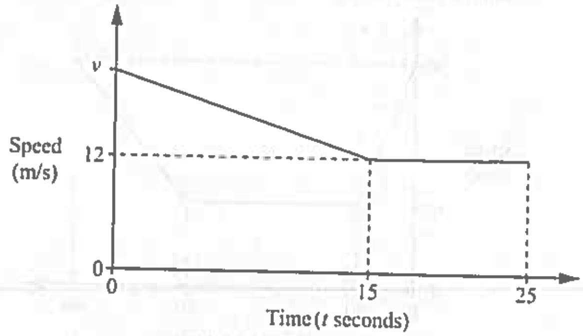
$60 = 12 \times (k - 20)$

$5 = k - 20$

$k = 25$ **Ans.**

19 (J2017 P1 Q18)

The diagram is the speed-time graph for 25 seconds of a car's journey.



The car slows down uniformly from a speed of v m/s to a speed of 12 m/s in 15 seconds. It then travels at constant speed for a further 10 seconds.

(a) The retardation of the car is 0.4 m/s². Calculate the value of v. [2]

(b) Calculate the distance travelled by the car from $t = 0$ to $t = 25$. [2]

Thinking Process

(a) Use deceleration $= \frac{\text{change in speed}}{\text{time taken}}$.

(b) ✎ Distance travelled = area under the speed-time graph.

Solution

(a) Retardation $= \frac{\text{change in speed}}{\text{time taken}}$

$0.4 = \frac{v-12}{15}$

$6 = v - 12$

$v = 18$ m/s **Ans.**

(b) Distance travelled

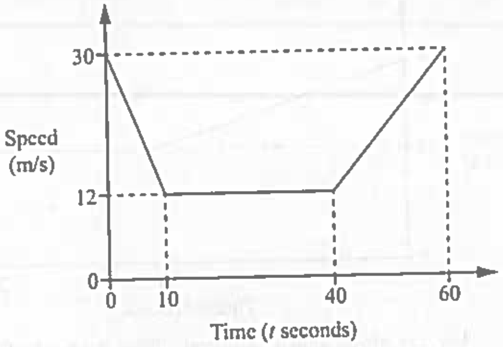
= area of triangle + area of rectangle

$= \frac{1}{2}(15)(6) + (12 \times 25)$

$= 45 + 300 = 345$ m **Ans.**

20 *(N2017/P1/Q24)*

The diagram is the speed-time graph of part of a train's journey.



(a) Calculate the speed when $t = 5$. [1]

(b) Calculate the acceleration. [1]

(c) Calculate the distance travelled from $t = 40$ to $t = 60$. [2]

Thinking Process

(a) To find the speed when $t = 5$ ✎ find deceleration for the 1st 10 seconds.

(b) Find the acceleration from $t = 40$ to $t = 60$ seconds.

(c) Find the area under the graph from $t = 40$ to $t = 60$ seconds.

Solution

(a) Deceleration from 0 to 10 seconds $= \frac{30-12}{10}$

$= \frac{18}{10}$ m/s^2

let v be the speed at $t = 5$ seconds,

$$\text{deceleration} = \frac{\text{change in speed}}{\text{time taken}}$$

$$\frac{18}{10} = \frac{30-v}{5}$$

$$9 = 30 - v$$

$v = 21$ m/s **Ans.**

(b) $\text{Acceleration} = \frac{\text{change in speed}}{\text{time taken}}$

$= \frac{30-12}{60-40}$

$= \frac{18}{20} = \frac{9}{10}$ m/s^2 **Ans.**

(c) Distance travelled = area of trapezium

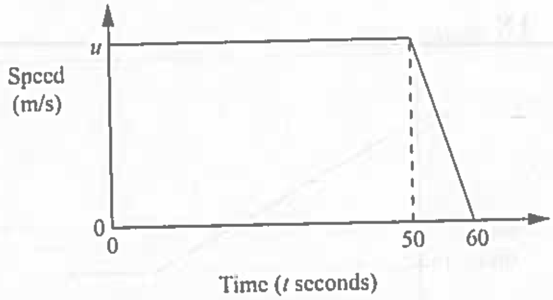
$= \frac{1}{2}(60-40)\times(12+30)$

$= \frac{1}{2}(20)(42) = 420$ m **Ans.**

21 *(J2018/P1/Q25)*

The diagram is the speed–time graph for 60 seconds of a train's journey.

At the beginning of this part of the journey the train is travelling at u m/s.



Giving each answer in its simplest form, find expressions in terms of u, for

(a) the deceleration for the last 10 seconds. [1]

(b) the speed when $t = 55$. [1]

(c) the distance travelled during these 60 seconds. [2]

Thinking Process

(a) To find deceleration ✎ calculate the gradient of the line from $t = 50$ to $t = 60$.

(b) To find speed ✎ use the time difference between 50 s to 55 s.

(c) To find the distance ✎ find the area under the graph from $t = 0$ to $t = 60$.

Solution

(a) Deceleration $= \frac{u}{60-50}$

$= \frac{u}{10}$ m/s^2 **Ans.**

(b) Time, $t = 55 - 50 = 5$ seconds.

$\therefore$ speed $= \frac{u}{10}\times 5$

$= 0.5u$ m/s **Ans.**

(c) Distance travelled = area of trapezium

$= \frac{1}{2}(u)(50+60)$

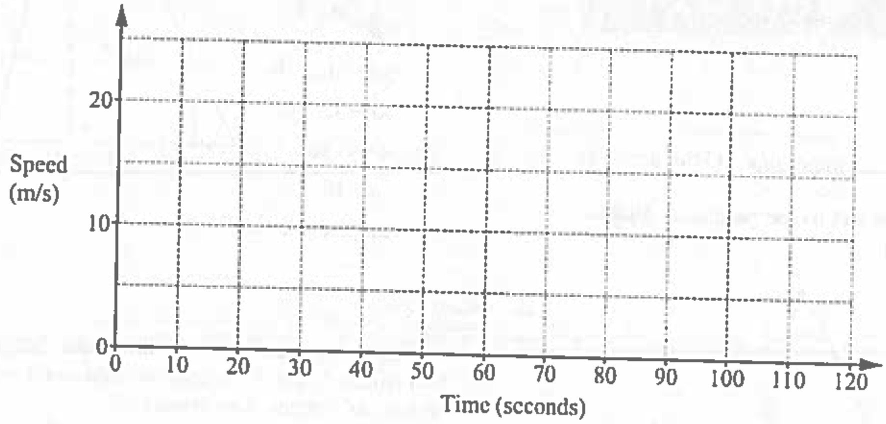
$= 55u$ m **Ans.**

22 (N2018/P1/Q24)

A train travels between two stations, starting and finishing at rest.

For this journey it
- accelerates uniformly, from rest, for the first 30 seconds until it reaches a speed of 20 m/s
- travels at a constant speed of 20 m/s for the next 60 seconds
- slows down uniformly for the last 20 seconds until it stops.

(a) On the grid, draw the speed–time graph for this journey.



[2]

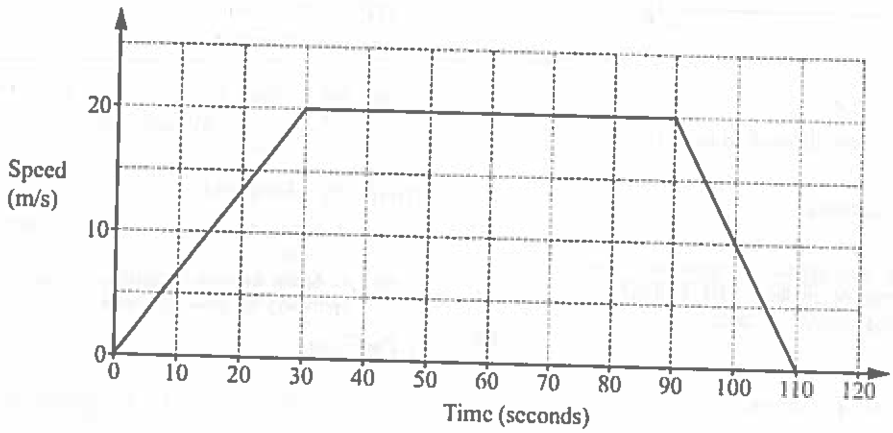
(b) Calculate the distance between the stations. [2]

Thinking Process

(a) Draw the graph as per given information.
(b) To find the distance ✐ find the area under the graph from $t = 0$ to $t = 110$.

Solution

(a)



(b) Distance travelled = area of trapezium

$= \frac{1}{2}(20)(60 + 110)$

$= (10)(170)$

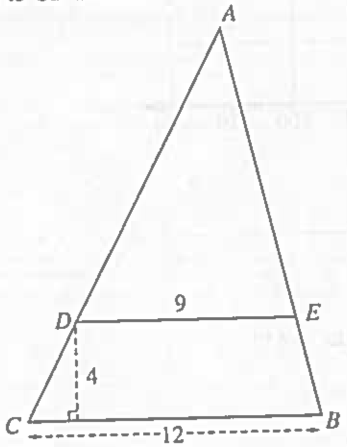
$= 1700$ m **Ans.**

Topic 9

Similarity and Congruency

1 (J2007 P1 Q15)

In the diagram, $BCDE$ is a trapezium, and the sides CD and BE are produced to meet at A.
CB = 12cm, DE = 9cm and the perpendicular distance from D to CB is 4cm.



Calculate

(a) the area of $BCDE$, [1]

(b) the perpendicular distance from A to CB. [2]

Thinking Process

(a) Apply the area of a trapezium.
(b) Consider two similar triangles ADE and ACB and apply rule of similarity, i.e. ratio of corresponding lengths is equal.

Solution

(a) As $BCDE$ is a trapezium.

$\therefore$ area of $BCDE = \frac{1}{2} \times 4(9+12)$

$= 2(21) = 42 \text{ cm}^2$ Ans.

(b) Let d be the perpendicular distance from A to CB. Therefore

height of $\Delta ACB = d$, and

height of $\Delta ADE = d-4$

$\frac{DE}{CB} = \frac{d-4}{d}$

$\frac{9}{12} = \frac{d-4}{d}$

$\frac{3}{4} = \frac{d-4}{d}$

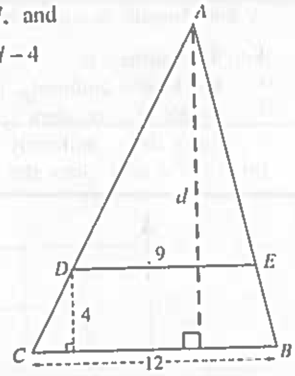
$3d = 4(d-4)$

$3d = 4d - 16$

$3d - 4d = -16$

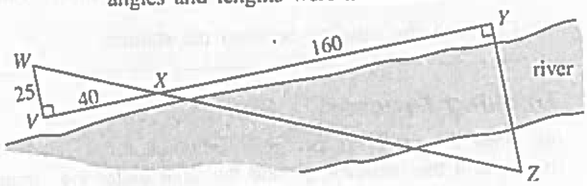
$d = 16$

$\therefore$ req. perp. distance = 16 cm Ans.



2 (N2007 P2 Q3b)

(b) In a surveying exercise to find the distance between two points, Y and Z, on opposite banks of a river, angles and lengths were measured.



WXZ and VXY are straight lines.

$W\hat{V}X = Z\hat{Y}X = 90$.

(i) Show that triangles VWX and YZX are similar. [2]

(ii) VW = 25m, VX = 40m and XY = 160m.
Calculate the distance YZ. [2]

Thinking Process

(b) (i) Look for the two equal angles in both triangles.

(ii) Apply rule of similarity, i.e. ratio of corresponding lengths is equal.

Solution

(b) (i) $\angle VXW = \angle YXZ$ (vert. opp angles)

$\angle WVX = \angle ZYX = 90°$ (given)

$\therefore$ ΔVWX and ΔYZX are similar Ans

(ii) $\frac{YZ}{VW} = \frac{YX}{VX}$

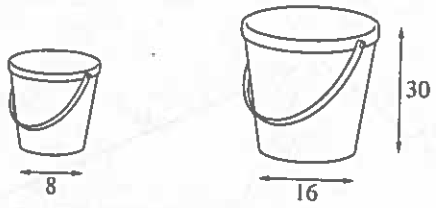
$\frac{YZ}{25} = \frac{160}{40}$

$YZ = \frac{160}{40} \times 25$

$YZ = 100$ m Ans

3 (J2009/P1/Q11)

Similar buckets are available in two sizes.
The large bucket has height 30 cm and base diameter 16 cm.
The small bucket has base diameter 8 cm.



(a) Find the height of the small bucket. [1]

(b) Given that the small bucket has volume 850 cm^3, find the volume of the large bucket. [2]

Thinking Process

(a) Apply rule of similarity, i.e. ratio of corresponding lengths is equal.

(b) To find volume compute the cube of the ratio of their linear dimensions.

Solution

(a) As the two buckets are similar

$\therefore \frac{\text{ht. of small bucket}}{\text{ht. of large bucket}} = \frac{8}{16}$

$\frac{\text{ht. of small bucket}}{30} = \frac{8}{16}$

ht. of small bucket $= \frac{8}{16} \times 30 = 15$ cm **Ans.**

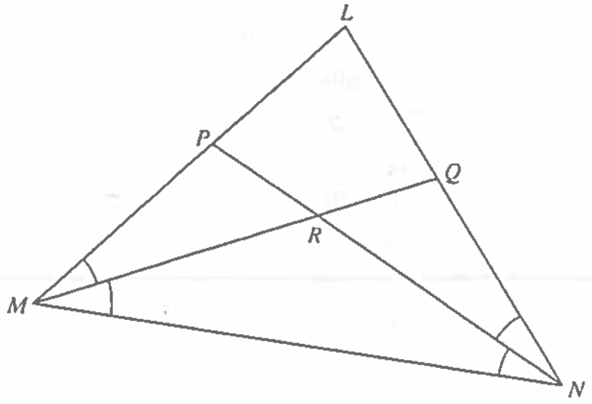
(b) $\frac{\text{vol. of large bucket}}{\text{vol. of small bucket}} = \left(\frac{16}{8}\right)^3$

$\frac{\text{vol. of large bucket}}{850} = 8$

vol. of large bucket $= 8 \times 850$

$= 6800$ cm^3 **Ans.**

4 (J2009/P1/Q19)



In the diagram, $L\hat{M}Q = Q\hat{M}N = M\hat{N}P = P\hat{N}L$.

(a) Show that triangles LMQ and LNP are congruent. [3]

(b) Show that $M\hat{P}N = M\hat{Q}N$. [1]

(c) The straight lines MQ and NP intersect at R. State the name of the special quadrilateral $LPRQ$. [1]

Thinking Process

(a) Observe that ΔLMN is an isosceles triangle. Prove $LM = LN$. Prove $\angle MLN$ is a common angle. Prove one more equal angle to satisfy ASA property.

(b) Prove that both angles are adjacent to two equal angles of the congruent triangles.

(c) Note that $LP = LQ$ and $RP = RQ$.

Solution

(a) $L\hat{M}Q = P\hat{N}L$ (given)

$Q\hat{M}N = M\hat{N}P$ (given)

$\Rightarrow L\hat{M}N = L\hat{N}M$

$\therefore$ ΔLMN is an isosceles triangle.

now,

$L\hat{M}Q = L\hat{N}P$ (given)

$M\hat{L}Q = N\hat{L}P$ (common angle)

$LM = LN$ (LMN is an isosceles Δ)

$\therefore$ $\Delta LMQ \equiv \Delta LNP$ (ASA) **(Shown).**

(b) As triangles LMQ and LNP are congruent.

$\therefore$ $L\hat{Q}M = L\hat{P}N$

now,

$M\hat{Q}N = 180° - L\hat{Q}M$ ($\angle$s on a straight line)

$M\hat{P}N = 180° - L\hat{P}N$ ($\angle$s on a straight line)

$= 180° - L\hat{Q}M$ (since $L\hat{P}N = L\hat{Q}M$)

$= M\hat{Q}N$

$\therefore$ $M\hat{P}N = M\hat{Q}N$. **Shown.**

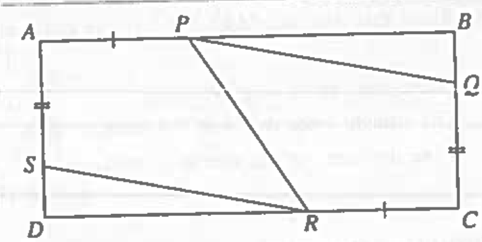
> Alternatively,
> In ΔPMR and ΔQNR
> $P\hat{M}R = Q\hat{N}R$ (given)
> $P\hat{R}M = Q\hat{R}N$ (vert. opp. $\angle$s)
> $\therefore$ $M\hat{P}R = N\hat{Q}R$
> or $M\hat{P}N = M\hat{Q}N$. **Shown.**

(c) $LP = LQ$ (since $\Delta LMQ \equiv \Delta LNP$)

$RP = RQ$

$\therefore$ $LPRQ$ is a Kite. **Ans.**

5 (N2009 P2 Q2)



$ABCD$ is a rectangle.
Points P, Q, R and S lie on AB, BC, CD and DA respectively such that $AP = CR$ and $QC = SA$.

(a) Giving reasons, show that
- (i) $PB = RD$, [1]
- (ii) triangle PBQ is congruent to triangle RDS, [3]
- (iii) $R\hat{P}Q = P\hat{R}S$. [3]

(b) State the special name of the quadrilateral $PQRS$. [1]

Thinking Process

(a) (i) $ABCD$ is a rectangle with $AP = CR$.
(ii) Prove that $BQ = SD$. Observe that triangles are congruent by SAS property.
(iii) PB is parallel to DR, $\angle BPR = \angle DRP$, and $\angle BPQ = \angle DRS$.

(b) Note that PQ is parallel to SR.

Solution

(a) (i) Given that $AB = DC$ and $AP = RC$

$\therefore \quad PB = AB - AP$
$= DC - RC$
$= RD$ **Shown.**

(ii) Given that, $BC = AD$ and $QC = AS$

$\Rightarrow \quad BQ = BC - QC$
$= AD - AS$
$= DS$

$\therefore \quad BQ = DS$

from part (a) (i): $PB = RD$

also $P\hat{B}Q = R\hat{D}S = 90°$

$\therefore \quad \Delta PBQ \equiv \Delta RDS$ (SAS) **Shown.**

(iii) $ABCD$ is a rectangle, therefore PB is parallel to DR.

$\Rightarrow B\hat{P}R = D\hat{R}P$ (alternate $\angle$s)

and $B\hat{P}Q = D\hat{R}S$ $(\Delta PBQ \equiv \Delta RDS)$

now,

$B\hat{P}Q + R\hat{P}Q = B\hat{P}R$

$R\hat{P}Q = B\hat{P}R - B\hat{P}Q$
$= D\hat{R}P - D\hat{R}S$
$= P\hat{R}S$

$\therefore \quad R\hat{P}Q = P\hat{R}S$ **Shown.**

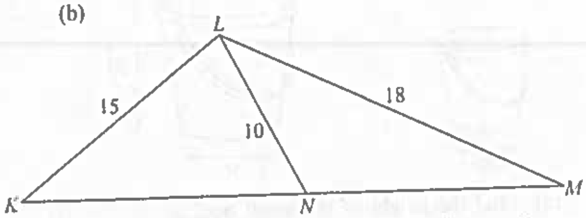
(b) From (a) (iii), $R\hat{P}Q = P\hat{R}S$

$\Rightarrow \quad PQ$ is parallel to SR

$\therefore \quad PQRS$ is a parallelogram. **Ans.**

6 (J2010 P2 Q7b)

(b)



In the diagram, triangle KLM is similar to triangle LNM.
$KL = 15$ cm, $LM = 18$ cm and $LN = 10$ cm.

- (i) Find KM. [2]
- (ii) Find KN. [2]
- (iii) P is the point on LM such that PN is parallel to LK.

Find $\dfrac{\text{the area of triangle } NPM}{\text{the area of trapezium } KLPN}$.

Give your answer as a fraction in its simplest form. [2]

Thinking Process

(b) (i) To find KM use properties of similar triangles, i.e. ratio of corresponding lengths is equal.
(ii) Apply rule of similarity to find MN. Subtract MN from KM to find KN.
(iii) Apply concept of area of similar triangles.

$$\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2.$$

Solution

(b) (i) ΔKLM is similar to ΔLNM

$$\Rightarrow \frac{KM}{LM} = \frac{KL}{LN}$$

$$\frac{KM}{18} = \frac{15}{10}$$

$$KM = \frac{15}{10} \times 18$$

$= 27$ cm. **Ans.**

(ii) ΔKLM is similar to ΔLNM

$$\Rightarrow \frac{KL}{LN} = \frac{LM}{NM}$$

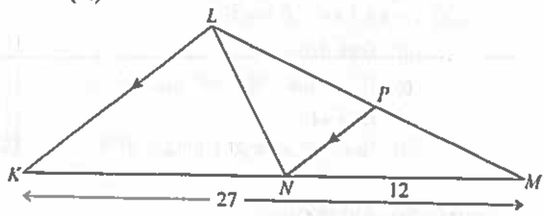
$$\frac{15}{10} = \frac{18}{NM}$$

$$NM = 18 \times \frac{10}{15}$$

$= 12$ cm.

$KN = KM - NM$
$= 27 - 12$
$= 15$ cm **Ans.**

(iii)



ΔNPM is similar to ΔKLM

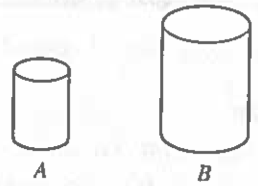
$$\Rightarrow \frac{\text{area of } \Delta NPM}{\text{area of } \Delta KLM} = \left(\frac{NM}{KM}\right)^2$$

$$= \left(\frac{12}{27}\right)^2 = \frac{16}{81}$$

$$\therefore \frac{\text{area of } \Delta NPM}{\text{area of } KLPN} = \frac{16}{81-16}$$

$$= \frac{16}{65} \quad \textbf{Ans.}$$

7 (J2011/P1/Q10)



These two cylinders are similar.
The ratio of their volumes is 8 : 27.
The height of cylinder A is 12 cm.
Find the height of cylinder B. [2]

Thinking Process

Apply rule of similarity, i.e. ratio of volumes of two objects is equal to the cube of the ratio of their linear lengths.

Solution

$$\frac{\text{volume of } A}{\text{volume of } B} = \left(\frac{\text{height of } A}{\text{height of } B}\right)^3$$

$$\frac{8}{27} = \left(\frac{12}{\text{height of } B}\right)^3$$

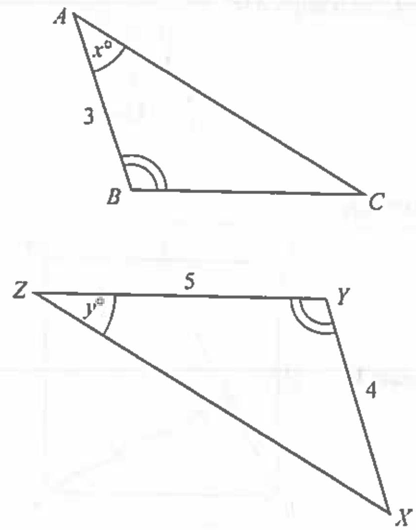
$$\left(\frac{2}{3}\right)^3 = \left(\frac{12}{\text{height of } B}\right)^3$$

$$\frac{2}{3} = \frac{12}{\text{height of } B}$$

$$\text{height of } B = 12 \times \frac{3}{2}$$

$$= 18 \text{ cm.} \quad \textbf{Ans.}$$

8 (N2011/P1/Q20)



The triangles ABC and XYZ are similar and $A\hat{B}C = X\hat{Y}Z$.

$B\hat{A}C = x°$, $Y\hat{Z}X = y°$ where $x \neq y$.

$AB = 3$ cm, $XY = 4$ cm and $YZ = 5$ cm.

(a) Express $A\hat{B}C$ in terms of x and y. [1]

(b) Find BC. [1]

(c) Write down the value of $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } XYZ}$. [1]

Thinking Process

(a) Recall, corresponding angles in similar triangles are equal. $\therefore \angle BCA = y°$

(b) Apply rule of similar figures: ratio of corresponding lengths is equal.

(c) Apply, $\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2$

Solution

(a) $B\hat{C}A = Y\hat{Z}X = y°$ (since Δs are similar)

consider ΔABC.

$$A\hat{B}C + x° + y° = 180° \quad (\angle \text{ sum of } \Delta)$$

$$\Rightarrow A\hat{B}C = 180° - x° - y° \quad \textbf{Ans.}$$

(b) $\dfrac{BC}{YZ} = \dfrac{AB}{XY}$

$$\frac{BC}{5} = \frac{3}{4}$$

$$BC = \frac{3}{4} \times 5$$

$$= \frac{15}{4} = 3\frac{3}{4} \text{ cm} \quad \textbf{Ans.}$$

(c) $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } XYZ} = \left(\dfrac{AB}{XY}\right)^2$

$= \left(\dfrac{3}{4}\right)^2$

$= \dfrac{9}{16}$ **Ans.**

9 *(N2011/P2/Q9)*

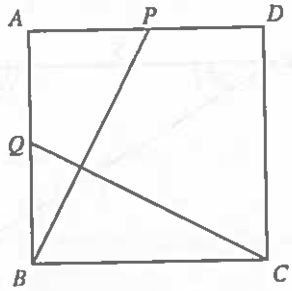


Diagram I

In **Diagram I**, $ABCD$ is a square.
P and Q are the midpoints of AD and AB respectively.

(a) Show that triangles APB and BQC are congruent. [3]

(b)

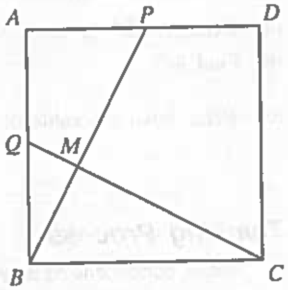


Diagram II

In **Diagram II**, QC and PB intersect at M.
Show that $B\hat{M}C = 90°$.
State your reasons clearly. [2]

(c)

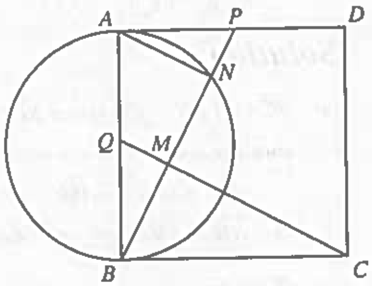


Diagram III

In **Diagram III**, the circle centre Q has diameter AB.
The circle intersects BP at N.

(i) State the reason why $A\hat{N}B = 90°$. [1]

(ii) Triangle BMQ is mapped onto triangle BNA by an enlargement.
Write down the centre and scale factor of the enlargement. [1]

(iii) Given that $QM = 3$ cm,

(a) find AN, [1]

(b) show that $MN = 6$ cm. [1]

(c) find MC, [1]

(d) find the area of triangle APB. [2]

Thinking Process

(a) Prove by SAS (side-angle-side) rule.

(b) Let $B\hat{C}Q = x°$, find $\angle MBC$ and prove.

(c) (i) Note that $\angle ANB$ is subtended by diameter AB.

(ii) Identify the centre. To find the scale factor find the ratio of the lengths of two corresponding sides.

(iii) (a) Apply rule of similarity, i.e. ratio of corresponding lengths is equal.

(b) Prove that triangles BMQ and ANP are congruent. find BM. Note that M is the mid-point of BN.

(c) To find MC consider the right triangle QBC. Find QB, BC and apply pythagoras to find QC.

(d) Apply area $= \frac{1}{2} \times \text{base} \times \text{height}$.

Solution

(a) $AB = BC$ ($ABCD$ is a square)

$AP = BQ$ ($\because$ P & Q are the mid-points)

$P\hat{A}B = Q\hat{B}C = 90°$

$\therefore \Delta APB \equiv BQC$ (SAS) **Shown.**

(b) Let $B\hat{C}Q = x°$

ΔAPB is congruent to ΔBQC

$\therefore A\hat{B}P = x°$

$\Rightarrow M\hat{B}C = 90° - x°$

In ΔBMC

$B\hat{M}C + 90° - x° + x° = 180°$ ($\angle$ sum of a Δ)

$\Rightarrow B\hat{M}C = 180° - 90° = 90°$ **Shown.**

(c) (i) The diameter AB subtends an angle $A\hat{N}B$ at the circumference of the circle.

$\therefore A\hat{N}B = 90°$

> Recall:
> Angle subtended by the diameter at the circumference is always 90°.

(ii) Centre: B, Scale factor: 2

(iii) (a) ΔBMQ is similar to ΔBNA

$\dfrac{BQ}{BA} = \dfrac{QM}{AN}$

$\dfrac{1}{2} = \dfrac{3}{AN}$

$AN = 6$ cm **Ans.**

(b) $AP = BQ$

$B\widehat{M}Q = A\widehat{N}P = 90°$

$M\hat{B}Q = N\hat{A}P$ (∠s in alternate segment)

∴ ΔBMQ is congruent to ΔANP

⇒ $BM = AN = 6$ cm

since M is the mid-point of BN

∴ $MN = 6$ cm **Shown.**

(c) In right triangle BMQ

$QM = 3$ cm, $BM = 6$ cm, $B\widehat{M}Q = 90°$

∴ $BQ = \sqrt{(3)^2 + (6)^2} = \sqrt{45}$

since $ABCD$ is a square,

∴ $BC = AB$

$= 2BQ = 2\sqrt{45}$

In right triangle QBC

$QC = \sqrt{(BQ)^2 + (BC)^2}$

$= \sqrt{(\sqrt{45})^2 + (2\sqrt{45})^2}$

$= \sqrt{45 + 180}$

$= \sqrt{225} = 15$ cm

$MC = QC - QM$

$= 15 - 3 = 12$ cm **Ans.**

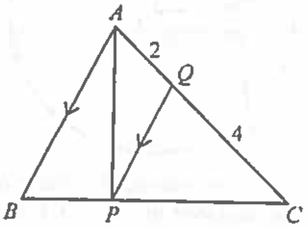
(d) $AP = BQ = \sqrt{45}$, $AB = 2\sqrt{45}$

Area of $\Delta APB = \frac{1}{2}(\sqrt{45})(2\sqrt{45})$

$= 45$ cm^2 **Ans.**

10 *(N2012/P1/Q18)*

In the diagram, the points P and Q lie on the sides BC and AC of triangle ABC.
AB is parallel to QP.
$AQ = 2$ cm and $QC = 4$ cm.
The area of triangle CPQ is 6 cm².



Find the area of
(a) triangle AQP, [1]
(b) triangle ABC, [1]
(c) triangle ABP. [1]

Thinking Process

(a) Note that ΔAQP and ΔCPQ share common height.

(b) Apply concept of area of similar Δs, i.e.

$$\frac{\text{area of } \Delta CPQ}{\text{area of } \Delta CBA} = \left(\frac{\text{length of } CQ}{\text{length of } QA}\right)^2$$

(c) Area of ΔABP = area of ΔABC − area of ΔCPQ − area of ΔAQP

Solution

(a) $\frac{\text{Area of } \Delta AQP}{\text{Area of } \Delta CPQ} = \frac{\frac{1}{2} \times AQ \times h}{\frac{1}{2} \times CQ \times h}$

$\frac{\text{Area of } \Delta AQP}{6} = \frac{2}{4}$

Area of $\Delta AQP = \frac{2}{4} \times 6$

$= 3$ cm^2 **Ans.**

(b) $QP \parallel AB$

∴ ΔABC is similar to ΔQPC

⇒ $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta QPC} = \left(\frac{AC}{QC}\right)^2$

$\frac{\text{Area of } \Delta ABC}{6} = \left(\frac{6}{4}\right)^2$

Area of $\Delta ABC = \frac{9}{4} \times 6$

$= \frac{27}{2} = 13.5$ cm^2 **Ans.**

(c) Area of ΔABP

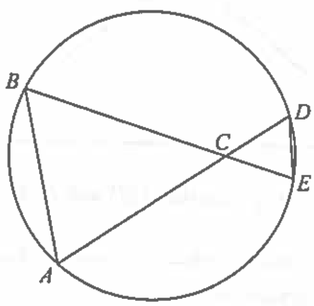
= area of ΔABC − area of ΔCPQ − area of ΔAQP

$= 13.5 - 6 - 3$

$= 4.5$ cm^2 **Ans.**

11 *(N2012/P2/Q4 b)*

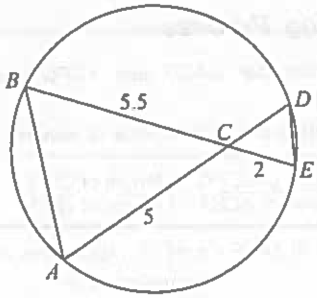
(b) A, B, D and E are points on a circle.



AD and BE intersect at C.

(i) Show that triangles ABC and EDC are similar.
Give your reasons. [2]

(ii)



Given that $AC = 5$ cm, $BC = 5.5$ cm and $CE = 2$ cm, find the length of the chord AD. [2]

Thinking Process

(b) (i) Apply rule of similar figures: the corresponding angles of two triangle are equal.

(ii) Apply rule of similarity: The ratio of corresponding lengths is equal.

Solution

(b) (i) $A\hat{B}C = E\hat{D}C$ (∠s in the same segment)

$B\hat{A}C = D\hat{E}C$ (∠s in the same segment)

$A\hat{C}B = E\hat{C}D$ (vert. opposite ∠s)

∴ ΔABC is similar to ΔEDC **Ans.**

(ii) $\frac{DC}{BC} = \frac{EC}{AC}$

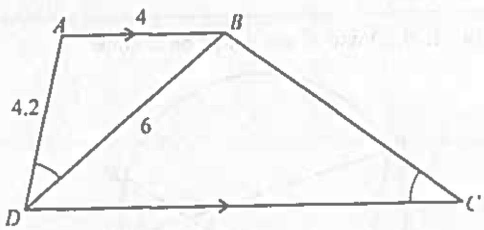
$\frac{DC}{5.5} = \frac{2}{5} \Rightarrow DC = \frac{2}{5} \times 5.5 = 2.2$ cm

∴ $AD = AC + CD$

$= 5 + 2.2 = 7.2$ **Ans.**

12 *(N2013/P1/Q26)*

In the diagram, AB is parallel to DC and $A\hat{D}B = B\hat{C}D$.



(a) Explain why triangles ABD and BDC are similar. [2]

(b) $AB = 4$ cm, $BD = 6$ cm and $AD = 4.2$ cm.

(i) Calculate BC. [2]

(ii) Write down the value of $\frac{\text{area of triangle } ABD}{\text{area of triangle } BDC}$. [1]

Thinking Process

(a) Prove that two angles of one triangle are equal to two angles of another triangle.

(b) (i) Apply rule of similarity: ratio of corresponding lengths is equal.

(ii) $\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\text{Length}_1}{\text{Length}_2}\right)^2$.

Solution

(a) $A\hat{D}B = B\hat{C}D$ (given)

$A\hat{B}D = B\hat{D}C$ (alt ∠s. $AB \parallel DC$)

since two corresponding angles are equal.

∴ ΔABD is similar to ΔBDC.

(b) (i) $\frac{BC}{AD} = \frac{BD}{AB}$

$BC = \frac{BD}{AB} \times AD$

$= \frac{6}{4} \times 4.2$

$= 6.3$ cm **Ans.**

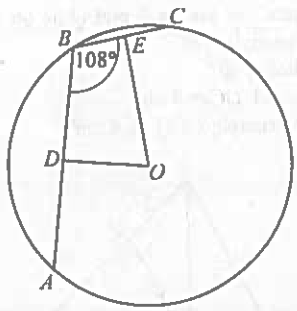
(ii) $\frac{\text{area of triangle } ABD}{\text{area of triangle } BDC} = \left(\frac{AB}{BD}\right)^2$

$= \left(\frac{4}{6}\right)^2$

$= \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ **Ans.**

13 *(N2013/P2/Q4)*

(a)

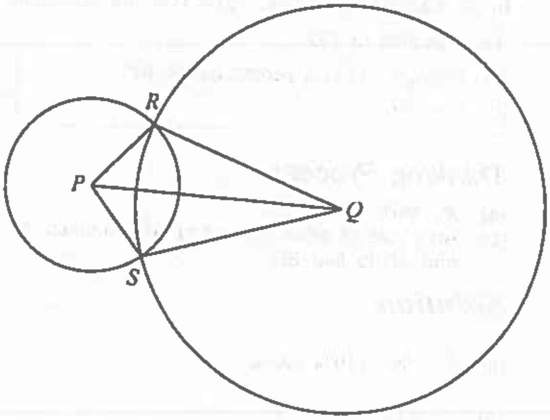


AB and BC are chords of a circle centre O. D is the midpoint of AB and E is the midpoint of BC.

$A\hat{B}C = 108°$.

Find $D\hat{O}E$ giving your reasons. [2]

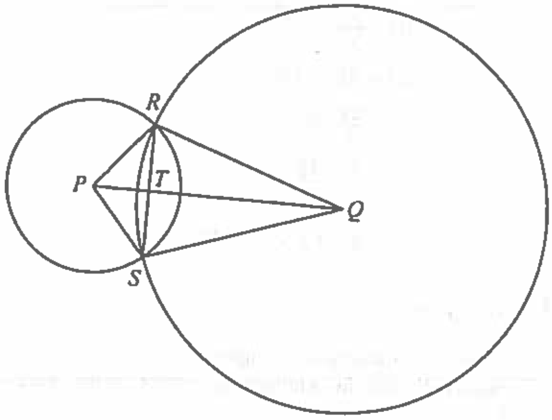
(b)



A circle centre P and a circle centre Q intersect at R and S.

(i) Show that triangle PRQ is congruent to triangle PSQ. [3]

(ii)



RS and PQ intersect at T.

(a) State the name of the special quadrilateral $PRQS$. [1]

(b) Find $P\hat{T}R$ [1]

Thinking Process

(a) To find $D\hat{O}E$ ∮ notice that $O\hat{E}B$ and $O\hat{D}B$ are right angles.

(b) (i) Observe that the triangles are congruent by SSS property.

(ii) (a) $PRQS$ is a kite.

Solution

(a) Since D and E are midpoints of AB and BC

$\Rightarrow O\hat{E}B = O\hat{D}B = 90°$ (line from circle center bisects the chord at 90°)

$\therefore D\hat{B}E + D\hat{O}E = 180°$

$108° + D\hat{O}E = 180°$

$D\hat{O}E = 180° - 108°$

$= 72°$ **Ans.**

(b) (i) $PR = PS$ (radii of circle)

$QR = QS$ (radii of circle)

PQ is common to both triangles

$\therefore \Delta PRQ \equiv \Delta PSQ$ (SSS) **Shown.**

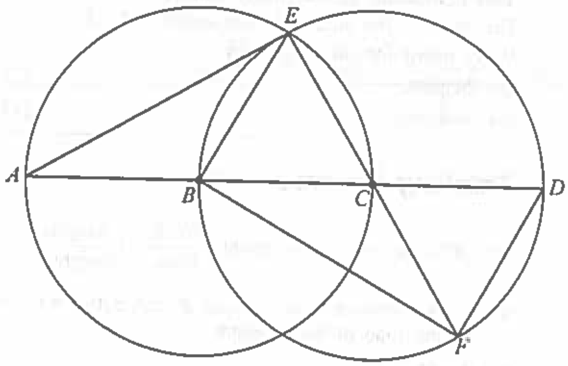
(ii) (a) $PRQS$ is a Kite. **Ans.**

(b) $P\hat{T}R = 90°$ **Ans.**

Note that diagonals of a kite intersect at right angles.

14 *(J2014/P2/Q11a)*

(a) The diagram shows two circles with equal radii. A, E and C are points on the circle, centre B. B, E, D and F are points on the circle, centre C. $ABCD$ is a straight line.



(i) Show that triangles AEC and FBE are congruent. [3]

(ii) State another triangle that is congruent to triangle AEC. [1]

(iii) Explain why EB is parallel to DF. [2]

(iv) Work out $A\hat{B}E$. [1]

Thinking Process

(a) (i) Note that both right angled triangles are congruent by RHS property.

(iii) Observe that $\angle EBF + \angle DFB = 180°$. The lines are parallel as their interior angles are supplementary.

(iv) Note that triangle EBC is an equilateral triangle.

Solution *with* TEACHER's COMMENTS

(a) (i) $AC = FE$ (diameter of circles)

$EC = BE$ (radii of two circles)

$A\hat{E}C = F\hat{B}E = 90°$ (rt. ∠ in semi-circle)

$\therefore \Delta AEC \equiv \Delta FBE$ (RHS). **Shown.**

Right-angled Hypotenuse Side Rule (RHS):
If the hypotenuse and one other side of the first right-angled triangle are equal to the hypotenuse and corresponding side of the second right-angled triangle, then the two triangles are congruent.

(ii) ΔBFD **Ans.**

(iii) EF and DB are diameters of the circle.

$E\hat{B}F = D\hat{F}B = 90°$ (rt. ∠ in semi-circle)

also, $E\hat{B}F + D\hat{F}B = 180°$

which is the property of supplementary angles. Thus EB and DF are parallel.

(iv) $EB = BC = EC$ (radii of circles)

$\Rightarrow$ ΔEBC is an equilateral triangle.

$\therefore$ $A\hat{B}E = 180° - 60°$

$= 120°$ **Ans.**

15 *(N2014/P1/Q8)*

Two bottles are geometrically similar.
The ratio of the areas of their bases is 1 : 4.
Write down the ratios of their

(a) heights, [1]

(b) volumes. [1]

Thinking Process

(a) Apply rule of similarity: $\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\text{height}_1}{\text{height}_2}\right)^2$

(b) To find ratio of their volumes compute the cube of the ratio of their heights.

Solution

(a) $\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\text{height}_1}{\text{height}_2}\right)^2$

$\frac{1}{4} = \left(\frac{\text{height}_1}{\text{height}_2}\right)^2$

$\frac{\text{height}_1}{\text{height}_2} = \frac{1}{2}$

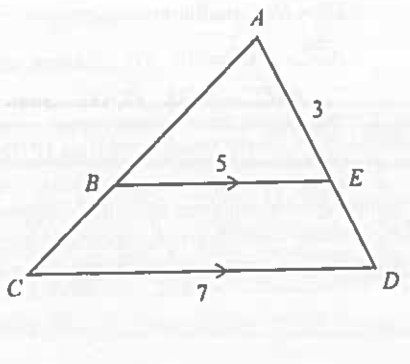
$\therefore$ ratio of heights $= 1:2$ **Ans.**

(b) $\frac{\text{volume}_1}{\text{volume}_2} = \left(\frac{1}{2}\right)^3$

$= \frac{1}{8}$

$\therefore$ ratio of their volumes $= 1:8$ **Ans.**

16 *(N2014/P1/Q13)*



In the diagram, $BE = 5$cm, $CD = 7$cm and $AE = 3$cm.
BE is parallel to CD.

(a) Express CD as a percentage of BE. [1]

(b) Find ED. [2]

Thinking Process

(a) Multiply by 100.

(b) Apply rule of similarity to find AD. Subtract AE from AD to find ED.

Solution

(a) $\frac{7}{5} \times 100 = 140\%$ **Ans.**

(b) $BE \parallel CD$

$\therefore$ ΔABE is similar to ΔACD

$\Rightarrow \frac{AD}{AE} = \frac{CD}{BE}$

$\frac{AD}{3} = \frac{7}{5}$

$AD = \frac{7}{5} \times 3 = \frac{21}{5}$

$\therefore$ $ED = AD - AE$

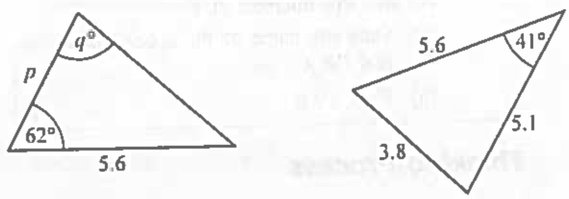
$= \frac{21}{5} - 3$

$= \frac{21-15}{5}$

$= \frac{6}{5} = 1.2$ cm **Ans.**

17 *(J2015/P1/Q10)*

These two triangles are congruent.
The lengths are in centimetres, correct to the nearest 0.1 cm.



Find p and q. [2]

Thinking Process

Understand that when two triangles are congruent, they will have exactly the same three sides and exactly the same three angles.

Solution

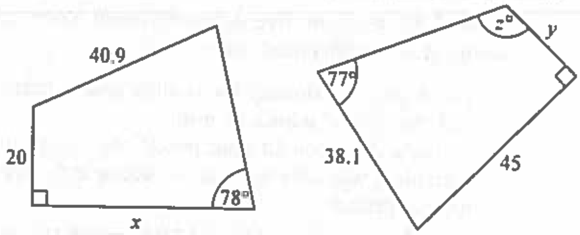
$p = 3.8$ cm **Ans.**

$q° + 62° + 41° = 180°$ (∠ sum of Δ)

$q° = 180° - 62° - 41°$

$= 77°$ **Ans.**

18 (N2015/P1/Q13)



These two quadrilaterals are congruent. The lengths are in millimetres.
Find the values of x, y and z. [3]

Thinking Process

Look for the equal corresponding angles in both quadrilaterals.

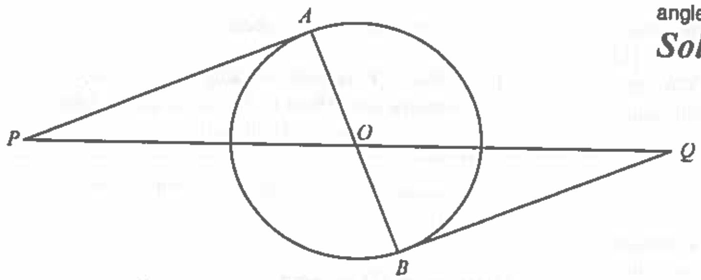
Solution

$x = 45$ mm. **Ans.**

$y = 20$ mm. **Ans.**

$z° + 77° + 78° + 90° = 360°$
$z° + 245° = 360°$
$z° = 115°$ **Ans.**

19 (J2016/P1/Q19)



AB is a diameter of the circle, centre O.
PA and QB are tangents to the circle at A and B respectively.
Prove that triangle PAO is congruent to triangle QBO.
Give a reason for each statement you make. [3]

Thinking Process

Observe that triangles are congruent by ASA property.

Solution

$OA = OB$ (radii of circle)

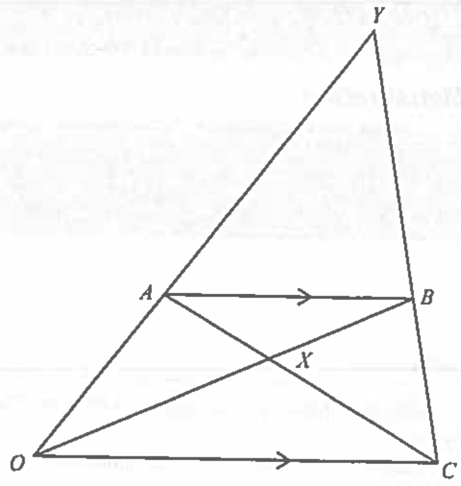
$\angle OAP = \angle OBQ = 90°$ (radius ⊥ tangent)

$\angle AOP = \angle BOQ$ (vert. opposite ∠s)

∴ $PAO \equiv QBO$ (ASA) **Proved.**

> **Angle-Side-Angle Rule (ASA):**
> Two triangles are congruent if any two angles and their included side are equal in both triangles.

20 (J2018/P1/Q8a)



OYC is a triangle.
A is a point on OY and B is a point on CY.
AB is parallel to OC. AC and OB intersect at X.
Prove that triangle ABX is similar to triangle COX.
Give a reason for each statement you make. [3]

Thinking Process

Prove that two angles of one triangle are equal to two angles of another triangle.

Solution

$A\hat{B}X = C\hat{O}X$ (alt ∠s. $AB \parallel OC$)

$B\hat{A}X = O\hat{C}X$ (alt ∠s. $AB \parallel DC$)

since two corresponding angles are equal.

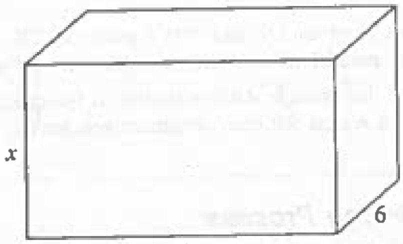
∴ ΔABX is similar to ΔCOX. **Proved.**

Topic 10

Mensuration

1 *(J2008 P1 Q7)*

The diagram shows a solid cuboid with base 10cm by 6cm.
The height of the cuboid is x centimetres.



(a) Find an expression, in terms of x, for the total surface area of the cuboid. [1]

(b) The total surface area of the cuboid is 376 cm^2. Form an equation in x and solve it to find the height of the cuboid. [2]

Thinking Process

(a) Total surface area = 2(*LxB*) + 2(*LxH*) + 2(*BxH*)

(b) To find height use the answer to part (a) to form an equation and solve for *x*.

Solution

(a) Total surface area $= 2(10\times 6) + 2(10\times x) + 2(6\times x)$

$= 2(60) + 20x + 12x$

$= (120 + 32x)\ \text{cm}^2$ **Ans.**

(b) Total surface area = 376

$\Rightarrow 120 + 32x = 376$

$32x = 376 - 120$

$x = \frac{256}{32} = 8$

$\therefore$ Height of the cuboid = 8 cm **Ans.**

2 *(J2008 P2 Q7)*

A, B, C, D and E are five different shaped blocks of ice stored in a refrigerated room.

(a) At 11 p.m. on Monday the cooling system failed, and the blocks started to melt.
At the end of each 24 hour period, the volume of each block was 12% less than its volume at the start of that period.

(i) Block A had a volume of 7500 cm^3 at 11 p.m. on Monday.
Calculate its volume at 11 p.m. on Wednesday. [2]

(ii) Block B had a volume of 6490 cm^3 at 11 p.m. on Tuesday.
Calculate its volume at 11 p.m. on the previous day. [2]

(iii) **Showing your working clearly**, find on which day the volume of Block C was half its volume at 11 p.m. on Monday. [2]

(b) **[The volume of a sphere is $\frac{4}{3}\pi r^3$.]**
[The surface area of a sphere is $4\pi r^2$.]
At 11 p.m. on Monday Block D was a **hemisphere** with radius 18 cm.
Calculate

(i) its volume, [2]

(ii) its **total** surface area. [2]

(c) As Block E melted, its shape was always **geometrically similar** to its original shape.
It had a volume of 5000 cm^3 when its height was 12 cm.
Calculate its height when its volume was 1080cm^3. [2]

Thinking Process

(a) (i) The volume decreased by 12% from Monday to Tuesday and then again 12% from Tuesday to Wednesday. Therefore solve accordingly.

(ii) The given volume of block B is 12% less. Find the actual volume.

(iii) The volume of Block C was 100% on Monday. On Tuesday it was 88% (12% less). On Wednesday it was further 12% less and so on. Keep on reducing the volume by 12% each day until the value of the volume is less than half the original value.

(c) Rule of similar figures:
ratio of volume of similar figures = ratio of cube of corresponding sides.

Solution

(a) (i) On monday, volume = 7500 cm^3

The volume decreases by 12%, hence the remaining volume is 100 − 12 = 88%.

On tuesday, volume = 88% of 7500

$= \frac{88}{100} \times 7500 = 6600$

On wednesday, volume = 88% of 6600

$= \frac{88}{100} \times 6600$

$= 5808$

≈ 5810 cm^3 (3 sf) **Ans.**

(ii) Let x be the actual volume of block B. the volume of block B has been reduced to 88% of its actual volume on tuesday.

$\Rightarrow$ 88% of $x = 6490$

$\frac{88}{100}x = 6490$

$x = 6490 \times \frac{100}{88} = 7375$

$\therefore$ Actual volume of block B on monday

$= 7375 \approx 7380 cm^3$ (3 sig. fig) **Ans.**

(iii) Let v be the volume of block C on monday.

On tuesday, vol. $= \frac{88}{100}v = 0.88v$

On wednesday, vol. $= \frac{88}{100}(0.88v) = 0.774v$

On thursday, vol. $= \frac{88}{100}(0.774v) = 0.681v$

On friday, vol. $= \frac{88}{100}(0.681v) = 0.599v$

On saturday, vol. $= \frac{88}{100}(0.599v) = 0.527v$

On sunday, vol. $= \frac{88}{100}(0.527v) = 0.464v$

$\therefore$ Volume reduces to half on Sunday **Ans.**

(b) (i) Volume of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$\therefore$ Vol. of block D $= \frac{1}{2}\left(\frac{4}{3}\pi(18)^3\right)$

$= \frac{2}{3} \times 3.142 \times 5832$

$= 12216.1$

≈ 12200 cm^3 **Ans.**

(ii) Total surface area of solid hemisphere D

$= \frac{1}{2}(4\pi r^2) + \pi r^2$

$= \frac{1}{2}(4\pi 18^2) + \pi 18^2$

$= 2036.016 + 1018.008$

$= 3054.024 \approx 3050$ cm^2 (3 sf) **Ans.**

(c) $\frac{\text{Volume before}}{\text{Volume after}} = \left(\frac{\text{height before}}{\text{height after}}\right)^3$

$\frac{5000}{1080} = \left(\frac{12}{h}\right)^3$

$\frac{125}{27} = \left(\frac{12}{h}\right)^3$

$\left(\frac{5}{3}\right)^3 = \left(\frac{12}{h}\right)^3$

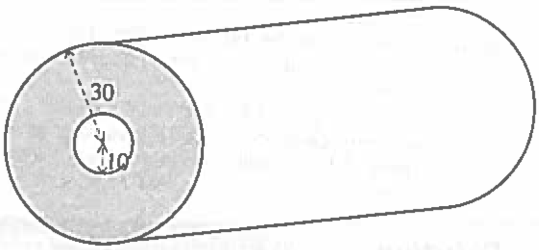
$\frac{5}{3} = \frac{12}{h}$

$5h = 36$ or $h = 7.2$

$\therefore$ required height = 7.2 cm **Ans.**

3 (N2008 P2 Q9)

(a) The diagram shows a roll of material. The material is wound onto a metal cylinder whose cross-section is a circle of radius 10 cm.
The shaded area shows the cross-section of the material on the roll.
The outer layer of material forms the curved surface of a cylinder of radius 30 cm.

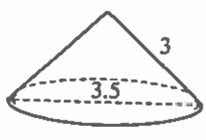


(i) Calculate, in square centimetres, the area of the cross-section of the material on the roll (shaded on the diagram). [2]

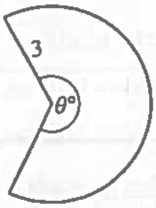
(ii) The material is 200 cm wide on the roll. Calculate, **in cubic metres**, the volume of the material. [2]

(iii) When unwound, the length of the material is 150 m.
Calculate the thickness of the material, giving your answer in millimetres. [2]

(b) The diagram shows a conical tent.
The diameter of the base is 3.5 m and the slant height is 3 m.

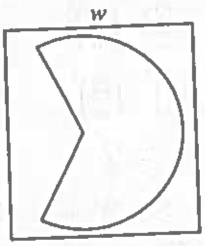


It is made from a flat piece of canvas that forms a sector of a circle of radius 3 m. The angle at the centre is θ.



(i) Show that $\theta = 210$. [3]

(ii) As shown, the required shape is cut from a rectangular piece of canvas of width w metres.



Given that w is a whole number find its least possible value.
Show all your working. [3]

Thinking Process

(a) (i) To find the shaded area subtract the area of small circle from the area of large circle.
(ii) Volume of the material = volume of complete cylinder − volume of small metal cylinder.
(iii) When unwound, the material would then be a cuboid. The height of the cuboid represents the thickness.

(b) (i) To find θ, find the arc length. Note that the arc length is actually the circumference of the base of the cone.
(ii) To find w find the horizontal distance from centre of sector to one of the vertices of the sector. Add the radius 3 to your answer. Round your final answer to a whole number.

Solution with TEACHER's COMMENTS

(a) (i) Shaded area = area of bigger circle − area of smaller circle

$= \pi(30)^2 - \pi(10)^2$

$= 800\pi$

$= 2513.6 \approx 2510$ (3sf) **Ans.**

(ii) Vol. of material

= vol. of big cylinder − vol. of small cylinder

$= \pi(30)^2(200) - \pi(10)^2(200)$

$= 160000\pi$

$= 502720 \text{ cm}^3$

$= 0.5027 \text{ m}^3$ $\boxed{\because 1\text{m}^3 = 1000000 \text{ cm}^3}$

$\approx 0.503 \text{ m}^3$ (3dp) **Ans.**

(iii) After unwinding, the material has the shape of a cuboid.

$\therefore$ Vol. of material $= l \times w \times h$

$0.5027 = 150 \times 2 \times h$

$h = 0.001676$ m

$= 1.676 \approx 1.68$ mm (3 sf)

$\therefore$ thickness of material = 1.68 mm **Ans.**

$\boxed{\text{Note that, } 1\text{m} = 1000 \text{ mm}}$

(b) (i) Circumference of base of conical tent

$= \pi d = 3.5\pi$ m.

$\Rightarrow$ arc length of sector $= 3.5\pi$

$\frac{\theta}{360}(2)(\pi)(3) = 3.5\pi$

$\frac{\theta}{60} = 3.5$

$\theta = 210°$ **Shown.**

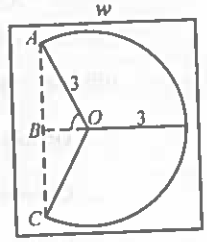
> When a sector is folded to make a cone, the arc of the sector becomes the base of the cone, and the radius of the sector becomes the slant height of the cone.

(ii) From the figure

$A\hat{O}C = 360° - 210°$

$= 150°$

$\therefore A\hat{O}B = \frac{150}{2} = 75°$



consider ΔAOB

$\cos A\hat{O}B = \frac{OB}{OA}$

$\Rightarrow \cos 75° = \frac{OB}{3}$

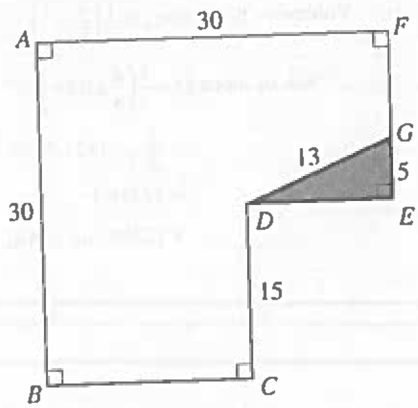
$OB = 0.77\dot{6}$ m

as w is a whole number

$\therefore$ least value of $w = 0.776 + 3$

$= 3.776 \approx 4.0$ m **Ans.**

4 (J2009/P1/Q22)



$ABCDEF$ represents an L-shaped piece of glass with $AB = AF = 30$ cm and $CD = 15$ cm.

The glass is cut to fit the window in a door and the shaded triangle DEG is removed.
$DG = 13$ cm and $EG = 5$ cm.

(a) Show that $DE = 12$ cm. [1]

(b) For the remaining piece of glass $ABCDGF$, find

(i) its perimeter, [2]

(ii) its area. [2]

(c) State the value of $\cos D\hat{G}F$. [1]

Thinking Process

(a) Apply Pythagoras Theorem.

(b) (i) Add all the sides of the remaining figure.

(ii) Note that the figure can be divided into a rectangle and a trapezium.

(c) $\cos(180-\theta) = -\cos\theta$

Solution

(a) $DG^2 = DE^2 + EG^2$

$(13)^2 = DE^2 + (5)^2$

$169 = DE^2 + 25$

$DE^2 = 169 - 25$

$DE^2 = 144$

$DE = 12$ cm. **Shown.**

(b) (i) $CD + EG + GF = AB$

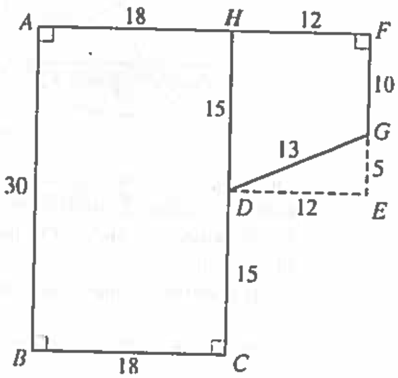
$15 + 5 + GF = 30 \Rightarrow GF = 10$ cm.

Perimeter $= AB + BC + CD + DG + GF + FA$

$= 30 + 18 + 15 + 13 + 10 + 30$

$= 116$ cm **Ans.**

(ii)



Area = area of rectangle $ABCH$
+ area of trapezium $DGFH$

$= (30 \times 18) + \frac{1}{2}(12)(15+10)$

$= 540 + 150$

$= 690$ cm^2. **Ans.**

Alternative Method :
Area = area of rectangle $ABCH$
+ area of rectangle $DEFH$
− area of triangle DEG

$= (30 \times 18) + (12 \times 15) - \frac{1}{2}(12)(5)$

$= 540 + 180 - 30 = 690$ cm^2. **Ans.**

(c) $\cos D\hat{G}F = \cos(180 - D\hat{G}E)$

$= -\cos D\hat{G}E$

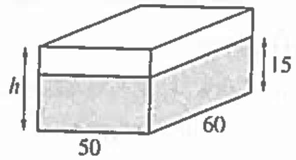
$= -\frac{5}{13}$ **Ans.**

5 *(J2009/P2 Q7)*

(a) When a solid rectangular wooden block of oak floats, 60% of its height is under water.

(i) What fraction of its height is above water? [1]

(ii) A block of oak has length 60 cm, breadth 50 cm and height h centimetres.

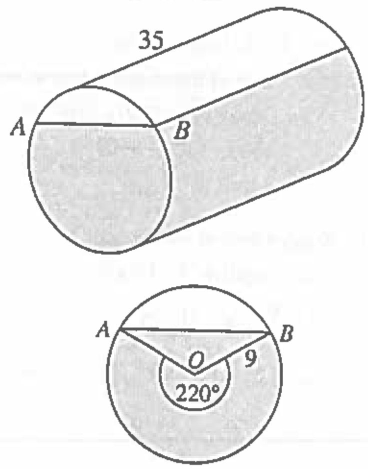


It floats with 15 cm of its height under water.

(a) Find the value of h. [1]

(b) In the diagram, the shaded region represents part of the surface area of the block that is in contact with the water.
Calculate the total surface area of the block that is in contact with the water. [2]

(b)



A solid cylinder, made from a different type of wood, floats in water.
The shaded region represents part of the surface of the cylinder that is in contact with the water.

The right hand diagram shows the circular cross-section of one end.
The centre of the circle is O and the water level reaches the points A and B on the circumference.
Reflex angle $AOB = 220°$.
The cylinder has radius 9 cm and length 35 cm.
Calculate

(i) the area of the **curved surface** of the cylinder that is in contact with the water, [2]

(ii) the surface area of **one end** of the cylinder that is in contact with the water, [4]

(iii) the distance between the water level AB and the top of the cylinder. [2]

Thinking Process

(a) (i) Express (100 − 60)% in fraction.
 (ii) (a) Observe that 60% now represent 15 cm. Find 100%.
 (b) Calculate area of the shaded region.

(b) (i) Curved surface area of shaded part of cylinder = arc length of major sector x height of cylinder.
 (ii) Surface area of one end = Area of major sector AB + area of triangle AOB.
 (iii) Find the perpendicular distance from O to AB and subtract it from the radius.

Solution

(a) (i) 60% of height is under water

$\therefore$ 40% of height is above water.

$$\text{fraction of height above water} = \frac{40}{100}$$

$$= \frac{2}{5} \quad \textbf{Ans.}$$

(ii) (a) 60% of height —— 15 cm

$$100\% \text{ of height} —— \frac{15}{60} \times 100 = 25 \text{ cm}$$

$\therefore$ $h = 25$ cm **Ans.**

(b) Total shaded surface area

$= \text{area of four walls} + \text{area of base}$

$= 2(50 \times 15) + 2(60 \times 15) + (50 \times 60)$

$= 1500 + 1800 + 3000$

$= 6300 \text{ cm}^2$ **Ans.**

(b) (i) Shaded curved surface area

$= \text{arc length } ACB \times \text{height}$

$$= \left(\frac{220}{360} \times 2\pi \times 9\right) \times 35$$

$= 1209.51 \approx 1210 \text{ cm}^2$ (3sf) **Ans.**

(ii) Shaded area of one end of cylinder

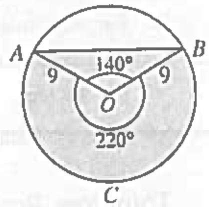
$= \text{area of sector } ACB + \text{area of triangle } AOB$

$$= \frac{220}{360}\pi(9)^2 + \frac{1}{2}(9)(9)\sin 140°$$

$= 155.529 + 26.033$

$= 181.562$

$\approx 182 \text{ cm}^2$ (3sf) **Ans.**



(iii) In ΔAOD

$$\cos 70° = \frac{OD}{9}$$

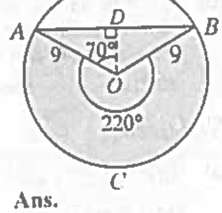
$OD = 9 \times \cos 70°$

$= 3.078$

$\therefore$ required distance

$= 9 - 3.078$

$= 5.92$ cm (3sf) **Ans.**

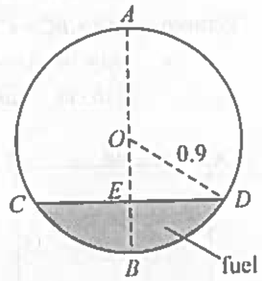


6 *(N2009 P2 Q7)*

(a) A fuel tank is a cylinder of diameter 1.8 m.

(i) The tank holds 25 000 litres when full.
Given that $1 \text{ m}^3 = 1000$ litres, calculate the length of the cylinder.
Give your answer in metres. [4]

(ii)



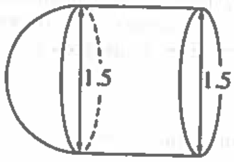
The diagram shows the cross-section of the cylinder, centre O, containing some fuel.
CD is horizontal and is the level of the fuel in the cylinder.
AB is a vertical diameter and intersects CD at E.
Given that E is the midpoint of OB,

(a) show that $E\hat{O}D = 60°$. [1]

(b) calculate the area of the segment BCD, [3]

(c) calculate the number of litres of fuel in the cylinder. [2]

(b) **[Volume of a sphere** $= \frac{4}{3}\pi r^3$**]**
A different fuel tank consists of a cylinder of diameter 1.5 m and a hemisphere of diameter 1.5 m at one end.



The volume of the cylinder is 10 times the volume of the hemisphere.
Calculate the length of the cylinder. [2]

Thinking Process

(a) (i) Volume of cylinder $= \pi r^2 h$.

(ii) (a) Find OE and apply $\cos\theta = \frac{\text{adj}}{\text{hyp}}$.

(b) Area of segment BCD = Area of sector $OCBD$ − Area of triangle OCD.

(c) Volume of fuel = Area of segment BCD × Length of cylinder.

(b) Volume of cylinder = 10(volume of hemisphere).

Solution

(a) (i) Radius of cylinder, $r = \frac{1.8}{2} = 0.9$ m

$1000 \text{ litres} = 1 \text{ m}^3$

$25000 \text{ litres} = \frac{1}{1000} \times 25000 = 25 \text{ m}^3$

∴ Volume of tank $= 25 \text{ m}^3$

volume of cylinder $= \pi r^2 h$

$25 = \pi(0.9)^2 h$

$h = \frac{25}{\pi(0.9)^2}$

$= \frac{25}{2.545} = 9.823$

∴ length of cylinder = 9.82 m (3sf) **Ans.**

(ii) (a) Given that E is the midpoint of OB,

$\Rightarrow OE = \frac{1}{2}OB$

$= \frac{1}{2}(0.9) = 0.45$ m.

In ΔEOD,

$\cos E\hat{O}D = \frac{OE}{OD}$

$= \frac{0.45}{0.9} = 0.5$

∴ $E\hat{O}D = 60°$ **Shown.**

(b) $C\hat{O}D = 2(60°) = 120°$

Area of segment BCD

$=$ area of sector $OCBD$ − area of ΔOCD

$= \frac{120}{360}(\pi)(0.9)^2 - \frac{1}{2}(0.9)(0.9)\sin 120°$

$= 0.84834 - 0.35074$

$= 0.49759 \approx 0.498 \text{ m}^2$ **Ans.**

(c) Volume of fuel in the cylinder

= area of segment BCD × length of cylinder

$= 0.49759 \times 9.823$

$= 4.8878 \text{ m}^3$

∴ number of litres of fuel in cylinder

$= 4.8878 \times 1000$

$= 4887.8 \approx 4890$ litres **Ans.**

(b) Volume of cylinder = 10(volume of hemisphere)

$\pi(0.75)^2 h = 10 \times \frac{1}{2}\left(\frac{4}{3}\pi(0.75)^3\right)$

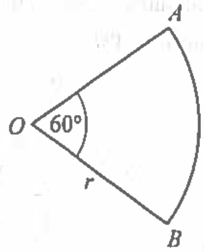
$0.5625h = 5(0.5625)$

$h = 5$

∴ length of the cylinder = 5 m **Ans.**

7 (J2010 P1 Q18)

OAB is the sector of a circle of radius r cm. $A\hat{O}B = 60°$.



Find, in its simplest form, an expression in terms of r and π for

(a) the area of the sector, [1]

(b) the perimeter of the sector. [2]

Thinking Process

(a) Apply formula for area of sector.

(b) Perimeter = $OA + OB$ + arc length AB.

Solution

(a) Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \pi r^2$

$= \frac{1}{6}\pi r^2$ **Ans.**

(b) Arc length $AB = \frac{60}{360} \times 2\pi r = \frac{1}{3}\pi r$

Perimeter of the sector $= OA + OB + \overparen{AB}$

$= r + r + \frac{1}{3}\pi r$

$= 2r + \frac{1}{3}\pi r$

$= r\left(\frac{6+\pi}{3}\right)$ **Ans.**

(b) [The volume of a pyramid $= \frac{1}{3} \times$ area of base $\times$ height]

Calculate

(i) the volume of the pyramid, [2]

(ii) the area of triangle BCD, [3]

(iii) the height of the pyramid when the base is triangle BCD. [3]

(c) An identical triangular pyramid is removed from each of the other 7 vertices of the cube to form the new solid shown in Diagram III.

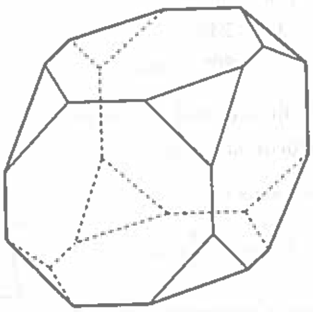


Diagram III

The original cube had 6 faces, 8 vertices and 12 edges.
For the new solid, write down the number of

(i) faces, [1]

(ii) vertices, [1]

(iii) edges. [1]

Thinking Process

(a) Look for the line that is perpendicular to the triangular base ABD.

(b) (ii) ΔBCD is an equilateral triangle. Use sine rule to find its area.

(iii) Use the formula for pyramid's volume to find the height.

(c) Carefully count the number of faces, vertices and edges from diagram III.

Solution

(a) AC is perpendicular to triangle ABD

$\therefore$ AC is the height of pyramid. **Ans.**

(b) (i) Vol. of pyramid $= \frac{1}{3} \times$ area of $\Delta ABD \times AC$

$$= \frac{1}{3} \times \left(\frac{1}{2}(5)(5)\right) \times 5$$
$$= \frac{1}{3} \times 12.5 \times 5$$
$$= 20.833$$
$$\approx 20.8 \text{ cm}^3 \text{ (3 sf)} \quad \textbf{Ans.}$$

(ii) Consider ΔABD

By pythagoras theorem,

$$BD^2 = (5)^2 + (5)^2$$
$$BD^2 = 50$$
$$BD = \sqrt{50}$$

ΔABD, ΔACD, and ΔABC are three identical triangles.

Hence ΔBCD is an equilateral triangle.

$$\therefore \quad \text{Area of } \Delta BCD = \frac{1}{2} \times \sqrt{50} \times \sqrt{50} \times \sin 60°$$
$$= \frac{1}{2} \times 50 \times 0.866$$
$$= 21.65$$
$$\approx 21.7 \text{ cm}^2 \text{ (3 sf)} \quad \textbf{Ans.}$$

(iii) Using the answers of (b) (i) & (ii),

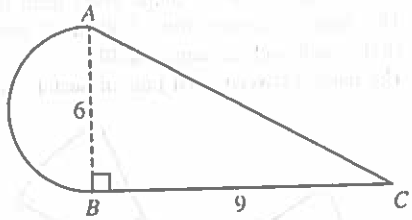
$$\text{Vol. of pyramid} = \frac{1}{3} \times \text{base area} \times \text{ht}$$
$$20.833 = \frac{1}{3} \times 21.65 \times \text{ht}$$
$$\text{ht} = \frac{20.833 \times 3}{21.65}$$
$$= 2.887$$
$$\approx 2.89 \text{ cm (3 sf)} \quad \textbf{Ans.}$$

(c) (i) Number of faces = 14 **Ans.**

(ii) Number of vertices = 24 **Ans.**

(iii) Number of edges = 36 **Ans.**

12 *(J2011 P1 Q6)*



ABC is a right-angled triangle with $AB = 6$ cm and $BC = 9$ cm.
A semicircle of diameter 6 cm is joined to the triangle along AB.
Find an expression, in the form $a + b\pi$, for the total area of the shape. [2]

Thinking Process

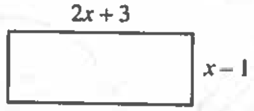
✎ Total area = area of triangle + area of semicircle.

Solution

$$\text{Total area} = \text{area of } \Delta ABC + \text{area of semicircle}$$
$$= \frac{1}{2} \times 9 \times 6 + \frac{1}{2}\pi(3)^2$$
$$= (27 + \frac{9}{2}\pi) \text{ cm}^2 \quad \textbf{Ans.}$$

13 *(J2011/P1/Q26)*

The diagram shows a rectangle with length $(2x+3)$ cm and width $(x-1)$ cm .



(a) The area of the rectangle is 12 cm².
Form an equation in x and show that it reduces to $2x^2 + x - 15 = 0$. [2]

(b) Solve $2x^2 + x - 15 = 0$. [2]

(c) Find the perimeter of the rectangle. [1]

Thinking Process

(a) ✎ Use area = length × width.

(b) ✎ Solve by grouping.

(c) To find the perimeter ✎ substitute the value of x found in (b) into length and width of the rectangle.

Solution

(a) Area of rectangle $= l \times w$

$\Rightarrow \; 12 = (2x+3)(x-1)$

$\Rightarrow \; 12 = 2x^2 + 3x - 2x - 3$

$\Rightarrow \; 2x^2 + x - 15 = 0$ **Shown.**

(b)
$$2x^2 + x - 15 = 0$$
$$2x^2 + 6x - 5x - 15 = 0$$
$$2x(x+3) - 5(x+3) = 0$$
$$(x+3)(2x-5) = 0$$
$$\Rightarrow x = -3 \text{ or } x = \frac{5}{2}$$
$\therefore \; x = -3$ or 2.5 **Ans.**

(c) Using $x = 2.5$ from part (b), we have,

length $= 2(2.5) + 3 = 8$ cm

width $= 2.5 - 1 = 1.5$ cm

$\therefore$ perimeter $= 2(l + w)$
$= 2(8 + 1.5)$
$= 2(9.5)$
$= 19$ cm **Ans.**

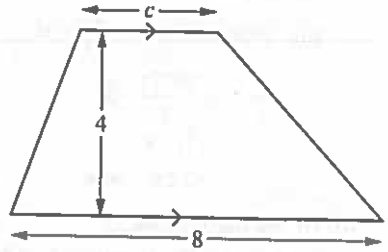
14 *(J2011/P2/Q2)*

(a) The formula for the area of a trapezium is

$$A = \frac{1}{2}h(c+d).$$

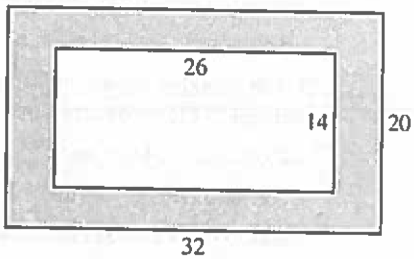
(i) Find an expression for c in terms of A, h and d. [2]

(ii)



The diagram shows a trapezium with dimensions given in centimetres.
The perpendicular distance between the parallel lines is 4 cm.
The area of the trapezium is 22 cm².
Find c. [1]

(b)



In the diagram, the shaded area represents a rectangular picture frame.
The outer rectangle is 32 cm by 20 cm.
The inner rectangle is 26 cm by 14 cm.
All measurements are given to the nearest centimetre.

(i) Calculate the lower bound of the perimeter of the outer rectangle. [2]

(ii) Calculate the upper bound of the area of the frame. [3]

Thinking Process

(a) (i) Make c the subject of formula.

(ii) To find c ✎ use the formula for the area of a trapezium.

(b) (i) To find the lower bound of the perimeter ✎ find the lowest possible length and width ✎ subtract 0.5 cm from each of the length and width of the rectangle.

(ii) To find the greatest area of the frame ✎ add 0.5 cm to the length and width of the outer rectangle, and subtract 0.5 from length and width of the inner rectangle.

Solution

(a) (i)
$$A = \frac{1}{2}h(c+d)$$
$$2A = h(c+d)$$
$$\frac{2A}{h} = c+d$$
$$c = \frac{2A}{h} - d$$ **Ans.**

(ii) From (a) (i). $c = \frac{2A}{h} - d$

$\Rightarrow c = \frac{2(22)}{4} - 8$

$= 11 - 8$

$= 3$ cm **Ans.**

(b) (i) For outer rectangle,

least possible length $= 32 - 0.5 = 31.5$ cm

least possible width $= 20 - 0.5 = 19.5$ cm

lower bound of the perimeter $= 2(31.5 + 19.5)$

$= 2(51)$

$= 102$ cm **Ans.**

(ii) Upper bound of the area of the outer rectangle $= (32 + 0.5) \times (20 + 0.5)$

$= 32.5 \times 20.5 = 666.25$ cm^2

Lower bound of the area of the inner rectangle $= (26 - 0.5) \times (14 - 0.5)$

$= 25.5 \times 13.5 = 344.25$ cm^2

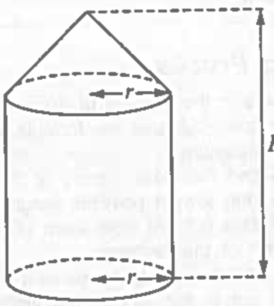
Upper bound of the area of the frame

$= 666.25 - 344.25 = 322$ cm^2 **Ans.**

Note that:
Upper bound or greatest area of the frame = greatest area of the outer rectangle − least area of the inner rectangle.

15 *(J2011/P2/Q11)*

[Volume of a cone $= \frac{1}{3}\pi r^2 h$]



The solid above consists of a cone with base radius r centimetres on top of a cylinder of radius r centimetres. The height of the cylinder is twice the height of the cone. The total height of the solid is H centimetres.

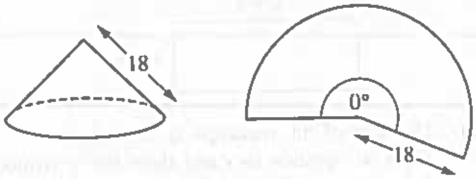
(a) Find an expression, in terms of π, r and H, for the volume of the solid.
Give your answer in its simplest form. [3]

(b) It is given that $r = 10$ and the height of the cone is 15 cm.

(i) Show that the slant height of the cone is 18.0 cm, correct to one decimal place. [2]

(ii) Find the circumference of the base of the cone. [2]

(iii) The curved surface area of the cone can be made into the shape of a sector of a circle with angle $\theta°$. Show that θ is 200, correct to the nearest integer.



[2]

(iv) Hence, or otherwise, find the total surface area of the solid. [3]

Thinking Process

(a) Volume of the solid = volume of cylinder + volume of cone.

(b) (i) Apply pythagoras theorem.

(ii) Circumference $= 2\pi r$.

(iii) Circumference of the base of cone = arc length of the sector.

(iv) To find the total surface area find the areas of the circular base, the curved surface of cylinder and the curved suface area of the cone.

Solution

(a) Let height of the cone $= h$ cm.

$\Rightarrow$ height of cylinder $= 2h$ cm

$\therefore h + 2h = H \Rightarrow h = \frac{1}{3}H$

Volume of solid

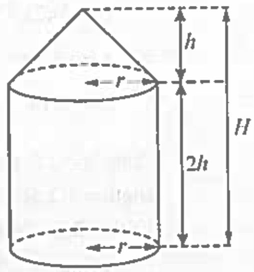
$= \pi r^2(2h) + \frac{1}{3}\pi r^2 h$

$= 2\pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \frac{7}{3}\pi r^2 h$

$= \frac{7}{3}\pi r^2(\frac{1}{3}H)$

$= \frac{7}{9}\pi r^2 H$ cm^3 **Ans.**



(b) (i) Applying pythagoras theorem.

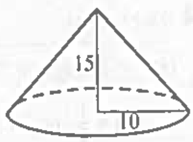
slant height

$= \sqrt{(10)^2 + (15)^2}$

$= \sqrt{325}$

$= 18.02$

≈ 18.0 cm **Shown.**



(ii) Circumference $= 2\pi r$

$= 2 \times \frac{22}{7} \times 10$

$= 62.8574$

≈ 62.9 cm **Ans.**

(iii) Arc length of sector = circumference of the base of cone

$$\frac{\theta}{360}\times 2\pi(18) = 2\pi(10)$$

$$\frac{\theta}{10} = 20$$

$$\theta = 200° \quad \textbf{Shown.}$$

(iv) Total surface area

$= \pi r^2 + 2\pi rh + \pi rl$

$= \pi(10)^2 + 2\pi(10)(30) + \pi(10)(18)$

$= 100\pi + 600\pi + 180\pi$

$= 880\pi$

$= 2764.96 \approx 2765 \text{ cm}^2$ **Ans.**

16 *(N2011/P2 Q3)*

(a)

Diagram I

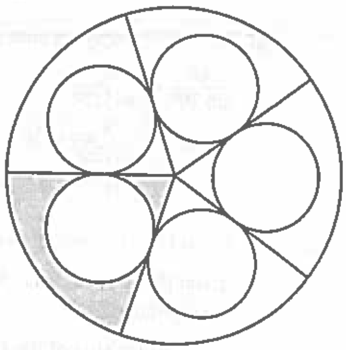


Diagram I shows one large circle and five identical small circles.
Each of the five radii shown is a tangent to two of the small circles.

(i) Describe the symmetry of the diagram. [1]

(ii) The radius of the large circle is R centimetres and the radius of each small circle is r centimetres. Each small circle is equal in area to the shaded region.
Find $R^2 : r^2$. [3]

(b)

Diagram II

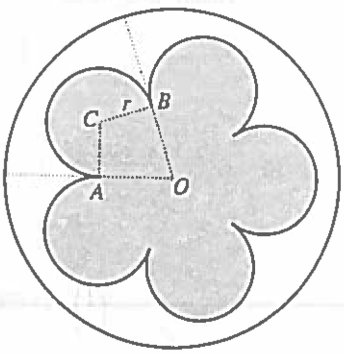


Diagram II shows the same large circle and arcs of the same small circles as in **Diagram I**.
C is the centre of one of the small circles.
This circle touches the adjacent circles at A and B.
O is the centre of the large circle.

(i) Show that reflex $A\hat{C}B = 252°$. [2]

(ii) The perimeter of the shaded region is $k\pi r$ centimetres. Calculate the value of k. [2]

Thinking Process

(a) (i) Understand the definition of line symmetry and rotational symmetry.

(ii) Find the area of the shaded region. Use the given condition to find the required ratio.

(b) (i) To find the reflex $\angle ACB$ ✐ find the interior angles of the quadrilateral $OACB$.

(ii) To find k ✐ find the perimeter of the shaded region. Compare it with the given perimeter.

Solution

(a) (i) The figure has 1 line of symmetry.

(ii) Area of one small circle $= \pi r^2$

The large circle is divided into 5 equal sectors, one of which is shaded.

$\therefore$ area of one sector $= \frac{1}{5}\pi R^2$

area of small circle = area of the shaded region

$$\pi r^2 = \frac{1}{5}\pi R^2 - \pi r^2$$

$$\frac{1}{5}\pi R^2 = 2\pi r^2$$

$$R^2 = 10r^2 \Rightarrow \frac{R^2}{r^2} = \frac{10}{1}$$

$\therefore$ $R^2 : r^2 = 10 : 1$ **Ans.**

(b) (i) $A\hat{O}B = \frac{360°}{5} = 72°$

$O\hat{A}C = O\hat{B}C = 90°$

> OA is tangent ⊥ to radius AC through point of contact A.

In quadriteral $OACB$,

$O\hat{A}C + A\hat{C}B + O\hat{B}C + A\hat{O}B = 360°$

$90° + A\hat{C}B + 90° + 72° = 360°$

$A\hat{C}B = 360° - 90° - 90° - 72°$

$= 108°$

$\therefore$ reflex $A\hat{C}B = 360° - 108°$

$= 252°$ **Shown.**

(ii) Length of arc $A\hat{C}B = \frac{252°}{360°}\times 2\pi r = \frac{7}{5}\pi r$

Total length of 5 arcs $= 5(\frac{7}{5}\pi r) = 7\pi r$

perimeter of the shaded region $= k\pi r$

$\Rightarrow$ $7\pi r = k\pi r$

$\Rightarrow$ $k = 7$ **Ans.**

17 *(N2011/P2 Q7)*

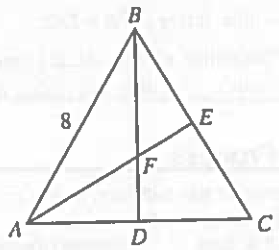


Diagram I

In **Diagram I**, ABC is an equilateral triangle of side 8 cm. D and E are the midpoints of AC and BC respectively.
BD and AE intersect at F.

(a) (i) Find the area of triangle ABC. [2]

(ii) Show that $A\hat{F}B = 120°$. [1]

(iii) Calculate AF. [2]

(b) [The volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height]

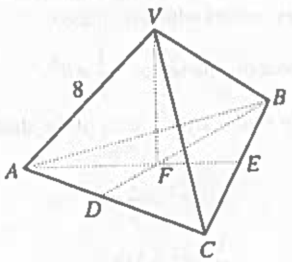


Diagram II

The equilateral triangle of side 8 cm in **Diagram I** forms the base of the triangular pyramid $VABC$ in **Diagram II**.
The vertex V is vertically above F.
$VA = VB = VC = 8$ cm.

(i) Calculate the surface area of the pyramid. [1]

(ii) Calculate the volume of the pyramid. [3]

(c) A pyramid P is geometrically similar to $VABC$ and its volume is $\frac{1}{64}$ of the volume of $VABC$.

(i) Find the length of an edge of P. [2]

(ii) A pyramid that is identical to P is removed from each of the four vertices of $VABC$. State the number of faces of the new solid. [1]

Thinking Process

(a) (i) Apply, area $= \frac{1}{2}ab\sin c$.

(ii) Note that AE and BD are angle bisectors of $\angle BAC$ and $\angle ABC$ respectively.

(iii) To find AF, apply sine rule.

(b) (i) Surface area = area of 4 equilateral triangles.

(c) (i) Use $\frac{Vol_1}{Vol_2} = \left(\frac{L_1}{L_2}\right)^3$

(ii) Count the number of faces of the new solid.

Solution with TEACHER's COMMENTS

(a) (i) ΔABC is an equilateral triangle.

$\therefore$ area of $\Delta ABC = \frac{1}{2}(8)(8)\sin 60°$

$= 27.7 \text{ cm}^2$ **Ans.**

(ii) $A\hat{B}F = B\hat{A}F = 30°$

> In an equilateral triangle, a median is also a bisector. So, in ΔABC, the two medians AE and BD are also bisectors of angles $\hat{A}$ & $\hat{B}$.

In ΔABF,

$A\hat{B}F + B\hat{A}F + A\hat{F}B = 180°$

$30° + 30° + A\hat{F}B = 180°$

$A\hat{F}B = 180° - 60°$

$= 120°$ **Ans.**

(iii) In ΔABF, applying sine rule.

$\frac{AF}{\sin 30°} = \frac{8}{\sin 120°}$

$AF = \frac{8}{\sin 120°} \times \sin 30°$

$= 4.619 \approx 4.62$ cm **Ans.**

(b) (i) From (a) (i). area of $\Delta ABC = 27.7 \text{ cm}^2$

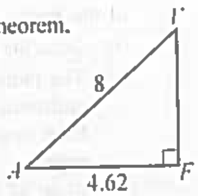
given that all four triangles of the pyramid are equilateral.

$\therefore$ surface area of the pyramid

$= 4(27.7) = 110.8 \approx 111 \text{ cm}^2$ **Ans.**

(ii) Consider ΔAFV.

applying Pythagoras Theorem.



$VF = \sqrt{(AV)^2 - (AF)^2}$

$= \sqrt{(8)^2 - (4.619)^2}$

$= \sqrt{42.665}$

$= 6.53$ cm

volume of pyramid $= \frac{1}{3}$(base area)(height)

$= \frac{1}{3}(27.7)(6.53)$

$= 60.3 \text{ cm}^3$ **Ans.**

(c) (i) $\frac{\text{Volume of } P}{\text{Volume of } VABC} = \left(\frac{\text{length}_P}{\text{length}_{VABC}}\right)^3$

$\frac{1}{64} = \left(\frac{\text{length}_P}{8}\right)^3$

$\left(\frac{1}{4}\right)^3 = \left(\frac{\text{length}_P}{8}\right)^3$

$\text{length}_P = 2$

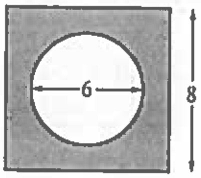
$\therefore$ length of one edge of $P = 2$ cm **Ans.**

(ii) Number of faces of new solid = 8 **Ans.**

18 (J2012/P1/Q6)

A circle of diameter 6 cm is cut from a square of side 8 cm.

Find an expression, in the form $a - b\pi$, for the shaded area. [2]



Thinking Process

Area of shaded region = area of square − area of circle.

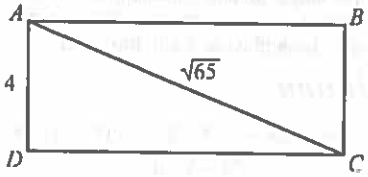
Solution

Shaded area = area of square − area of circle

$= (8)^2 - \pi(3)^2$

$= (64 - 9\pi)\text{ cm}^2$ **Ans.**

19 (J2012/P1/Q15)



$ABCD$ is a rectangle with $AC = \sqrt{65}$ cm and $AD = 4$ cm.

Calculate the area of $ABCD$. [3]

Thinking Process

To find area apply Pythagoras theorem to find DC.

Solution

By Pythagoras Theorem,

$AC^2 = AD^2 + DC^2$

$(\sqrt{65})^2 = (4)^2 + DC^2$

$DC^2 = 65 - 16$

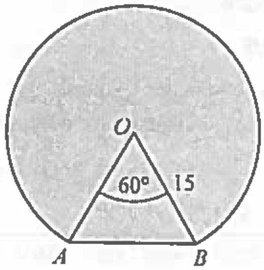
$DC^2 = 49 \Rightarrow DC = 7$

$\therefore$ area of $ABCD = 4 \times 7 = 28\text{ cm}^2$ **Ans.**

20 (J2012/P2/Q10)

The diagram shows a major segment of a circle with centre O and radius 15 cm.

A and B are two points on the circumference such that $A\hat{O}B = 60°$.



(a) Calculate

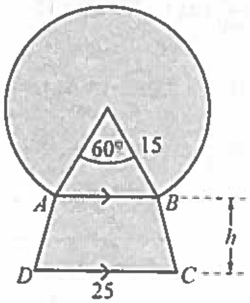
(i) the area of the major segment, [4]

(ii) the perimeter of the major segment. [2]

(b) **Shape I** is formed by joining this segment to a trapezium, $ABCD$, along AB.

AB is parallel to DC, $DC = 25$ cm and the perpendicular height of the trapezium is h cm.

The area of the trapezium is 248 cm².

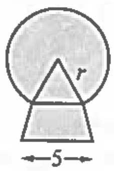


Shape I

Calculate h. [2]

(c) **Shape II** is geometrically similar to **Shape I**.

The longest side of the trapezium in **Shape II** is 5 cm.



Shape II

(i) Find the radius, r, of the segment in **Shape II**. [1]

(ii) Find the total area of **Shape II**. [3]

Thinking Process

(a) (i) Area of major segment = area of major sector + area of triangle AOB.

(ii) Perimeter = major arc length + AB.

(b) To find h Apply formula for area of a trapezium.

(c) (i) Apply rule of similarity, i.e. ratio of corresponding lengths is equal.

(ii) Find the area of shape I. Then apply rule of similar figures, i.e. ratio of area of similar figures = ratio of squares of corresponding lengths.

Solution

(a) (i) Area of major segment

$= \text{area of major sector} + \text{area of } \Delta AOB$

$= \frac{300}{360}(3.142)(15)^2 + \frac{1}{2}(15)(15)\sin 60°$

$= 589.125 + 97.428$

$= 686.553 \approx 687 \text{ cm}^2$ **Ans.**

(ii) In ΔAOB, using cosine rule,

$AB = \sqrt{15^2 + 15^2 - 2(15)(15)\cos 60°}$

$= \sqrt{450 - 225}$

$= \sqrt{225} = 15$ cm.

perimeter of the major segment

$= \text{major arc length} + AB$

$= \frac{300}{360}(2)(3.142)(15) + 15$

$= 78.55 + 15$

$= 93.55 \approx 93.6$ cm **Ans.**

(b) Area of trapezium $= \frac{1}{2} \times h(AB + DC)$

$248 = \frac{1}{2} \times h(15 + 25)$

$248 = \frac{40}{2} h$

$h = 248 \times \frac{2}{40} = 12.4$ cm **Ans.**

(c) (i) $\frac{\text{radius of segment in shape II}}{\text{radius of segment in shape I}} = \frac{5}{25}$

$\Rightarrow \frac{r}{15} = \frac{5}{25}$

$r = \frac{5}{25} \times 15$

$= 3$ cm **Ans.**

(ii) Total area of shape I

= area of major segment + area of trapezium

$= 686.55 + 248$

$= 934.55 \text{ cm}^2$

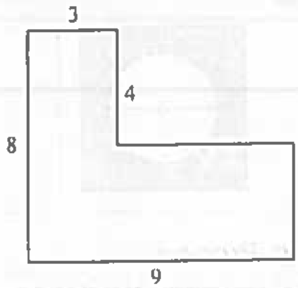
$\frac{\text{total area of shape II}}{\text{total area of shape I}} = \left(\frac{5}{25}\right)^2$

$\frac{\text{total area of shape II}}{934.55} = \frac{1}{25}$

total area of shape II $= 37.4 \text{ cm}^2$ **Ans.**

21 *(N2012/P1/Q3)*

The diagram shows an L-shaped piece of card. The measurements are in centimetres and all the angles are right-angles.



(a) Calculate the perimeter of this card. [1]

(b) Square pieces, each of side 2 cm, are cut from this card.
Find the greatest number of squares that can be obtained. [1]

Thinking Process

(a) Find the two unknown sides, then add up all the sides to find perimeter.

(b) Find how many squares can be cut lengthwise and breadthwise from the card.

Solution

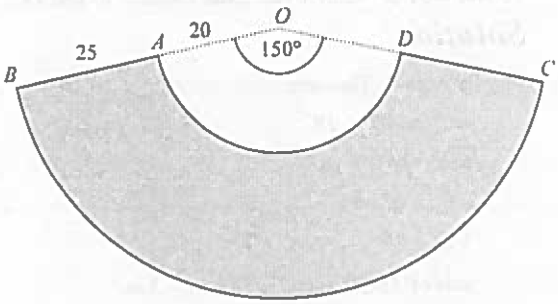
(a) Perimeter $= 9 + 8 + 3 + 4 + (9 - 3) + (8 - 4)$

$= 24 + 6 + 4$

$= 34$ cm **Ans.**

(b) Greatest number of squares obtained = 10 **Ans.**

22 *(N2012/P2/Q5)*



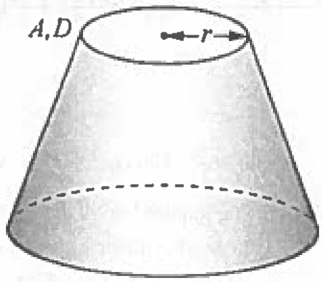
AD and BC are arcs of circles with centre O.
A is a point on OB, and D is a point on OC.
$OA = 20$ cm and $AB = 25$ cm.

$A\hat{O}D = 150°$.

(a) Calculate the perimeter of the shaded shape $ABCD$. [3]

(b) Calculate the area of the shaded shape $ABCD$. [3]

(c) The shape $ABCD$ is used to make a lampshade by joining AB and DC.



Calculate the radius, r cm, of the circular top of the lampshade. [2]

Thinking Process

(a) Perimeter of the shaded region = AB + arc length $BC + CD$ + arc length AD.

(b) Area of the shaded region = area of major sector − area of minor sector.

(c) To find r, note that arc length AD is the circumference of the top of the lampshade.

Solution

(a) Perimeter of the shaded region $ABCD$

$= AB + \overset{\frown}{BC} + CD + \overset{\frown}{DA}$

$= 25 + (\frac{150}{360} \times 2 \times \pi \times 45) + 25 + (\frac{150}{360} \times 2 \times \pi \times 20)$

$= 25 + 117.825 + 25 + 52.3667$

$= 220.192 \approx 220$ cm (3sf) **Ans.**

(b) Area of the shaded region $ABCD$

= area of sector OBC − area of sector OAD

$= (\frac{150}{360} \times \pi \times 45^2) - (\frac{150}{360} \times \pi \times 20^2)$

$= 2651.063 - 523.667$

$= 2127.396 \approx 2130\text{ cm}^2$ (3sf) **Ans.**

(c) Circumference of the top of the lampshade

= Arc length AD

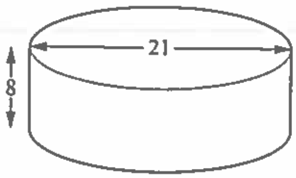
$\Rightarrow \quad 2\pi r = \frac{150}{360}(2)(\pi)(20)$

$r = \frac{150}{360}(20)$

$= \frac{25}{3} = 8.33$ cm **Ans.**

23 *(N2012/P2/Q7)*

A cylindrical, open container has a diameter of 21 cm and height of 8 cm.



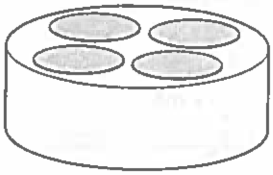
(a) (i) Calculate the total external surface area of this container. [3]

(ii) A manufacturer receives an order for 30 000 containers.
He needs an extra 150 cm² of material for each container to cover wastage.
Calculate the area of material needed to make these containers.
Give your answer in square metres. [2]

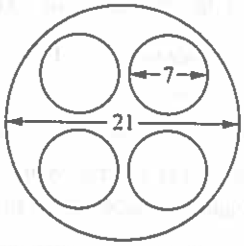
[The Surface area of a sphere is $4\pi r^2$]

[The Volume of a sphere is $\frac{4}{3}\pi r^3$]

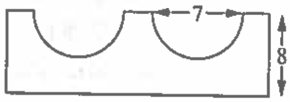
(b) A circular top that can hold 4 hemispherical bowls can be placed on the container.



Container and Top



Top



Cross-section

The top is a circle of diameter 21 cm with four circular holes of diameter 7 cm.
A hemispherical bowl of diameter 7 cm fits into each hole.
The cross-section shows two of these bowls.

(i) Calculate the inside curved surface area of one of these hemispherical bowls. [1]

(ii) Calculate the total surface area of the top of the container, including the inside curved surface area of each bowl. [3]

(iii) With the top and the 4 bowls in place, calculate the volume of water required to fill the container. [3]

Thinking Process

(a) (i) To find total external surface area find the areas of the circular base and the area of curved surface.
(ii) Add 150 cm^2 to total external surface area of one container. Multiply it by 30000.
(b) (ii) The circular top is a circle with four circular holes. Therefore total surface area of the top = area of top circle + area of 4 hemispheres − area of 4 circles.
(iii) Subtract the volume of 4 hemispheres from the total volume of the cylinder.

Solution

(a) (i) Total external surface area

$= \pi r^2 + 2\pi rh$

$= \pi(\frac{21}{2})^2 + 2\pi(\frac{21}{2})(8)$

$= \pi(\frac{441}{4} + 168)$

$= \frac{1113}{4}\pi$

$= \frac{1113}{4} \times 3.142$

$= 874.26 \approx 874 \text{ cm}^2$ (3sf) **Ans.**

(ii) Area of material needed for one container

$= 874.26 \text{ cm}^2$

material needed to cover wastage $= 150 \text{ cm}^2$

Total area of material required for one container $= 874.26 + 150 = 1024.26 \text{ cm}^2$

$\therefore$ Total area of material needed for 30000 containers $= 1024.26 \times 30000$

$= 30727800 \text{ cm}^2$

$= 3072.78 \text{ m}^2$

$\approx 3070 \text{ m}^2$ (3sf) **Ans.**

Recall: $1 \text{ m} = 100 \text{ cm}$
$(1 \text{ m})^2 = (100 \text{ cm})^2$
$1 \text{ m}^2 = 10000 \text{ cm}^2$

(b) (i) Surface area of one hemispherical bowl

$= \frac{1}{2}(4\pi r^2)$

$= \frac{1}{2} \times 4 \times \pi \times (\frac{7}{2})^2$

$= \frac{49}{2}\pi$

$= 76.979 \approx 77 \text{ cm}^2$ **Ans.**

(ii) Total surface area of the top

$= \pi(\frac{21}{2})^2 + 4\left(\frac{1}{2} \times 4\pi(\frac{7}{2})^2\right) - 4\left(\pi(\frac{7}{2})^2\right)$

$= \frac{441}{4}\pi + 98\pi - 49\pi$

$= \frac{637}{4}\pi$

$= 500.363 \approx 500 \text{ cm}^2$ (3sf) **Ans.**

(iii) Volume required to fill the container

= volume of cylinder − volume of 4 bowls

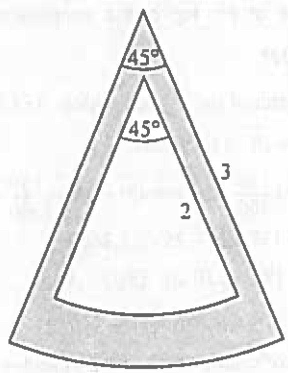
$= \pi(\frac{21}{2})^2(8) - 4 \times \frac{1}{2}\left(\frac{4}{3}\pi(\frac{7}{2})^3\right)$

$= 882\pi - \frac{343}{3}\pi$

$= \frac{2303}{3}\pi$

$= 2412 \approx 2410 \text{ cm}^3$ (3sf) **Ans.**

24 *(J2013/P1/Q17)*



The diagram shows part of an earring.
It is in the shape of a sector of a circle of radius 3 cm and angle 45°, from which a sector of radius 2 cm and angle 45° has been removed.

(a) Calculate the shaded area.
Give your answer in the form $\frac{a\pi}{b}$, where a and b are integers and as small as possible. [2]

(b) The earring is cut from a sheet of silver.
The mass of 1 cm^3 of the silver sheet is 1.6 g.
By taking the value of π to be 3, estimate the mass of the earring. [1]

Thinking Process

(a) Shaded area = area of bigger sector − area of smaller sector.
(b) Use the answer found in (a) to find the mass.

Solution

(a) Shaded area

$= \text{area of bigger sector} - \text{area of smaller sector}$

$= \frac{45}{360}(\pi)(3)^2 - \frac{45}{360}(\pi)(2)^2$

$= \frac{45}{360}(\pi)(9-4)$

$= \frac{5\pi}{8}$ cm^2 **Ans.**

(b) Area of earring $= \frac{5\pi}{8}$

$= \frac{5(3)}{8} = \frac{15}{8}$ cm^2

mass of 1 cm^2 of silver $= 1.6$ g

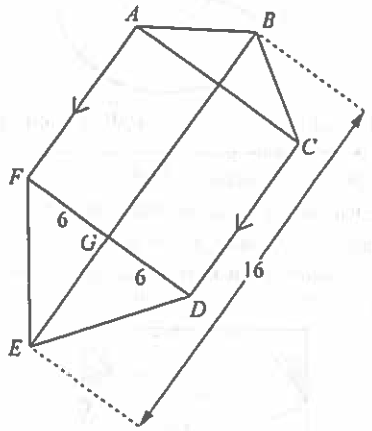
mass of $\frac{15}{8}$ cm^2 of silver $= 1.6 \times \frac{15}{8}$

$= 3$g

$\therefore$ mass of the earring $= 3$g **Ans.**

25 *(J2013/P2/Q4)*

ABCDEF is a hexagon with *BE* as its only line of symmetry.
AF is parallel to *CD* and *DF* intersects *BE* at *G*.
$BE = 16$cm and $DG = GF = 6$cm.
The area of the hexagon *ABCDEF* is 138 cm².



(a) Calculate *AF*. [2]

(b) The area of the hexagon *ABCDEF* is four times the area of the triangle *DEF*.

(i) Find *EG*. [2]

(ii) Find *EG* : *GB*, giving your answer in the form *m* : *n* where *m* and *n* are integers. [2]

Thinking Process

(a) Observe that *ABEF* is a trapezium. Apply area of trapezium $= \frac{1}{2}(h)(a+b)$.

(b) (i) Form an equation by using the given information and solve it for *EG*.

(ii) First find *GB*, then reduce the ratio *EG* : *GB* to its simplest form.

Solution

(a) Area of trapezium $ABEF = \frac{1}{2}(GF)(AF+BE)$

$\frac{138}{2} = \frac{1}{2}(6)(AF+16)$

$69 = 3AF + 48$

$3AF = 21$

$AF = 7$ cm **Ans.**

(b) (i) Area of $ABCDEF = 4 \times$ area of ΔDEF

$138 = 4(\frac{1}{2} \times DF \times EG)$

$138 = 4(\frac{1}{2} \times 12 \times EG)$

$138 = 24EG$

$EG = 5.75$ cm **Ans.**

(ii) $GB = BE - EG$

$= 16 - 5.75 = 10.25$

$EG : GB$

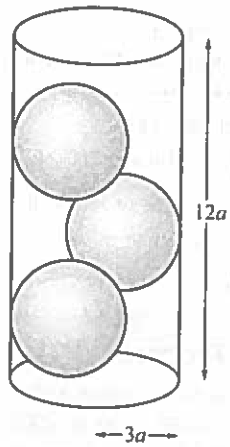
$= 5.75 : 10.25$

$= 575 : 1025$

dividing by 25

$= 23 : 41$ **Ans.**

26 *(N2013/P1/Q16)*



[Volume of a sphere $= \frac{4}{3}\pi r^3$]

Three spheres, each of radius $2a$ cm are placed inside a cylinder of radius $3a$ cm and height $12a$ cm.
Water is poured into the cylinder to fill it completely.
The volume of water is $k\pi a^3$ cm^3.
Find the value of k. [3]

Thinking Process

Volume of water = volume of cylinder − volume of 3 spheres.

Solution

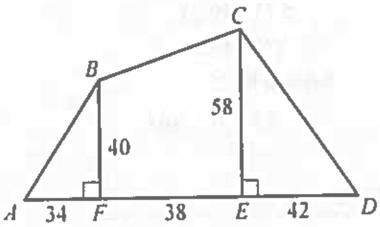
$$\text{Volume of 3 spheres} = 3\left(\frac{4}{3}\pi(2a)^3\right)$$
$$= 4\pi(8a^3)$$
$$= 32\pi a^3 \text{ cm}^3$$

$$\text{volume of cylinder} = \pi r^2 h$$
$$= \pi(3a)^2(12a)$$
$$= \pi(9a^2)(12a)$$
$$= 108\pi a^3$$

volume of water = vol. of cylinder −vol. of 3 spheres

$$k\pi a^3 = 108\pi a^3 - 32\pi a^3$$
$$k\pi a^3 = 76\pi a^3$$
$$k = 76 \quad \textbf{Ans.}$$

27 *(N2013/P2 Q1)*



ABCD is a level field.

F and *E* are points on *AD* such that *BF* and *CE* are perpendicular to *AD*.

$BF = 40$m and $CE = 58$m.

$AF = 34$m, $FE = 38$m and $ED = 42$m.

(a) Calculate the area of the field. [3]

(b) Calculate the length of *BC*. [2]

(c) Calculate $C\hat{D}E$. [2]

Thinking Process

(a) Area of the field = area of ΔABF + area of trapezium *BCEF* + area of ΔCDE.

(b) To find *BC* ✐ draw a perpendicular from *B* to *CE*. Apply pythagoras theorem.

(c) ✐ Apply $\tan C\hat{D}E = \frac{CE}{ED}$.

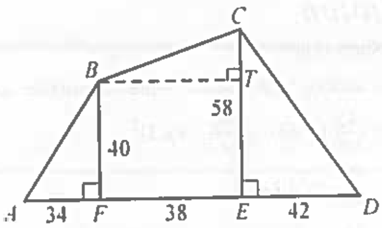
Solution

(a) Area of the field

= area of ΔABF + area of trapezium *BCEF* + area of ΔCDE

$$= \frac{1}{2}(34)(40) + \frac{1}{2}(38)(40+58) + \frac{1}{2}(42)(58)$$
$$= 680 + 1862 + 1218$$
$$= 3760 \text{ m}^2 \quad \textbf{Ans.}$$

(b)



In ΔBCT, $BT = 38$ m, $TC = 58 - 40 = 18$ m

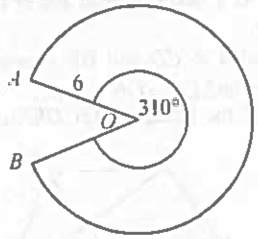
applying pythagoras theorem.

$$BC^2 = BT^2 + TC^2$$
$$= (38)^2 + (18)^2$$
$$= 1768$$
$$\therefore \quad BC = 42.047 \approx 42.0 \text{ m} \quad \textbf{Ans.}$$

(c) In ΔCDE,

$$\tan C\hat{D}E = \frac{58}{42}$$
$$C\hat{D}E = 54.09 \approx 54.1° \quad \textbf{Ans.}$$

28 *(N2013/P2 Q8)*



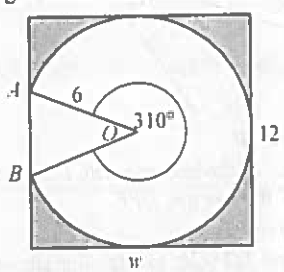
The diagram shows a sector *AOB* of a circle with centre 0 and radius 6 cm.

The angle of the sector is 310°.

(a) Calculate the total perimeter of the sector. [3]

(b) Calculate the area of the sector. [2]

(c) This sector is cut from a rectangular piece of card of height 12 cm and width *w* cm.



One edge of the rectangular piece of card passes through *A* and *B*.

The other edges are tangents to the circle.

(i) Calculate the value of *w*. [3]

(ii) When the sector is cut out, the triangle *AOB* is retained.

The rest of the rectangular piece of card, shown shaded, is discarded as waste.

Calculate the percentage of the rectangular piece of card that is discarded as waste. [4]

Thinking Process

(a) Perimeter = $OA + OB +$ arc length AB.

(b) Apply, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$.

(c) (i) To find w Draw a perpendicular from O to AB and find its length. Add the radius 6 to your answer.

(ii) Calculate the shaded area and express it as a percentage of the total area of rectangle.

Solution

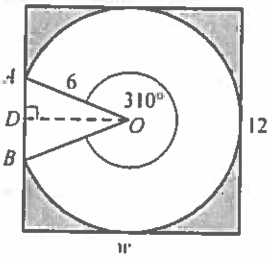
(a) Perimeter of the sector $= \overset{\frown}{AB} + OA + OB$

$$= \frac{310^\circ}{360^\circ} \times 2\pi(6) + 6 + 6$$
$$= 32.4673 + 12$$
$$= 44.467$$
$$\approx 44.5 \text{ cm (3sf)} \quad \textbf{Ans.}$$

(b) Area of the sector $= \frac{310^\circ}{360^\circ} \times \pi(6)^2$

$$= 31\pi$$
$$= 97.4 \text{ cm}^2 \text{ (3sf)} \quad \textbf{Ans.}$$

(c) (i) From the figure,



$A\hat{O}B = 360^\circ - 310^\circ$
$= 50^\circ$

ΔAOB is isosceles

$\therefore \quad A\hat{O}D = \frac{50}{2} = 25^\circ$

consider ΔAOD,

$$\cos A\hat{O}D = \frac{OD}{OA}$$
$$\cos 25^\circ = \frac{OD}{6}$$
$$OD = 6\cos 25^\circ = 5.44 \text{ cm}$$

$\therefore \quad w = 5.44 + 6 = 11.44 \quad$ **Ans.**

(ii) Area of shaded region

= area of rectangle − area of sector − area of ΔAOB

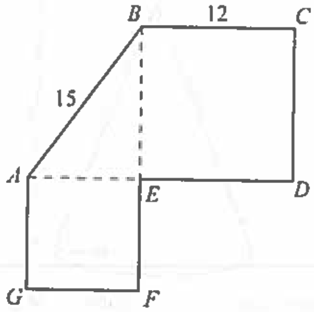
$$= (12 \times 11.44) - 97.4 - \frac{1}{2}(6)(6)\sin 50^\circ$$
$$= 137.28 - 97.4 - 13.789$$
$$= 26.091 \text{ cm}^2$$

$\therefore$ percentage of card discarded

$$= \frac{26.091}{137.28} \times 100 = 19\% \quad \textbf{Ans.}$$

29 *(J2014 P1 Q22)*

Shape $ABCDEFG$ is made from two squares and a right-angled triangle.
$AB = 15$ cm and $BC = 12$ cm.



(a) Find the length AG. [2]

(b) Find the total area of the shape. [2]

Thinking Process

(a) Apply Pythagoras theorem to triangle ABE.

(b) Total area consists of two squares and one triangle.

Solution

(a) Since $BCDE$ is a square.

$\therefore \quad BE = BC = 12$ cm.

using pythagoras theorem on ΔAEB.

$$AE = \sqrt{AB^2 - BE^2}$$
$$= \sqrt{15^2 - 12^2}$$
$$= \sqrt{225 - 144}$$
$$= \sqrt{81} = 9 \text{ cm}$$

$\therefore \quad AG = AE$
$= 9$ cm **Ans.**

(b) Total area of the shape

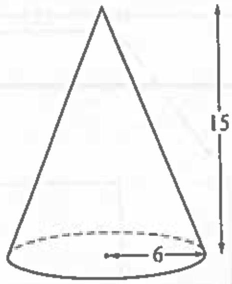
= area of $AEFG$ + area of ΔABE + area of $BCDE$

$$= 9^2 + \frac{1}{2}(9)(12) + 12^2$$
$$= 81 + 54 + 144$$
$$= 279 \text{ cm}^2 \quad \textbf{Ans.}$$

30 (J2014/P2/Q9)

[Volume of a cone $= \frac{1}{3}\pi r^2 h$]

[Curved surface area of a cone $= \pi r l$]



The diagram shows a solid cone of height 15 cm and base radius 6 cm.

(a) Show that the slant height of the cone is 16.2 cm, correct to one decimal place. [1]

(b) Calculate the total surface area of the cone. [3]

(c) Calculate the volume of the cone. [2]

(d) The cone is made from wood.
The mass of 1 m³ of the wood is 560 kg.
Calculate the mass of the cone in grams. [2]

(e) Another cone is made of the same material and is geometrically similar to the first.
The mass of the second cone is double the mass of the first.
(i) Calculate the height of the second cone. [2]
(ii) Calculate the total surface area of the second cone. [2]

Thinking Process

(a) use Pythagoras' theorem.

(b) Total surface area = curved surface area + area of circular base.

(d) To find the mass Express 1 m³ into cm³. Convert 560 kg to grams.

(e) (i) Apply rule of similar figures:

$$\frac{\text{mass } A}{\text{mass } B} = \left(\frac{\text{length } A}{\text{length } B}\right)^3$$

(ii) Ratio of area of similar figures = ratio of squares of corresponding lengths.

Solution

(a) Using Pythagoras theorem.

$$\text{slant height} = \sqrt{6^2 + 15^2} = \sqrt{261} = 16.155 \approx 16.2 \text{ cm} \quad \textbf{Shown.}$$

(b) Total surface area $= \pi r^2 + \pi r l$

$$= \pi(6)^2 + \pi(6)(16.155) = 132.93\pi = 417.61 \approx 418 \text{ cm}^2 \quad \textbf{Ans.}$$

(c) Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(6)^2(15) = 180\pi = 565.487 \approx 565 \text{ cm}^3 \text{ (3sf)} \quad \textbf{Ans.}$$

(d) Volume of cone in m³ $= \frac{565.487}{1000\,000} = 5.65487 \times 10^{-4}$ m³

1 m³ —— 560 kg

5.65487×10^{-4} m³ —— $5.65487 \times 10^{-4} \times 560 = 0.3167$ kg

∴ mass of cone in grams $= 0.3167 \times 1000 = 316.7 \text{ g} \approx 317 \text{ g}$ **Ans.**

> Note that: 1 m = 100 cm
> 1 m³ = 1000000 cm³
> 1 kg = 1000 g

(e) (i)

$$\frac{\text{mass of 1st cone}}{\text{mass of 2nd cone}} = \left(\frac{\text{height of 1st cone}}{\text{height of 2nd cone}}\right)^3$$

$$\frac{1}{2} = \left(\frac{15}{\text{height of 2nd cone}}\right)^3$$

$$\sqrt[3]{\frac{1}{2}} = \frac{15}{\text{height of 2nd cone}}$$

$$0.7937 = \frac{15}{\text{height of 2nd cone}}$$

$$\text{height of 2nd cone} = \frac{15}{0.7937} = 18.899 \approx 18.9 \text{ cm} \quad \textbf{Ans.}$$

(ii)

$$\frac{\text{area of 2nd cone}}{\text{area of 1st cone}} = \left(\frac{\text{height of 2nd cone}}{\text{height of 1st cone}}\right)^2$$

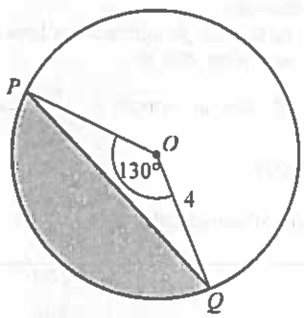
$$\frac{\text{area of 2nd cone}}{417.61} = \left(\frac{18.899}{15}\right)^2$$

$$\text{area of 2nd cone} = \left(\frac{18.899}{15}\right)^2 \times 417.61 = 662.93 \approx 663 \text{ cm}^2 \quad \textbf{Ans.}$$

31 (J2014/P2/Q11b)

(b) P and Q are points on the circle centre O with radius 4 cm.

$P\hat{O}Q = 130°$.



(i) Calculate the area of triangle POQ. [2]

(ii) Calculate the area of the major segment, shown unshaded in the diagram. [3]

Thinking Process

(b) (i) Recall, area of $\Delta = \frac{1}{2}ab\sin c$.

(ii) Find reflex angle POQ. Find area of major sector and add it to the area of triangle POQ.

Solution

(b) (i) Area of $\Delta POQ = \frac{1}{2} \times 4 \times 4 \times \sin 130°$

$= 6.128 \approx 6.13 \text{ cm}^2$ **Ans.**

(ii) Reflex angle $P\hat{O}Q = 360° - 130° = 230°$

Area of major sector $POQ = \frac{230}{360} \times \pi \times (4)^2$

$= 32.114 \text{ cm}^2$

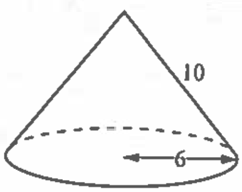
Area of major segment

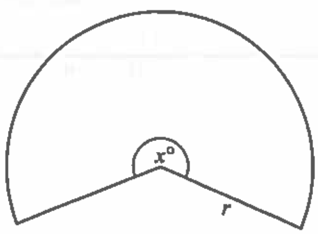
$=$ area of ΔPOQ + area of major sector POQ

$= 6.128 + 32.114$

$= 38.242 \approx 38.2 \text{ cm}^2$ **Ans.**

32 (N2014/P1/Q14)





A hollow cone has a base radius 6 cm and slant height 10 cm.
The curved surface of the cone is cut, and opened out into the shape of a sector of a circle, with angle $x°$ and radius r cm.

(a) Write down the value of r. [1]

(b) Calculate x. [2]

Thinking Process

(a) Note that r is the slant height of hollow cone.

(b) To find x Apply the formula for arc length. Note that the arc length of the sector is actually the circumference of the base of the hollow cone.

Solution with TEACHER's COMMENTS

(a) r = slant height of cone

$= 10$ cm **Ans.**

(b) Circumference of base of cone

= arc length of sector

$\Rightarrow \quad 2\pi(6) = \frac{x°}{360}(2)(\pi)(10)$

$12\pi = \frac{x°}{18}\pi$

$x° = 12 \times 18 = 216°$ **Ans.**

> When a sector is folded to make a cone, the arc of the sector becomes the base of the cone, and the radius of the sector becomes the slant height of the cone.

33 (N2014/P1/Q15)

[The volume of a sphere is $\frac{4}{3}\pi r^3$]

20 spheres, each of radius 3 cm, have a total volume of $k\pi$ cm³.

(a) Find the value of k. [1]

(b) The spheres are inside an open cylinder, with radius 6 cm.
The cylinder stands on a horizontal surface and contains enough water to cover the spheres.
Calculate the change in depth of the water when the spheres are taken out of the cylinder. [2]

Thinking Process

(a) Find the volume of 20 spheres and equate it to the given volume to find k.

(b) Equate the volume of 20 spheres to the expression for volume of cylinder and solve for d.

Solution

(a) Volume of 20 spheres $= 20(\frac{4}{3}\pi r^3)$

$\Rightarrow \quad k\pi = 20\left(\frac{4}{3}\pi(3)^3\right)$

$k\pi = 20(36\pi)$

$k = 720$ **Ans.**

(b) Volume of 20 spheres $= k\pi$

$$= 720\pi \text{ cm}^3$$

let d be the drop in water level.

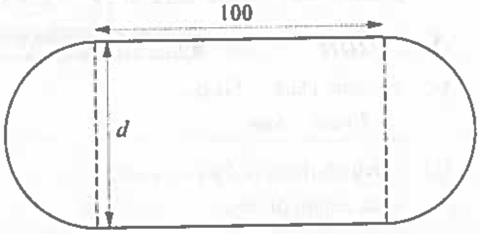
$$\therefore\ 720\pi = \pi(6)^2 d$$

$$720 = 36d$$

$$d = \frac{720}{36} = 20$$

$\therefore$ change in depth in water level = 20 cm **Ans.**

34 (N2014/P2 Q5)



The diagram shows the perimeter of a 400 m running track.

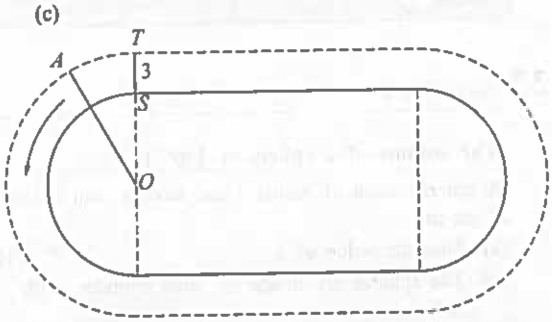
It consists of a rectangle measuring 100 m by d metres and two semicircles of diameter d metres.

The length of each semicircular arc is 100 m.

(a) Calculate d. [2]

(b) Calculate the total area of the region inside the running track. [3]

(c)



S is the starting point and finishing point for the 400 m race for a runner in the inside lane.

A runner in an outer lane is always 3 m from the inner perimeter.

The runner in the outer lane starts at A, runs 400 m and finishes at T.

$TS = 3$ m.

(i) Calculate the length of the arc TA. [3]

(ii) O is the centre of a semi-circular part of the track.

Calculate $A\hat{O}T$. [2]

Thinking Process

(a) ✎ Arc Length of half circle $= \frac{1}{2}\pi d$.

(b) Total surface area = area of two semi-circle + area of rectangle.

(c) (i) To find TA ✎ subtract the total length of outer lane from 400 m.

(ii) ✎ Use, arc length $= \frac{\theta}{360} \times 2\pi r$.

Solution

(a) Length of semicircular arc $= \frac{1}{2}(\pi d)$

$$\Rightarrow \quad 100 = \frac{1}{2}\pi d$$

$$d = \frac{200}{\pi}$$

$$= 63.662 \approx 63.7 \text{ m} \quad \textbf{Ans.}$$

(b) Radius of semi-circle. $r = \frac{1}{2}(d)$

$$= \frac{1}{2}\left(\frac{200}{\pi}\right) = \frac{100}{\pi} \text{ m}$$

Total area of the region

= area of 2 semi circles + area of rectangle

$$= 2\left(\frac{1}{2}(\pi)\left(\frac{100}{\pi}\right)^2\right) + (100)\left(\frac{200}{\pi}\right)$$

$$= \frac{10000}{\pi} + \frac{20000}{\pi}$$

$$= \frac{30000}{\pi} = 9549.3 \approx 9550 \text{ m}^2 \quad \textbf{Ans.}$$

(c) (i) Radius of outer lane. $r = \left(\frac{100}{\pi} + 3\right)$ m

peremeter of outer lane $= 2(\pi r) + 100 + 100$

$$= 2\pi\left(\frac{100}{\pi} + 3\right) + 200$$

$$= 200 + 6\pi + 200$$

$$= (400 + 6\pi) \text{ m}$$

since the runner runs 400 m.

$$\therefore \quad TA = (400 + 6\pi) - 400$$

$$= 6\pi = 18.85 \text{ m} \quad \textbf{Ans.}$$

(ii) Arc length $AT = \frac{A\hat{O}T}{360^\circ} \times 2\pi r$

$$6\pi = \frac{A\hat{O}T}{360} \times 2\pi\left(\frac{100}{\pi} + 3\right)$$

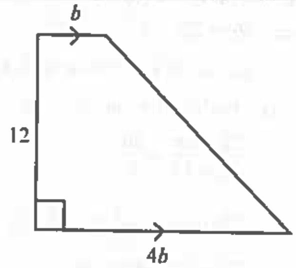
$$6\pi = \frac{A\hat{O}T}{360} \times (200 + 6\pi)$$

$$A\hat{O}T = 6\pi \times \left(\frac{360}{200 + 6\pi}\right)$$

$$= 18.85 \times 1.645$$

$$= 31^\circ \quad \textbf{Ans.}$$

35 *(J2015/P1/Q3)*



The diagram shows a trapezium with lengths in centimetres.
The area of the trapezium is 120 cm².
Find the value of b. [2]

Thinking Process

To find b ✎ Apply formula for area of a trapezium.

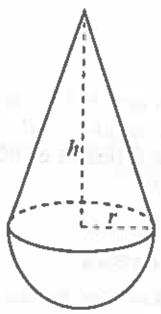
Solution

$$\text{Area of trapezium} = \frac{1}{2} \times 12(b + 4b)$$
$$120 = \frac{1}{2} \times 12(5b)$$
$$120 = 30b$$
$$b = \frac{120}{30} = 4 \text{ cm} \quad \textbf{Ans.}$$

36 *(J2015/P1/Q24)*

[Volume of a cone $= \frac{1}{3}\pi r^2 h$,
curved surface area of a cone $= \pi r l$]

[Volume of a sphere $= \frac{4}{3}\pi r^3$,
surface area of a sphere $= 4\pi r^2$]



The solid is formed from a hemisphere of radius r cm fixed to a cone of radius r cm and height h cm.
The volume of the hemisphere is one third of the volume of the solid.

(a) Find h in terms of r. [2]

(b) The slant height of the cone can be written as $r\sqrt{k}$ cm, where k is an integer.
Find the value of k. [2]

(c) Find an expression, in terms of r and π, for the total surface area, in cm², of the solid. [1]

Thinking Process

(a) Volume of hemisphere = $\frac{1}{3}$ (volume of cone + volume of hemisphere).

(b) Apply pythagoras theorem.

(c) Total surface area = curved surface area of cone + surface area of hemisphere.

Solution

(a) Vol. of hemisphere $= \frac{1}{3}$(vol. of cone + vol. of hemisphere)

$$\Rightarrow \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{3}\left(\frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\right)$$
$$\frac{2}{3}\pi r^3 = \frac{1}{3}\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$
$$\frac{2}{3}\pi r^3 = \frac{1}{9}\pi r^2 h + \frac{2}{9}\pi r^3$$
$$\frac{2}{3}\pi r^3 = \frac{1}{9}\pi r^2(h + 2r)$$
$$6r = h + 2r$$
$$h = 6r - 2r$$
$$h = 4r \quad \textbf{Ans.}$$

(b) Using Pythagoras theorem.

$$(r\sqrt{k})^2 = h^2 + r^2$$
$$r^2 k = h^2 + r^2$$
$$k = \frac{h^2 + r^2}{r^2}$$

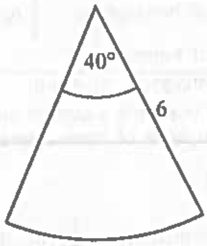
from part (a). $h = 4r$

$$\therefore \quad k = \frac{(4r)^2 + r^2}{r^2}$$
$$= \frac{16r^2 + r^2}{r^2}$$
$$= \frac{17r^2}{r^2} = 17 \quad \textbf{Ans.}$$

(c) Slant height of cone, $l = r\sqrt{17}$

$$\text{Total surface area} = \pi r l + \frac{1}{2}(4\pi r^2)$$
$$= \pi r(r\sqrt{17}) + 2\pi r^2$$
$$= \pi r^2\sqrt{17} + 2\pi r^2$$
$$= \pi r^2(\sqrt{17} + 2) \text{ cm}^2 \quad \textbf{Ans.}$$

37 *(J2015/P2 Q4b)*



The angle of a sector of a circle, radius 6 cm, is 40°.

(i) The area of the sector is $k\pi$ cm^2.
Find the value of k. [2]

(ii) Find an expression, in terms of π, for the perimeter of the sector.
Give your answer in the form $(a+b\pi)$ centimetres. [2]

(iii) A geometrically similar sector has perimeter $(72+n\pi)$ centimetres.
Find the value of n. [1]

Thinking Process

(i) To find k apply area of sector $= \frac{\theta}{360°} \times \pi r^2$

(ii) Perimeter of a sector = $2r$ + arc length.

(iii) Find the radius of new sector, then apply rule of similarity, i.e. ratio of corresponding lengths is equal.

Solution

(i) Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$$\Rightarrow \quad k\pi = \frac{40°}{360°} \times \pi(6)^2$$

$$k = \frac{40°}{360°} \times 36$$

$$= 4 \quad \textbf{Ans.}$$

(ii) Perimeter of the sector = arc length + 6 + 6

$$= \frac{40°}{360°} \times 2\pi(6) + 6 + 6$$

$$= \frac{1}{9} \times 12\pi + 12$$

$$= \frac{4}{3}\pi + 12$$

$$= (12 + \frac{4}{3}\pi) \text{ cm} \quad \textbf{Ans.}$$

(iii) Perimeter of a sector = $2r$ + arc length
given perimeter of the sector = $72 + n\pi$

$\Rightarrow \quad 2r = 72 \quad \Rightarrow \quad r = 36$

$\therefore$ radius of the similar sector = 36 cm
since both sectors are similar.

$$\therefore \quad \frac{72+n\pi}{\frac{4}{3}\pi+12} = \frac{36}{6}$$

$$72 + n\pi = 6(\frac{4}{3}\pi + 12)$$

$$72 + n\pi = 8\pi + 72$$

$$n\pi = 8\pi$$

$$n = 8 \quad \textbf{Ans.}$$

38 *(N2015 P2 Q2)*

A is the point (8, 7), B is the point (–2, 11) and C is the point (1, 7).

(a) Calculate the area of triangle ABC. [2]

(b) Calculate the length of AB. [2]

(c) Calculate the perimeter of triangle ABC. [2]

(d) Calculate $B\hat{A}C$. [2]

Thinking Process

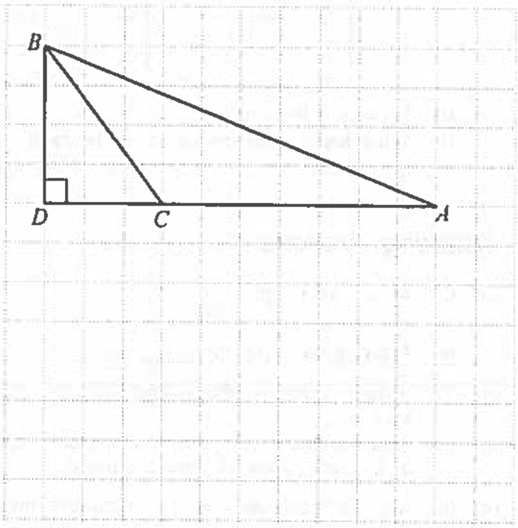
(a) Draw triangle ABC on the grid and use it to find the area.

(b) Length $= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

(c) Add up all sides of the triangle ABC.

(d) Draw a horizontal line from C to a point vertically below B. Apply $\tan\theta = \frac{\text{opp}}{\text{adj}}$ to the triangle formed.

Solution



(a) Area of ΔABC

$= \frac{1}{2} \times 7 \times 4$

$= 14 \text{ units}^2$ **Ans.**

(b) Length of AB

$= \sqrt{(-2-8)^2 + (11-7)^2}$

$= \sqrt{100+16}$

$= \sqrt{116}$

$= 10.77 \approx 10.8$ units **Ans.**

(c) Applying pythagoras theorem on ΔBCD

Length of $BC = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{25}$

$= 5$ units

length of $CA = 7$ units

perimeter of $\Delta ABC = AB + BC + CA$

$= 10.77 + 5 + 7$

$= 22.77 \approx 22.8$ units **Ans.**

(d) Consider ΔABD.

$\tan B\hat{A}D = \frac{4}{10}$

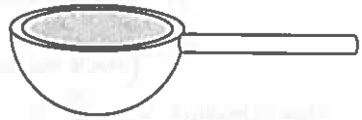
$B\hat{A}D = 21.8°$

$\therefore \quad B\hat{A}C = 21.8°$ **Ans.**

39 *(N2015 P2 Q4)*

[The volume of a sphere is $\frac{4}{3}\pi r^3$]

(a)

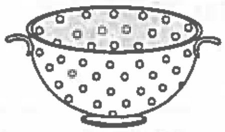


A spoon used for measuring in cookery consists of a hemispherical bowl and a handle.
The internal volume of the hemispherical bowl is 20 cm³.
The handle is of length 5 cm.

(i) Find the internal radius of the hemispherical bowl. [2]

(ii) The hemispherical bowl of a geometrically similar spoon has an internal volume of 50 cm³. Find the length of its handle. [2]

(b) **[The surface area of a sphere is $4\pi r^2$]**



An open hemisphere of radius 5.5 cm is used to make a metal kitchen strainer.
50 holes are cut out of the curved surface.
Assume that the piece of metal removed to make each hole is a circle of radius 1.5 mm.
Calculate the external surface area that remains. [3]

Thinking Process

(a) (i) To find internal radius ✐ substitute the value of internal volume into the formula and solve.

(ii) ✐ Rule of similar figures: ratio of volume of similar figures = ratio of cube of corresponding sides.

(b) External surface area = surface area of hemisphere − area of 50 circular holes.

Solution

(a) (i) Volume of hemisphere $= \frac{1}{2}(\frac{4}{3}\pi r^3)$

$20 = \frac{2}{3}\pi r^3$

$r^3 = \frac{20 \times 3}{2\pi}$

$r^3 = 9.5493$

$r = 2.1216$

$\therefore$ internal radius $= 2.1216 \approx 2.12$ cm **Ans.**

(ii) $\dfrac{\text{Volume}_1}{\text{Volume}_2} = \left(\dfrac{\text{handle length}_1}{\text{handle length}_2}\right)^3$

$\dfrac{20}{50} = \left(\dfrac{5}{\text{handle length}_2}\right)^3$

$\dfrac{2}{5} = \dfrac{125}{(\text{handle length}_2)^3}$

$(\text{handle length}_2)^3 = 125 \times \dfrac{5}{2}$

$(\text{handle length}_2)^3 = 312.5$

$\text{handle length}_2 = 6.786$

$\therefore$ length of handle = 6.79 cm **Ans.**

(b) External surface area

= surface area of hemisphere − area of 50 circles

$= \frac{1}{2}(4\pi(5.5)^2) - 50(\pi(0.15)^2)$

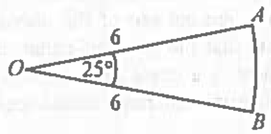
$= 60.5\pi - 1.125\pi$

$= 59.375\pi$

$= 186.532 \approx 187 \text{ cm}^2$ **Ans.**

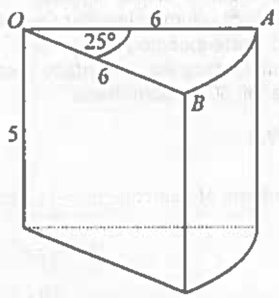
40 *(N2015 P2 Q8)*

(a) OAB is a sector of a circle, centre O, radius 6 cm. $A\hat{O}B = 25°$.



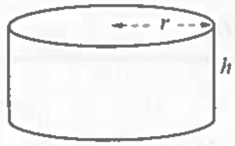
(i) Calculate the length of the arc AB. [2]
(ii) Calculate the area of the sector OAB. [2]

(b) The sector OAB from part (a) is the cross-section of a slice of cheese.
The slice has a height of 5 cm.



(i) Calculate the volume of this slice of cheese. [1]
(ii) Calculate the total surface area of this slice of cheese. [3]
(iii) Another 25° slice of cheese has 3 times the height and twice the radius.
Calculate its volume. [2]

(c) A dairy produces cylindrical cheeses, each with a volume of 800 cm³.
The height h cm and the radius r cm can vary.



(i) Express h in terms of r. [1]
(ii) What happens to the height if the radius is doubled? [1]

Thinking Process

(a) (i) Apply, arc length $= \dfrac{\theta}{360°} \times 2\pi r$

(ii) Apply, area of sector $= \dfrac{\theta}{360°} \times \pi r^2$

(b) (i) Volume = cross-sectional area × height of slice.

(ii) Total surface area = area of 2 sectors + area of 2 sides + area of curved surface.

(c) (i) Volume of cylinder $= \pi r^2 h$. Substitute given volume into it and make h the subject.

(ii) Replace r by $2r$ and analyse the change in height.

Solution

(a) (i) Length of arc $AB = \dfrac{25°}{360°} \times 2\pi(6)$

$= 2.618 \approx 2.62$ cm. **Ans.**

(ii) Area of sector $OAB = \dfrac{25°}{360°} \times \pi(6)^2$

$= 7.854$

$\approx 7.85 \text{ cm}^2$ (3sf) **Ans.**

(b) (i) Volume of slice

= cross-sectional area × height of slice

$= 7.854 \times 5$

$= 39.27 \approx 39.3 \text{ cm}^3$ **Ans.**

(ii) Total surface area

= 2(area of sector) + 2(area of sides) + curved area of slice

$= 2(7.854) + 2(6 \times 5) + 2.618 \times 5$

$= 15.708 + 60 + 13.09$

$= 88.798 \approx 88.8 \text{ cm}^2$ **Ans.**

(iii) New height = 3(5) = 15 cm

new radius = 2(6) = 12 cm

Volume of slice

= area of sector × height of slice

$= \left(\dfrac{25°}{360°} \times \pi(12)^2\right) \times 15$

$= 471.239 \approx 471 \text{ cm}^3$ (3sf) **Ans.**

(c) (i) Volume of cylinder $= \pi r^2 h$

$800 = \pi r^2 h$

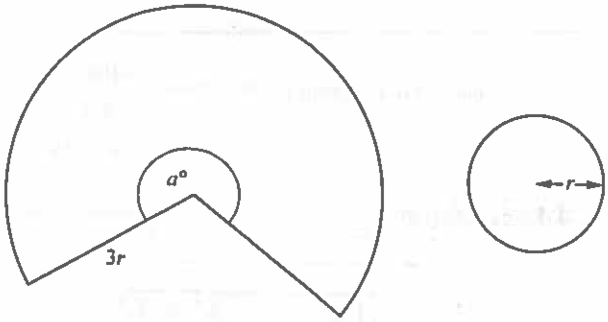
$h = \dfrac{800}{\pi r^2}$ **Ans.**

(ii) New radius = $2r$

$$\text{new height} = \frac{800}{\pi(2r)^2}$$
$$= \frac{800}{4\pi r^2}$$
$$= \frac{1}{4}\left(\frac{800}{\pi r^2}\right) = \frac{1}{4}h$$

$\therefore$ the height is reduced to $\frac{1}{4}$th its actual height if radius is doubled. **Ans.**

41 (J2016/P1/Q24)



The diagram shows a sector of a circle with radius $3r$ cm and angle $a°$ and a circle with radius r cm.
The ratio of the area of the sector to the area of the circle with radius r cm is 8 : 1.

(a) Find the value of a. [3]

(b) Find an expression, in terms of π and r, for the perimeter of the sector. [2]

Thinking Process

(a) Divide the area of sector by the area of circle and equate it to the given ratio.

(b) Perimeter = $3r + 3r$ + arc length of the sector.

Solution

(a) $$\frac{\text{Area of sector}}{\text{area of circle}} = \frac{8}{1}$$

$$\frac{\frac{a°}{360} \times \pi(3r)^2}{\pi r^2} = \frac{8}{1}$$

$$\frac{9a°}{360} = 8$$

$$\frac{a°}{40} = 8$$

$a° = 8 \times 40 = 320°$ **Ans.**

(b) Perimeter of the sector = length of arc + $3r + 3r$

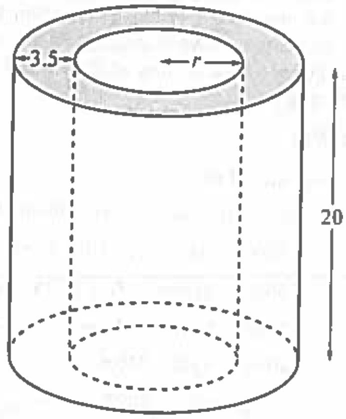
$$= \frac{320°}{360°} \times 2\pi(3r) + 6r$$

$$= \frac{16\pi r}{3} + 6r \quad \textbf{Ans.}$$

42 (J2016/P2/Q11)

[Volume of a cone = $\frac{1}{3}\pi r^2 h$]

(a)

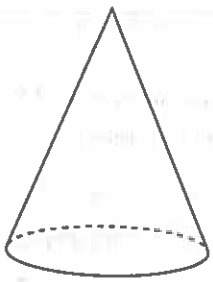


Solid I

Solid I is a cylinder with a small cylinder removed from its centre, as shown in the diagram.
The height of each cylinder is 20 cm and the radius of the small cylinder is r cm.
The radius of the large cylinder is 3.5 cm greater than the radius of the small cylinder.
The volume of Solid I is 3000 cm³.

(i) Calculate r. [4]

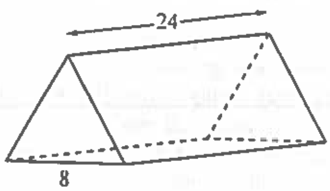
(ii) Solid II is a cone with volume of 3000 cm³.
The perpendicular height of the cone is twice its radius.



Solid II

Which solid is the taller and by how much? [4]

(b) The diagram shows a triangular prism of length 24 cm.
Its cross-section is an equilateral triangle with sides 8 cm.



Calculate the total surface area of the prism. [4]

Thinking Process

(a) (i) Volume of the solid = volume of big cylinder − volume of small cylinder.

(ii) Find the radius of the cone and use it to find the height of the cone. Then find the difference between heights of two solids.

(b) Total surface area = area of 2 triangles + area of 3 rectangles.

Solution

(a) (i) Volume of solid

= vol. of big cylinder − vol. of small cylinder

$\Rightarrow \quad 3000 = \pi(r+3.5)^2(20) - \pi(r)^2(20)$

$3000 = 20\pi(r^2+7r+12.25) - 20\pi r^2$

$3000 = 20\pi r^2 + 140\pi r + 245\pi - 20\pi r^2$

$140\pi r = 3000 - 245\pi$

$r = \dfrac{3000-245\pi}{140\pi} = 5.07$ cm **Ans.**

(ii) Let the radius of the cone = p cm

$\Rightarrow$ height of cone, $h = 2p$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad 3000 = \frac{1}{3}\pi p^2(2p)$

$3000 = \frac{2}{3}\pi p^3$

$p^3 = 3000 \times \frac{3}{2\pi}$

$p^3 = 1432.394$

$p = 11.273$

$\Rightarrow$ height of the cone, $h = 2(11.273)$

$= 22.55$

difference in heights $= 22.55 - 20 = 2.55$

$\therefore$ Solid II is taller by 2.55 cm **Ans.**

(b) Total surface area $= 2(\text{area of } \Delta) + 3(24 \times 8)$

$= 2\left(\frac{1}{2}(8)(8)\sin 60^\circ\right) + 3(192)$

$= 55.424 + 576$

$= 631.424$

≈ 631 cm^2 (3sf) **Ans.**

43 *(N2016/P1/Q21)*

[The volume of a sphere is $\frac{4}{3}\pi r^3$.]

During a storm, raindrops fall into a cylinder which stands on horizontal ground.
The cylinder was empty before the storm started.
The cylinder has radius 20 mm.
Each raindrop is a sphere of radius 2 mm.
After the storm, the depth of water in the cylinder is 16 mm.
Calculate the number of raindrops that fell into the cylinder. [3]

Thinking Process

To find the number of raindrops ⁄⁄ divide the volume of water by the volume of raindrop.

Solution

Volume of water in the cylinder $= \pi r^2 h$

$= \pi(20)^2(16)$

$= 6400\pi$ mm^3

Volume of one raindrop $= \frac{4}{3}\pi r^3$

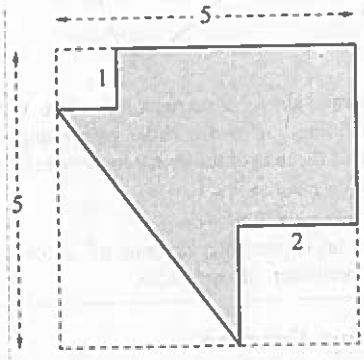
$= \frac{4}{3}\pi(2)^3$

$= \frac{32}{3}\pi$ mm^3

$\therefore$ number of raindrops in the cylinder $= \dfrac{6400\pi}{\frac{32}{3}\pi}$

$= 600$ **Ans.**

44 *(N2016/P1/Q23)*



The diagram shows a square piece of card, from which a triangle and two small squares are removed.
All lengths on the diagram are in centimetres.

(a) Calculate the area of the shaded card. [2]

(b) Calculate the perimeter of the shaded card. [2]

Thinking Process

(a) To find the shaded area ⁄⁄ subtract the total area of 2 small squares and triangle from the area of larger square.

(b) To find the perimeter ⁄⁄ add up all the sides of the shaded figure.

Solution

(a) Area of the shaded card

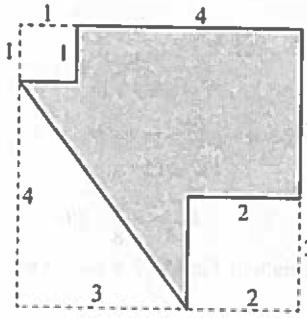
= area of larger square − (area of 2 small squares + area of triangle)

$= (5\times5) - \left((1\times1) + (2\times2) + (\frac{1}{2}\times3\times4)\right)$

$= 25 - (1+4+6)$

$= 25 - 11 = 14$ cm^2 **Ans.**

(b)



Using Pythagoras theorem,

slant height of triangle $= \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$ cm

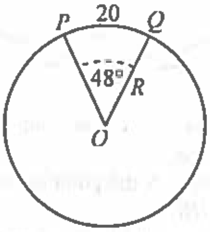
Perimeter of the shaded card

$= 5 + 2 + 2 + 3 + 4 + 1 + 1$

$= 18$ cm **Ans.**

45 *(N2016/P2 Q8)*

(a) P and Q are points on the circumference of a circle, centre O, radius R cm.

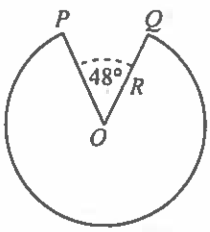


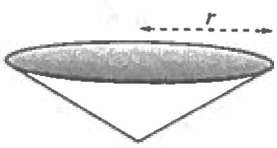
The minor arc $PQ = 20$ cm and $P\hat{O}Q = 48°$.

(i) Show that $R = 23.9$, correct to one decimal place. [3]

(ii) Calculate the area of the minor sector POQ. [2]

(iii) The minor sector POQ is removed from the circle and the remaining major sector is shaped to form an open cone of radius r cm.

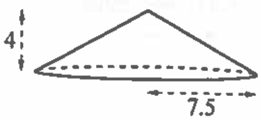




Calculate r. [2]

(b) **[The curved surface area of a cone is πrl, where l is the slant height]**

Cone A has radius 7.5 cm and height 4 cm.



Cone A

(i) Calculate the curved surface area of cone A. [3]

(ii) Cone B is geometrically similar to cone A. The ratio

curved surface area of cone A : curved surface area of cone B is 64 : 25 .

Find the height of cone B. [2]

Thinking Process

(a) (i) To find R ✎ Apply, arc length $= \frac{\theta}{360°} \times 2\pi r$

(ii) Apply, area of sector $= \frac{\theta}{360°} \times \pi r^2$

(iii) Take note that the arc length of the major sector is equal to the circumference of the base of the cone.

(b) (i) To find the curved surface area ✎ find slant height by using Pythagoras theorem.

(ii) ✎ Ratio of area of similar figures = ratio of squares of corresponding lengths.

Solution

(a) (i) Arc length $PQ = \frac{48°}{360°} \times 2\pi R$

$20 = \frac{4\pi}{15} \times R$

$R = 20 \times \frac{15}{4\pi}$

$= 23.87 \approx 23.9$ cm **Shown.**

(ii) Area of minor sector POQ

$= \frac{48°}{360°} \times \pi \times (R)^2$

$= \frac{48°}{360°} \times \pi \times (23.87)^2$

$= 238.667$

≈ 239 cm^2 **Ans.**

(iii) Reflex angle $P\hat{O}Q = 360° - 48°$

$= 312°$

Circumference of base of cone

= arc length of major sector POQ

$\Rightarrow \quad 2\pi r = \frac{312°}{360°} \times 2\pi(23.87)$

$r = \frac{312°}{360°} \times 23.87$

$r = 20.7$ cm **Ans.**

(b) (i) Using Pythagoras theorem.

slant height $= \sqrt{4^2 + 7.5^2}$

$= \sqrt{72.25} = 8.5$ cm

curved surface area of cone $= \pi rl$

$= \pi(7.5)(8.5)$

$= 200.27$

≈ 200 cm^2 **Ans.**

(ii) $\dfrac{\text{curved surface area of cone A}}{\text{curved surface area of cone B}} = \left(\dfrac{\text{height of cone A}}{\text{height of cone B}}\right)^2$

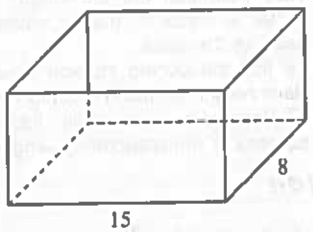
$\dfrac{64}{25} = \left(\dfrac{4}{\text{height of cone B}}\right)^2$

$\dfrac{8}{5} = \dfrac{4}{\text{height of cone B}}$

height of cone B $= 4 \times \dfrac{5}{8}$

$= 2.5$ cm **Ans.**

46 (J2017/P1/Q22)



A container is made out of thin material in the shape of a cuboid with an open top.
The container has length 15 cm and width 8 cm.
The volume of the container is 720 cm³.

(a) Calculate the height, h cm, of the container. [2]

(b) Calculate the surface area of the outside of the container. [2]

(c) Liquid is poured into the container.
The liquid fills 60% of the container.
Calculate the height of the liquid in the container. [1]

Thinking Process

(a) ☞ Apply formula, Volume of cuboid = $L \times B \times H$

(b) Surface area = $(L \times B) + 2(L \times H) + 2(B \times H)$

(c) To find the height ☞ find 60% of the volume of the container.

Solution

(a) Volume of cuboid $= l \times b \times h$

$720 = 15 \times 8 \times h$

$h = \dfrac{720}{15 \times 8} = 6$ cm. **Ans.**

(b) Surface area $= (l \times w) + 2(l \times h) + 2(w \times h)$

$= (15 \times 8) + 2(15 \times 6) + 2(8 \times 6)$

$= 120 + 180 + 96$

$= 396$ cm² **Ans.**

(c) Volume of liquid $= \dfrac{60}{100} \times 720$

$= 432$ cm³

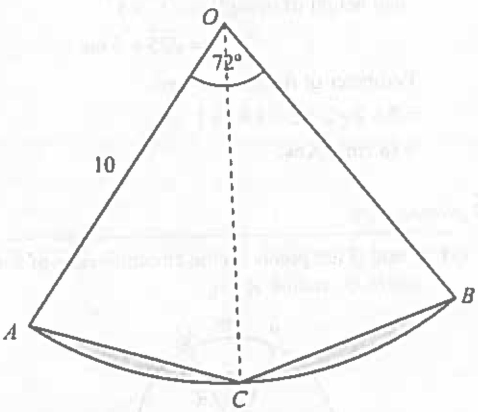
Volume of liquid $= l \times b \times h$

$432 = 15 \times 8 \times h$

$h = \dfrac{432}{15 \times 8} = 3.6$

∴ height of liquid = 3.6 cm **Ans.**

47 (J2017/P2/Q12)



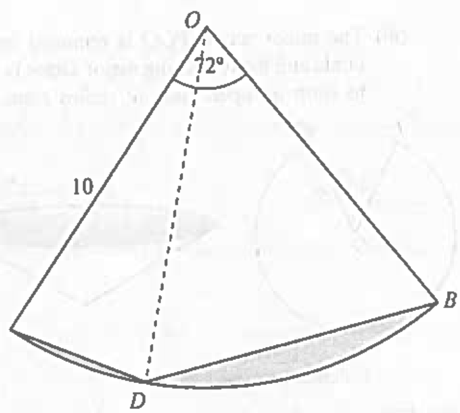
OAB is a sector of a circle, centre O, and radius 10 cm.
$A\hat{O}B = 72°$ and C is the point on the arc AB such that OC bisects $A\hat{O}B$.

(a) Calculate the perimeter of sector OAB. [3]

(b) (i) Calculate the area of sector OAB. [2]

(ii) Calculate the total shaded area. [3]

(c)



D is the point on the arc AB such that $A\hat{O}D : D\hat{O}B = 1 : 2$.
Gavin says that the shaded area on this diagram is the same as the shaded area calculated in part (b)(ii).
Is he correct? Show your working. [4]

Thinking Process

(a) Perimeter = $OA + OB$ + arc length AB.
(b) (ii) To find total shaded area subtract the area of two triangles from the area of sector OAB.
(c) Calculate the shaded areas of two sectors. Add them together and compare with the answer to part (b) (ii).

Solution

(a) Perimeter of sector OAB

$= OA + OB + \text{arc length } \overset{\frown}{ACB}$

$= 10 + 10 + \frac{72}{360} \times 2\pi(10)$

$= 32.566 \approx 32.6$ cm **Ans.**

(b) (i) Area of sector OAB

$= \frac{72}{360} \times \pi \times (10)^2$

$= 62.832 \approx 62.8 \text{ cm}^2$ **Ans.**

(ii) Total shaded area

$=$ area of sector $OAB - 2$(area of ΔOAC)

$= 62.832 - 2\left(\frac{1}{2}(10)(10)\sin 36\right)$

$= 62.832 - 58.779$

$= 4.05$ **Ans.**

(c) Given that, $A\hat{O}D : D\hat{O}B = 1:2$

$\Rightarrow A\hat{O}D = \frac{1}{3} A\hat{O}B$

$= \frac{1}{3}(72°) = 24°$

$\therefore D\hat{O}B = 72° - 24° = 48°$

Shaded area of sector AOD

$=$ area of sector $AOD -$ area of ΔAOD

$= \frac{24}{360}(\pi)(10)^2 - \frac{1}{2}(10)(10)\sin 24°$

$= 20.944 - 20.337 = 0.607 \text{ cm}^2$

Shaded area of sector DOB

$=$ area of sector $DOB -$ area of ΔDOB

$= \frac{48}{360}(\pi)(10)^2 - \frac{1}{2}(10)(10)\sin 48°$

$= 41.888 - 37.157 = 4.731 \text{ cm}^2$

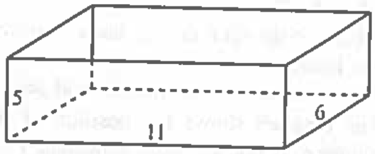
Total shaded area $= 0.607 + 4.731$

$= 5.338 \approx 5.34 \text{ cm}^2$

The shaded area is different as compared to the shaded area found in (b)(ii).

$\therefore$ Gavin is wrong. **Ans.**

48 *(N2017/P1/Q18)*



An open rectangular tray has inside measurements

length 11 cm width 6 cm height 5 cm.

(a) Calculate the total surface area of the four sides and base of the inside of the tray. [2]

(b) Cubes are placed in the tray and a lid is placed on top.
Each cube has an edge of 2 cm.
Find the maximum number of cubes that can be placed in the tray. [1]

Thinking Process

(a) Total surface area = $(L \times W) + 2(L \times H) + 2(W \times H)$
(b) Consider the length, width and height of the tray to find the number of cubes that can fill the tray.

Solution

(a) Total surface area $= (11 \times 6) + 2(11 \times 5) + 2(6 \times 5)$

$= 66 + 110 + 60$

$= 236 \text{ cm}^2$ **Ans.**

(b) Number of cubes that fill the length = 5

Number of cubes that fill the width = 3

Number of cubes that fill the height = 2

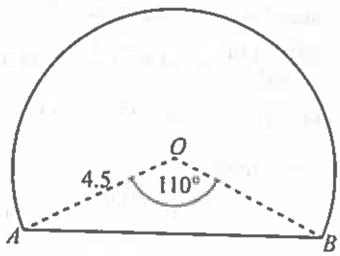
$\therefore$ Number of cubes that can be placed

$= 5 \times 3 \times 2 = 30$ **Ans.**

49 *(N2017/P2/Q9)*

(a) The ventilation shaft for a tunnel is in the shape of a cylinder.
The cylinder has radius 0.4 m and length 15 m.
Calculate the volume of the cylinder. [2]

(b) The diagram shows the cross-section of the tunnel.

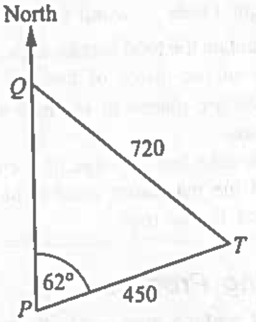


The cross-section of the tunnel is a major segment of a circle, centre O.

The radius of the circle is 4.5 m and $A\hat{O}B = 110°$.
Calculate the area of the cross-section of the tunnel. [4]

(c) The length of the tunnel is 1750 m.
A car drives through the tunnel at an average speed of 45 km/h.
Work out the time the car takes to travel through the tunnel.
Give your answer in minutes and seconds. [2]

(d) The diagram shows the position of the tunnel entrance, T, and two road junctions, P and Q, on horizontal ground.



Q is due north of P and T is on a bearing of 062° from P.
$PT = 450$ m and $QT = 720$ m.
Calculate the bearing of T from Q. [4]

Thinking Process

(a) ✐ Volume of cylinder $= \pi r^2 h$.
(b) ✐ Cross-section area = area of major sector + area of triangle OAB.
(c) Apply, speed $= \dfrac{\text{distance}}{\text{time}}$ ✐ convert the given speed to m/s.
(d) To find the bearing ✐ apply sine rule to find angle $P\hat{Q}T$.

Solution

(a) Volume of the cylinder $= \pi r^2 h$

$$= \pi(0.4)^2(15)$$
$$= 7.54 \text{ m}^3 \text{ (3 sf)} \quad \textbf{Ans.}$$

(b) Area of the cross-section of tunnel

= area of sector + area of ΔOAB

$$= \frac{360° - 110°}{360°} \times \pi(4.5)^2 + \frac{1}{2}(4.5)(4.5)\sin 110°$$

$$= 44.1786 + 9.5144 = 53.7 \text{ m}^2 \text{ (3 sf)} \quad \textbf{Ans.}$$

(c) Average speed = 45 km/h

$$= 45 \times \frac{1000}{3600} = 12.5 \text{ m/s}$$

now, Total time $= \dfrac{\text{total distance}}{\text{average speed}}$

$$= \frac{1750}{12.5} = 140 \text{ seconds}$$
$$= 2 \text{ minutes } 20 \text{ seconds}$$

∴ the car takes 2 minutes 20 seconds to travel through the tunnel. **Ans.**

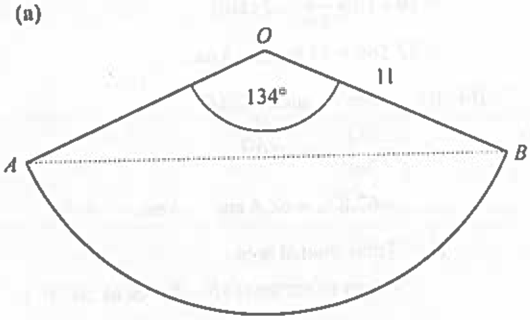
(d) Using sine rule,

$$\frac{\sin P\hat{Q}T}{450} = \frac{\sin 62°}{720}$$
$$\sin P\hat{Q}T = \frac{\sin 62°}{720} \times 450$$
$$= 0.55184$$
$$\Rightarrow P\hat{Q}T = 33.5°$$

∴ bearing of T from $Q = 180° - 33.5°$
$= 146.5°$ **Ans.**

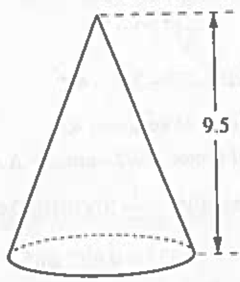
50 *(J2018 P2 Q5)*

(a)



OAB is a sector of a circle, centre O, radius 11 cm.
$A\hat{O}B = 134°$.
(i) Calculate the length of the arc AB. [2]
(ii) Calculate the shortest distance from O to the line AB. [2]

(b) [Volume of a cone $= \frac{1}{3}\pi r^2 h$]
[Curved surface area of a cone $= \pi r l$]



A cone has height 9.5 cm and volume 115 cm³.
(i) Show that the radius of the base of the cone is 3.4 cm, correct to 1 decimal place. [2]
(ii) Calculate the curved surface area of the cone. [3]

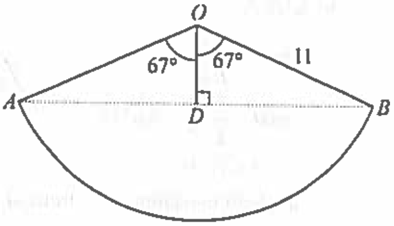
Thinking Process

(a) (ii) Draw a perpendicular line from O to AB and find its length.
(b) (i) To find the radius ✐ equate the formula for volume of cone to 115
(ii) To find the curved surface area ✐ find the slant height by using pythagoras theorem.

Solution

(a) (i) Length of the arc $AB = \frac{134°}{360°} \times 2 \times \pi \times 11$
$= 25.73$ cm **Ans.**

(ii)



In ΔODB, $\cos 67° = \frac{OD}{11}$

$OD = \cos 67° \times 11 = 4.30$

$\therefore$ Shortest distance from O to AB
$= 4.30$ cm **Ans.**

(b) (i) Volume of a cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow 115 = \frac{1}{3}(\pi)(r^2)(9.5)$

$\Rightarrow r^2 = 115 \times \frac{3}{9.5\pi}$

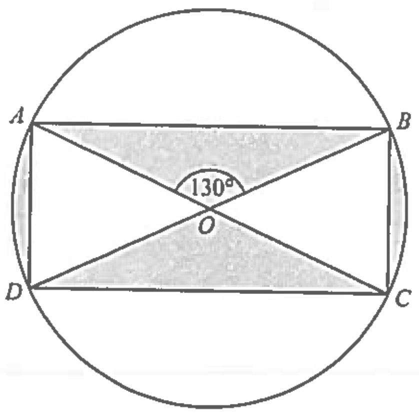
$\Rightarrow r^2 = 11.56 \Rightarrow r = 3.4$ cm **Shown.**

(ii) Using Pythagoras theorem.

slant height, $l = \sqrt{3.4^2 + 9.5^2}$
$= \sqrt{101.81} = 10.1$ cm

Curved surface area $= \pi r l$
$= \pi \times 3.4 \times 10.1$
$= 107.88$
≈ 108 cm^2 **Ans.**

51 *(N2018/P2/Q5)*



AC and BD are diameters of the circle, centre O.
$AC = 12$ cm and $A\hat{O}B = 130°$.

(a) Calculate the area of triangle AOB. [2]

(b) Calculate the area of the sector AOD. [2]

(c) Calculate the percentage of the area of the circle that is shaded. [4]

Thinking Process

(a) Apply, area of $\Delta = \frac{1}{2}ab\sin c$

(b) Apply, area of sector $= \frac{\theta}{360°} \times \pi r^2$

(c) Compute the area of the shaded region and express it as a percentage of the area of circle.

Solution

(a) Area of $\Delta AOB = \frac{1}{2} \times 6 \times 6 \times \sin 130°$
$= 13.79$ cm^2 **Ans.**

(b) $A\hat{O}D = 180° - 130° = 50°$

$\therefore$ Area of the sector $AOD = \frac{50°}{360°} \times \pi(6)^2$
$= 15.71$ cm^2 **Ans.**

(c) Area of shaded segment AD
$=$ area of sector AOD $-$ area of ΔAOD
$= 15.71 - \left(\frac{1}{2} \times 6 \times 6 \times \sin 50°\right)$
$= 15.71 - 13.79 = 1.92$ cm^2

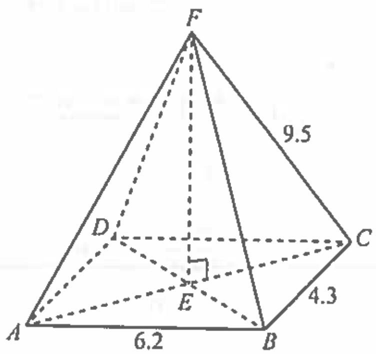
Total shaded area
$= 2(\text{area of } \Delta AOB) + 2(\text{area of shaded segment } AD)$
$= 2(13.79) + 2(1.92) = 31.42$ cm^2

Area of circle $= \pi(6)^2 = 113.1$ cm^2

$\therefore$ required percentage $= \frac{31.42}{113.1} \times 100$
$= 27.8\%$ **Ans.**

52 *(N2018/P2/Q9)*

[Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height]



The diagram shows a pyramid with a rectangular, horizontal base.

Vertex F of the pyramid is vertically above the centre of the base, E.
$AB = 6.2$ cm and $BC = 4.3$ cm.
The length of each sloping edge of the pyramid is 9.5 cm.

(a) Show that the height, EF, of the pyramid is 8.72 cm, correct to 3 significant figures. [4]
(b) Calculate the volume of the pyramid. [2]
(c) Calculate angle AFB. [3]
(d) Calculate the angle of elevation of F from the midpoint of AB. [2]

Thinking Process

(a) To find EF ∮ find EC ∮ apply Pythagoras theorem on triangle ABC.
(c) To find angle AFB ∮ apply Cosine rule.
(d) Draw a line from F to the midpoint of AB.

Apply $\tan\theta = \frac{\text{opp.}}{\text{adj.}}$ to the triangle formed.

Solution

(a) In ΔABC, using Pythagoras theorem.

$$AC = \sqrt{(6.2)^2 + (4.3)^2}$$
$$= \sqrt{56.93}$$
$$= 7.545 \text{ cm}$$
$$EC = \frac{1}{2}AC$$
$$= \frac{1}{2}(7.545) = 3.77 \text{ cm}$$

Now, in ΔCEF, using Pythagoras theorem,

$$EF = \sqrt{(CF)^2 - (EC)^2}$$
$$= \sqrt{(9.5)^2 - (3.77)^2}$$
$$= \sqrt{76.037}$$
$$= 8.72 \text{ cm (3sf)} \quad \textbf{Shown.}$$

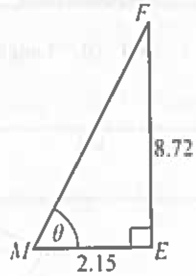
(b) Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

$$= \frac{1}{3} \times (6.2 \times 4.3) \times 8.72$$
$$= 77.4917 \approx 77.5 \text{ cm}^3 \quad \textbf{Ans.}$$

(c) In ΔAFB, using cosine rule,

$$\cos A\hat{F}B = \frac{(9.5)^2 + (9.5)^2 - (6.2)^2}{2(9.5)(9.5)}$$
$$= \frac{90.25 + 90.25 - 38.44}{180.5}$$
$$= \frac{142.06}{180.5} = 0.7870$$
$$\therefore \quad A\hat{F}B = 38.1^\circ \quad \textbf{Ans.}$$

(d) Let M be the mid-point of AB.



$$\Rightarrow EM = \frac{1}{2}BC$$
$$= \frac{1}{2}(4.3) = 2.15 \text{ cm}$$

In ΔMEF,

$$\tan\theta = \frac{EF}{EM}$$
$$\Rightarrow \tan\theta = \frac{8.72}{2.15} = 4.0558$$
$$\Rightarrow \theta = 76.1^\circ$$

$\therefore$ angle of elevation of F from $M = 76.1^\circ$ **Ans.**

Topic 11

Symmetry

1 *(J2008/P1/Q11)*

The following list gives the names of six shapes.

Square	Rectangle	Equilateral triangle
Kite	Trapezium	Parallelogram

From this list, write down the name of the shape which always has

(a) rotational symmetry of order 3, [1]

(b) rotational symmetry of order 2 and exactly 2 lines of symmetry, [1]

(c) one line of symmetry only. [1]

Thinking Process

(a) Understand the definition of rotational symmetry.

(b) Remember that line of symmetry is the axis of reflection.

Solution

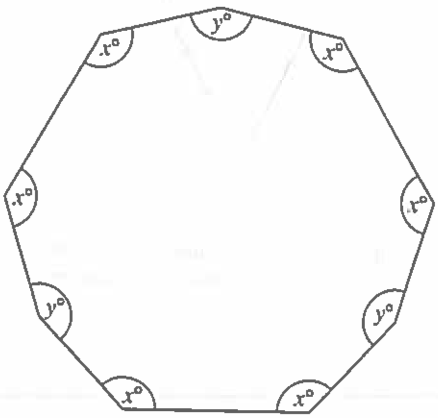
(a) Equilateral triangle. **Ans.**

(b) Rectangle. **Ans.**

(c) Kite. **Ans.**

2 *(J2009/P2/Q4a)*

(a) In the diagram, the 9-sided polygon has 6 angles of $x°$ and 3 angles of $y°$.



(i) For this polygon, state

(a) the number of lines of symmetry, [1]

(b) the order of rotational symmetry. [1]

(ii) (a) Show that the sum of the interior angles of a 9-sided polygon is 1260°. [1]

(b) Find an expression for y in terms of x. [2]

(c) Given also that $y = 12 + x$, find x. [2]

Thinking Process

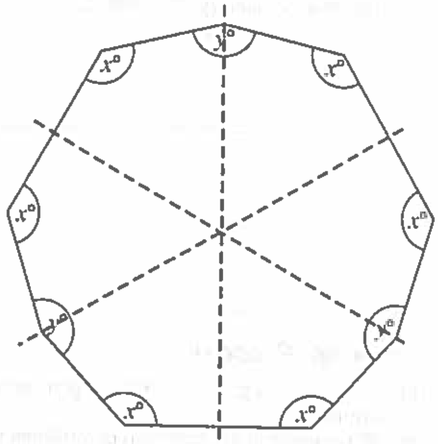
(a) (i) (a) & (b) Understand the definition of line symmetry and rotational symmetry.

(ii) (a) Interior angle sum $= (n-2)180$

(b) To find the required expression ✐ Add up all the interior angles of polygon.

(c) Solve the given equation and the equation found in (b) above simultaneously for x.

Solution



(a) (i) (a) Number of lines of symmetry = 3

(b) Order of rotational symmetry = 3

(ii) (a) Sum of interior angles $= (n-2)180$

$= (9-2)180$

$= 1260°$ **Shown.**

(b) Sum of interior angles of the polygon

$= 6x + 3y$

given sum of interior angles = 1260°

$\therefore\ 6x + 3y = 1260$

$2x + y = 420$

$y = 420 - 2x$ **Ans.**

(c) $y = 12 + x$

From (ii) (b), $y = 420 - 2x$

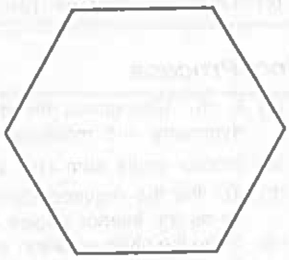
$\Rightarrow\ 12 + x = 420 - 2x$

$3x = 408$

$x = 136$ **Ans.**

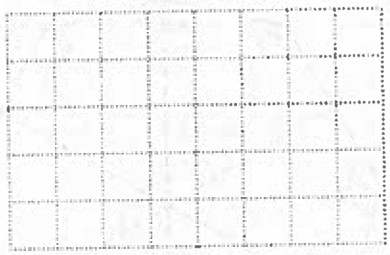
3 *(J2010/P1/Q7)*

(a) On the regular hexagon below, draw all the lines of symmetry.



[1]

(b) On the grid below, draw a quadrilateral with rotational symmetry of order 2.



[1]

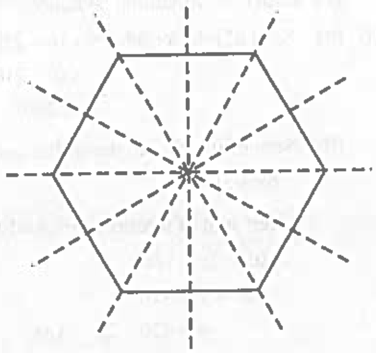
Thinking Process

(a) Note that a regular *n*-sided polygon has *n* lines of symmetry.

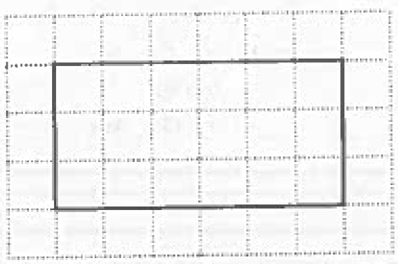
(b) Understand the definition of rotational symmetry.

Solution

(a)



(b)



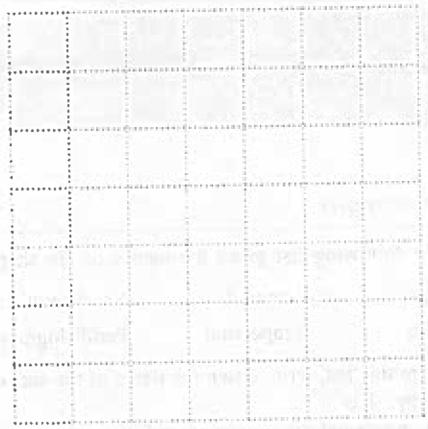
Note that you can also draw a Parallelogram or a Rhombus.

4 *(J2012/P1/Q3)*

(a) On the grid below, draw a quadrilateral with

no rotational symmetry

and just 1 line of symmetry.



[1]

(b) Complete this description.

A parallelogram has rotational symmetry of order ....and .... lines of symmetry. [1]

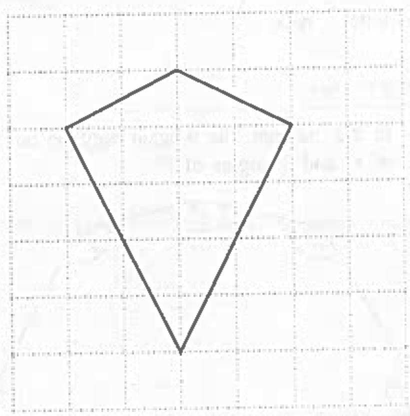
Thinking Process

(a) Understand the definition of rotational symmetry and line symmetry.

(b) Note that a parallelogram has rotational symmetry of order 2, but no line symmetry.

Solution

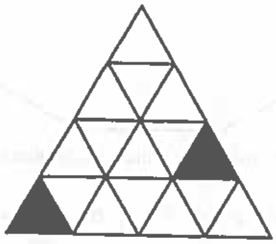
(a)



(b) A parallelogram has rotational symmetry of order ......2....and .....0..... lines of symmetry.

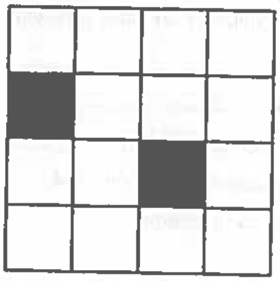
5 *(N2013/P1/Q5)*

(a) In the diagram, two small triangles are shaded. Shade one more small triangle, so that the diagram will then have one line of symmetry.



[1]

(b) In the diagram, two small squares are shaded. Shade two more small squares, so that the diagram will then have rotational symmetry of order 2.



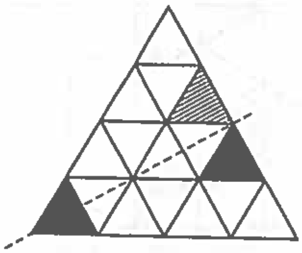
[1]

Thinking Process

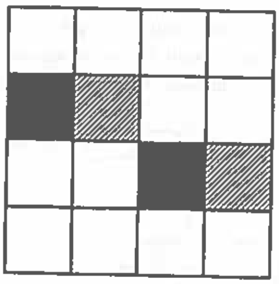
(a) Remember that line of symmetry is the axis of reflection.

Solution

(a)

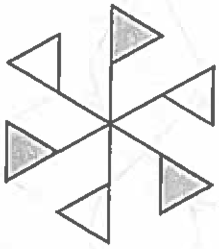


(b)



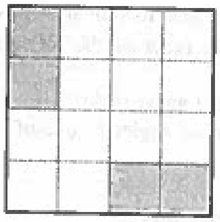
6 *(J2014/P1/Q6)*

(a) Complete the description of the pattern below.



The pattern has rotational symmetry of order .... and .... lines of symmetry. [1]

(b) Shade in two more small squares in this shape to make a pattern with exactly 2 lines of symmetry.



[1]

Thinking Process

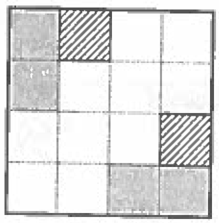
(a) Understand the definition of line symmetry and rotational symmetry.

(b) Remember that line of symmetry is the axis of reflection.

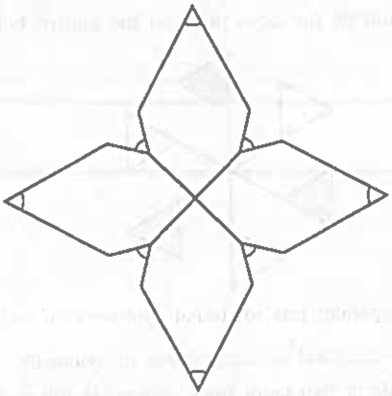
Solution

(a) The pattern has rotational symmetry of order ....3........ and ....0........ lines of symmetry.

(b)



7 (N2014 P1 Q19)



The diagram shows a figure made from four identical hexagons.
It has both line and rotational symmetry.

(a) What is the order of the rotational symmetry? [1]

(b) Each marked angle is 60°.
Find the other angles in one of the hexagons. [3]

Thinking Process

(b) Use the given information and the symmetry of the figure to find the unknown angles.

Solution

(a) Order of the rotational symmetry = 4 **Ans.**

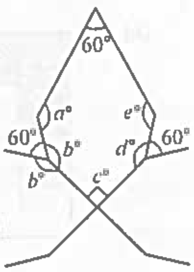
(b) The figure has rotational symmetry of order 4

$\therefore \quad c° = 90°$

$b° + b° + 60° = 360°$ (∠s sum at a point)

$2b° = 300°$

$b° = 150°$



since, $d° = b°$,

$\therefore \quad d° = 150°$

sum of interior angles of a hexagon $= (6-2)180°$

$= 720°$

$\Rightarrow 60° + a° + 150° + 90° + 150° + e° = 720°$

$\Rightarrow 450° + a° + e° = 720°$

$450° + a° + a° = 720°$ (since $a° = e°$)

$2a° = 270°$

$a° = 135°$

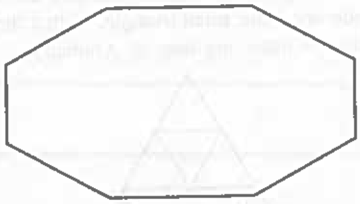
$\Rightarrow e° = 135°$

$\therefore$ other angles are:

135°, 150°, 90°, 150°, 135° **Ans.**

8 (J2015 P2 Q10 b,c,d)

(b)



State the number of lines of symmetry of the octagon above. [1]

(c) The cross-section of a prism is an equilateral triangle.
State the number of planes of symmetry of the prism. [1]

(d) Name two special quadrilaterals that have exactly 2 lines of symmetry and also rotational symmetry of order 2. [2]

Solution

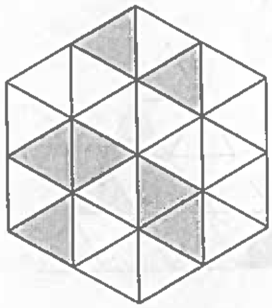
(b) No. of lines of symmetry = 2

(c) No. of planes of symmetry = 4.

(d) Rectangle and Rhombus.

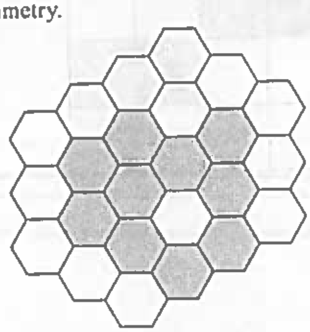
9 (N2015 P1 Q7)

(a) In the diagram, seven small triangles are shaded. Shade two more small triangles, so that the diagram will then have rotational symmetry of order 3.



[1]

(b) In the diagram, ten small hexagons are shaded. Shade one more small hexagon, so that the diagram will then have exactly one line of symmetry.



[1]

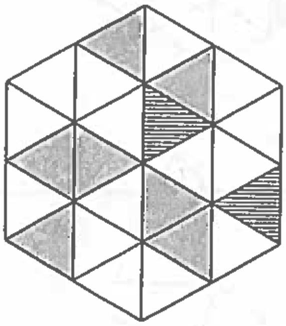
Thinking Process

(a) Understand the definition of rotational symmetry.

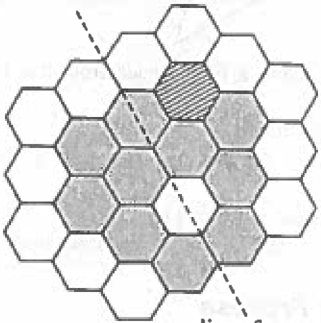
(b) Note that figure with 1 line of symmetry has only 1 axis of reflection.

Solution

(a)



(b)

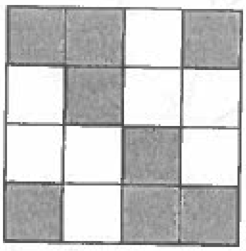


10 *(J2016/P1/Q2)*

(a) Complete this description.

A rectangle has rotational symmetry of order .... and .... lines of symmetry. [1]

(b) Shade 4 more small squares in the shape below to make a pattern with rotational symmetry of order 4.



[1]

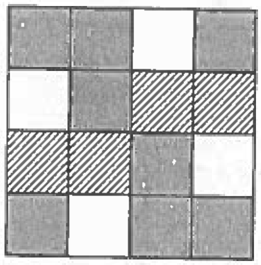
Thinking Process

Understand the definition of line symmetry and rotational symmetry.

Solution

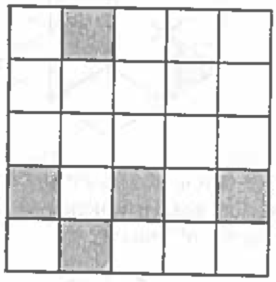
(a) A rectangle has rotational symmetry of order ...2.... and ......2. lines of symmetry.

(b)



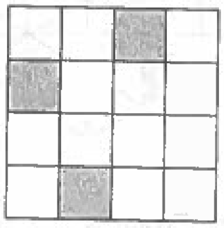
11 *(N2016/P1/Q3)*

(a)



In the diagram, five small squares are shaded. Shade **one more** small square, so that the diagram has exactly one line of symmetry. [1]

(b)



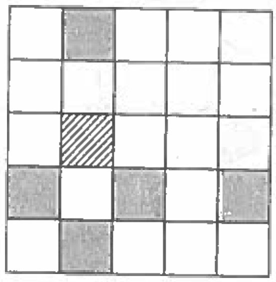
In the diagram, three small squares are shaded. Shade **one more** small square, so that the diagram has rotational symmetry of order 4. [1]

Thinking Process

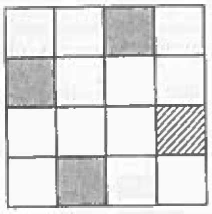
Understand the definition of line symmetry and rotational symmetry.

Solution

(a)

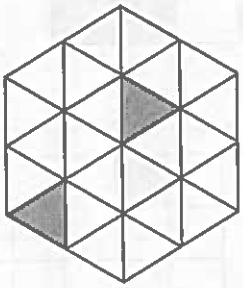


(b)



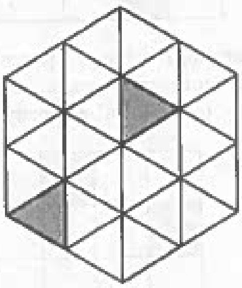
12 (J2017 P1 Q2)

(a) Shade one more small triangle in the shape below to make a pattern with one line of symmetry.



[1]

(b) Shade two more small triangles in the shape below to make a pattern with rotational symmetry of order 2.



[1]

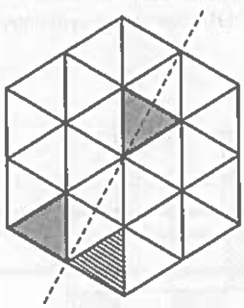
Thinking Process

(a) Remember that line of symmetry is the axis of reflection.

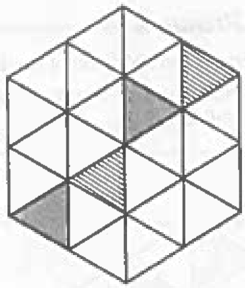
(b) Understand the definition of rotational symmetry.

Solution

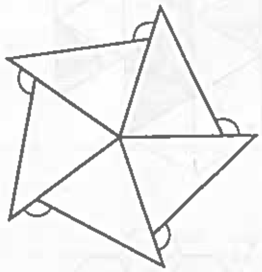
(a)



(b)



13 (N2017/P1/Q15)



The diagram shows a figure made from five identical triangles.

The figure has rotational symmetry.

(a) Write down the order of rotational symmetry. [1]

(b) Each marked angle is 110°.
Find the angles of one of the triangles. [2]

Thinking Process

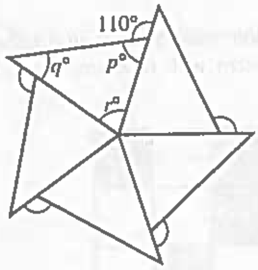
(a) Understand the concept of rotational symmetry.

(b) To find the angles of one triangle ∲ divide 360° by 5 to find the angle at the center. Find the angle adjacent to exterior angle, then use both angles to find the 3rd angle.

Solution

(a) Order of rotational symmetry = 5 **Ans.**

(b)



$r° = \frac{360°}{5} = 72°$ (∠ sum at a point)

$p° = 180° - 110°$ (∠s on a straight line)

$= 70°$

$p° + q° + r° = 180°$ (∠ sum of a Δ)

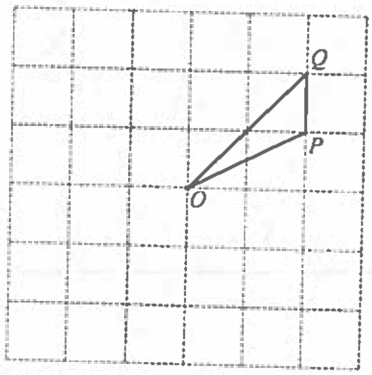
$q° = 180° - p° - r°$

$= 180° - 70° - 72° = 38°$

∴ angles are. 70°, 38°, 72° **Ans.**

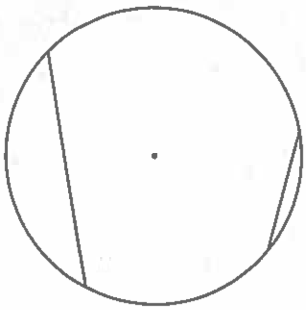
14 *(J2018/P1 Q7)*

(a) Triangle OPQ is part of a figure that has rotational symmetry of order 2 about the point O. Complete the figure.



[1]

(b) The diagram shows a circle, its centre, and two chords. Add **one** chord, to give a diagram that has one line of symmetry.



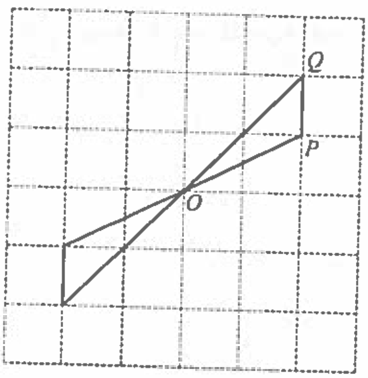
[1]

Thinking Process

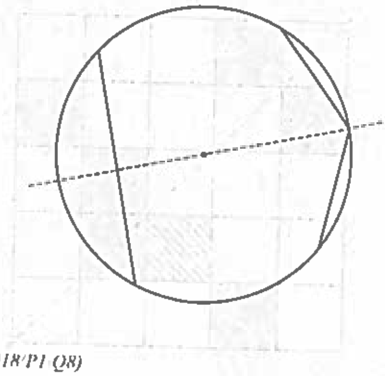
✐ Understand the definition of rotational symmetry and line symmetry.

Solution

(a)



(b)

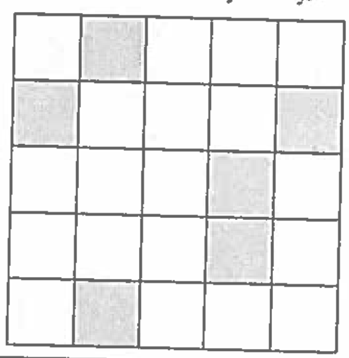


15 *(N2018/P1 Q8)*

(a) The diagram shows part of a figure that has rotational symmetry of order 4 about the point O.
Complete the figure.

[1]

(b) In the diagram, six small squares are shaded. Shade **one** more small square to give a diagram that has exactly one line of symmetry.



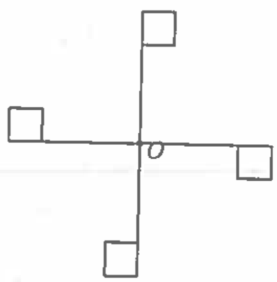
[1]

Thinking Process

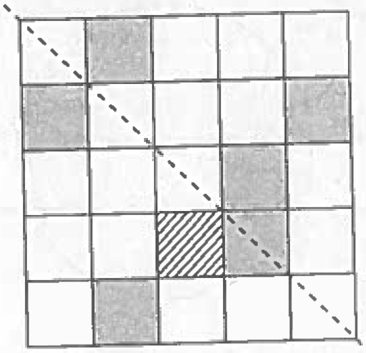
(a) Understand the definition of rotational symmetry.
(b) ✐ Remember that line of symmetry is the axis of reflection.

Solution

(a)



(b)

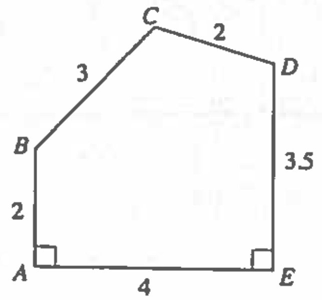


Topic 12

Loci and Geometrical Constructions

1 *(N2009 P2 Q9)*

(a) The diagram shows the side $ABCDE$ of a building.

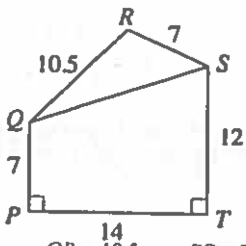


$AB = 2$ m, $BC = 3$ m, $CD = 2$ m, $DE = 3.5$ m and $EA = 4$ m.
AB and DE are vertical.
AE is horizontal.

(i) Using a scale of 2 cm to 1 m, construct an accurate scale drawing of $ABCDE$. [3]

(ii) Measure and write down $A\hat{B}C$. [1]

(b) The diagram shows the pentagon $PQRST$.



$PQ = 7$ cm, $QR = 10.5$ cm, $RS = 7$ cm, $ST = 12$cm and $TP = 14$ cm.

$Q\hat{P}T = P\hat{T}S = 90°$.

(i) Explain why the shapes $ABCDE$ and $PQRST$ are not geometrically similar. [1]

(ii) Show that $QS^2 = 221$. [2]

(iii) Calculate $Q\hat{R}S$. [3]

(iv) Calculate $R\hat{Q}S$. [2]

Thinking Process

(a) (i) Convert the dimensions into cm. Draw the diagram by using ruler and compass.
(ii) Measure the angle from your drawing by using protractor.

(b) (i) consider the ratio of sides of the two figures.
(ii) Apply pythagoras theorem.
(iii) Apply cosine rule.
(iv) Apply sine rule.

Solution

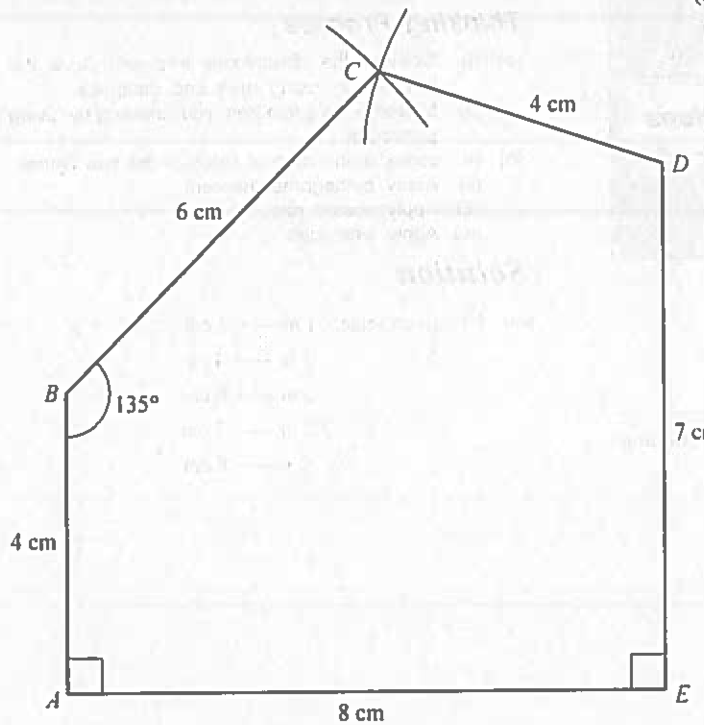
(a) (i) Given scale: 1 m —— 2 cm

$\Rightarrow$ 2 m —— 4 cm

3 m —— 6 cm

3.5 m —— 7 cm

4 m —— 8 cm



(ii) $A\hat{B}C = 135°$ **Ans.**

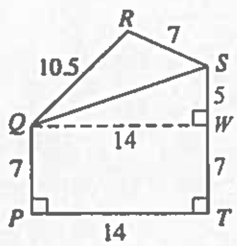
(b) (i) Considering the ratio of corresponding sides in centimetres, we have,

$\frac{14}{400} = 0.035$, $\frac{7}{200} = 0.035$,

$\frac{10.5}{300} = 0.035$, $\frac{12}{350} = 0.03429$

$\therefore$ The two shapes are not similar as their ratio of sides is not equal. **Ans.**

(ii) Applying pythagoras theorem on ΔQSW,



$QS^2 = QW^2 + WS^2$

$= 14^2 + 5^2$

$= 221$ **Shown.**

(iii) Applying cosine rule in ΔQRS,

$$\cos Q\hat{R}S = \frac{RQ^2 + RS^2 - QS^2}{2(RQ)(RS)}$$

$$= \frac{(10.5)^2 + (7)^2 - 221}{2(10.5)(7)}$$

$$= -\frac{61.75}{147}$$

$\therefore$ $Q\hat{R}S = 114.84° \approx 115°$ **Ans.**

(iv) Applying sine rule in ΔRQS,

$$\frac{\sin R\hat{Q}S}{RS} = \frac{\sin Q\hat{R}S}{QS}$$

$$\frac{\sin R\hat{Q}S}{7} = \frac{\sin 114.84°}{\sqrt{221}}$$

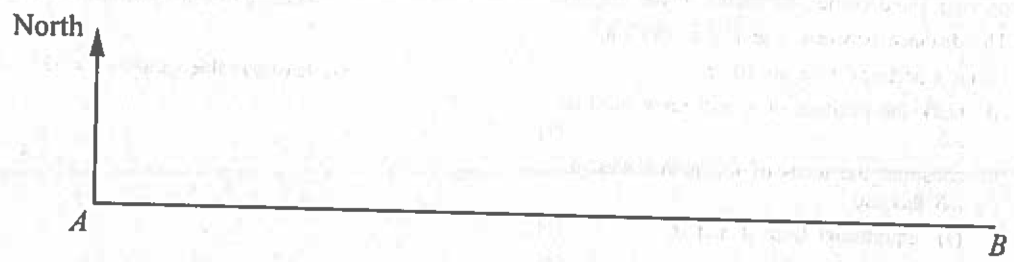
$$\sin R\hat{Q}S = \frac{\sin 114.84°}{\sqrt{221}} \times 7$$

$$= 0.42731$$

$\therefore$ $R\hat{Q}S = 25.297° \approx 25.3°$ **Ans.**

2 *(J2010/P1/Q26)*

A map is drawn to a scale of 1 cm to 5 km.
The diagram below shows the positions of two radio masts A and B on the map.



(a) A third radio mast, C, is north of the line AB.
It is 40 km from A and 50 km from B.
Using ruler and compasses, construct triangle ABC. [2]

(b) A house D, inside the triangle, is more than 35 km from B and closer to B than to A.
Shade the region on your diagram that represents the possible positions of the house D. [3]

Thinking Process

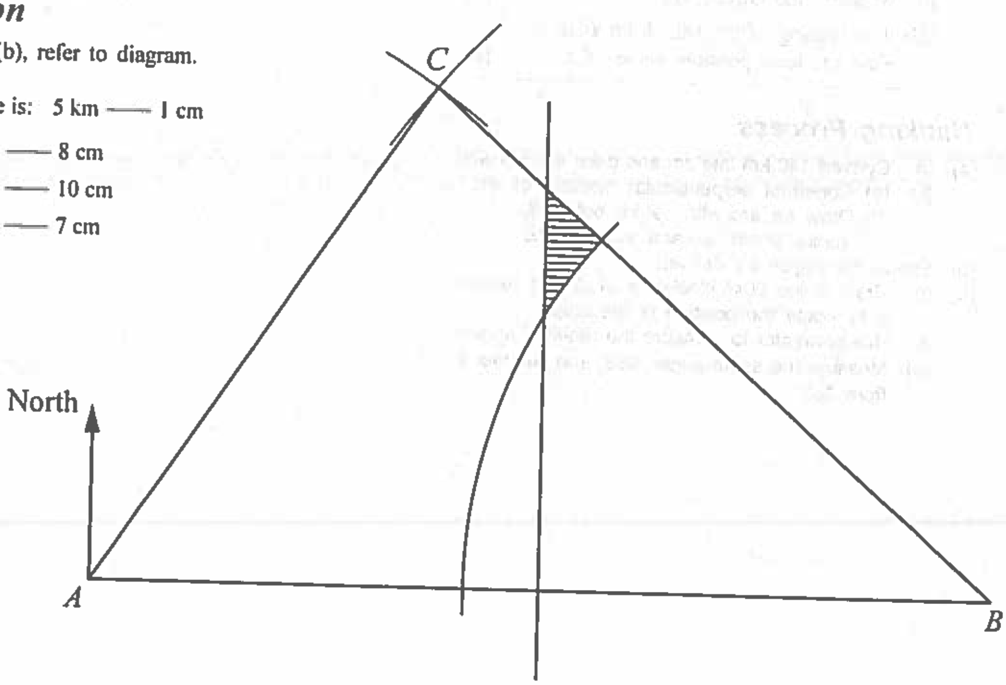
(a) Construct ΔABC according to the given dimensions.
(b) Locus 1: Draw a circle of radius 7 cm with centre at B.
Locus 2: Construct perpendicular bisector of line AB.

Solution

For (a) & (b), refer to diagram.

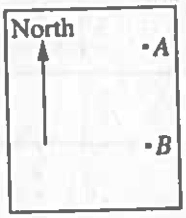
Given scale is: 5 km —— 1 cm

∴ 40 km —— 8 cm
50 km —— 10 cm
35 km —— 7 cm



3 (N2010 P2 Q6)

Answer this question on a new page.



A and B are two coastguard stations with A due north of B.
On your new blank page, mark A in a position near the top right hand corner, as shown in the diagram.

(a) The distance between A and B is 140 km.
Using a scale of 1 cm to 10 km,
(i) mark the position of B and draw the line AB, [1]
(ii) construct the locus of the points west of AB that are
(a) equidistant from A and B, [1]
(b) 90 km from B. [1]

(b) A ship, S, lies to the west of AB and is
I nearer to A than B,
II within 90 km of B.
On your diagram, shade the region in which the ship is situated. [1]

(c) It is also known that the bearing of the ship from A is 204°.
(i) On your diagram, mark the two extreme positions, S_1 and S_2, of the ship. [2]
(ii) Measure the angle S_1BS_2. [1]
(iii) The bearing of the ship from B is $x°$. Find the least possible value of x. [1]

Thinking Process

(a) (i) Convert 140 km into cm and draw the line AB.
(ii) (a) Construct perpendicular bisector of AB.
(b) Draw an arc with centre being B, radius 9 cm, towards west of AB.

(b) Shade the region as defined.

(c) (i) Draw a line 204° clockwise at A and extend it to locate the position of the ship.
(ii) Use protractor to measure the required angle.
(iii) Measure the acute angle ABS_2 and subtract it from 360°.

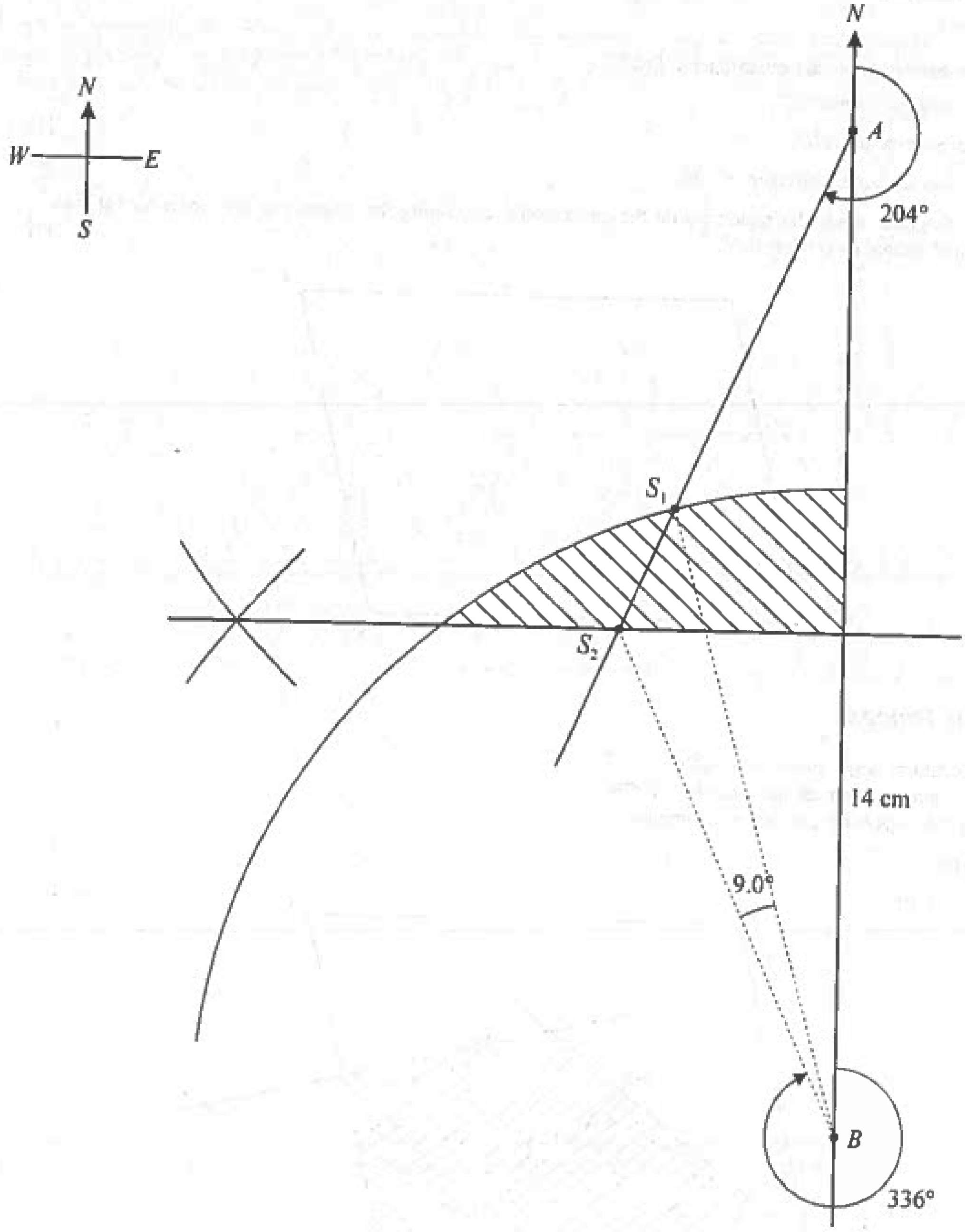
Solution

Given scale: 10 km = 1 cm
$\therefore$ 140 km = 14 cm
90 km = 9 cm

Refer to figure on the next page for parts (a) (i), (ii), (b) and (c) (i).

(c) (ii) $\angle S_1BS_2 = 9.0°$ **Ans**

(iii) The bearing of the ship from B is least when it is at S_2.
$\angle ABS_2 = 24°$
$\therefore$ bearing of ship S_2 from $B = 360° - 24°$
$= 336°$
$\Rightarrow$ least possible value of $x = 336$ **Ans**


N
W
E
S
N
A
204°
S_1
S_2
14 cm
9.0°
B
336°

4 *(J2011/P1/Q18)*

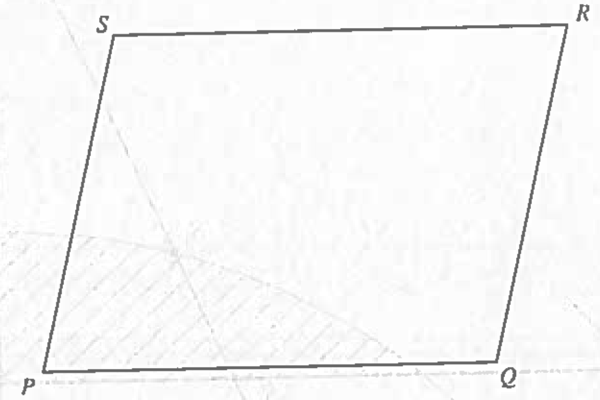
The diagram below shows the quadrilateral $PQRS$.

(a) On the diagram, construct

(i) the bisector of $S\hat{P}Q$. [1]

(ii) the perpendicular bisector of QR. [1]

(b) On the diagram, shade the region inside the quadrilateral containing the points that are closer to PQ than to PS and nearer to Q than to R. [1]



Thinking Process

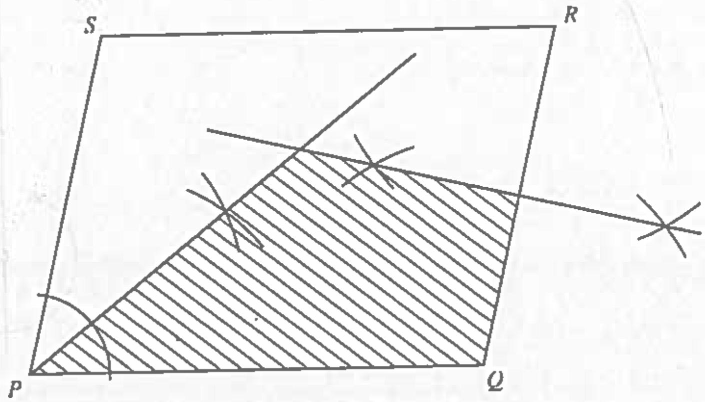
(a) (i) Construct angle bisector of $S\hat{P}Q$.

(ii) Construct perpendicular bisector of QR.

(b) Shade the region as per given information.

Solution

(a) (i), (ii), & (b)



5 *(N2011/P1/Q26)*

The diagram on next page shows triangle ABC.

(a) The point D is on the opposite side of AC to B.

$AD = 6$ cm and $CD = 8$ cm.

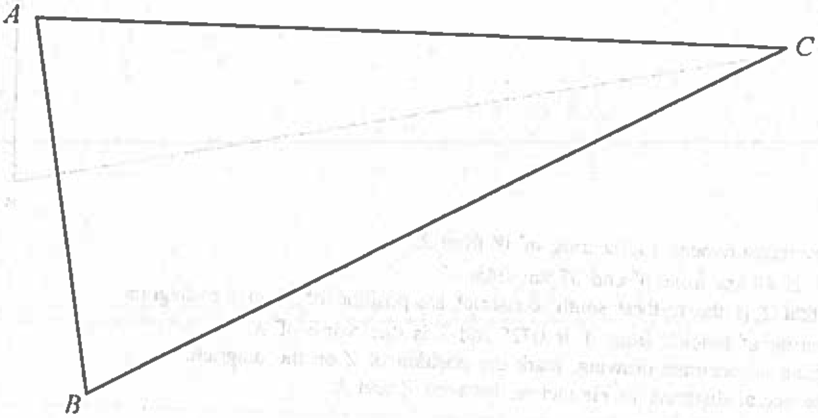
Construct triangle ADC. [1]

(b) On the diagram, construct the locus of points **inside** the quadrilateral $ABCD$ that are

(i) 2.5 cm from AC, [1]

(ii) equidistant from AB and BC. [1]

(c) The points P and Q are 2.5 cm from AC and equidistant from AB and BC.
Mark and label P and Q.
Measure PQ. [1]



Thinking Process

(a) Construct ΔADC according to the given dimensions.

(b) (i) Construct two parallel lines on both sides of AC, 2.5 cm away.

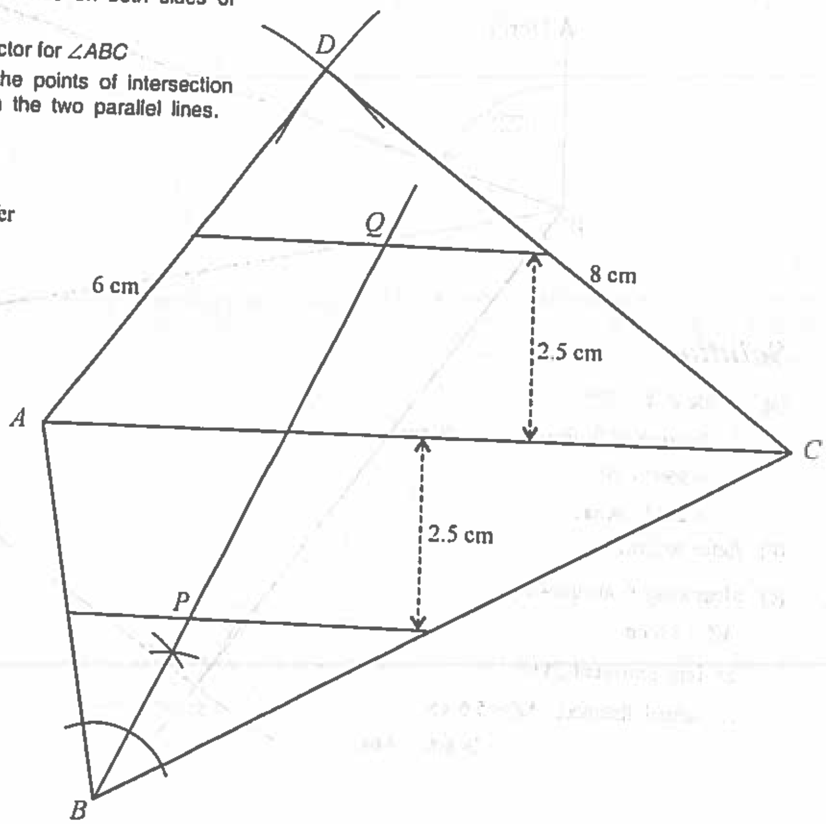
(ii) Construct angle bisector for $\angle ABC$.

(c) Note that P and Q are the points of intersection of the angle bisector with the two parallel lines. Measure PQ.

Solution

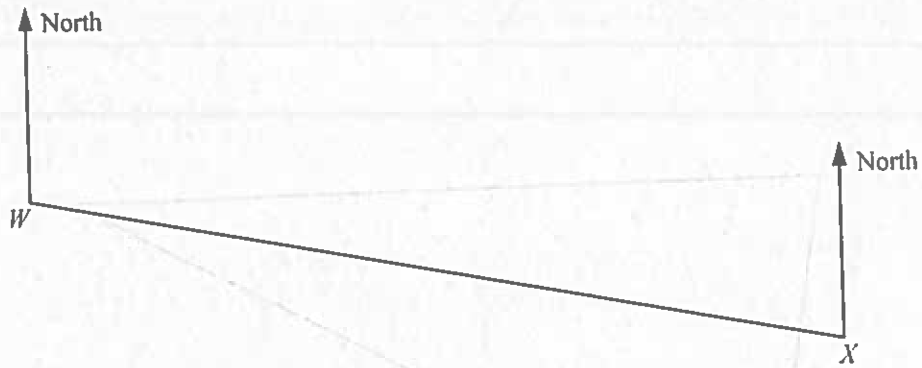
For (a), (b) (i), (ii) & (c), refer to diagram.

$PQ = 5.5$ cm **Ans.**



6 (J2012/P2/Q4)

The scale diagram shows the position of two hotels, W and X, drawn to a scale of 1 cm to 5 km.



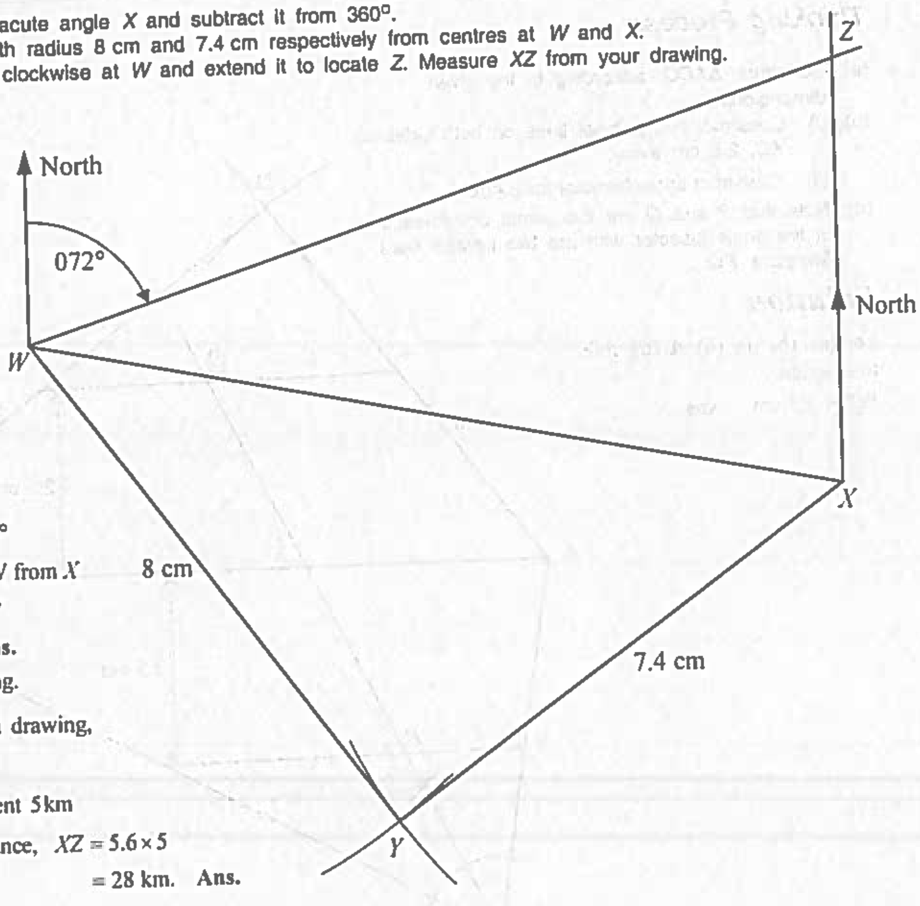
(a) Find, by measurement, the bearing of W from X. [1]

(b) Hotel Y is 40 km from W and 37 km from X.
Given that Y is the furthest south, construct the position of Y on the diagram. [2]

(c) The bearing of hotel Z from W is 072° and Z is due North of X.
By making an accurate drawing, mark the position of Z on the diagram.
Find the actual distance, in kilometres, between Z and X. [3]

Thinking Process

(a) Measure the acute angle X and subtract it from 360°.
(b) Draw arcs with radius 8 cm and 7.4 cm respectively from centres at W and X.
(c) Draw a line 72° clockwise at W and extend it to locate Z. Measure XZ from your drawing.



Solution

(a) Acute $\angle X = 79°$

$\therefore$ bearing of W from X

$= 360° - 79°$

$= 281°$ **Ans.**

(b) Refer to drawing.

(c) Measuring from drawing,

$XZ = 5.6$ cm

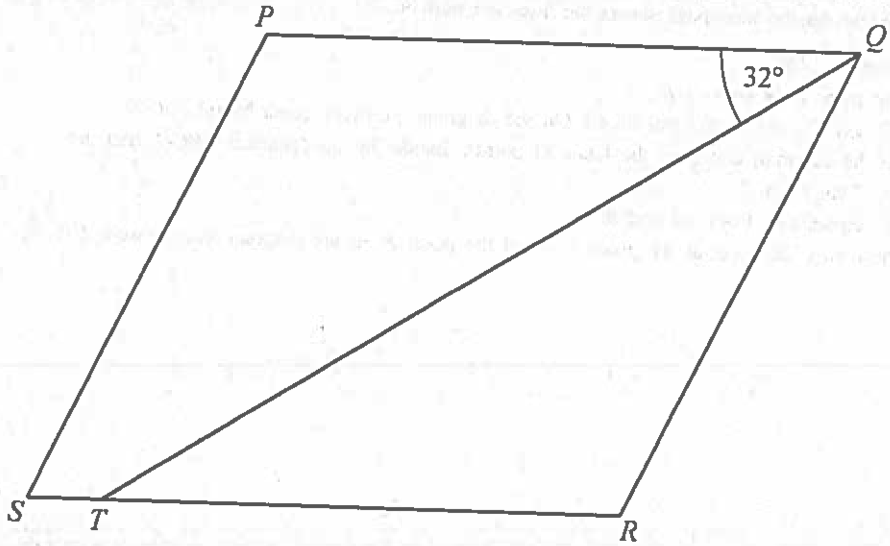
as 1cm represent 5km

$\therefore$ actual distance, $XZ = 5.6 \times 5$

$= 28$ km. **Ans.**

7 (J2012/P2/Q11 b)

(b)



$PQRS$ is a parallelogram. QT is the bisector of $P\hat{Q}R$ and $P\hat{Q}T = 32°$.

(i) Giving a reason for your answer, find

(a) $Q\hat{T}R$, [1]

(b) $S\hat{P}Q$. [1]

(ii) On the diagram, construct the locus of points inside the parallelogram $PQRS$ which are

I 4 cm from PS,

II 5 cm from R. [2]

(iii) The point V is inside $PQRS$,
less than 4 cm from PS,
less than 5 cm from R,
nearer to QR than PQ.

Shade the region containing the possible positions of V. [1]

Thinking Process

(b) (i) (a) $\angle PQT = \angle QTR$ (alternate angles).

(b) $\angle SPQ + \angle PQR = 180°$ (interior angles).

Solution

(b) (i) (a) $Q\hat{T}R = 32°$

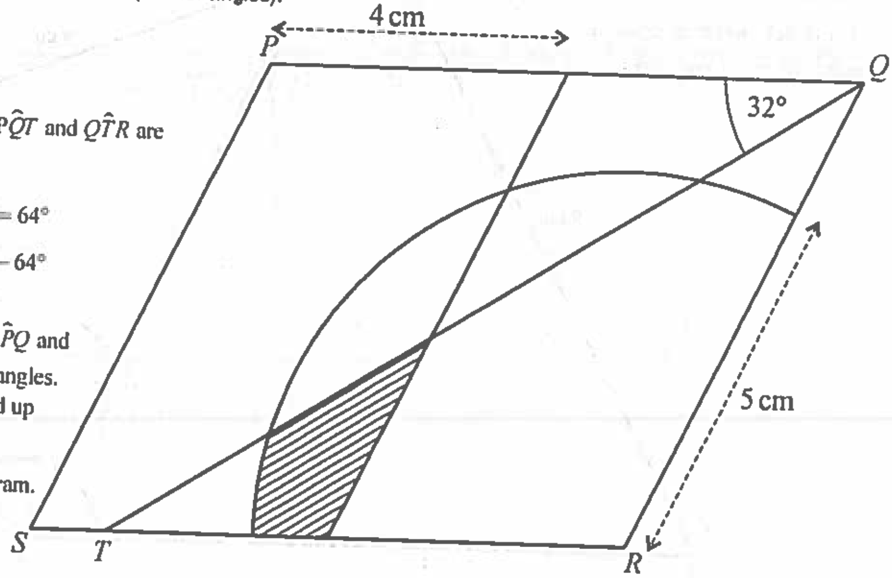
reason: because $P\hat{Q}T$ and $Q\hat{T}R$ are alternate angles.

(b) $P\hat{Q}R = 32° + 32° = 64°$

$\therefore \quad S\hat{P}Q = 180° - 64°$
$= 116°$

reason: because $S\hat{P}Q$ and $P\hat{Q}R$ are interior angles. Interior angles add up to 180°.

(ii) & (iii) See diagram.



8 (N2012 P1 Q27)

The diagram on the next page shows the lines AB and BC.

(a) Measure $A\hat{B}C$. [1]

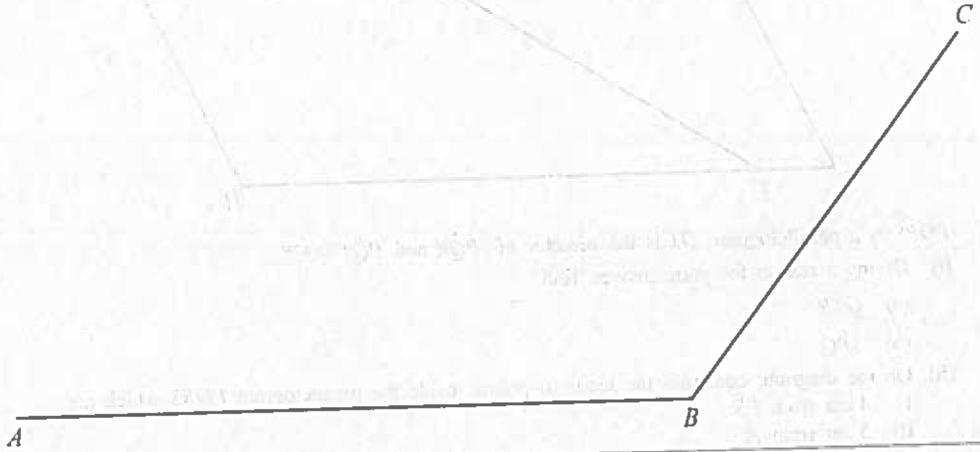
(b) The point D is above AB.
AD and CD are each equal to AB. On the diagram, construct quadrilateral $ABCD$. [1]

(c) On the diagram, construct the locus of points, **inside** the quadrilateral $ABCD$, that are

(i) 7 cm from C, [1]

(ii) equidistant from AB and BC. [1]

(d) These two loci meet at the point P. Label the point P on the diagram and measure DP. [1]



Thinking Process

(a) Measure the angle with a protractor.

(b) Draw two arcs with center A and C and radius equal to the length of AB. The two arcs intersect at D.

(c) (i) Draw an arc of radius 7 cm with center at C.

(ii) Draw an angle bisector of angle ABC.

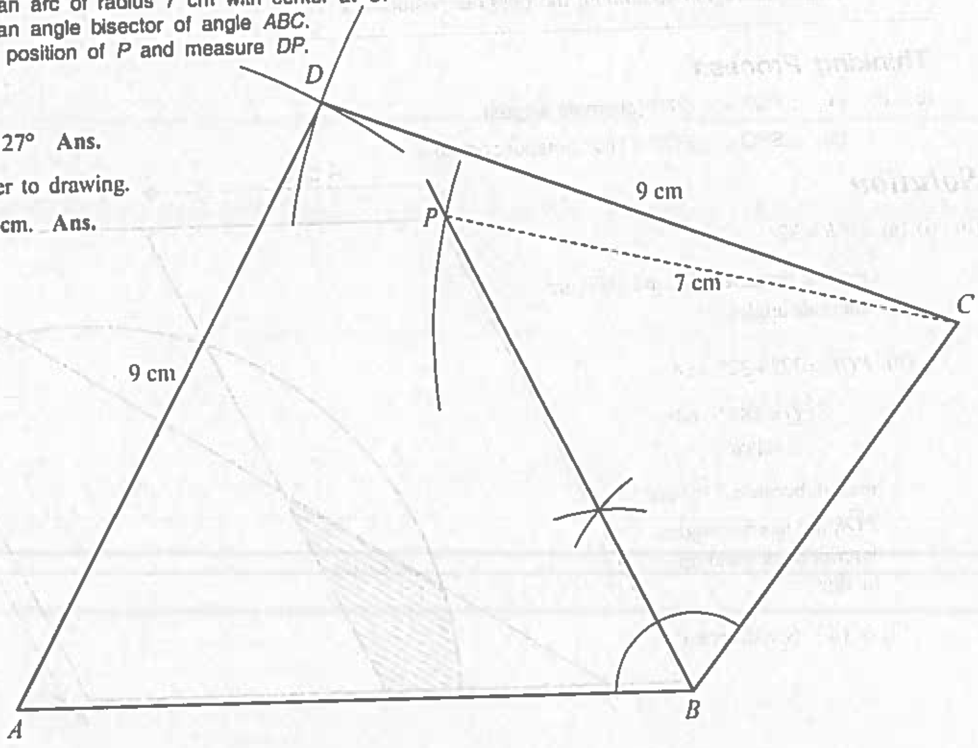
(d) Locate the position of P and measure DP.

Solution

(a) $\angle ABC = 127°$ **Ans.**

(b) & (c) Refer to drawing.

(d) $DP = 2.2$ cm. **Ans.**

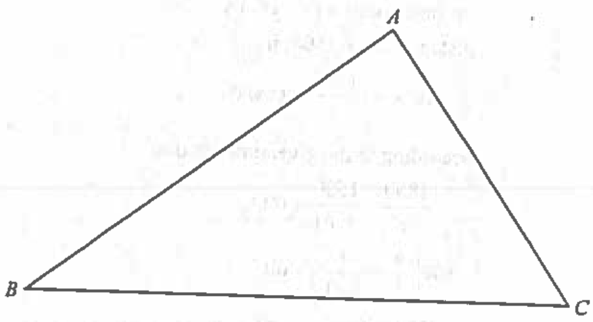


9 *(J2013/P1/Q12)*

The diagram below shows triangle ABC.

(a) Construct the perpendicular bisector of AB. [1]

(b) Shade the region inside the triangle containing points that are closer to A than to B and more than 6 cm from C. [2]

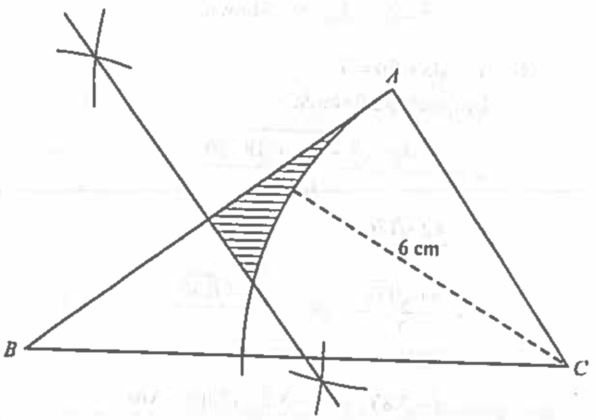


Thinking Process

(b) Contruct an arc of radius 6 cm with centre at C. Shade the region as defined.

Solution

(a) & (b) Refer to figure.



10 *(J2013/P2 Q8)*

The scale diagram shows the positions, A and B, of two buoys.

B is due South of A and $AB = 1500$ m.

(a) Write down the scale of the diagram. [1]

(b) A third buoy is positioned at C which is due East of B and 1800 m from A.

Mark the position of C on the diagram. [2]

(c) Calculate the actual distance BC.

Give your answer correct to the nearest metre. [2]

(d) A boat travels from C to A at an average speed of x m/s.

A second boat travels from B to A at an average speed 1 m/s faster than the first boat. It takes the first boat 1 **minute** longer to reach A than the second boat.

Write down an equation in x and show that it simplifies to $x^2 - 4x - 30 = 0$. [3]

(e) Solve $x^2 - 4x - 30 = 0$, giving each answer correct to two decimal places. [3]

(f) How long did it take the first boat to reach A? Give your answer in seconds. [1]

Thinking Process

(a) To write the scale ✐ measure the length AB in cm.

(b) Using the scale, convert 1800 m into cm. Draw an arc with centre at A towards east of B. Draw a line from B towards east of B. The two lines meet at C.

(c) To calculate BC ✐ apply Pythagoras theorem.

(d) Use, time $= \frac{\text{distance}}{\text{speed}}$. Make two expressions for the time taken by the two boats.
To form an equation ✐ note that the difference of the two times taken is equal to 60 seconds.

(e) Solve by quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(f) Substitute the value of x found into the expression of time taken by the first boat.

Solution

(a) Length of AB in the scale drawing = 10 cm

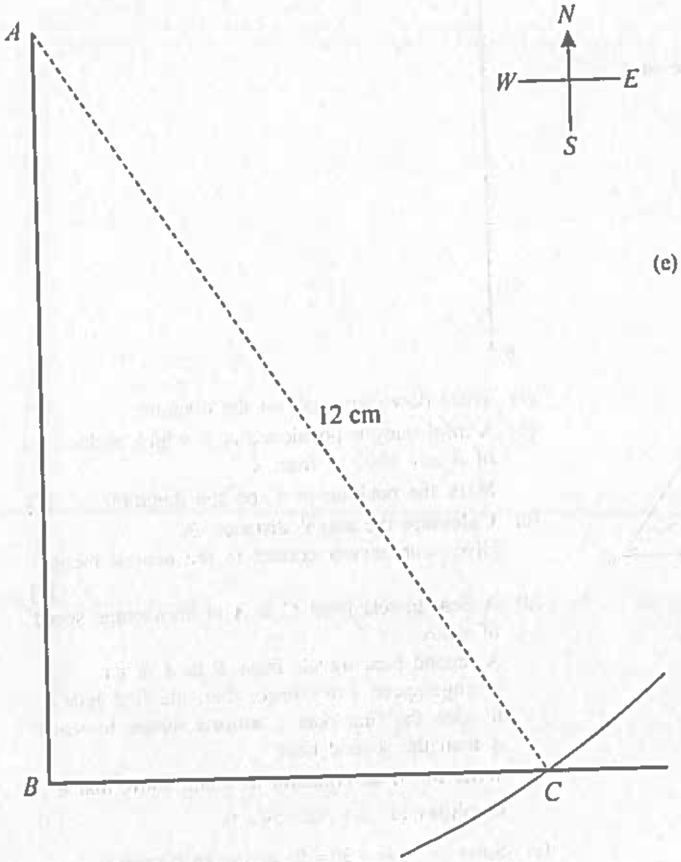
Actual length of $AB = 1500$ m

10 cm —— 1500 m

1 cm —— 150 m

∴ required scale is: 1 cm to 150 m **Ans.**

(b) Refer to drawing.



(c) Applying Pythagoras theorem on ΔABC,

$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{(1800)^2 - (1500)^2}$$
$$= \sqrt{990000}$$
$$= 994.99$$
$$\approx 995 \text{ m (3sf)}$$ **Ans.**

(d) From C to A.

average speed $= x$ m/s

distance $AC = 1800$ m

$\therefore$ time $= \frac{\text{distance}}{\text{speed}} = (\frac{1800}{x})$ seconds.

From B to A.

average speed $= (x+1)$ m/s

distance $AB = 1500$ m

$\therefore$ time $= (\frac{1500}{x+1})$ seconds

according to the given information

$$\frac{1800}{x} - \frac{1500}{x+1} = 60$$
$$300\left(\frac{6}{x} - \frac{5}{x+1}\right) = 60$$
$$\frac{6(x+1) - 5x}{x(x+1)} = \frac{60}{300}$$
$$\frac{6x + 6 - 5x}{x^2 + x} = \frac{1}{5}$$
$$\frac{x+6}{x^2+x} = \frac{1}{5}$$
$$5x + 30 = x^2 + x$$
$$x^2 - 4x - 30 = 0$$ **Shown.**

(e) $x^2 - 4x - 30 = 0$

by quadratic formula,

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-30)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{136}}{2}$$
$$= \frac{4 + \sqrt{136}}{2} \quad \text{or} \quad = \frac{4 - \sqrt{136}}{2}$$
$$= 7.83 \quad \text{or} \quad = -3.83$$

$\therefore$ $x = 7.83$ or -3.83 (2dp) **Ans.**

(f) From (e), average speed of boat is.

$x = 7.83$ m/s.

$\therefore$ time taken by first boat to

reach $A = \frac{1800}{7.83}$

$= 229.885$

≈ 230 seconds **Ans.**

11 *(N2013/P1/Q24)*

The diagram at the bottom of the page shows triangle *ABC*.

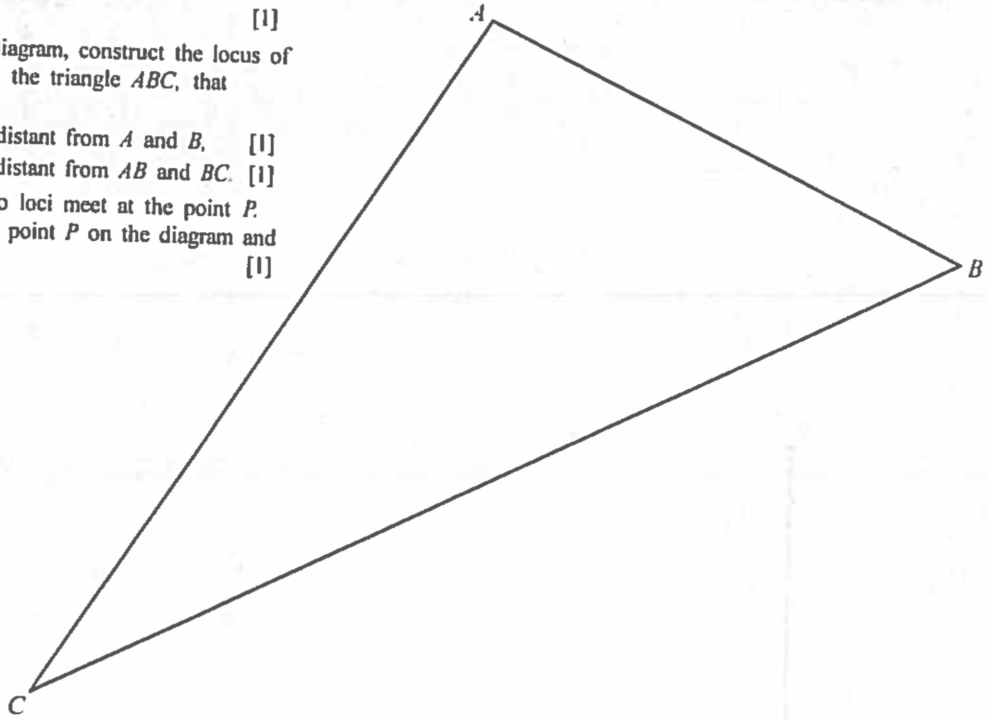
(a) Measure [1]

(b) On the diagram, construct the locus of points, inside the triangle *ABC*, that are

(i) equidistant from *A* and *B*, [1]

(ii) equidistant from *AB* and *BC*. [1]

(c) These two loci meet at the point *P*. Label the point *P* on the diagram and measure *CP*. [1]



Thinking Process

(a) Measure the angle by using a protractor.

(b) (i) Construct a perpendicular bisector of *AB*.

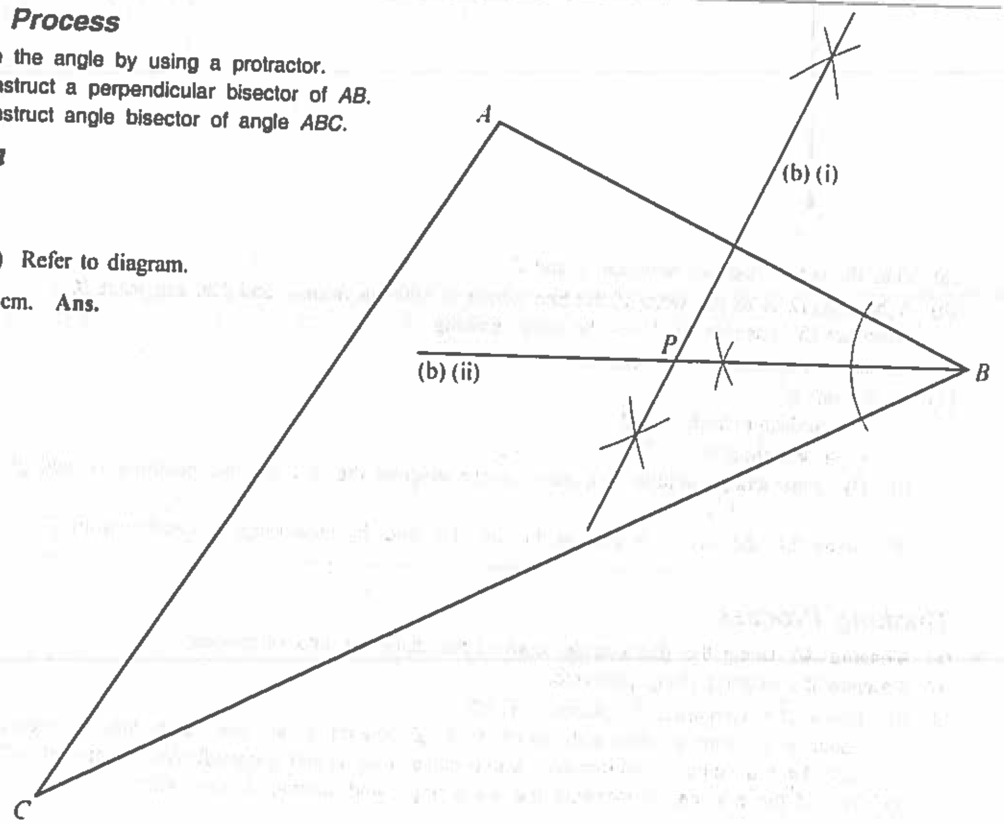
(ii) Construct angle bisector of angle *ABC*.

Solution

(a)

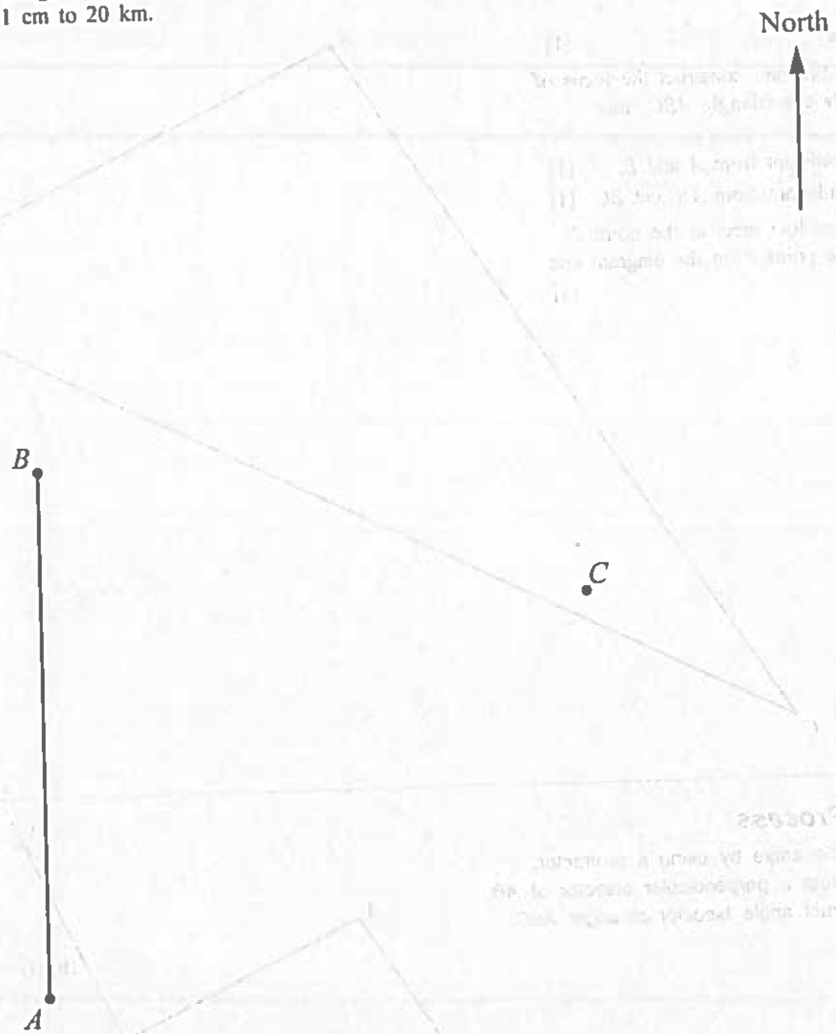
(b) (i) & (ii) Refer to diagram.

(c) $CP = 10$ cm. **Ans.**



12 *(J2014/P2/Q1)*

The scale drawing shows three airfields, *A*, *B* and *C*, with *B* due north of *A*.
The scale is 1 cm to 20 km.



(a) Find the actual distance between *A* and *B*. [1]

(b) A beacon, *D*, is to the west of the line *AB*. It is 100 km from *A* and 120 km from *B*.
Construct the position of *D* on the scale drawing. [2]

(c) Measure the bearing of *C* from *B*. [1]

(d) An aircraft is
- equidistant from *A* and *C*,
- 90 km from *B*.

(i) By constructing suitable loci, mark on the diagram the two possible positions, *P* and *Q*, of the aircraft. [3]

(ii) Given that the aircraft is east of the line *AB*, find, by measuring, its bearing from *C*. [1]

Thinking Process

(a) Measure *AB*. Using the given scale, convert the distances into kilometres.

(c) Measure the bearing using protractor.

(d) (i) Locus of I: Construct ⊥ bisector of *AC*.
Locus of II: Draw a circle with centre at *B*. ∴ convert 90 km into cm to find the radius of the circle.
Look for the points of intersection where circle from *B* and perpendicular bisector of *AC* meet.

(ii) To find the bearing ∴ measure the acute angle and subtract it from 360°.

Solution

(a) By measurement. $AB = 7$ cm

Given scale is : 1 cm —— 20 km

7 cm —— $20 \times 7 = 140$ km

∴ actual distance between A and $B = 140$ km. **Ans.**

(b) Refer to drawing.

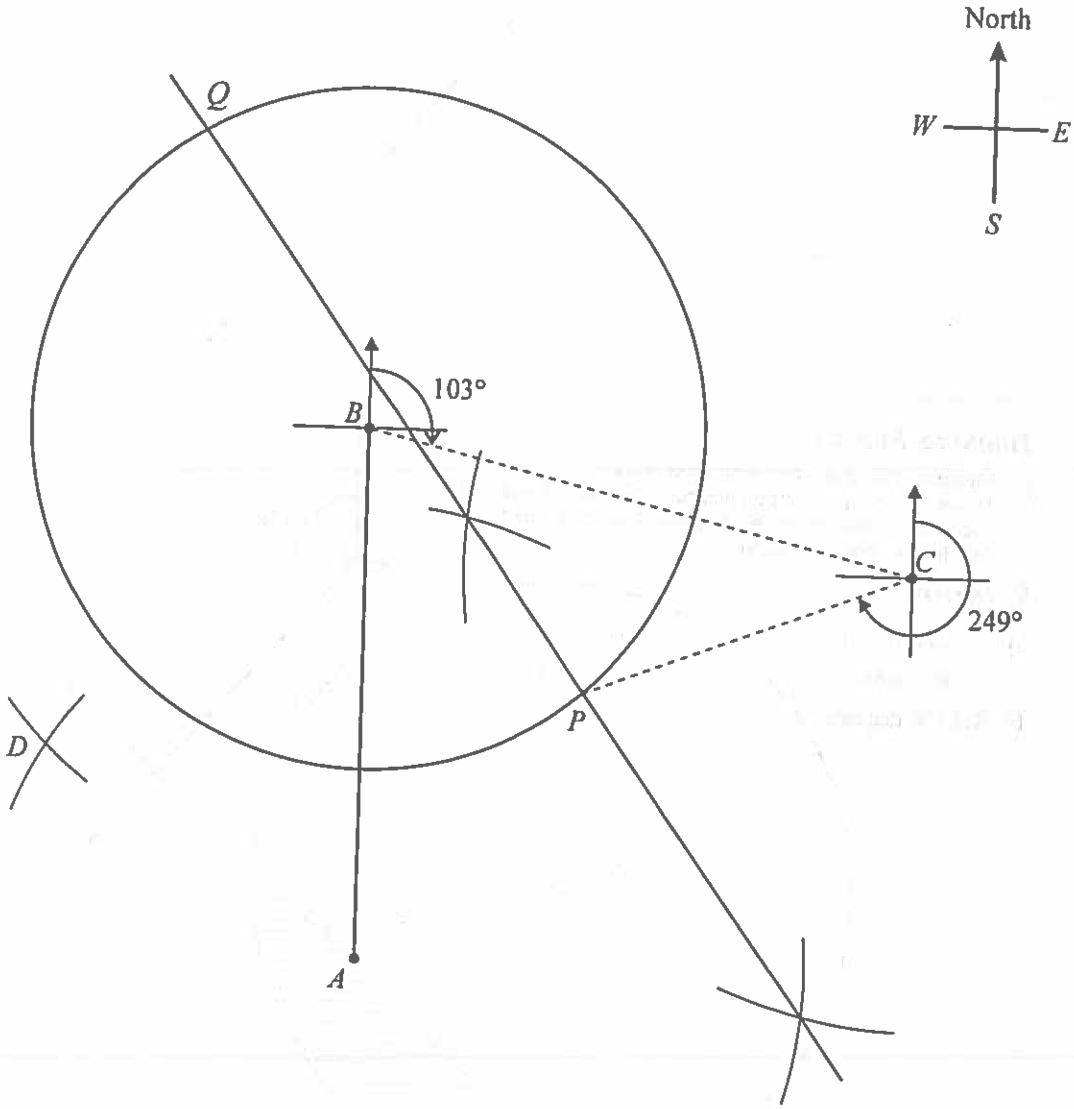
(c) Bearing of C from $B = 103°$ **Ans.**

(d) (i) Refer to drawing.

(ii) From drawing, the possible position of aircraft due east of AB is at point P.

∴ Bearing of P from C

$= 249°$ **Ans.**



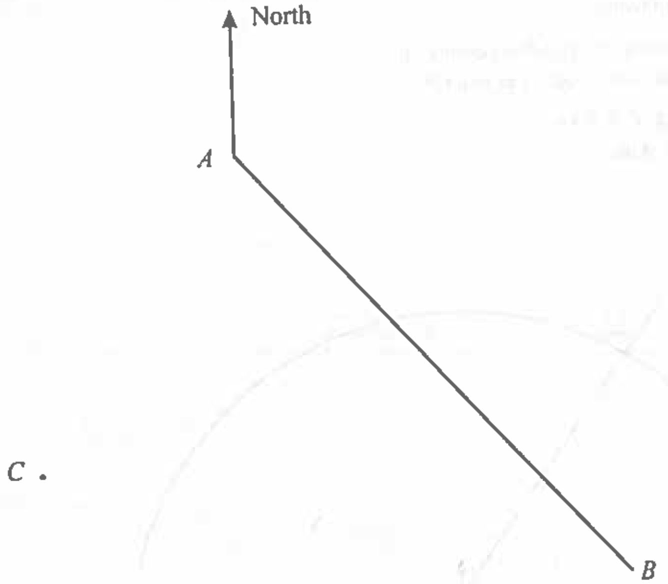
13 (N2014/P1/Q24)

The diagram shows the positions, on a map, of three boats A, B and C.
The map has a scale of 1 cm to 1 km.

(a) Find the bearing of B from A. [1]

(b) A fourth boat, D, is
- closer to B than to A,
- less than 4 km from C.

By drawing appropriate loci find, and shade, the region in which D is situated. [3]



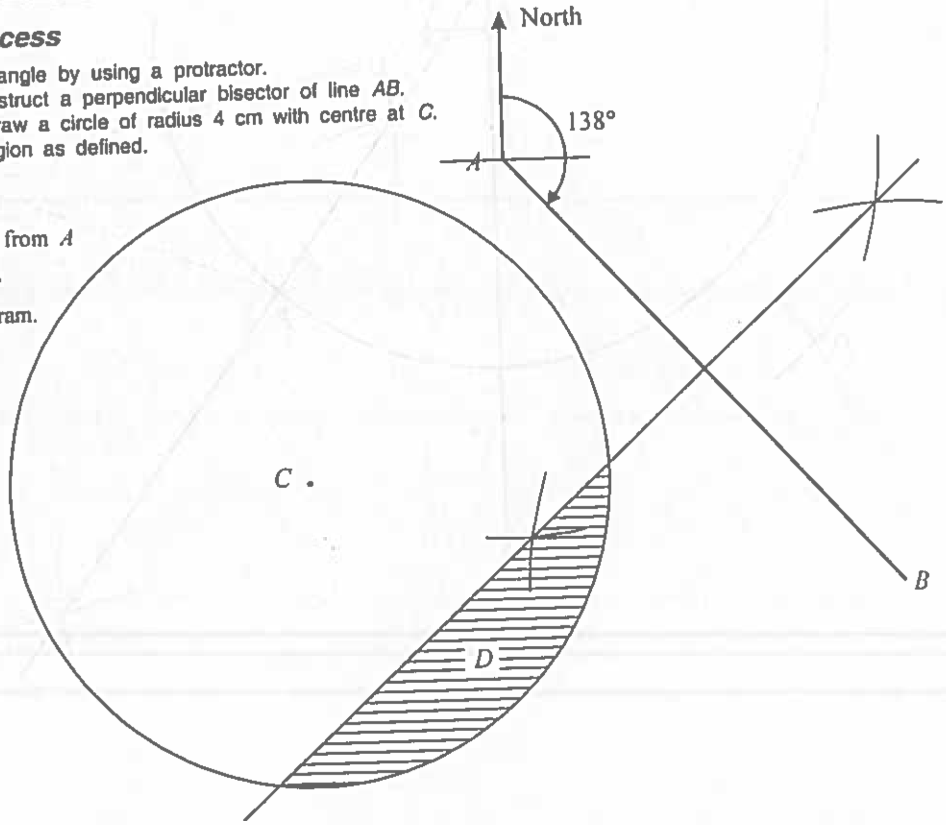
Thinking Process

(a) Measure the angle by using a protractor.
(b) Locus 1: Construct a perpendicular bisector of line AB.
Locus 2: Draw a circle of radius 4 cm with centre at C.
Shade the region as defined.

Solution

(a) Bearing of B from A
$= 138°$ **Ans.**

(b) Refer to diagram.



14 *(N2014/P2/Q6)*

ABCD is a field in the shape of a trapezium.

$A\hat{B}C = 56°$, $B\hat{A}D = 104°$ and the distance between the parallel sides of the field is 90 m.

(a) Using a scale of 1 cm to 20 m, draw a plan of the field.
AB has been drawn for you.

[4]

(b) Find the actual distance *CD*.

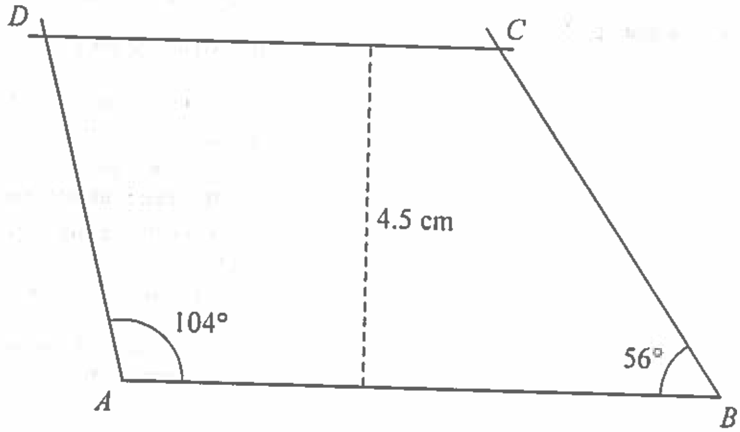
[2]

Thinking Process

(a) Draw the given angles using a protractor. Construct a parallel line 4.5 cm from *AB*.
(b) Measure *CD* on the diagram. Then using the given scale, change *CD* into meters.

Solution

(a)



(b) From drawing $CD = 6.1$ cm

$\therefore$ actual distance $CD = 6.1 \times 20 = 122$ m **Ans.**

15 *(J2015/P1/Q14)*

In triangle ABC, $AB = 5$ cm and $AC = 6$ cm.

(a) Construct triangle ABC. [2]
Line BC is drawn for you.

(b) Measure $B\hat{A}C$ in your triangle. [1]

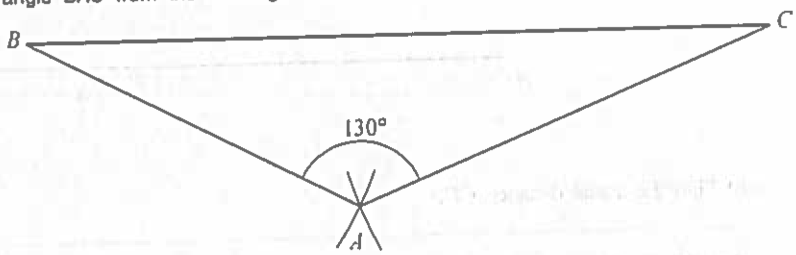
Thinking Process

(a) Using compass and ruler, draw the triangle according to given dimensions.
(b) Using a protractor, Measure angle *BAC* from the drawing.

Solution

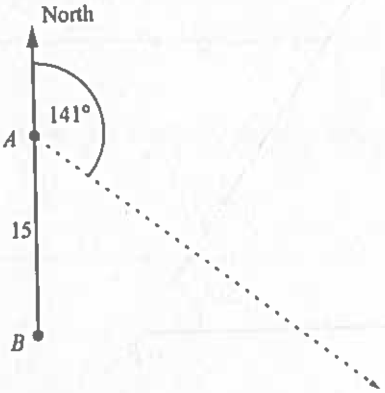
(a) Refer to drawing.

(b) From the drawing,
$B\hat{A}C = 130°$ Ans.



16 *(J2015/P2/Q8)*

Two ports, A and B, are 15 km apart and B is due south of A.
A boat sails from A on a bearing of 141°.



(a) State the bearing of A from the boat. [1]

(b) Calculate the shortest distance between the boat and B. [2]

(c) When the boat is due east of B, calculate its distance from A. [2]

(d) The scale drawing, drawn to a scale of 1 cm to 2 km, shows A, B and a third port, C. (see next page).

(i) When the boat has travelled 24 km, it stops at the point X.
Mark and label X on the diagram. [2]

(ii) A second boat is located

I less than 12 km from A

II nearer to BC than to BA.

Shade the region in which this second boat must lie. [3]

(iii) The point Y is the position of the second boat when it is as far as possible from X.
Mark and label Y on the diagram and hence find the maximum possible distance between the two boats. [2]

Thinking Process

(a) Draw north on the dotted line to find the bearing.

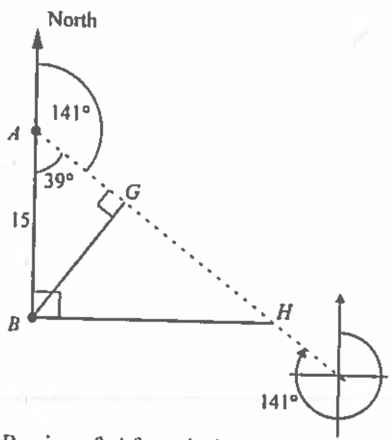
(b) Shortest distance is the perpendicular distance from boat to port B.

(c) Draw a horizontal line from boat to B. Use the triangle formed to find the distance.

(d) (i) To mark X ⇒ Construct the locus of boat. Convert 24 km into cm.

(ii) I — An arc of radius 6 cm with centre at A.
II — An angle bisector of angle ABC.
Shade the region as defined.

Solution



(a) Bearing of A from the boat

$= 180° + 141°$

$= 321°$ **Ans.**

(b) Consider ΔABG.

$\angle BAG = 180° - 141°$

$= 39°$

$\sin 39° = \dfrac{BG}{15}$

$BG = 15\sin 39°$

$= 9.439 \approx 9.44$

$\therefore$ shortest distance from the boat to B

$= 9.44$ km **Ans.**

(c) Consider ΔABH.

$\cos 39° = \dfrac{15}{AH}$

$AH = \dfrac{15}{\cos 39°}$

$= 19.3$

$\therefore$ distance of boat from $A = 19.3$ km **Ans.**

(d) (i) 2 km —— 1 cm

24 km —— $\frac{1}{2} \times 24 = 12$ cm

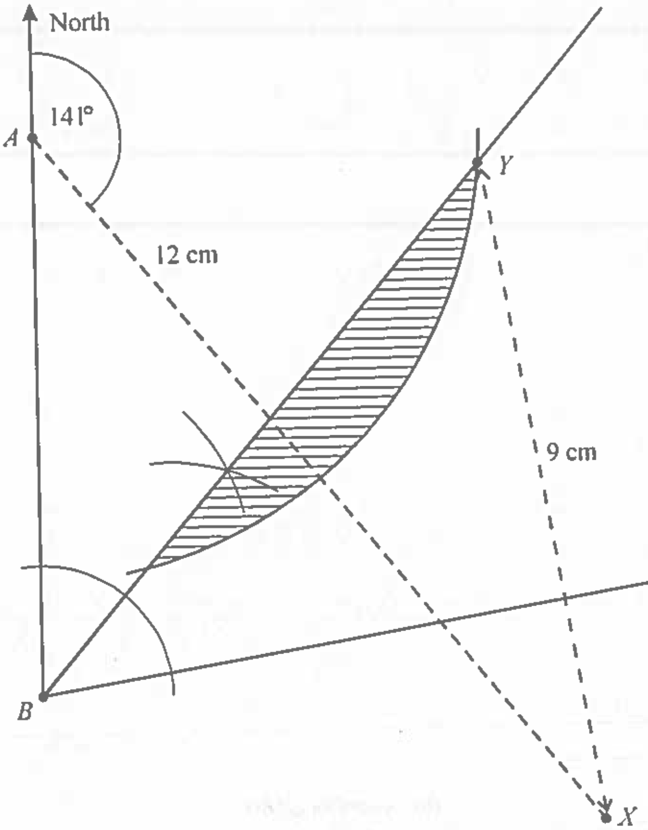
see figure for point X.

(ii) Refer to figure

(iii) Refer to figure for point Y.

From figure, length of $XY = 9$ cm.

$\therefore$ Actual distance between the two boats $= 9 \times 2 = 18$ km **Ans.**



17 (N2015/P1/Q21)

The diagram shows the positions of three ships A, B and C.
It is drawn to a scale of 1 cm to 20 km.

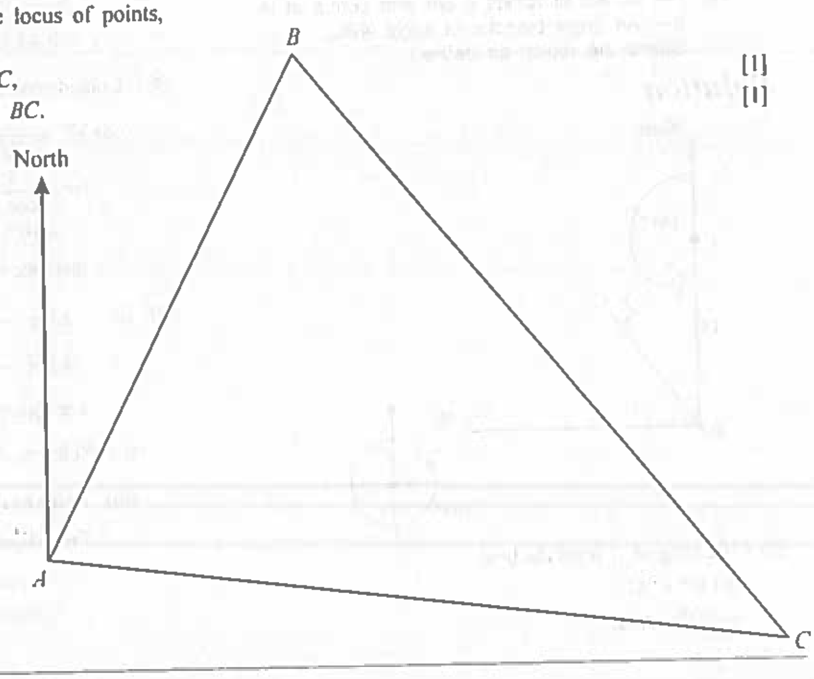
(a) Find, by measurement, the bearing of C from A. [1]

(b) On the diagram construct the locus of points, inside triangle ABC, that are
  (i) equidistant from B and C, [1]
  (ii) equidistant from AB and BC. [1]

(c) A ship D is
- equidistant from B and C, and
- equidistant from AB and BC.

Label the position of D on the diagram and find the actual distance of D from A. [1]

Scale: 1 cm to 20 km



Thinking Process

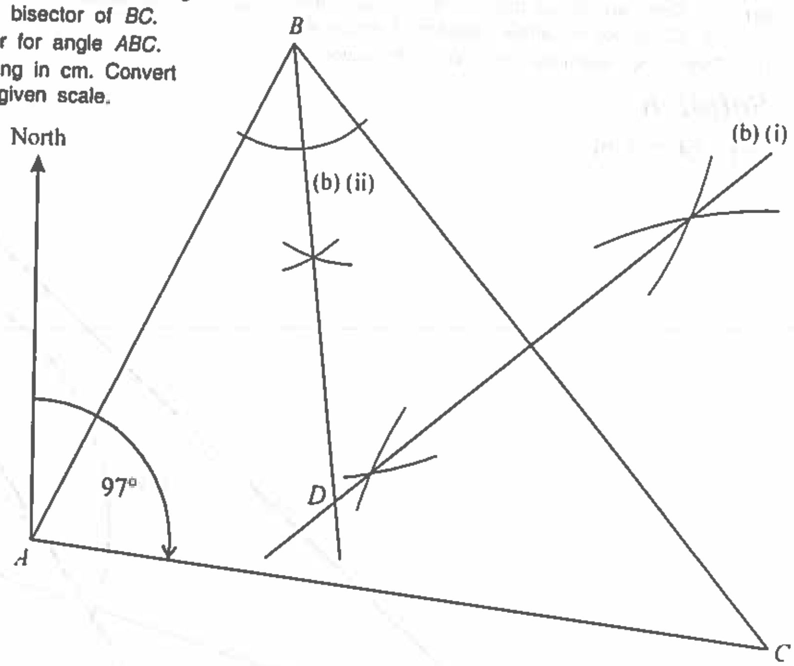
(a) Use protractor to measure the required bearing.
(b) (i) Construct perpendicular bisector of *BC*.
(ii) Construct angle bisector for angle *ABC*.
(c) Measure *DA* from the drawing in cm. Convert it into km according to the given scale.

Solution

(a) Bearing of C from A
$= 097°$ **Ans.**

(b) (i), (ii) Refer to drawing.

(c) From drawing $AD = 4$ cm
$\therefore$ actual distance from
D to $A = 4 \times 20$
$= 80$ km **Ans.**

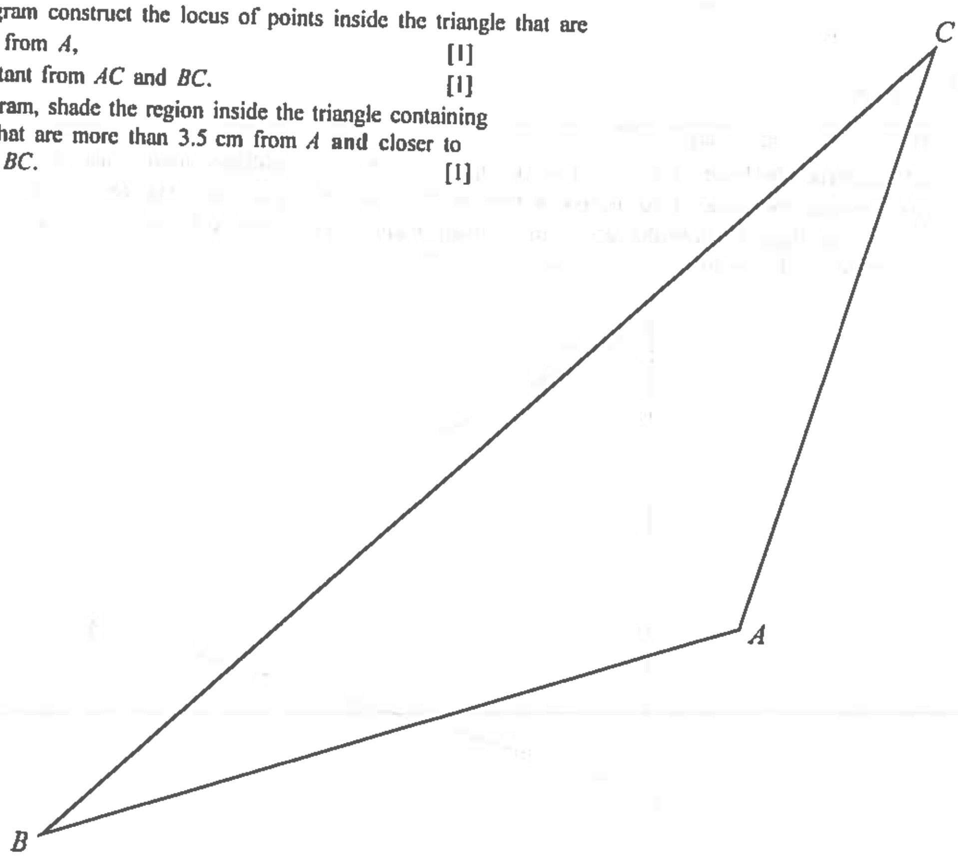


18 *(J2016/P1/Q12)*

The diagram below shows triangle *ABC*.

(a) On the diagram construct the locus of points inside the triangle that are
(i) 3.5 cm from *A*, [1]
(ii) equidistant from *AC* and *BC*. [1]

(b) On the diagram, shade the region inside the triangle containing the points that are more than 3.5 cm from *A* and closer to *AC* than to *BC*. [1]

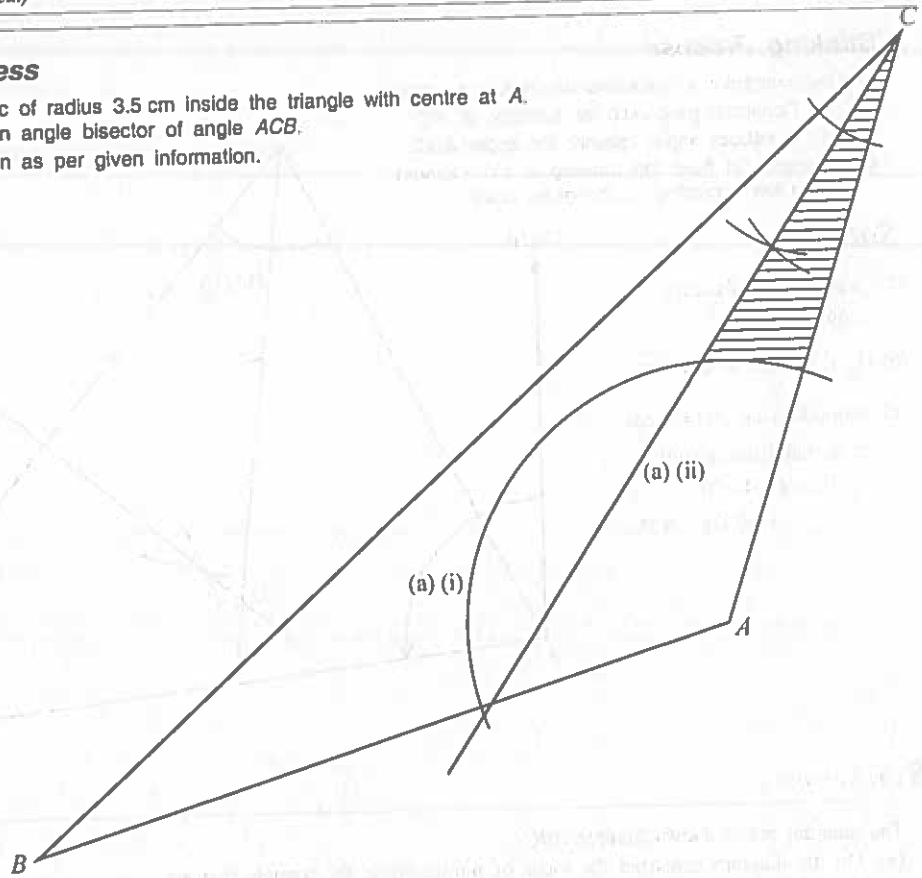


Thinking Process

(a) (i) Draw an arc of radius 3.5 cm inside the triangle with centre at A.
(ii) Construct an angle bisector of angle ACB.
(b) Shade the region as per given information.

Solution

(a) (i), (ii) and (b)



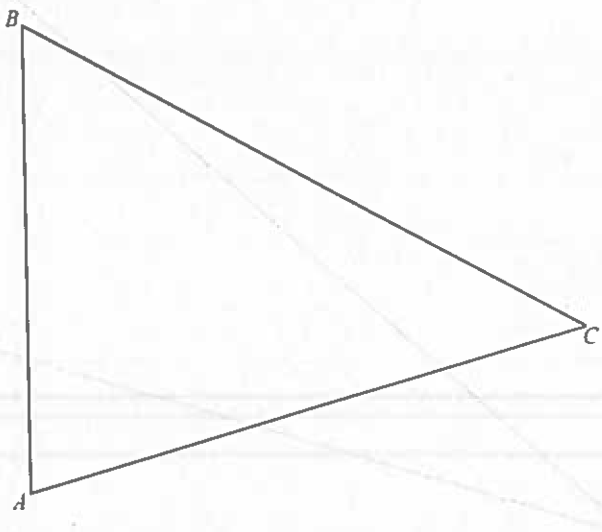
19 *(N2016/P1/Q22)*

The diagram shows triangle ABC.

(a) Construct the locus of points, **inside** triangle ABC, that are equidistant from A and B. [1]

(b) Construct the locus of points, **inside** triangle ABC, that are equidistant from AB and BC. [1]

(c) On the diagram, shade the region **inside** triangle ABC which contains the points that are nearer to A than to B and nearer to BC than AB. [1]

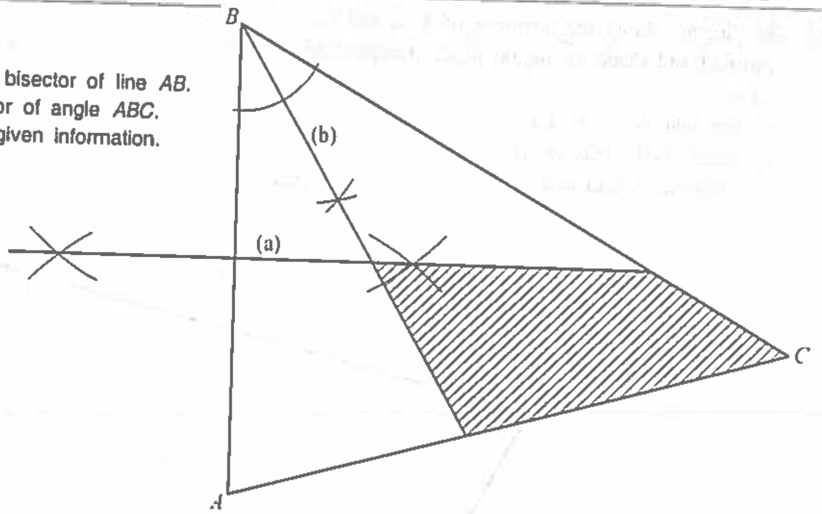


Thinking Process

(a) Construct a perpendicular bisector of line AB.
(b) Construct an angle bisector of angle ABC.
(c) Shade the region as per given information.

Solution

(a), (b) & (c)



20 *(J2017/P2/Q10)*

The diagram shows the position of point A.

Point B is 8 cm from A on a bearing of 062°. Point C is 6.5 cm from A on a bearing of 194°.

(a) (i) Find and label B and C. [3]

Point D is the point on BC that is the shortest distance from A.

(ii) Find and label D. [1]

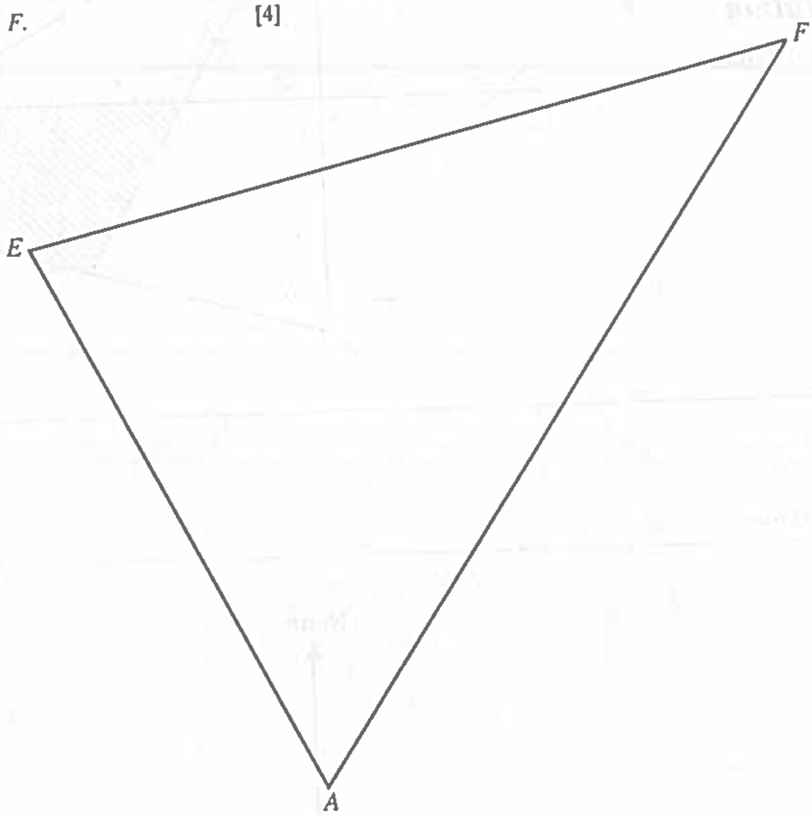
(iii) Measure AD. [1]

(iv) By taking measurements, find the ratio $CD : DB$.
Give your answer in the form $1 : n$. [2]

(v) The area of triangle ADB is w cm².
Giving your answer in terms of w, find the area of triangle ADC. [1]

(b) The diagram shows the positions of A, E and F.
Construct and shade the region inside triangle AEF that is

- less than 6 cm from E
- nearer to AF than to AE
- nearer to A than to F. [4]



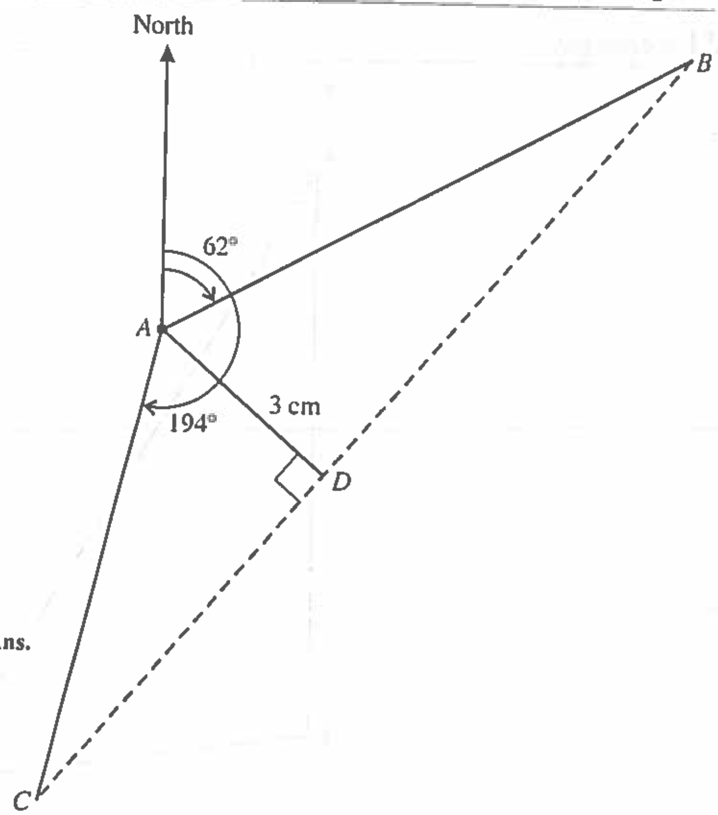
Thinking Process

(a) (i) To label B ✎ draw a line 8 cm long and 62° clockwise at A.
To label C ✎ draw a line 6.5 cm long and 194° clockwise at A.
(ii) Note that shortest distance is the perpendicular distance from A to BC.
(iv) To find the ratio ✎ measure CD and DB with a ruler.
(v) To find the area of triangle ✎ note that both triangles share common height.

(b) Locus 1: Draw an arc of radius 6 cm with centre at E.
Locus 2: Construct angle bisector of angle A.
Locus 3: Construct perpendicular bisector of line AF.
Shade the region as per given instructions.

Solution

(a) (i) & (ii) Refer to diagram.



(iii) $AD = 3$ cm. **Ans.**

(iv) $CD = 5.8$ cm, $DB = 7.4$ cm

$CD : DB$
$= 5.8 : 7.4$
$= 1 : \frac{7.4}{5.8}$
$= 1 : 1.28$ **Ans.**

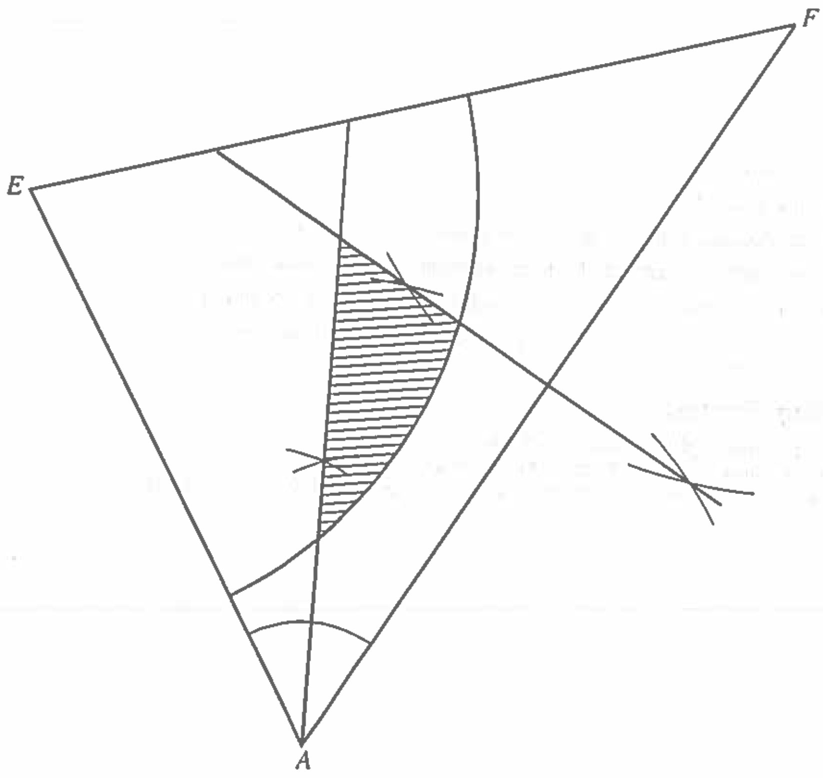
(v) $\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ADB} = \frac{\frac{1}{2} \times CD \times AD}{\frac{1}{2} \times DB \times AD}$

$\frac{\text{Area of } \Delta ADC}{w} = \frac{CD}{DB}$

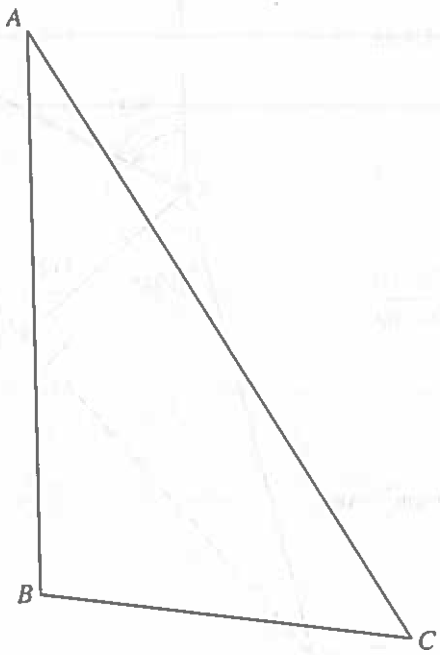
$\text{Area of } \Delta ADC = \frac{1}{1.28} \times w$

$= 0.781w \text{ cm}^2$ **Ans.**

(b)



21 *(N2017/P1/Q23)*



The diagram shows the triangle ABC.

(a) Measure angle ABC. [1]

(b) On the diagram, construct the perpendicular bisector of AB. [1]

(c) On the diagram, construct the locus of points that are 5 cm from C. [1]

(d) The points P and Q lie on the perpendicular bisector of AB and are 5 cm from C.
Mark and label the points P and Q on the diagram and measure PQ. [1]

Thinking Process

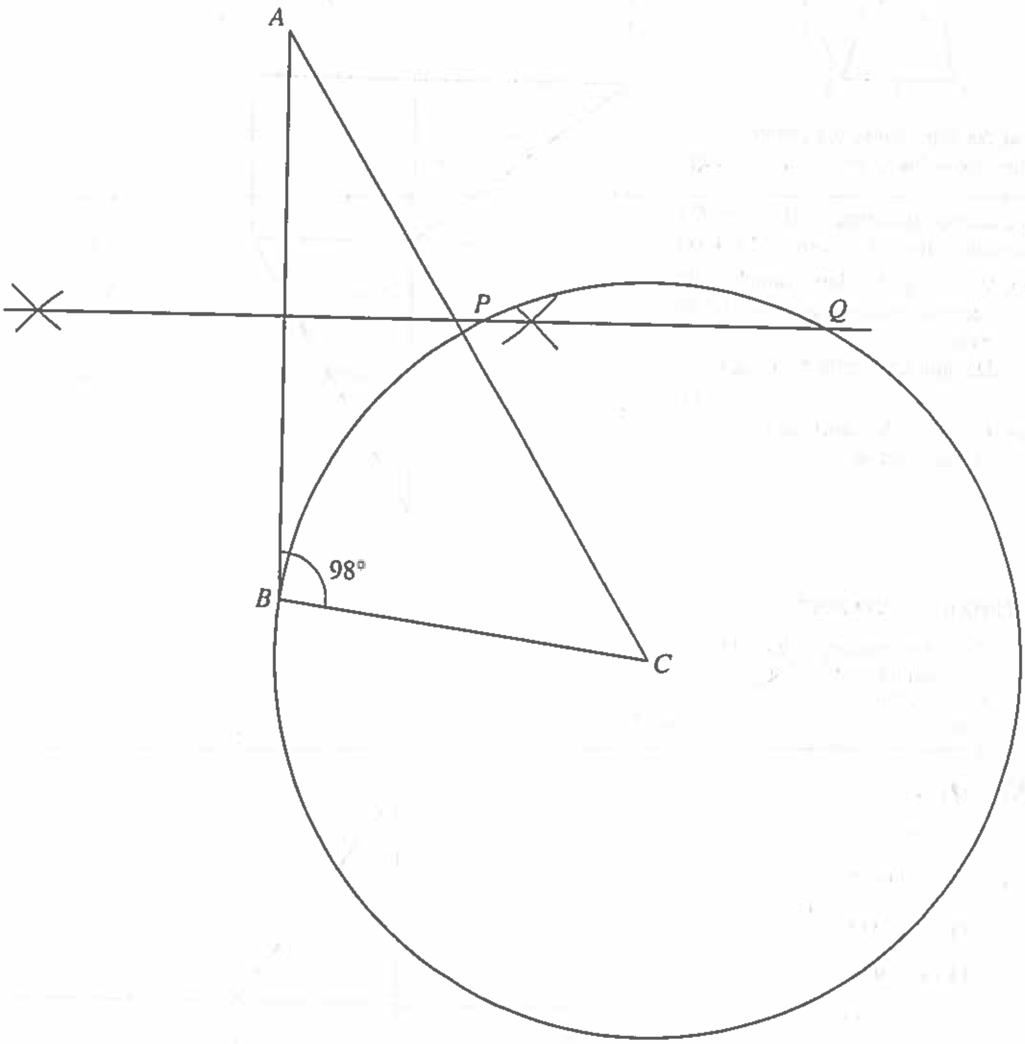
(a) Measure the angle by using a protractor.
(c) Draw a circle of radius 5 cm with centre at C.
(d) Look for the points of intersection where the circle and the perpendicular bisector meet.

Solution

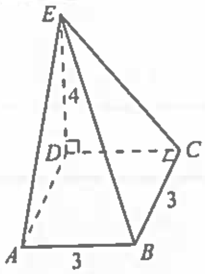
(a) Angle $ABC = 98°$ **Ans.**

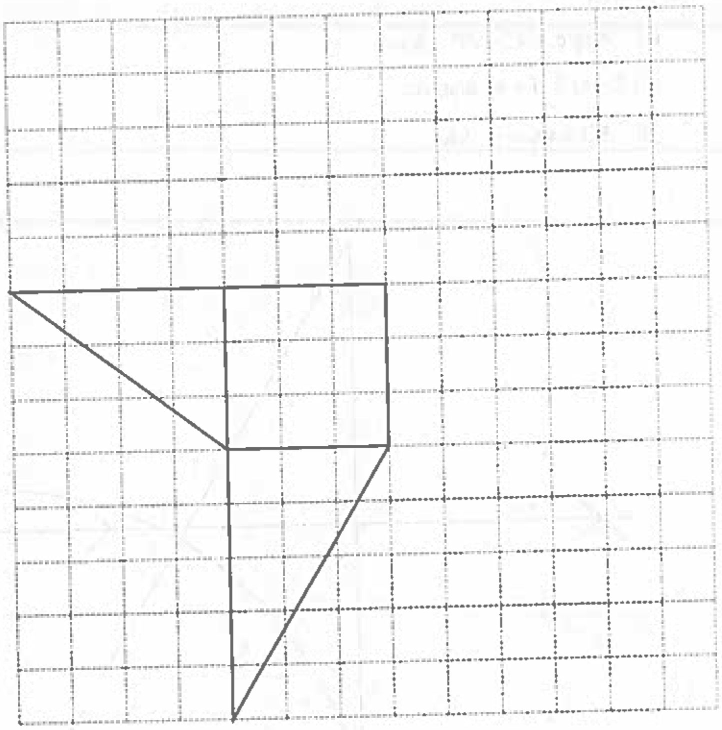
(b) & (c) Refer to diagram.

(d) $PQ = 4.6$ cm **Ans.**



22 *(J2018/P1/Q17)*





The diagram shows a pyramid.
The square base, $ABCD$, has an edge of 3 cm.
The base is horizontal, and vertex E is vertically above D, where $ED = 4$ cm.

(a) On the grid below, complete the accurate drawing of a net of the pyramid.
Do not draw outside the grid. [2]

(b) Calculate the total surface area of the pyramid. [2]

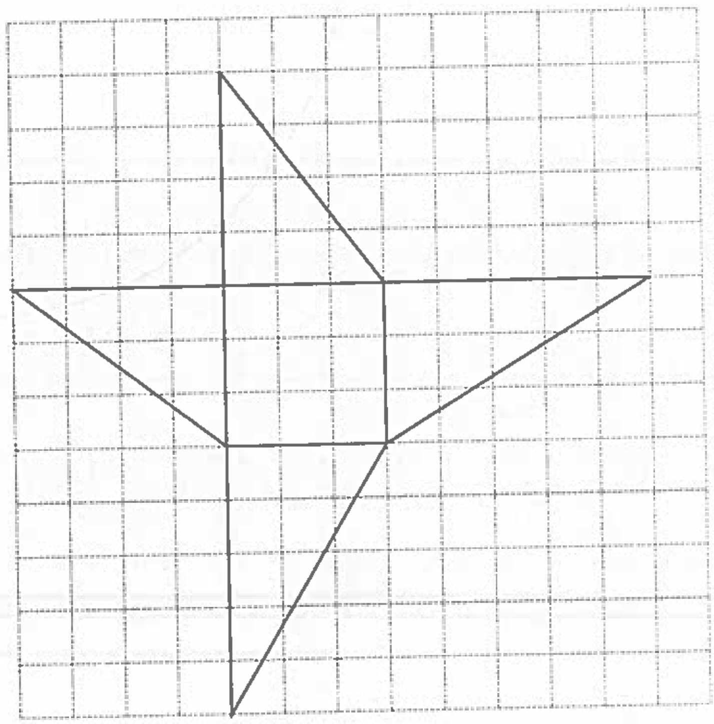
Thinking Process

(a) Draw the missing 2 triangles by referring to the diagram of the pyramid.

(b) Total surface area = Area of square + area of 4 triangles

Solution

(a) Refer to grid.



(b) Total surface area

$= 2\left(\frac{1}{2} \times 3 \times 4\right) + 2\left(\frac{1}{2} \times 3 \times 5\right) + (3 \times 3)$

$= 12 + 15 + 9$

$= 36 \text{ cm}^2$ **Ans.**

23 *(J2018/P1/Q19)*

(a) On the diagram, construct the perpendicular bisector of AB. [1]

(b) On the diagram, construct the locus of points inside triangle ABC, that are

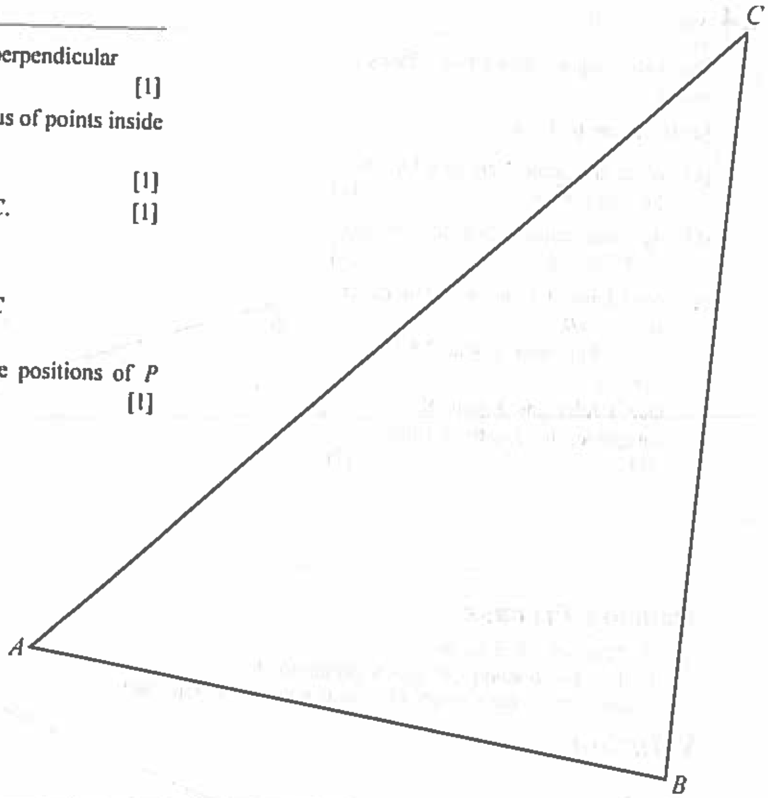
(i) 7 cm from C, [1]

(ii) equidistant from AB and AC. [1]

(c) P is any point which is

equidistant from A and B
and more than 7 cm from C
and nearer to AC than AB.

Find the extremes of the possible positions of P and label them P_1 and P_2. [1]



Thinking Process

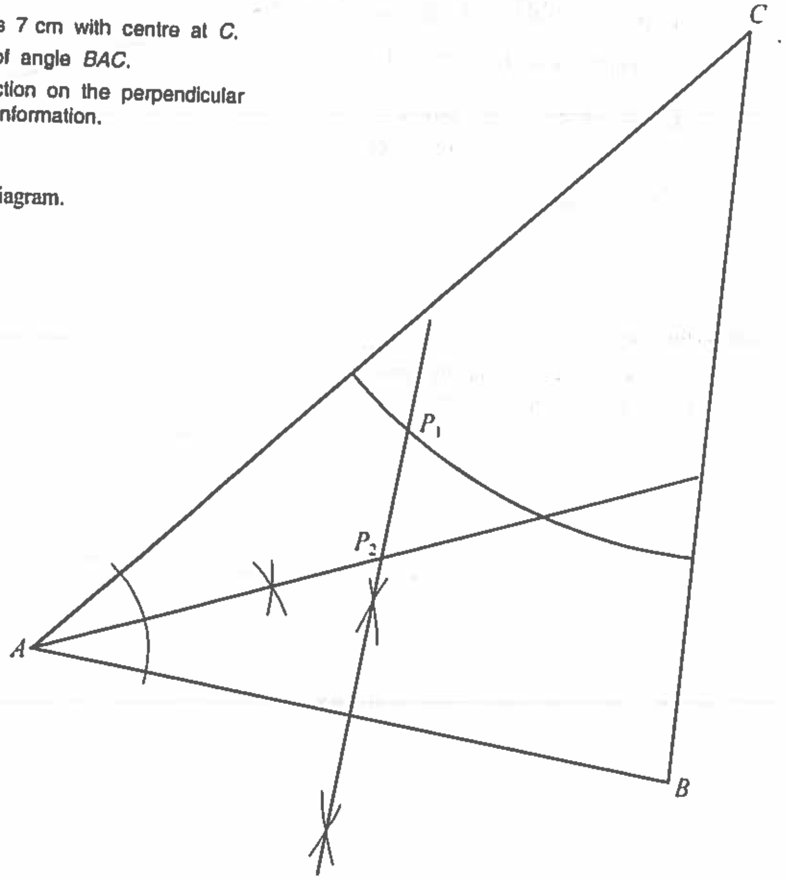
(b) (i) Construct an arc of radius 7 cm with centre at C.

(ii) Construct angle bisector of angle BAC.

(c) Locate the 2 points of intersection on the perpendicular bisector of AB as per given information.

Solution

(a), (b)(i), (b)(ii) & (c) Refer to diagram.



24 (N2018/P1/Q13)

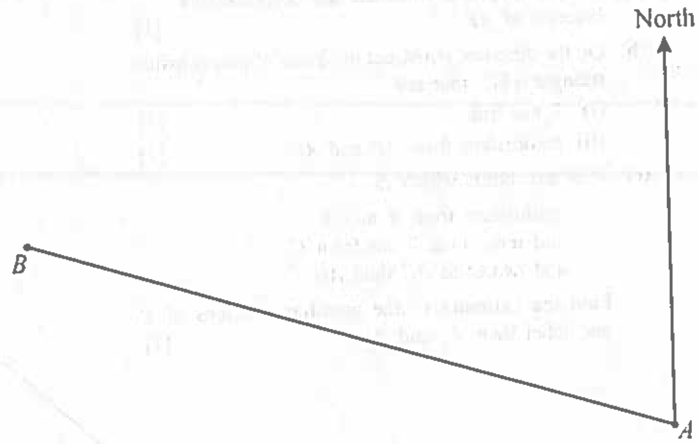
The scale diagram shows two islands at A and B.

Scale: 2 cm to 1 km

(a) Write the scale 2 cm to 1 km in the form $1 : n$. [1]

(b) By measurement, find the bearing of B from A. [1]

(c) An island at C is on the northern side of AB.
It is 3 km from A and 2.5 km from B.
Use a ruler and a pair of compasses to construct triangle ABC. [2]



Thinking Process

(a) Convert 1 km to cm

(b) To find the bearing use a protractor to measure the acute angle at A and subtract it from 360°.

Solution

(a) 2 cm —— 1 km

2 cm —— 100000 cm

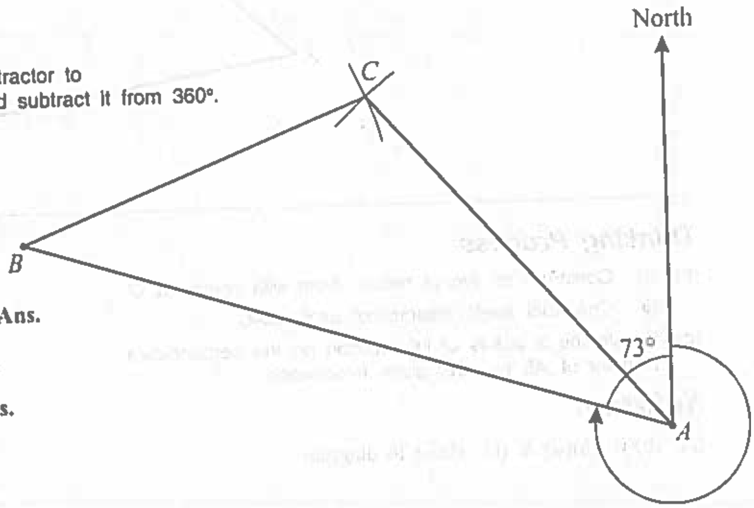
1 cm —— $\frac{100000}{2} = 50000$ cm

$\therefore$ required scale is. $1:50000$ **Ans.**

(b) Bearing of B from $A = 360° - 73°$

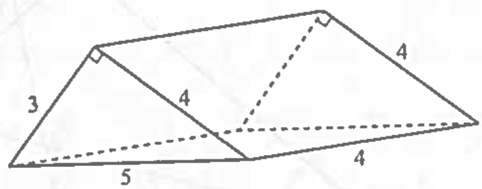
$= 287°$ **Ans.**

(c) Refer to figure.



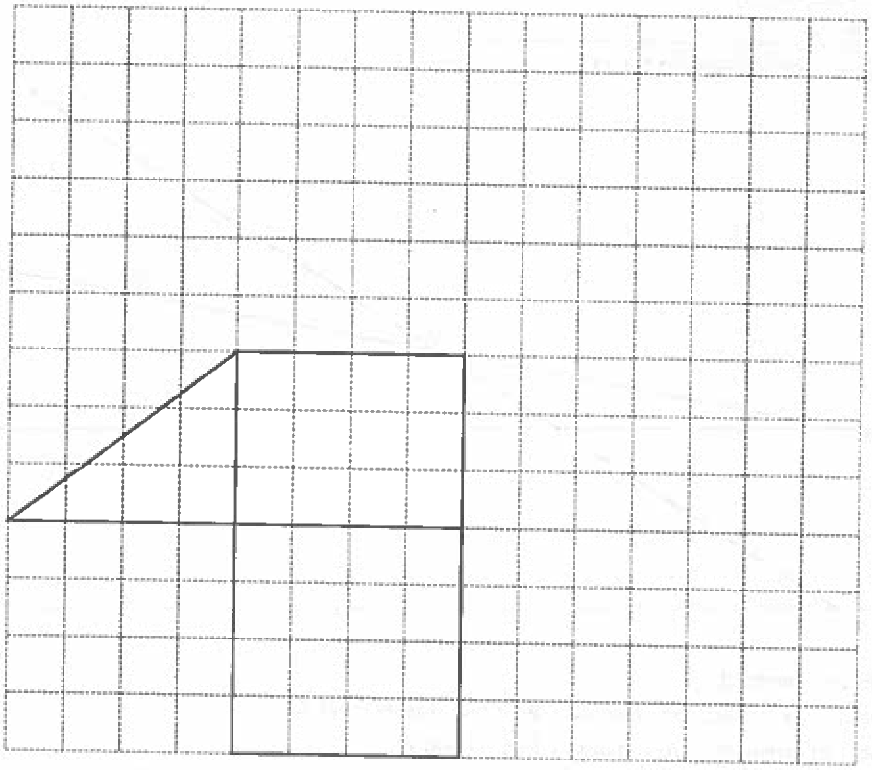
25 (N2018/P1/Q14)

The diagram shows a triangular prism.
The measurements are in centimetres.



(a) On the grid below, complete the accurate drawing of a net of the prism.
Do not draw outside the grid. [2]

(b) Find the total surface area of the prism. [2]



Thinking Process

(a) Draw the remaining 2 sides by referring to the diagram of prism.
(b) Total surface area = sum of the areas of all of the sides.

Solution

(a) Refer to grid

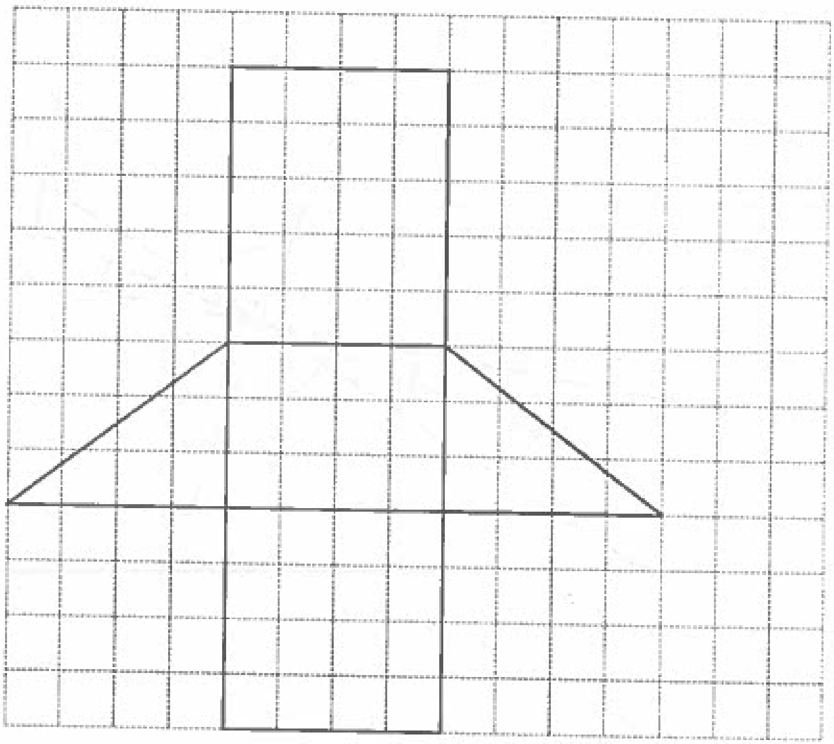
(b) Total surface area

$= 2\left(\frac{1}{2} \times 3 \times 4\right)$

$+ (4 \times 4) + (4 \times 3) + (5 \times 4)$

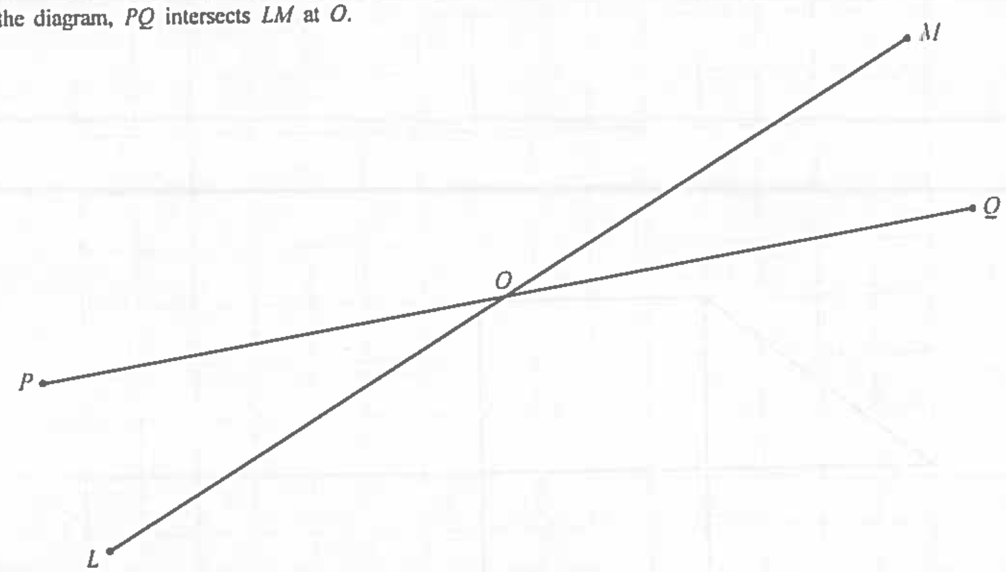
$= 12 + 16 + 12 + 20$

$= 60 \text{ cm}^2$ Ans.



26 *(N2018/P1/Q19)*

In the diagram, *PQ* intersects *LM* at *O*.



On the diagram, construct

(a) the locus of points that are equidistant from the lines *PQ* and *LM*, [2]

(b) the locus of points that are equidistant from *O* and *Q*. [1]

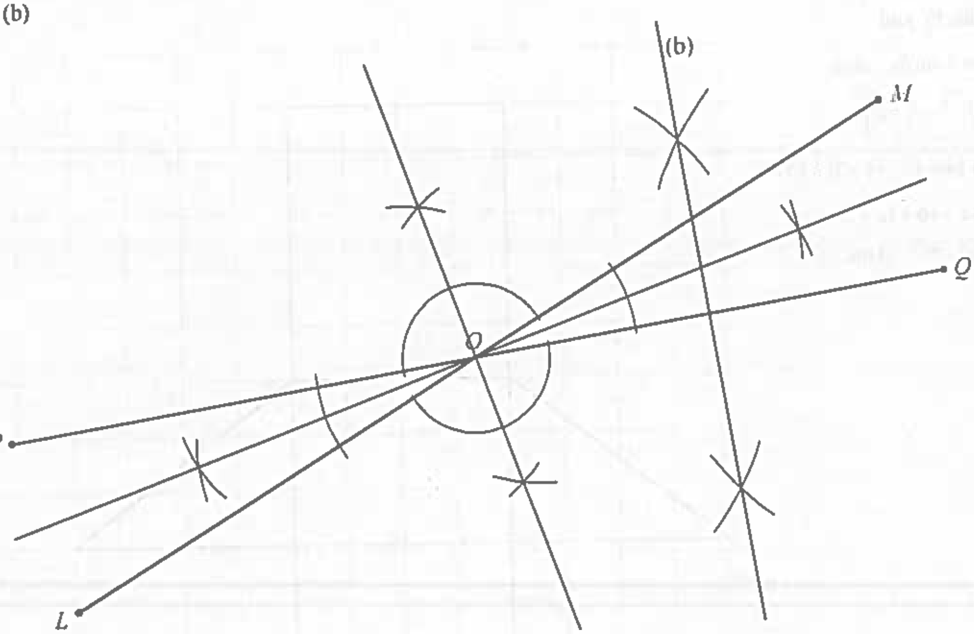
Thinking Process

(a) Construct four angle bisectors at angles *POL*, *POM*, *MOQ* and *LOQ*.

(b) Construct a perpendicular bisector of line *OQ*

Solution

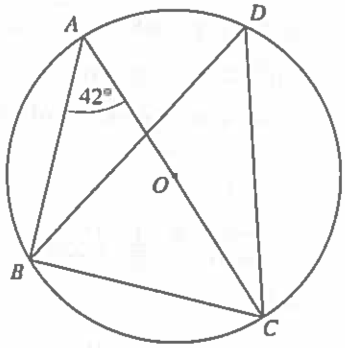
(a) & (b)



Topic 13

Angles and Circle Properties

1 (J2008 P1 Q9)



The diagram shows a circle, centre O, passing through A, B, C and D.
AOC is a straight line and $B\hat{A}C = 42°$.
Find

(a) $B\hat{D}C$. [1]

(b) $A\hat{B}C$. [1]

(c) $A\hat{C}B$. [1]

Thinking Process

(a) Note that $\angle BAC$ and $\angle BDC$ are angles in the same segment.

(b) AC is the diameter, therefore triangle ABC is a right angled triangle with $\angle B = 90°$

(c) Apply sum of angles in a triangle to find $\angle ACB$.

Solution

(a) $B\hat{D}C = B\hat{A}C = 42°$ **Ans.**

(∠s in the same segment)

(b) Since AC is the diameter

$\therefore \quad A\hat{B}C = 90°$ **Ans.**

(c) Consider ΔABC

$A\hat{C}B + A\hat{B}C + B\hat{A}C = 180°$

sum of interior angles of a triangle is 180°

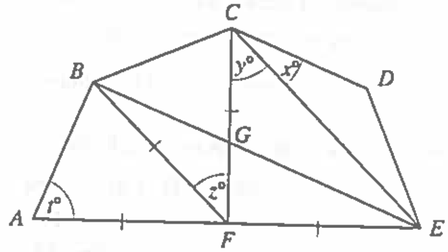
$A\hat{C}B + 90° + 42° = 180°$

$A\hat{C}B = 180° - 90° - 42° = 48°$ **Ans.**

2 (J2008 P2 Q4)

(a) Show that each interior angle of a regular octagon is 135°. [2]

(b)



In the diagram, AB, BC, CD and DE are four adjacent sides of a regular octagon.
$FA = FB = FC = FE$.
CF meets BE at G.

(i) Calculate

(a) x, [1]

(b) y, [1]

(c) z, [1]

(d) t. [1]

(ii) Write down the special name given to the quadrilateral $BCEF$. [1]

(iii) Given that $FC = 10$ cm, calculate CE. [2]

(iv) (a) Show that ΔCGE is similar to ΔFGB. [1]

(b) Find $\dfrac{\text{the area of } \Delta CGE}{\text{the area of } \Delta FGB}$. [1]

Thinking Process

(a) One interior angle $= \dfrac{(n-2)180}{n}$

(b) (i) (a) CDE is an isosceles triangle, therefore $D\hat{C}E = D\hat{E}C$

(b) CF bisects angle BCD.

(c) BFC is an isosceles triangle, therefore $F\hat{B}C = F\hat{C}B$

(d) BFA is an isosceles triangle, therefore $F\hat{A}B = F\hat{B}A$

(ii) Note that BF is parallel to CE.

(iii) CFE is a right angled triangle. Therefore apply pythagoras theorem.

(iv) (a) Prove that all the three angles of the two triangles are equal.

(b) Ratio of area of similar figures = Ratio of squares of corresponding lengths.

Solution

(a) One interior angle $= \frac{(n-2)180}{n}$

number of sides in an octagon, $n = 8$

$\therefore$ one interior angle in an octagon

$= \frac{(8-2)180}{8} = \frac{1080}{8} = 135°$ Shown.

(b) (i) (a) CDE is an isosceles triangle

$\therefore\ C\hat{D}E + D\hat{E}C + E\hat{C}D = 180$

$135 + x + x = 180$

$2x = 45$

$x = 22.5°$ Ans.

(b) CF bisects $B\hat{C}D$

$\Rightarrow\ F\hat{C}D = \frac{1}{2}(B\hat{C}D)$

$y + x = \frac{1}{2}(135)$

$y + 22.5 = 67.5$

$y = 45°$ Ans.

Alternative Solution:

F is the midpoint of AE.

$\therefore$ GF is perpendicular to AE

consider isosceles ΔCFE

$90 + y + y = 180$

$y = 45°$ Ans.

(c) CF bisects $B\hat{C}D$, and ΔBCF is isosceles

$\Rightarrow\ F\hat{B}C = F\hat{C}B = \frac{1}{2}(135) = 67.5$

now, $67.5 + 67.5 + z = 180$

sum of interior angles of a Δ is $180°$

$\therefore\ z = 180 - 67.5 - 67.5$

$= 45°$ Ans.

Alternative Solution:

BF is parallel to CE.

$\therefore\ z = y$ (alternate angles)

or $z = 45°$ Ans.

(d) BF bisects $A\hat{B}C$,

$\Rightarrow\ F\hat{B}A = \frac{1}{2}(135) = 67.5$

as ΔAFB is isosceles,

$\therefore\ t = F\hat{B}A = 67.5°$ Ans.

(ii) As BF is parallel to CE,

$\therefore$ $BCEF$ is a Trapezium Ans.

(iii) CFE is a right angled triangle.

$FC = FE = 10\text{cm}$ (given)

applying pythagoras theorem.

$FC^2 + FE^2 = CE^2$

$(10)^2 + (10)^2 = CE^2$

$CE^2 = 200$

$CE = 14.14$

$\approx 14.1\text{cm}$ (3 sf) Ans.

(iv) (a) $C\hat{G}E = F\hat{G}B$ (vert. opp. angles)

$G\hat{F}B = G\hat{C}E$ (alt. $\angle$s. $BF \parallel EC$)

$G\hat{B}F = G\hat{E}C$ (alt. $\angle$s)

$\therefore$ ΔCGE is similar to ΔFGB Shown.

(b) $BF = CF = 10$ cm (given)

$\frac{\text{area of } \Delta CGE}{\text{area of } \Delta FGB} = \left(\frac{CE}{BF}\right)^2$

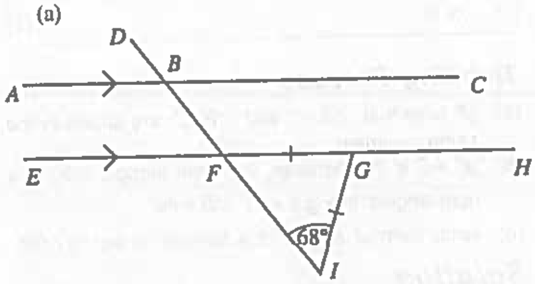
from (b) (iii) $(CE)^2 = 200$

$\therefore\ \frac{\text{area of } \Delta CGE}{\text{area of } \Delta FGB} = \frac{(CE)^2}{(BF)^2}$

$= \frac{200}{(10)^2} = \frac{2}{1}$ Ans.

3 *(N2008/P1/Q20)*

(a)



ABC and $EFGH$ are parallel lines.
The line DI intersects AC at B and EH at F.

$F\hat{I}G = 68°$ and $FG = GI$.

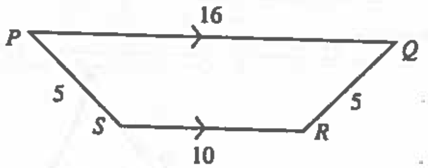
Find

(i) $B\hat{F}G$, [1]

(ii) $F\hat{G}I$. [1]

(iii) $D\hat{B}A$. [1]

(b)



PQRS is a trapezium.
$PS = QR = 5$cm, $PQ = 16$cm and $SR = 10$cm.
Find the area of the trapezium. [2]

Thinking Process

(a) (i) To find $B\hat{F}G$ ✎ find $G\hat{F}I$.

(ii) ✎ $\Delta G\hat{F}I$ is an isosceles triangle.

(iii) To find $D\hat{B}A$ ✎ find $F\hat{B}C$.

(b) Using pythagoras theorem, find the height of the trapezium first. Then use formula to find the area.

Solution

(a) (i) ΔGFI is an isosceles triangle,

$\therefore\ G\hat{F}I = 68°$

$B\hat{F}G + G\hat{F}I = 180°$ (∠s on a straight line)

$B\hat{F}G + 68° = 180°$

$B\hat{F}G = 112°$ **Ans.**

(ii) Consider ΔGFI.

$F\hat{G}I = 180° - 68° - 68°$ (∠ sum of ΔGFI)

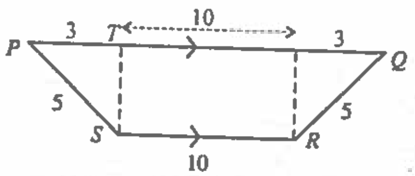
$= 44°$ **Ans.**

(iii) $C\hat{B}F = G\hat{F}I = 68°$ (corresponding ∠s)

$D\hat{B}A = C\hat{B}F$ (vert. opp. angles)

$= 68°$ **Ans.**

(b)



Applying pythagoras theorem on ΔPST.

$ST^2 + 3^2 = 5^2$

$ST^2 = 25 - 9 = 16$

$ST = 4$

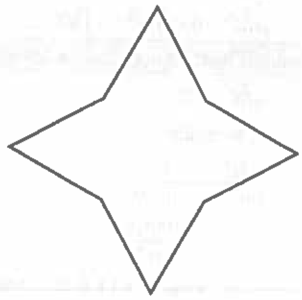
$\therefore$ height of trapezium = 4 cm.

area of trapezium $= \frac{1}{2}(4)(10+16)$

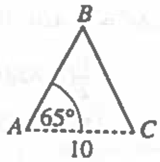
$= 52\ \text{cm}^2$ **Ans.**

4 *(N2008 P2 Q2)*

Eight straight paths in a level garden form this shape with rotational symmetry of order four.



(a)



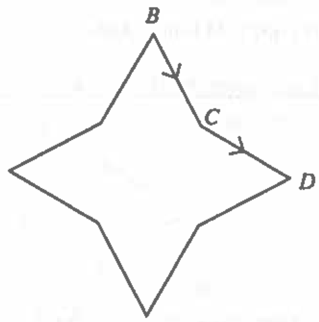
The two paths shown, AB and BC, form part of the isosceles triangle ABC.
$AC = 10$m and angle $BAC = 65°$.
Calculate

(i) the length of the path AB, [2]

(ii) the area of triangle ABC, [2]

(iii) the area of garden enclosed by all 8 paths. [2]

(b)



Ada walked along the paths BC and CD.

(i) Calculate $B\hat{C}D$. [2]

(ii) After walking in the direction BC, Ada turned to walk in the direction CD.
State the value of the angle through which she turned. [1]

Thinking Process

(a) (i) Apply sine rule ✎ find angle ABC.

(ii) Area of $\Delta = \frac{1}{2} \times a \times b \times \sin C$

(iii) ✎ Total area consists of four triangles and a square in the middle.

(b) (i) Subtract the reflex angle BCD from 360°.

(ii) Extend the line BC through C and find the angle through which Ada turned.

Solution

(a) (i) ΔABC is isosceles,

$\therefore\ B\hat{A}C = B\hat{C}A = 65°$

$A\hat{B}C + 65° + 65° = 180°$

sum of interior angles of a $\Delta = 180°$

$\therefore\ A\hat{B}C = 50°$

using sine rule

$$\frac{AB}{\sin 65} = \frac{10}{\sin 50}$$

$$AB = \frac{10\sin 65}{\sin 50}$$

$$= 11.83 \approx 11.8 \text{ m} \quad \textbf{Ans.}$$

(ii) Area of $\Delta ABC = \frac{1}{2}(AB)(AC)\sin 65°$

$$= \frac{1}{2}(11.83)(10)(0.9063)$$

$$= 53.608 \approx 53.6 \text{ m}^2 \quad \textbf{Ans.}$$

(iii) All the four corner triangles are identical to ΔABC

$\therefore$ area of four triangles

$= 4(53.608) = 214.432 \text{ m}^2$

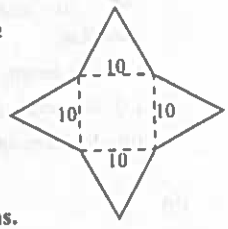
area of middle square

$= (10)^2 = 100 \text{ m}^2$

$\therefore$ total area of garden

$= 214.432 + 100$

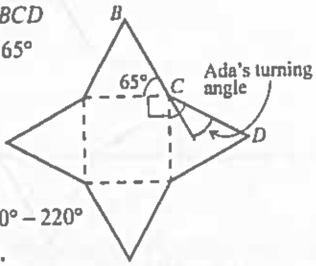
$= 314.432 \approx 314 \text{ m}^2$ **Ans.**



(b) (i) Reflex angle BCD

$= 65° + 90° + 65°$

$= 220°$



$\therefore\ B\hat{C}D = 360° - 220°$

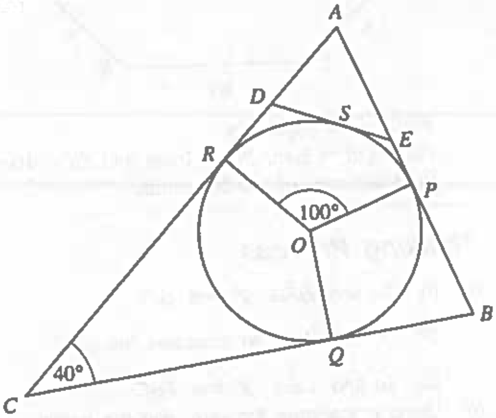
$= 140°$ **Ans.**

(ii) The angle which Ada turned $= 180° - B\hat{C}D$

$= 180° - 140°$

$= 40°$ **Ans.**

5 (N2008/P2/Q5)



The diagram shows a circle, centre O.
The lines AB, BC and CA touch the circle at P, Q and R respectively.

(a) (i) Explain why $C\hat{Q}O = 90$. [1]

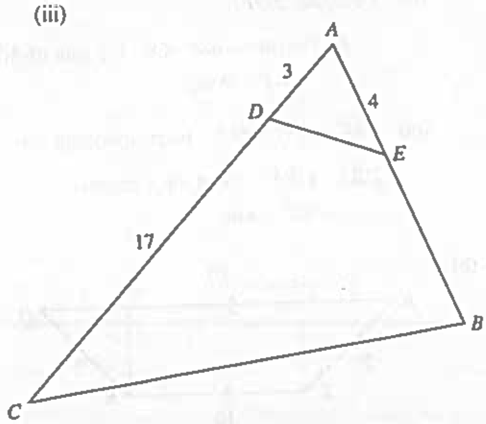
(ii) Given that $A\hat{C}B = 40$, find $R\hat{O}Q$. [1]

(b) The line DE touches the circle at S.
The triangles ABC and ADE are similar.

(i) Write down the value of $A\hat{E}D$. [1]

(ii) Given that $R\hat{O}P = 100$, find $R\hat{O}S$. [2]

(iii)



Given also that AD = 3cm, CD = 17cm and AE = 4cm, calculate BE. [2]

Thinking Process

(a) (i) OQ is the radius and CB is tangent to the circle.

(ii) $R\hat{O}Q + R\hat{C}Q = 180°$.

(b) (i) To find the angle AED look for the corresponding sides of both triangles.

(ii) Draw OS and find angle RDS which is an exterior angle of triangle ADE. Note that angle RDS + angle ROS = 180°.

(iii) Apply rule of similarity: ratio of corresponding lengths is equal.

Solution with TEACHER's COMMENT

(a) (i) CQ is tangent to the circle. The radius OQ of circle meets the tangent CQ at Q. Therefore $C\hat{Q}O = 90$.

> **Property:**
> A tangent to a circle is perpendicular to the radius drawn to the point of contact.

(ii) $R\hat{C}Q + R\hat{O}Q = 180$

$\therefore \quad R\hat{O}Q = 180 - 40° = 140°$ **Ans.**

(b) (i) ΔABC and ΔADE are similar (given)

$\therefore \quad A\hat{E}D = A\hat{C}B = 40°$ **Ans.**

(ii) AR and AP are tangents to the circle.

$\therefore \quad R\hat{A}P + R\hat{O}P = 180$

$R\hat{A}P = 180° - 100° = 80°$

$R\hat{D}E = D\hat{A}E + A\hat{E}D$ (ext.∠ of a Δ = sum of opp. interior ∠s.)

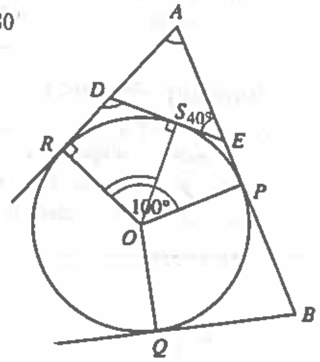
$= 80° + 40° = 120°$

line DE is also a tangent to the circle at S.

$\therefore \quad R\hat{O}S + R\hat{D}S = 180$

$R\hat{O}S = 180° - 120°$

$= 60°$ **Ans.**



(iii) ΔABC and ΔADE are similar (given).

$$\therefore \frac{AB}{AD} = \frac{AC}{AE}$$

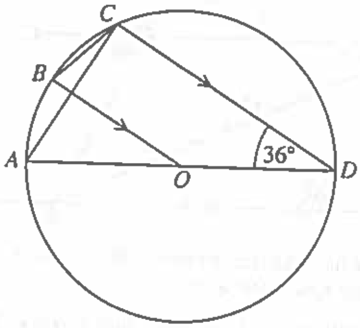
$$\frac{AB}{3} = \frac{3+17}{4}$$

$$AB = \frac{20}{4} \times 3 = 15$$

$BE = AB - AE$

$= 15 - 4 = 11$ cm **Ans.**

6 (J2009/P1/Q14)



The diagram shows a circle, centre O, passing through A, B, C and D.

AOD is a straight line, BO is parallel to CD and $C\hat{D}A = 36°$.

Find

(a) $B\hat{O}A$, [1]

(b) $B\hat{C}A$, [1]

(c) $D\hat{C}B$, [1]

(d) $O\hat{B}C$. [1]

Thinking Process

(a) $\angle BOA = \angle CDA$

(b) $\angle BCA = \frac{1}{2}\angle BOA$

(c) $\angle DCB = \angle DCA + \angle BCA$. AD is the diameter therefore $\angle DCA = 90°$

(d) $\angle OBC + \angle DCB = 180°$

Solution

(a) $B\hat{O}A = C\hat{D}A = 36°$ (corresponding ∠s) **Ans.**

(b) $B\hat{O}A = 2(B\hat{C}A)$ (∠ at centre is twice ∠ at circumference)

$\therefore \quad B\hat{C}A = \frac{1}{2} B\hat{O}A$

$= \frac{1}{2}(36°) = 18°$ **Ans.**

(c) $D\hat{C}A = 90°$ (∠ in semi-circle)

Now,

$D\hat{C}B = D\hat{C}A + B\hat{C}A$

$= 90° + 18° = 108°$ **Ans.**

(d) BO is parallel to CD (given)

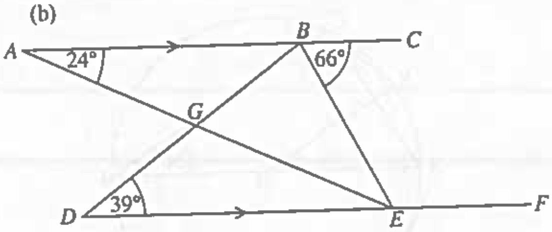
$\therefore \quad D\hat{C}B + O\hat{B}C = 180°$ (Interior ∠s between || lines)

$108° + O\hat{B}C = 180°$

$O\hat{B}C = 180° - 108° = 72°$ **Ans.**

7 *(J2009/P2/Q4 b)*

(b)



In the diagram, the lines ABC and DEF are parallel. AE meets DB at G.

$B\hat{A}E = 24°$, $C\hat{B}E = 66°$ and $B\hat{D}E = 39°$.

Calculate

(i) $F\hat{E}B$, [1]

(ii) $B\hat{E}A$, [1]

(iii) $A\hat{G}D$. [1]

Thinking Process

(b) (i) $BC \parallel EF$, therefore $\angle FEB$ and $\angle CBE$ are supplementary.

(ii) Note that $\angle CBE$ is an exterior angle of ΔABE.

(iii) Note that $\angle AGD$ is an exterior angle of ΔABG.

Solution with TEACHER's COMMENTS

(b) (i) Since $AC \parallel DF$

$\therefore\ C\hat{B}E + F\hat{E}B = 180°$ — interior angles between ∥ lines

$66° + F\hat{E}B = 180°$

$F\hat{E}B = 180° - 66°$

$= 114°$ **Ans.**

(ii) $B\hat{A}E + B\hat{E}A = C\hat{B}E$ — Ext. ∠ of a Δ = sum of opp. int ∠s

$24° + B\hat{E}A = 66°$

$B\hat{E}A = 66° - 24°$

$= 42°$ **Ans.**

(iii) $A\hat{B}D = B\hat{D}E = 39°$ (alt ∠s)

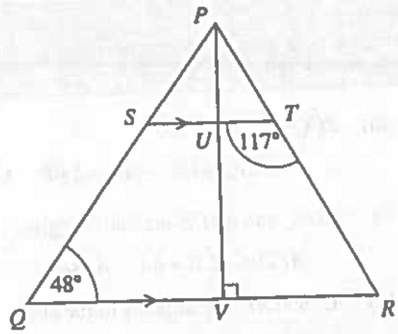
$A\hat{G}D = B\hat{A}G + A\hat{B}D$ — Ext. ∠ of a Δ = sum of opp. int ∠s

$= 24° + 39°$

$= 63°$ **Ans.**

8 *(N2009/P1/Q18)*

The diagram shows the triangle PQR.
The points S and T lie on the lines PQ and PR respectively.
The line ST is parallel to the line QR.



(a) $S\hat{T}R = 117°$ and $S\hat{Q}R = 48°$. Find $Q\hat{P}R$. [1]

(b) U and V are points on ST and QR respectively.

PUV is a straight line with $2PU = UV$ and $P\hat{V}R = 90°$.

Find

(i) $PU : PV$, [1]

(ii) the ratio of the area of triangle PQR to the area of the trapezium $STRQ$. [2]

Thinking Process

(a) Angle $PST = 48°$. Also observe that angle STR is an exterior angle for triangle SPT.

(b) (i) Note that $PV = PU + UV$.

(ii) Apply concept of area of similar triangles.

Solution

(a) $P\hat{S}T = 48°$ (corresponding ∠s)

$S\hat{T}R = P\hat{S}T + S\hat{P}T$

$117° = 48° + S\hat{P}T$

$S\hat{P}T = 117° - 48° = 69°$

since $Q\hat{P}R = S\hat{P}T$

$\therefore\ Q\hat{P}R = 69°$ **Ans.**

(b) (i) $2PU = UV$

$\Rightarrow \dfrac{PU}{UV} = \dfrac{1}{2}$

$\therefore\ PU : PV = 1:3$ **Ans.**

(ii) $\dfrac{\text{Area of } \Delta PST}{\text{Area of } \Delta PQR} = \left(\dfrac{PU}{PV}\right)^2$

$= \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

$\therefore$ Area of ΔPQR : Area of $STRQ$

$= 9 : 9 - 1$

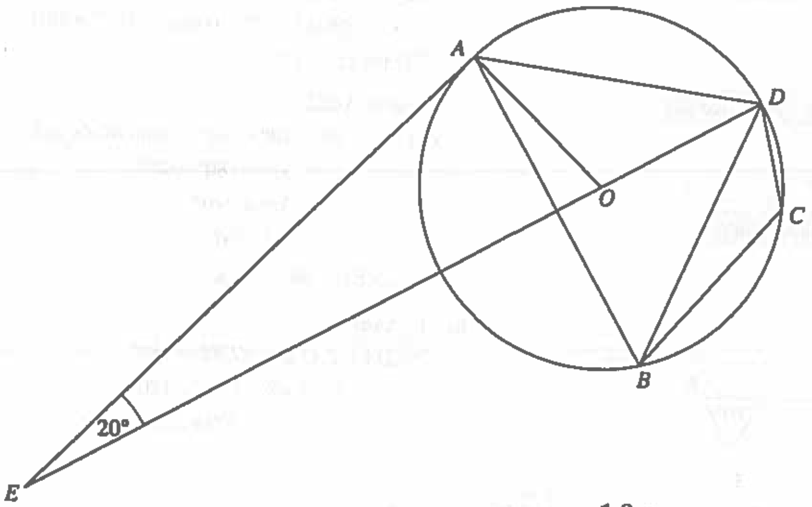
$= 9 : 8$ **Ans.**

9 (N2009/P1/Q20)

The quadrilateral $ABCD$ has its vertices on the circumference of a circle.

AE is a tangent to the circle and $A\hat{E}D = 20°$.
The centre of the circle, O, lies on the straight line DE.

(a) Find $A\hat{D}O$. [2]



(b) Given that DE is the perpendicular bisector of AB and $D\hat{B}A = 55°$,

(i) write down $B\hat{A}D$, [1]

(ii) find $B\hat{C}D$. [1]

Thinking Process

(a) To find $A\hat{D}O$ ✎ note that $O\hat{A}E = 90°$. Find $A\hat{O}E$ and apply: ∠at centre = 2 × ∠at circumference.

(b) (i) ✎ Triangle BAD is an isosceles triangle.
(ii) ✎ $ABCD$ is a cyclic quadrilateral.

Solution

(a) Radius OA meets the tangent EA at A.

$\therefore \quad O\hat{A}E = 90°$

In ΔOAE,

$A\hat{O}E = 180° - 90° - 20°$ (∠ sum of a Δ)
$= 70°$

$\therefore \quad A\hat{D}O = \frac{1}{2} \times A\hat{O}E$ (∠ at centre is 2 × ∠ at circumference)

$= \frac{1}{2} \times 70° = 35°$ **Ans.**

(b) (i) Given that, DE is perp. bisector of AB,

$\Rightarrow \quad \Delta ABD$ is isosceles

$\therefore \quad B\hat{A}D = D\hat{B}A = 55°$ **Ans.**

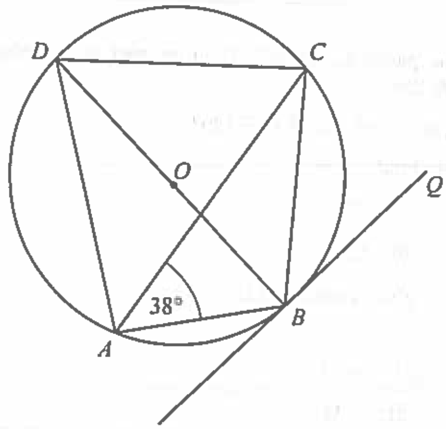
(ii) $ABCD$ is a cyclic quadrilateral

$\therefore \quad B\hat{C}D + B\hat{A}D = 180°$

$B\hat{C}D + 55° = 180°$

$B\hat{C}D = 125°$ **Ans.**

10 (J2010/P1/Q15)



The diagram shows a circle, centre O, passing through A, B, C and D.

BOD is a straight line and $B\hat{A}C = 38°$.

The line BQ is a tangent to the circle at B.

Find

(a) $D\hat{A}C$, [1]

(b) $D\hat{B}C$, [1]

(c) $C\hat{B}Q$. [1]

Thinking Process

(a) DB is the diameter. Therefore $\angle DAB = 90°$.

(b) $\angle DAC = \angle DBC$ (angles in the same segment).

(c) $\angle CDB = \angle CBQ$ (∠s in alternate segment)

Solution with TEACHER's COMMENT

(a) $\angle DAB = 90°$ [right ∠ in semi-circle]

$\therefore \quad \angle DAC + 38° = 90°$

$\angle DAC = 90° - 38° = 52°$ **Ans.**

(b) $\angle DBC = \angle DAC$ [∠s in the same segment]

$\therefore \quad \angle DBC = 52°$ **Ans.**

(c) In ΔDBC,

$\angle DCB = 90°$ [right ∠ in semi-circle]

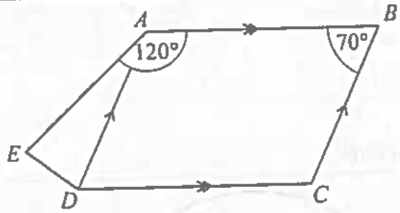
$\therefore \quad \angle CDB = 90° - \angle DBC$

$= 90° - 52° = 38°$

$\angle CBQ = \angle CDB$ [∠s in alt. segment]

$\therefore \quad \angle CBQ = 38°$ **Ans.**

11 *(J2010/P2/Q2)*



The parallelogram $ABCD$ forms part of the pentagon $ABCDE$.

$A\hat{B}C = 70°$ and $B\hat{A}E = 120°$.

(a) Find

(i) $B\hat{C}D$, [1]

(ii) $E\hat{A}D$. [1]

(b) $E\hat{D}C$ is twice $A\hat{E}D$.

Find

(i) $A\hat{E}D$, [3]

(ii) $E\hat{D}A$. [1]

Thinking Process

(a) (i) Use the fact that $AB // DC$ and consider $\angle ABC$.

(ii) Take note that $\angle BCD = \angle BAD$.

(b) (i) Find $\angle ADE$ in terms of $\angle AED$ and consider the angle sum of ΔAED.

(ii) To find $\angle EDA$ consider the angle sum of a triangle.

Solution

(a) (i) $\angle BCD + \angle ABC = 180°$ (int. ∠s between ∥ lines, $AB \parallel DC$)

$\angle BCD + 70° = 180°$

$\angle BCD = 110°$ **Ans.**

(ii) $\angle BCD = \angle DAB = 110°$ (opp. ∠s of a parallelogram)

$\angle EAD = 120° - \angle DAB$

$= 120° - 110° = 10°$ **Ans.**

(b) (i) Let $\angle AED = x°$

$\Rightarrow \quad \angle EDC = 2x°$ (given)

$\angle EDC = \angle EDA + \angle ADC$

$2x° = \angle EDA + 70°$ (since $\angle ADC = 70°$)

$\angle EDA = 2x° - 70°$

consider ΔAED

$x° + 2x° - 70° + 10° = 180°$ (sum of ∠s in Δ)

$3x° = 180° + 60°$

$3x° = 240°$

$x° = 80°$

$\therefore \quad \angle AED = 80°$ **Ans.**

(ii) In ΔAED,

$\angle EDA + \angle AED + \angle EAD = 180°$

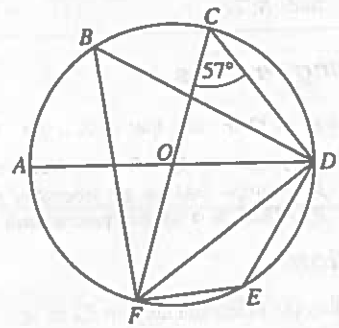
$\angle EDA + 80° + 10° = 180°$

$\angle EDA = 90°$ **Ans.**

12 *(N2010/P1/Q23)*

In the diagram, A, B, C, D, E and F lie on the circle, centre O.

AD and FC are diameters, and $F\hat{C}D = 57°$.



Find

(a) $D\hat{E}F$, [1]

(b) $F\hat{B}D$, [1]

(c) $C\hat{F}D$, [1]

(d) $A\hat{O}F$. [1]

Thinking Process

(a) Consider the cyclic quadrilateral $CDEF$.

(b) $F\hat{B}D$ and $F\hat{C}D$ are angles in the same segment.

(c) Note that CF is the diameter and $C\hat{D}F$ is a right-angle.

(d) Note that $F\hat{O}D = 2F\hat{B}D$. Also $A\hat{O}F$ and $F\hat{O}D$ lie on one straight line

Solution

(a) $CDEF$ is a cyclic quadrilateral

$\therefore \quad \angle DEF = 180° - \angle FCD$
$= 180° - 57°$
$= 123°$ **Ans.**

(b) $\angle FBD = \angle FCD$ (∠s in same segment)
$= 57°$ **Ans.**

(c) In ΔCFD,

$\angle CDF = 90°$ (rt. ∠ in semi-circle)

$\therefore \quad \angle CFD = 90° - 57°$
$= 33°$ **Ans.**

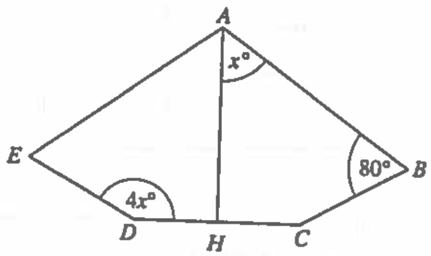
(d) $\angle FOD = 2\angle FBD$ (∠ at centre is 2 × ∠ at circumference)
$= 2(57°) = 114°$

$\angle AOF + \angle FOD = 180°$ (∠s on a straight line)
$\angle AOF = 180° - 114°$
$= 66°$ **Ans.**

13 *(N2010/P2/Q3)*

(a) Calculate the interior angle of a regular 10-sided polygon. [2]

(b)

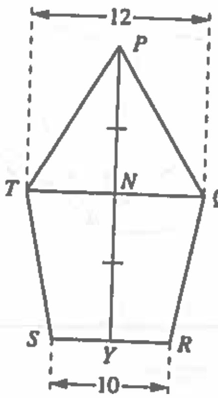


AH is the line of symmetry of the pentagon $ABCDE$.

$H\hat{A}B = x°$, $A\hat{B}C = 80°$ and $E\hat{D}H = 4x°$.

Find x. [3]

(c)



PY is the line of symmetry of the pentagon $PQRST$.

PY and TQ intersect at N.

$PN = NY$.

$TQ = 12$ cm and $SR = 10$ cm.

(i) Given that $PY = 2h$ centimetres, find an expression, in terms of h, for the area of the trapezium $QRST$. [2]

(ii) Given that the area of $PQRST$ is 221 cm², calculate h. [2]

Thinking Process

(a) Apply one interior angle $= \frac{(n-2)180}{n}$.

(b) To find x ∮ apply sum of angles in a quadrilateral = 360°.

(c) (i) To find area of the trapezium ∮ understand that NY is the height of the trapezium.

(ii) Area of $PQRST$ = area of trapezium $QRST$ + area of triangle PQR.

Solution

(a) One interior angle $= \frac{(10-2)180}{10}$
$= \frac{1440}{10} = 144°$ **Ans.**

(b) Since AH is the line of symmetry of $ABCDE$,

$\Rightarrow \quad A\hat{H}C = 90°$ and $B\hat{C}H = E\hat{D}H = 4x°$

In quadrilateral $ABCH$,

$x° + 80° + 4x° + 90° = 360°$
$5x° + 170° = 360°$
$5x° = 190°$
$x° = 38°$ **Ans.**

> Note that:
> Sum of interior angles in a quadrilateral = 360°

(c) (i) Since PY is the line of symmetry of $PQRST$,

$\Rightarrow \quad NY \perp SR$

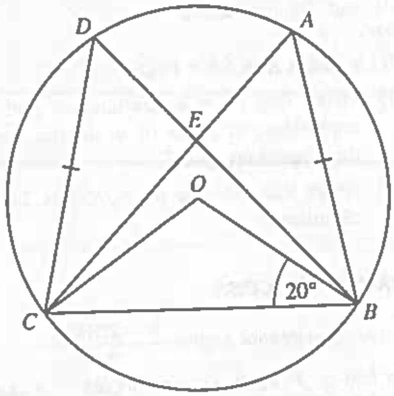
also $NY = h \quad (\because PN = NY)$

$\therefore \quad$ area of $QRST = \frac{1}{2}(h)(10+12)$
$= 11h$ cm² **Ans.**

(ii) Area of $PQRST$ = area of trapezium $QRST$ + area of triangle PQT

$\Rightarrow \quad 221 = 11h + \frac{1}{2}(12)(h)$
$221 = 11h + 6h$
$221 = 17h$
$h = 13$ cm **Ans.**

14 (J2011/P1/Q22)



Points A, B, C and D lie on the circumference of a circle, centre O, and $AB = CD$.
AC and BD intersect at E. $O\hat{B}C = 20°$.

(a) Calculate $B\hat{O}C$. [1]

(b) Calculate $C\hat{A}B$. [1]

(c) Show that triangles AEB and DEC are congruent. [3]

Thinking Process

(a) ΔBOC is isosceles ($OB = OC$ = radius of circle).

(b) Apply, $\angle$ at centre is $2 \times \angle$ at circumference.

(c) Prove, $\angle CDB = \angle BAC$, $AB = CD$ and $\angle DEC = \angle AEB$.

Solution with TEACHER'S COMMENTS

(a) $\angle OBC = \angle OCB = 20°$ (base $\angle$ of isosceles Δ)

$\therefore \angle BOC = 180° - 20° - 20°$
$= 140°$ **Ans.**

(b) $\angle CAB = \frac{1}{2}\angle COB$ ($\angle$ at centre is $2 \times \angle$ at circumference)

$\therefore \angle CAB = \frac{1}{2} \times 140° = 70°$ **Ans.**

(c) $AB = CD$ (given)

$\angle CAB = \angle BDC$ ($\angle$s in same segment)

$\angle DEC = \angle AEB$ (vert. opp angles)

$\therefore \Delta AEB \equiv \Delta DEC$ (AAS) **Proved.**

AAS (angle-angle-side) Property:
Two triangles are congruent if two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle.

15 (N2011/P1/Q9)

Each interior angle of a regular polygon is p times each exterior angle.
Find an expression, in terms of p, for

(a) an exterior angle, [1]

(b) the number of sides of the polygon. [1]

Thinking Process

(a) Exterior angle + Interior angle = 180°

(b) Apply ext. $\angle = \frac{360°}{\text{no. of sides}}$

Solution

(a) For any polygon, $\angle ext. = 180° - \angle int.$

given that, $\angle int = p(\angle ext.)$

$\Rightarrow \angle ext. = 180° - p(\angle ext.)$

$\Rightarrow \angle ext. + p(\angle ext.) = 180°$

$\Rightarrow \angle ext.(1 + p) = 180°$

$\Rightarrow \angle ext. = \frac{180°}{1+p}$ **Ans.**

(b) $\angle ext. = \frac{360°}{n}$

$\Rightarrow \frac{180°}{1+p} = \frac{360°}{n}$

$180°n = 360°(1 + p)$

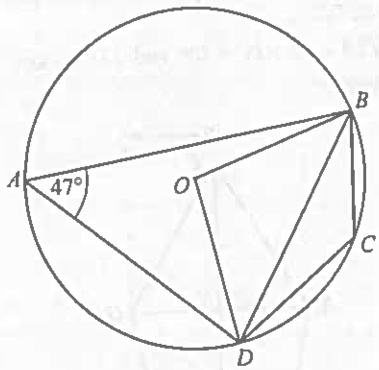
$n = \frac{360°(1+p)}{180°}$

$= 2(1 + p)$

$\therefore$ number of sides $= 2(1 + p)$ **Ans.**

16 (N2011/P1/Q14)

In the diagram, the points A, B, C and D lie on the circle, centre O. $B\hat{A}D = 47°$.



Find

(a) $B\hat{O}D$, (b) $B\hat{C}D$, (c) $O\hat{B}D$. [3]

Thinking Process

(a) Apply $\angle$ at centre $= 2 \times \angle$ at circumference.

(b) $ABCD$ is a cyclic quadrilateral.

(c) $\angle OBD = \angle ODB$ as ΔOBD is isosceles.

Solution

(a) $B\hat{O}D = 2 \times 47° = 94°$ **Ans.**

(∠ at centre is 2 × ∠ at circumference)

(b) $B\hat{C}D = 180° - 47°$

$= 133°$ **Ans.**

(opp. ∠s of cyclic quad. are supplementary)

(c) $O\hat{B}D = O\hat{D}B$ (base ∠s of isosceles Δ)

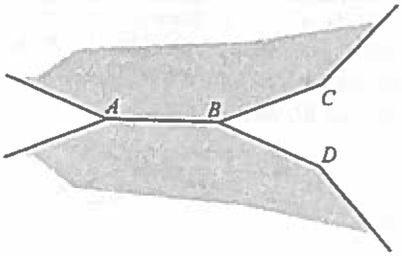
$O\hat{B}D + O\hat{D}B + B\hat{O}D = 180°$ (sum of ∠s in Δ)

$2O\hat{B}D + 94° = 180°$

$O\hat{B}D = \frac{180° - 94°}{2}$

$= 43°$ **Ans.**

17 (J2012/P1/Q16)



The diagram shows parts of two identical regular 12-sided polygons.

(a) Calculate angle ABC. [2]

(b) A different regular shape will fit exactly into the space at B. Name this shape. [1]

Thinking Process

(a) Note that angle ABC is one interior angle of the regular 12-sided polygon.

(b) Note that BCD is an equilateral triangle.

Solution with TEACHER's COMMENTS

(a) One interior angle $= \frac{(n-2)180}{n}$

$\angle ABC = \frac{(12-2)180}{12}$

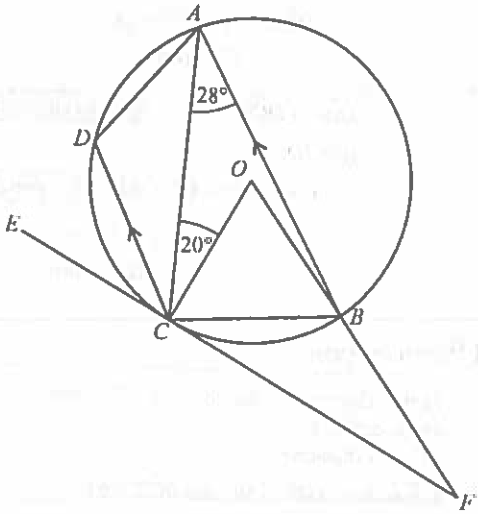
$= \frac{1800}{12} = 150°$ **Ans.**

(b) The space BCD is an Equilateral triangle. **Ans.**

Since both polygons are regular
$\Rightarrow BC = BD$ and $\angle ABC = \angle ABD$
now,
$\angle ABC + \angle ABD + \angle CBD = 360°$ (∠s at a point)
$150° + 150° + \angle GBD = 360°$
$\angle GBD = 360° - 300° = 60°$
∴ BCD is an equilateral triangle.

18 (J2012/P2/Q11 a)

(a)



A, B, C and D are points on the circumference of a circle, centre O.

$C\hat{A}B = 28°$, $A\hat{C}O = 20°$ and CD is parallel to BA.

EF is a tangent to the circle at C and OBF is a straight line.

Find

(i) $C\hat{O}B$, [1]

(ii) $O\hat{F}C$, [1]

(iii) $O\hat{C}B$, [1]

(iv) $D\hat{C}E$, [2]

(v) $A\hat{D}C$. [2]

Thinking Process

(a) (i) Apply: ∠ at centre is 2 × ∠ at circumference.

(ii) Note that $\angle OCF$ is a right-angle.

(iii) $\angle BCF = \angle BAC$ (∠s in alternate segment).

(iv) $\angle OCE$ is a right-angle. Find $\angle ACD$.

(v) $\angle DCE = \angle DAC$ (∠s in alternate segment).

Solution with TEACHER's COMMENTS

(a) (i) $C\hat{O}B = 2 \times 28°$ (∠ at centre is 2 × ∠ at circumference)

$= 56°$ **Ans.**

(ii) $O\hat{C}F = 90°$ (radius ⊥ tangent)

∴ $O\hat{F}C = 90° - 56° = 34°$ **Ans.**

(iii) $B\hat{C}F = 28°$ (∠s in alternate segment)

∴ $O\hat{C}B = 90° - 28° = 62°$ **Ans.**

(iv) $A\hat{C}D = 28°$ (alternate angles)

$O\hat{C}E = 90°$ (radius ⊥ tangent)

$\therefore \quad D\hat{C}E = 90° - 20° - 28°$

$= 42°$ **Ans.**

(v) $D\hat{A}C = D\hat{C}E = 42°$ (∠s in alt. segment)

In ΔADC,

$A\hat{D}C + 28° + 42° = 180°$ (∠ sum of a Δ)

$A\hat{D}C = 180° - 70°$

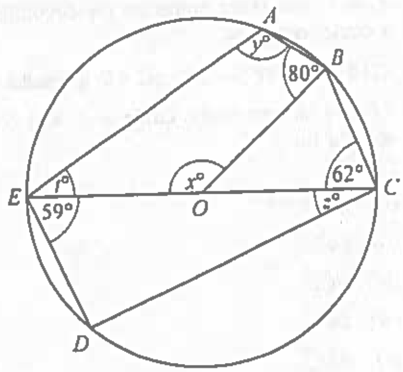
$= 110$ **Ans.**

19 *(N2012/P1/Q23)*

In the diagram, the points A, B, C, D and E lie on the circle centre O.

EC is a diameter.

$O\hat{B}A = 80°$, $D\hat{E}C = 59°$ and $B\hat{C}E = 62°$.



Find

(a) x, [1]

(b) y, [1]

(c) z, [1]

(d) t. [1]

Thinking Process

(a) ✎ ΔOBC is isosceles. ($OB = OC$ = radius of circle).

(b) ✎ Note that $ABCE$ is a cyclic quadrilateral.
✎ opposite angles are supplementary.

(c) ✎ Angle EDC is 90° since EC is a diameter.

(d) ✎ $ABCE$ is a cyclic quadrilateral. ✎ opposite angles are supplementary.

Solution *with* TEACHER's COMMENTS

(a) ΔOBD is an isosceles Δ. ($OB = OC$ = radius of circle)

$\Rightarrow \quad O\hat{C}B = O\hat{B}C = 62°$

$\therefore \quad x = 62° + 62°$ (ext. ∠ of Δ = sum of 2 int. opp ∠s.)

$= 124°$ **Ans.**

(b) $ABCE$ is a cyclic quadrilateral.

$y + 62° = 180°$ (opp. ∠s of a cyclic quad. are supplementary)

$y = 180° - 62°$

$= 118°$ **Ans.**

(c) In ΔECD,

$E\hat{D}C = 90°$ (right ∠ in semi-circle)

$\therefore \quad z = 90° - 59°$

$= 31°$ **Ans.**

(d) In cyclic quadrilateral $ABCE$.

$A\hat{B}C = 80° + 62° = 142°$

$\therefore \quad t = 180° - 142°$ (opp. ∠s of cyclic quad.)

$= 38°$ **Ans.**

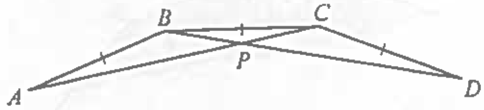
20 *(N2012/P1/Q24)*

A regular polygon has an interior angle of 160°.

(a) Find the number of sides of the polygon. [2]

(b) The diagram shows three sides AB, BC and CD of the regular polygon.

AC and BD meet at P.



(i) Calculate $B\hat{C}A$. [1]

(ii) Calculate $D\hat{P}C$. [1]

Thinking Process

(a) ✎ find Interior angle $= \dfrac{180°(n-1)}{n}$.

(b) (i) ✎ $B\hat{A}C = B\hat{C}A$ since triangle ABC is isosceles.

(ii) Find $D\hat{C}P$. Apply: sum of ∠s in a Δ.

Solution *with* TEACHER's COMMENTS

(a) $160 = \dfrac{180(n-2)}{n}$

$160n = 180n - 360$

$20n = 360$

$n = 18$

$\therefore$ number of sides of the polygon = 18 **Ans.**

(b) (i) ΔABC is an isosceles Δ.

$\therefore \quad B\hat{C}A = \dfrac{180° - 160°}{2}$

$= 10°$ **Ans.**

(ii) In ΔCDP,

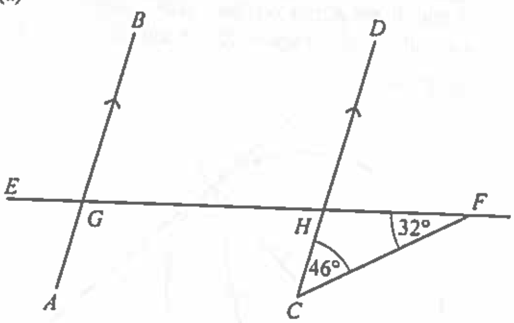
$D\hat{C}P = 160° - 10° = 150°$

$\therefore \quad D\hat{P}C = 180° - 10° - 150°$ (∠ sum of a Δ)

$= 20°$ **Ans.**

21 (N2012/P2/Q4 a)

(a)



AB and CD are parallel.
$EGHF$ is a straight line.
$H\hat{C}F = 46°$ and $H\hat{F}C = 32°$.

(i) Find $C\hat{H}F$ [1]

(ii) Find $G\hat{H}D$ [1]

(iii) Find $H\hat{G}B$ [1]

Thinking Process

(a) (i) To find $C\hat{H}F$ Apply sum of ∠s in a Δ.

(ii) $G\hat{H}D = C\hat{H}F$ (vert. opp. angles).

(iii) $BG \parallel DH$. interior ∠s between parallel lines are supplementary.

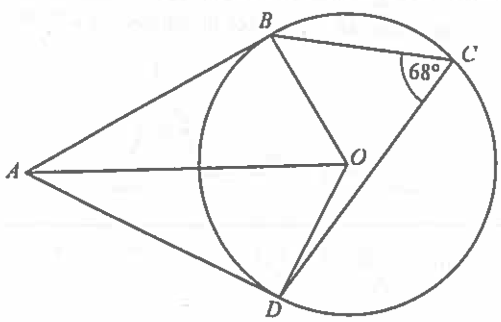
Solution

(a) (i) $C\hat{H}F = 180° - 32° - 46°$ (sum of ∠s in a Δ)
$= 102°$ **Ans.**

(ii) $G\hat{H}D = C\hat{H}F$ (vert. opposite ∠s)
$= 102°$ **Ans.**

(iii) BG is parallel to DH.
$\therefore\ H\hat{G}B + G\hat{H}D = 180°$ (interior ∠s between ∥ lines)
$\Rightarrow H\hat{G}B = 180° - 102°$
$= 78°$ **Ans.**

22 (J2013/P1/Q23)



B, C and D are points on the circle, centre O.
BA and DA are tangents to the circle at B and D.

(a) Show that triangles ABO and ADO are congruent. [3]

(b) What type of special quadrilateral is $ABOD$? [1]

(c) Angle $BCD = 68°$. Find angle BAD. [2]

Thinking Process

(a) Prove $AB = AD$ and $OB = OD$. Observe that angle ODA and angle OBA are right angles.

(b) Notice that $OB = OD$ and $AB = AD$.

(c) Apply ∠ at centre = 2∠ at circumference to find ∠BOD. Note that ∠BOD and ∠BAD are supplementary.

Solution

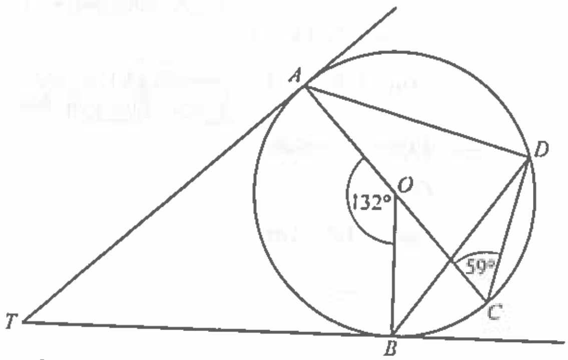
(a) $OD = OB$ (radii of circle)
$AB = AD$ (tangents from ext. source)
$\angle ABO = \angle ADO = 90°$ (radius ⊥ tangent)
$\therefore\ \Delta ABO \equiv \Delta ADO$ (SAS) **Shown.**

(b) Kite. **Ans.**

(c) $\angle BOD = 2 \times 68°$ (∠ at centre = 2 × ∠ at circumference)
$= 136°$

AB and AD are tangents to the circle
$\therefore\ \angle BAD + \angle BOD = 180°$
$\angle BAD = 180° - \angle BOD$
$= 180° - 136°$
$= 44°$ **Ans.**

23 (N2013/P1/Q22)



In the diagram, the points A, B, C and D lie on the circle centre O.
TA and TB are tangents touching the circle at A and B respectively.

$A\hat{O}B = 132°$, $A\hat{C}D = 59°$ and AOC is a straight line.

(a) Find $A\hat{T}B$. [1]

(b) Find $B\hat{D}A$. [1]

(c) Find $B\hat{D}C$. [1]

(d) Find $O\hat{B}D$. [1]

Thinking Process

(a) $\angle AOB + \angle ATB = 180°$.

(b) $\angle BDA = \frac{1}{2}\angle AOB$. ($\angle$ at centre is twice $\angle$ at circle centre).

(c) $\angle CDA = 90°$ (right $\angle$ in semi-circle)

(d) To find $\angle OBD$ find $\angle DAC$. Let OC and BD meet at E. Find $\angle OEB$ which is an exterior angle of triangle ADE.

Solution

(a) $A\hat{T}B + A\hat{O}B = 180°$ (tangents from ext. source)

$A\hat{T}B = 180° - A\hat{O}B$

$= 180° - 132° = 48°$ **Ans.**

(b) $B\hat{D}A = \frac{1}{2} \times A\hat{O}B$ ($\angle$ at centre is $2 \times \angle$ at circumference)

$= \frac{1}{2} \times 132° = 66°$ **Ans.**

(c) $A\hat{D}C = 90°$ (right $\angle$ in semi-circle)

$\therefore$ $B\hat{D}C = 90° - B\hat{D}A$

$= 90° - 66°$

$= 24°$ **Ans.**

(d) Let BD and OC meets at point E.

$D\hat{A}C = 180° - 90° - 59°$ ($\angle$ sum of a Δ)

$= 31°$

$O\hat{E}B = D\hat{A}C + B\hat{D}A$ (ext. $\angle$ of a Δ = sum of 2 int. opp $\angle$s.)

$= 31° + 66° = 97°$

$A\hat{O}B = O\hat{E}B + O\hat{B}E$ (ext. $\angle$ of a Δ = sum of 2 int. opp $\angle$s.)

$\Rightarrow$ $132° = 97° + O\hat{B}E$

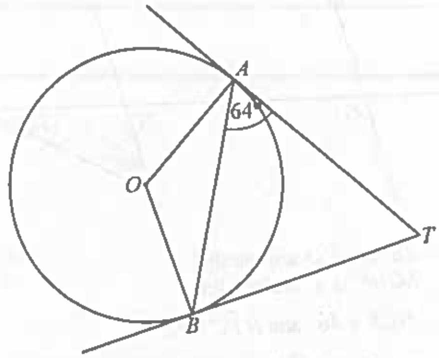
$O\hat{B}E = 35°$

$\therefore$ $O\hat{B}D = 35°$ **Ans.**

24 (J2014/P1/Q8)

A and B are points on the circle, centre O.
TA and TB are tangents to the circle.

$B\hat{A}T = 64°$.



(a) What special type of triangle is triangle ABT? [1]

(b) Work out $A\hat{O}B$. [1]

Thinking Process

(a) Note that $TA = TB$ (tangents from external source)

(b) $\angle ATB + \angle AOB = 180°$ (tangents from external source).

Solution

(a) ABT is an isosceles triangle.

(b) $\angle ATB = 180° - 2(64°)$

$= 180° - 128° = 52°$

TA and TB are tangents to the circle

$\therefore$ $\angle ATB + \angle AOB = 180°$

$\angle AOB = 180° - \angle ATB$

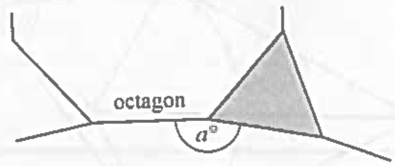
$= 180° - 52°$

$= 128°$ **Ans.**

25 (J2014/P1/Q18)

(a) Find the size of the interior angle of a regular octagon. [1]

(b) A regular octagon, an equilateral triangle and a regular n-sided polygon fit together at a point.



(i) An interior angle of the regular n-sided polygon is $a°$.
Find a. [1]

(ii) Find the value of n. [2]

Thinking Process

(a) Apply, one interior angle $= \frac{(n-2)180}{n}$.

(b) (i) Sum of angles at a point is equal to 360°.

Solution

(a) Number of sides in an octogan, $n = 8$

$\therefore$ Interior angle $= \frac{(8-2)180°}{8}$

$= \frac{1080°}{8} = 135°$ **Ans.**

(b) (i) $a° + 60° + 135° = 360°$ (∠s sum at a point)

$a° = 360° - 60° - 135°$

$= 165°$ **Ans.**

(ii) $a° = \frac{(n-2)\times 180°}{n}$

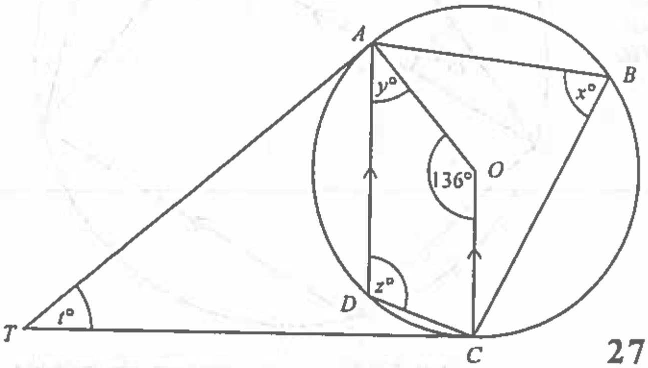
$165° = \frac{(n-2)\times 180°}{n}$

$165°n = 180°n - 360°$

$15°n = 360°$

$n = 24$ **Ans.**

26 *(N2014/P1/Q20)*



In the diagram, A, B, C and D lie on the circle, centre O.
CO is parallel to DA.
The tangents to the circle at A and C meet at T.
$A\hat{O}C = 136°$

(a) Find x. [1]
(b) Find y. [1]
(c) Find z. [1]
(d) Find t. [1]

Thinking Process

(a) $\angle AOC = 2x°$. (∠ at centre is twice ∠ at the circumference).

(b) AD is parallel to OC. Angles between parallel lines are supplementary.

(c) Note that $ABCD$ is a cyclic quadrilateral.

(d) $t° + \angle AOC = 180°$.

Solution

(a) $x° = \frac{1}{2}\times 136°$ (∠ at centre is 2 × ∠ at circumference)

$= 68°$ **Ans.**

(b) AD is parallel to OC.

$\therefore$ $y° + 136° = 180°$ (interior ∠s between ∥ lines)

$y° = 180° - 136°$

$= 44°$ **Ans.**

(c) $ABCD$ is a cyclic quadrilateral.

$x° + z° = 180°$ (opp. ∠s of a cyclic quad. are supplementary)

$z° = 180° - x°$

$= 180° - 68°$

$= 112°$ **Ans.**

(d) TA and TC are tangents to the circle.

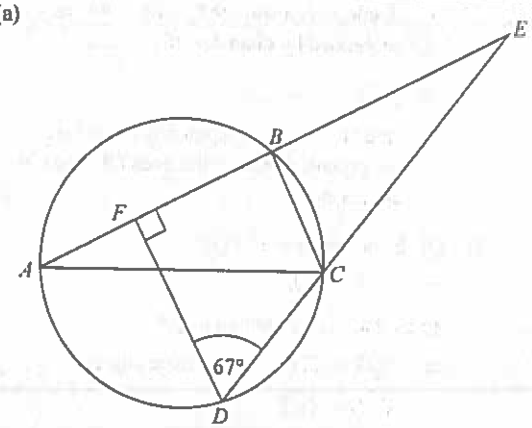
$\therefore$ $t° + 136° = 180°$

$t° = 180° - 136°$

$= 44°$ **Ans.**

27 *(N2014/P2/Q2)*

(a)



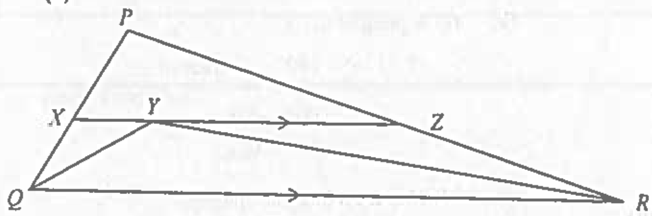
A, B, C and D are points on the circumference of the circle and AC is a diameter.
$AFBE$ and DCE are straight lines.
DF is perpendicular to AE and $C\hat{D}F = 67°$.

(a) (i) Find $A\hat{E}D$. [1]

(ii) Find $C\hat{B}E$, giving a reason for your answer. [1]

(iii) Explain why DF is parallel to CB. [1]

(b)



In the triangle PQR, the bisectors of $P\hat{Q}R$ and $P\hat{R}Q$ intersect at Y.

The straight line XYZ is parallel to QR.

Prove that the perimeter of triangle $PXZ = PQ + PR$. [3]

Thinking Process

(a) (i) To find $A\hat{E}D$ ✎ find $F\hat{E}D$. Note that ΔFED is a right angled triangle.

(ii) Note that $A\hat{B}C = 90°$ and $C\hat{B}E$ is adjacent to $A\hat{B}C$.

(iii) ✎ Angle DFB is equal to angle CBE which satisfies the property of corresponding angles.

(b) To find the perimeter ✎ Prove that triangle XYQ and triangle ZYR are issoceles triangles.

Solution

(a) (i) Consider right angled $\Delta F\hat{E}D$

$F\hat{E}D = 180° - 90° - 67°$

$= 23°$

$\therefore \quad A\hat{E}D = 23°$ **Ans.**

(ii) $C\hat{B}E = 90°$ because $A\hat{B}C = 90°$. Angle subtended at the circumference by the diameter is always 90°. $A\hat{B}C = 90°$ as it is subtended by diameter AC.

(iii) $D\hat{F}B = C\hat{B}E = 90°$

since the two corresponding angles are congruent, therefore the lines DF and CB are parallel.

(b) QY is the bisector of $P\hat{Q}R$

$\Rightarrow \quad Y\hat{Q}R = X\hat{Q}Y$

given that XZ is parallel to QR

$\Rightarrow \quad Y\hat{Q}R = X\hat{Y}Q$ (alternate angles)

$\therefore \quad X\hat{Y}Q = X\hat{Q}Y$

$\Rightarrow \quad \Delta XYQ$ is an issoceles triangle.

$\therefore \quad XY = XQ$ .... (1)

similarly,

RY is the bisector of $P\hat{R}Q$

$\Rightarrow \quad Y\hat{R}Q = Z\hat{R}Y$

XZ is parallel to QR

$\Rightarrow \quad Y\hat{R}Q = Z\hat{Y}R$ (alternate angles)

$\therefore \quad Z\hat{R}Y = Z\hat{Y}R$

$\Rightarrow \quad \Delta ZRY$ is an issoceles triangle.

$\therefore \quad YZ = ZR$ .... (2)

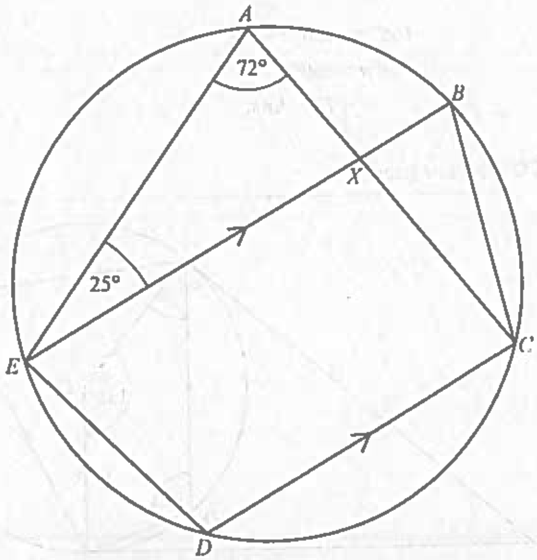
perimeter of $\Delta PXZ = PX + XY + YZ + PZ$

using results (1) and (2),

$= PX + XQ + ZR + PZ$

$= PQ + PR$ **Proved.**

28 (J2015/P2/Q4 a)



A, B, C, D and E are five points on the circumference of a circle.

EB is parallel to DC, $E\hat{A}C = 72°$ and $A\hat{E}B = 25°$.

X is the intersection of AC and EB.

Find

(i) $E\hat{B}C$, [1]

(ii) $C\hat{X}B$, [1]

(iii) $E\hat{D}C$, [1]

(iv) $A\hat{C}D$. [1]

Thinking Process

(i) ✎ $E\hat{B}C$ and $E\hat{A}C$ are angles in the same segment.

(ii) To find $C\hat{X}B$ Find $A\hat{X}E$. Apply sum of angles in a triangle
(iii) $ACDE$ is a cyclic quadrilateral. opposite angles are supplementary.
(iv) EB is parallel to DC. Apply rule of alternate angles to find angle ACD.

Solution

(i) $E\hat{B}C = E\hat{A}C$ (∠s in the same segment)
$= 72°$

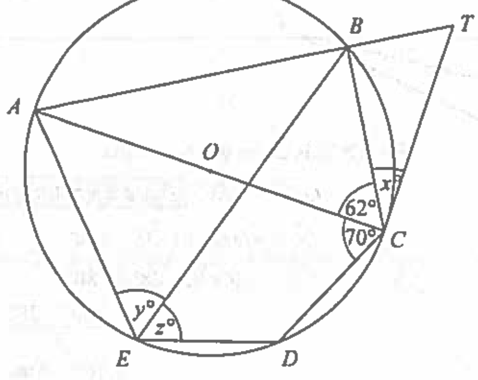
(ii) In ΔAXE,
$A\hat{X}E + 25° + 72° = 180°$ (∠ sum of a Δ)
$A\hat{X}E = 180° - 25° - 72°$
$= 83°$

$C\hat{X}B = A\hat{X}E$ (vert. opp ∠s)
$= 83°$ **Ans.**

(iii) $ACDE$ is a cyclic quadrilateral,
$\therefore\ E\hat{D}C + 72° = 180°$ (opp. ∠s of cyclic quad.)
$E\hat{D}C = 180° - 72°$
$= 108°$ **Ans.**

(iv) EB is parallel to DC.
$\Rightarrow\ B\hat{X}C = D\hat{C}X = 83°$ (alternate ∠s)
$\therefore\ A\hat{C}D = 83°$ **Ans.**

29 *(N2015/P1/Q17)*



In the diagram, A, B, C, D and E lie on the circle, centre O.
AC is a diameter.
The tangent to the circle at C meets the line AB produced at T.
$A\hat{C}B = 62°$ and $A\hat{C}D = 70°$.

(a) Find x. [1]
(b) Find y. [1]
(c) Find z. [1]

Thinking Process

(a) Note that $\angle ACT$ is a right angle.
(b) $\angle AEB$ and $\angle ACB$ are angles in the same segment.
(c) $BCDE$ is a cyclic quadrilateral opposite angles are supplementary.

Solution

(a) $A\hat{C}T = 90°$ (radius ⊥ tangent)
$\therefore\ x° + 62° = 90°$
$x° = 90° - 62°$
$= 28°$ **Ans.**

(b) $A\hat{E}B = A\hat{C}B$ (∠s in the same segment)
$\therefore\ y° = 62°$ **Ans.**

(c) $BCDE$ is a cyclic quadrilateral.
$z° + 70° + 62° = 180°$ (opp. ∠s of a cyclic quad. are supplementary)
$z° + 132° = 180°$
$z° = 180° - 132°$
$= 48°$ **Ans.**

30 *(N2015/P1/Q19)*

All the angles of a polygon are either 155° or 140°.
There are twice as many angles of 155° as 140°.
Find the number of sides of the polygon. [3]

Thinking Process

Find the exterior angles associated with 155° and 140°. Then use the given information to find the number of exterior angles of the polygon. Subsequently find the number of sides of the polygon

Solution *with* TEACHER'S COMMENTS

Exterior angle associated with $155° = 180° - 155°$
$= 25°$

Exterior angle associated with $140° = 180° - 140°$
$= 40°$

Let the number of 40° exterior angles of the polygon be x. Then, the number of 25° exterior angles will be $2x$.

sum of exterior angles of a polygon = 360°
$\Rightarrow\ 40x + 25(2x) = 360$
$40x + 50x = 360$
$90x = 360$
$x = 4$

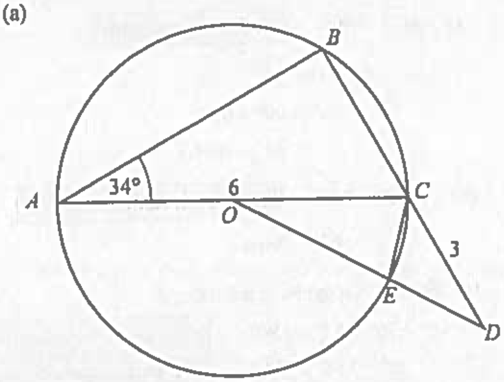
$\therefore$ number of 40° exterior angles = 4
and number of 25° exterior angles = 2(4) = 8
total number of exterior angles of the polygon
= 4 + 8 = 12
$\therefore$ number of sides of polygon = 12 **Ans.**

Note that,

- A polygon has the same number of exterior angles as it has interior angles.
- A polygon has the same number of exterior angles as it has sides.

31 (N2015/P2/Q3)

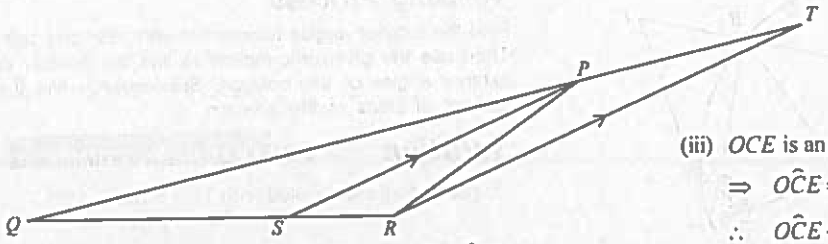
(a)



AC is a diameter of the circle, centre O.
BCD and OED are straight lines.
$AC = 6$ cm and $CD = 3$ cm.
$B\hat{A}C = 34°$.

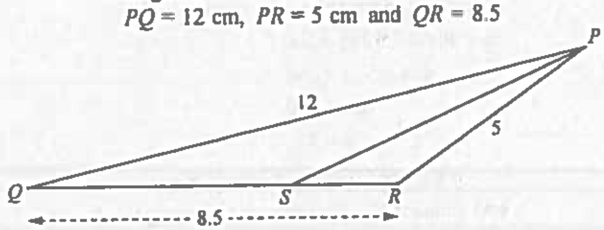
(i) Explain why $B\hat{C}A = 56°$. [1]
(ii) Find $C\hat{O}D$. [2]
(iii) Find $O\hat{C}E$. [1]

(b)



In the diagram, PS is the bisector of $Q\hat{P}R$.
QPT and QSR are straight lines.
RT is parallel to SP.

(i) Explain why $PT = PR$. [2]
(ii) This diagram shows part of the above diagram.
$PQ = 12$ cm, $PR = 5$ cm and $QR = 8.5$



It is given that $\dfrac{PQ}{PR} = \dfrac{QS}{SR}$.

Find SR. [3]

Thinking Process

(a) (i) Observe that AC is the diameter and therefore angle ABC is a right angle.
(ii) Observe that triangle COD is isosceles. Hence $\angle COD = \angle CDO$.
(iii) COE is an isosceles triangle.

(b) (i) Observe that PTR is an isosceles triangle. Prove $\angle PRT = \angle PTR$.
(ii) Substitute the values of PQ, PR and QS to find SR.

Solution

(a) (i) Given that AC is the diameter.

$\therefore \quad A\hat{B}C = 90°$

In ΔABC,

$B\hat{C}A + B\hat{A}C = 90°$

$B\hat{C}A + 34° = 90°$

$B\hat{C}A = 56°$

(ii) $OC = CD = 3$ cm (radius of circle)

$\therefore$ OCD is an isosceles triangle.

$C\hat{O}D + C\hat{D}O = B\hat{C}A$ (Ext. ∠ of a Δ = sum of opp. int ∠s.)

$2C\hat{O}D = 56 \quad (\because C\hat{O}D = C\hat{D}O)$

$C\hat{O}D = \dfrac{56}{2} = 28°$ **Ans.**

(iii) OCE is an isosceles triangle.

$\Rightarrow O\hat{C}E = O\hat{E}C$ (base ∠s of isosceles Δ)

$\therefore O\hat{C}E + O\hat{E}C + C\hat{O}E = 180°$

$2O\hat{C}E + 28° = 180°$

$O\hat{C}E = \dfrac{180° - 28°}{2}$

$= 76°$ **Ans.**

(b) (i) SP is parallel to RT

$\Rightarrow S\hat{P}R = P\hat{R}T$ (alternate ∠s)

$Q\hat{P}S = P\hat{T}R$ (corresponding ∠s)

PS is the angle bisector of QPR

$\Rightarrow Q\hat{P}S = S\hat{P}R$

$\therefore P\hat{R}T = P\hat{T}R$

$\Rightarrow \Delta PRT$ is isosceles.

$\therefore PT = PR$

(ii) $\frac{PQ}{PR} = \frac{QS}{SR}$

$\frac{12}{5} = \frac{8.5 - SR}{SR}$

$12SR = 5(8.5 - SR)$

$12SR = 42.5 - 5SR$

$17SR = 42.5$

$SR = 2.5$ Ans.

32 (J2016/P2/Q4)

(a) Triangle ABC has sides $AB = 8$ cm, $AC = 7$ cm and $BC = 12$ cm.

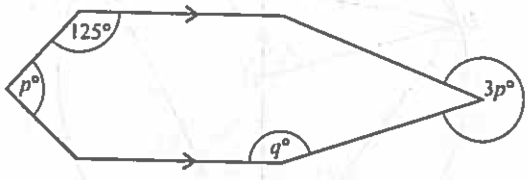
(i) Use a ruler and compasses to construct triangle ABC.
Side AB has been drawn for you.

[2]

(ii) Measure $B\hat{A}C$. [1]

(b) Calculate the interior angle of a regular 12-sided polygon. [2]

(c)

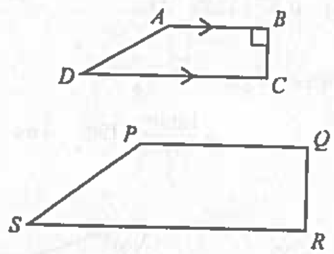


The diagram shows a hexagon with two parallel sides and one horizontal line of symmetry.

(i) Calculate p. [1]

(ii) Calculate q. [2]

(d)



Trapezium $PQRS$ is similar to trapezium $ABCD$.
AB is parallel to DC and $A\hat{B}C = 90°$.

$DC = 2AB$, $BC = \frac{1}{2}AB$ and $PQ = \frac{3}{4}DC$.

Given that $BC = x$ cm, find an expression, in terms of x, for the area of $PQRS$. [3]

Thinking Process

(a) (ii) Measure the angle by using a protractor.

(b) ✐ Apply, one interior angle $= \frac{(n-2)180}{n}$.

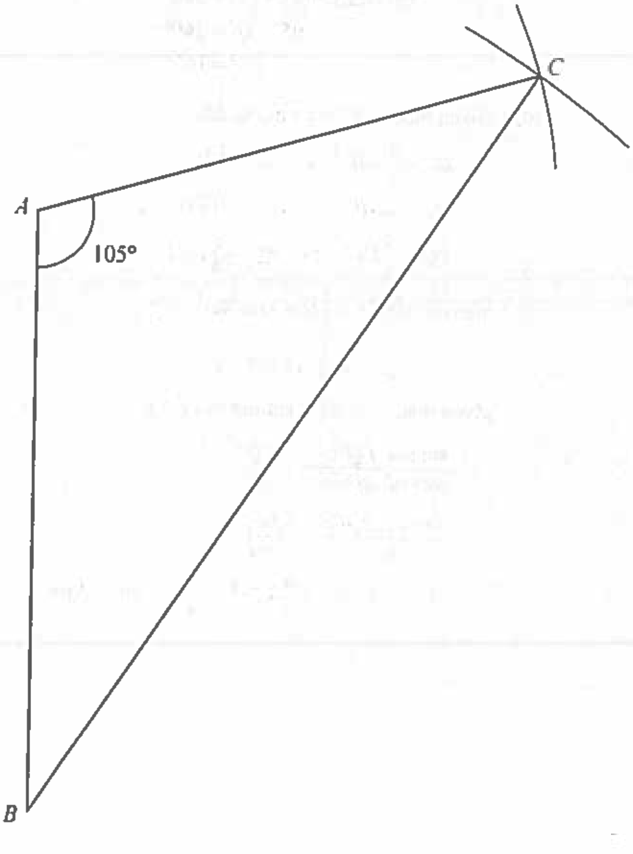
(c) (i) Draw the line of symmetry and consider the interior angles between parallel lines.

(ii) To find $q°$ ✐ apply sum of angles in a quadrilateral = 360°.

(d) Find the lengths of AB, BC and DC. Use the concept of area of similar figures to find the area of $PQRS$.

Solution

(a) (i)

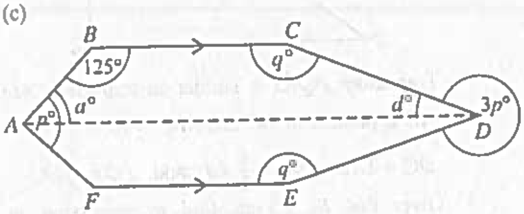


(ii) $B\hat{A}C = 105°$ Ans.

(b) Interior angle $= \frac{(12-2)180°}{12}$

$= \frac{1800°}{12} = 150°$ Ans.

(c)



(i) $a° + 125° = 180°$ (interior ∠s between ∥ lines)

$a° = 180° - 125°$
$= 55°$

$\therefore\ p° = 2 \times 55° = 110°$ Ans.

(ii) $\angle CDE = 360° - 3p°$
$= 360° - 3(110°) = 30°$

since AD is the line of symmetry.

$\therefore\ d° = \frac{30}{2} = 15°$

also, $\angle BCD = q°$

now, consider quadrilateral $ABCD$,

$a° + 125° + q° + d° = 360°$
$55° + 125° + q° + 15° = 360°$
$195° + q° = 360°$
$q° = 165°$ Ans.

(d) Given that, $BC = x$ cm, we have,

$BC = \frac{1}{2}AB \Rightarrow AB = 2BC = 2x$

$DC = 2AB \Rightarrow DC = 2(2x) = 4x$

$PQ = \frac{3}{4}DC \Rightarrow PQ = \frac{3}{4}(4x) = 3x$

area of $ABCD = \frac{1}{2}(x)(2x+4x)$

$= \frac{1}{2}(x)(6x) = 3x^2$

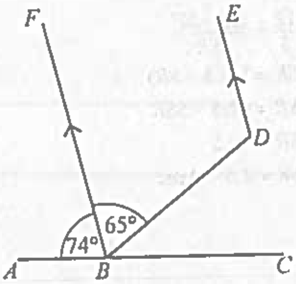
given that, $ABCD$ is similar to $PQRS$

$\frac{\text{area of } PQRS}{\text{area of } ABCD} = \left(\frac{PQ}{AB}\right)^2$

$\frac{\text{area of } PQRS}{3x^2} = \left(\frac{3x}{2x}\right)^2$

area of $PQRS = \frac{9}{4} \times 3x^2 = \frac{27}{4}x^2$ cm² Ans.

33 (N2016/P1/Q5)



In the diagram, ABC is a straight line and BF is parallel to DE.

$F\hat{B}A = 74°$ and $D\hat{B}F = 65°$.

(a) Find $C\hat{B}D$. [1]

(b) Find reflex $B\hat{D}E$. [1]

Thinking Process

(a) Apply, Sum of angles on a straight line = 180°

(b) ∵ BF is parallel to DE. Interior angles between parallel lines are supplementary.

Solution

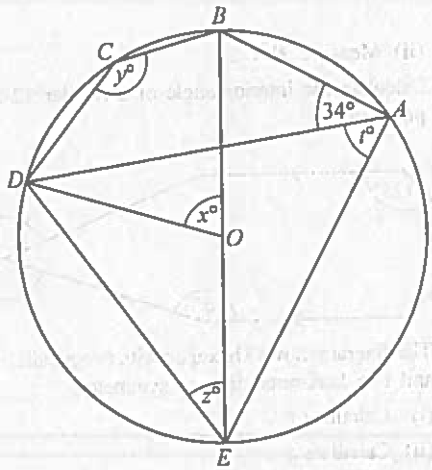
(a) $C\hat{B}D = 180° - 65° - 74°$
$= 41°$ Ans.

(b) $B\hat{D}E + 65° = 180°$ (interior ∠s between ∥ lines)

$B\hat{D}E = 180° - 65°$
$= 115°$

$\therefore$ reflex $B\hat{D}E = 360° - 115°$
$= 245°$ Ans.

34 (N2016/P1/Q24)



In the diagram, A, B, C, D and E lie on the circle, centre O.

BOE is a straight line.

$D\hat{A}B = 34°$.

(a) Find x. [1]

(b) Find y. [1]

(c) Find z. [1]

(d) Find t. [1]

Thinking Process

(a) $x° = 2\angle BAD$. ($\angle$ at centre is twice $\angle$ at the circumference).

(b) Note that $ABCD$ is a cyclic quadrilateral.

(c) Apply, angles in the same segment.

(d) $\angle BAE = 90°$ (right $\angle$ in semi-circle)

Solution

(a) $x° = 2 \times 34°$ ($\angle$ at centre is $2 \times \angle$ at circumference)

$= 68°$ **Ans.**

(b) $ABCD$ is a cyclic quadrilateral,

$y° + 34° = 180°$ (opp. $\angle$s of a cyclic quad. are supplementary)

$y° = 180° - 34°$

$= 146°$ **Ans.**

(c) $z° = B\hat{A}D$ ($\angle$s at the same segment)

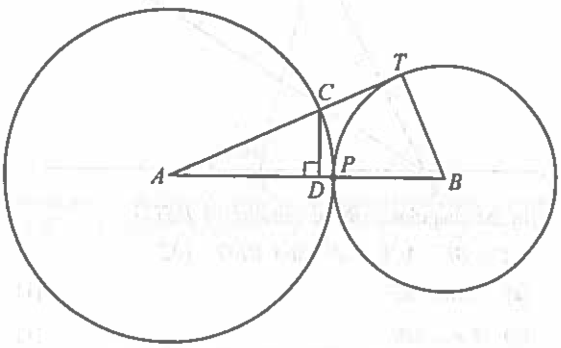
$= 34°$ **Ans.**

(d) $B\hat{A}E = 90°$ (right $\angle$ in semi-circle)

$\therefore$ $t° = 90° - 34°$

$= 56°$ **Ans.**

35 *(N2016/P2/Q5)*



In the diagram, A and B are the centres of two circles that touch at P.
The line ACT touches the small circle at T and intersects the large circle at C.

D is the point on AB such that $C\hat{D}A = 90°$.

(a) Complete the following, to show that triangle ACD is similar to triangle ABT.

In triangle ACD and triangle ABT

angle DAC = angle .... (same angle)

angle CDA = angle .... (..................................)

angle ACD = angle .... (two angles in a triangle are equal,so the third angles are equal)

Because the three pairs of angles are equal, the triangles are similar. [2]

(b) Given that the radii of the circles are 7 cm and 3 cm, calculate CD. [3]

Thinking Process

(a) Apply rules of similar triangles to fill in the missing angles.

(b) Apply rule of similarity: ratio of corresponding lengths is equal.

Solution

(a) angle DAC = angle ..TAB.. (same angle)

angle CDA = angle ..BTA.. (.....radius ⊥ tangent..)

angle ACD = angle ..ABT.. (two angles in a triangle are equal,so the third angles are equal)

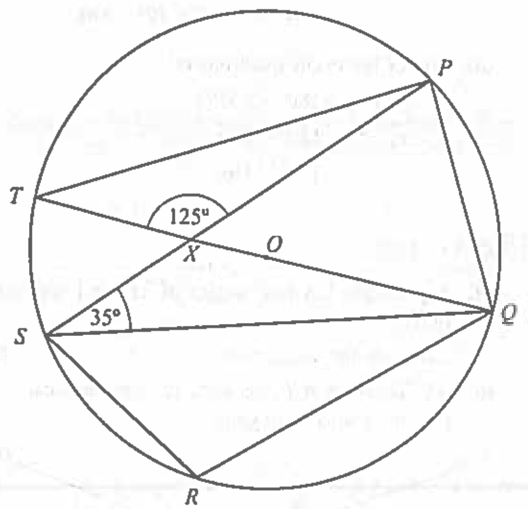
(b) ΔACD is similar to ΔABT.

$$\frac{CD}{BT} = \frac{AC}{AB}$$

$$CD = \frac{AC}{AB} \times BT$$

$$= \frac{7}{10} \times 3 = 2.1 \text{ cm} \quad \textbf{Ans.}$$

36 *(J2017/P1/Q17)*



In the diagram, P, Q, R, S and T lie on the circle.
QT is a diameter of the circle, centre O.
X is the point of intersection of PS and QT.
$P\hat{X}T = 125°$ and $P\hat{S}Q = 35°$.

(a) Complete the following statement with a geometrical reason.

$P\hat{T}Q = 35°$ because .... [1]

(b) Find $P\hat{Q}T$. [1]

(c) Find $S\hat{P}Q$. [1]

(d) Find $S\hat{R}Q$. [1]

Thinking Process

(a) $P\hat{T}Q$ and $P\hat{S}Q$ are angles in the same segment.

(b) Note that QT is the diameter and $Q\hat{P}T$ is a right-angle.

(c) To find $S\hat{P}Q$ find $T\hat{P}X$. Note that angle TPX + angle QPX = 90°.

(d) Consider the cyclic quadrilateral $PQRS$.

Solution

(a) $P\hat{T}Q = 35°$ because $P\hat{S}Q = 35°$. $P\hat{T}Q$ and $P\hat{S}Q$ are angles in the same segment.

(b) In ΔPQT,

$\angle QPT = 90°$ (rt. ∠ in semi-circle)

$\therefore \angle PQT = 90° - 35°$

$= 55°$ **Ans.**

(c) In ΔPXT,

$35° + 125° + \angle TPX = 180°$ (sum of ∠s in Δ)

$\angle TPX = 180° - 35° - 125°$

$= 20°$

$\angle TPX + \angle SPQ = 90°$ (rt. ∠ in semi-circle)

$\angle SPQ = 90° - \angle TPX$

$= 90° - 20° = 70°$ **Ans.**

(d) $PQRS$ is a cyclic quadrilateral

$\therefore \angle SRQ = 180° - \angle SPQ$

$= 180° - 70°$

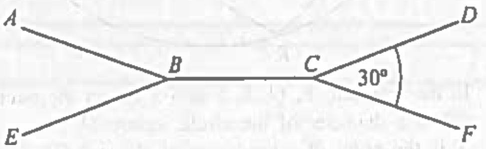
$= 110°$ **Ans.**

37 *(J2017/P1/Q19)*

(a) A pentagon has four angles of $2x°$ and one angle of $x°$.

Calculate the value of x. [2]

(b) $ABCD$ and $EBCF$ are parts of two identical regular n-sided polygons.



The two polygons are joined together along edge BC.

Angle $DCF = 30°$.

Calculate the value of n. [2]

Thinking Process

(a) To find x apply, sum of interior angles in a polygon = $(n - 2) \times 180°$.

(b) Find exterior angle of any one polygon, then apply, exterior angle = $\frac{360}{n}$.

Solution

(a) Sum of interior angles of a pentagon

$= (5-2) \times 180° = 540°$

$\therefore 2x° + 2x° + 2x° + 2x° + x° = 540°$

$9x° = 540°$

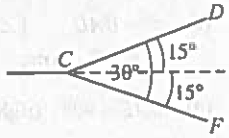
$x° = 60°$ **Ans.**

(b) Exterior angle of the polygon $= \frac{30°}{2} = 15°$

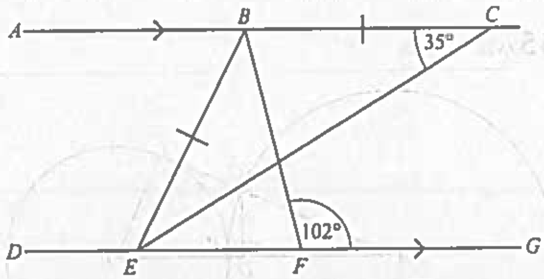
$\therefore 15° = \frac{360°}{n}$

$n = \frac{360°}{15°}$

$= 24$ **Ans.**



38 *(N2017/P1/Q7)*



In the diagram, ABC is parallel to $DEFG$.

$BC = BE$, $A\hat{C}E = 35°$ and $B\hat{F}G = 102°$.

(a) Find $C\hat{B}F$. [1]

(b) Find $A\hat{B}E$. [1]

Thinking Process

(a) $AC // DG$, therefore $\angle CBF$ and $\angle GFB$ are supplementary.

(b) BCE is an isosceles triangle, so $B\hat{C}E = B\hat{E}C$.

Solution

(a) $\angle CBF + 102° = 180°$ (int. ∠s between || lines)

$\angle CBF = 78°$ **Ans.**

(b) ΔCBE is isosceles.

$\therefore\ \angle BCE = \angle BEC = 35°$

$\angle CBE = 180° - 35° - 35°$ ($\angle$sum of a Δ)

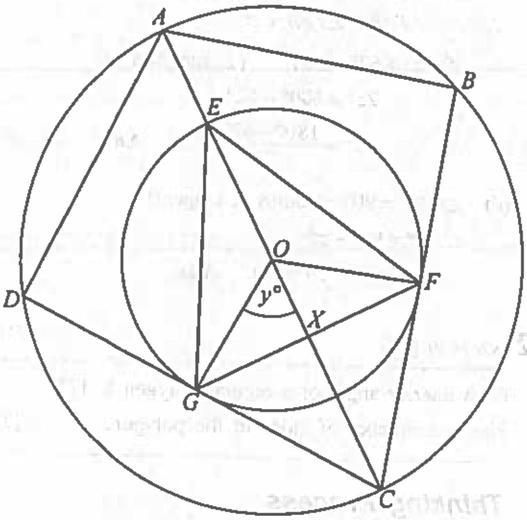
$= 110°$

$\angle ABE = 180° - \angle CBE$ ($\angle$s on a straight line)

$= 180° - 110°$

$= 70°$ Ans.

39 *(N2017/P2/Q8)*



The diagram shows two circles each with centre O.
A, B, C and D are points on the circumference of the large circle.
E, F and G are points on the circumference of the small circle.
CGD and CFB are tangents to the small circle.
Lines $AEOC$ and FG intersect at 90° at X.
$G\hat{O}X = y°$.

(a) Find each of these angles, as simply as possible, in terms of y.
Give reasons for your answers.

(i) $G\hat{E}O$ [2]

(ii) $G\hat{C}X$ [2]

(iii) $D\hat{A}B$ [2]

(b) Complete the sentence.
Triangle EGC is congruent to triangle .... [1]

(c) Prove that triangle ADC is similar to triangle OGC.
Give a reason for each statement you make. [2]

(d) What special type of quadrilateral is $AOGD$? [1]

(e) Find the ratio

(i) area of triangle OGC : area of triangle ADC, [1]

(ii) area of triangle OGC : area of quadrilateral $ABCD$. [1]

Thinking Process

(a) (i) $\angle GEO = \frac{1}{2}y°$. ($\angle$ at centre is twice $\angle$ at the circumference).

(ii) Note that $\angle OGC = 90°$ (radius $\perp$ tangent).

(iii) AC is the diameter. Therefore $\angle ADC = 90°$.

(c) Note that both triangles have their corresponding angles equal.

(d) Observe that OG is parallel to AD.

(e) (i) Apply rule of similarity: $\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{L_1}{L_2}\right)^2$

(ii) Note that triangles ABC and ADC are equal in area.

Solution

(a) (i) $G\hat{E}O = \frac{1}{2}y$.

because, angle at the centre is twice the angle at the circumference. Ans.

(ii) $G\hat{C}X = 90° - y$

because, angle between the radius OG and the tangent GC is 90°. Ans.

(iii) $D\hat{A}C = 90° - G\hat{C}X$

$= 90° - (90° - y) = y$

$D\hat{A}B = 2(D\hat{A}C) = 2y$

because, angle $A\hat{D}C = 90°$, since angle in a semicircle is always 90°. Ans.

(b) ΔEGC is congruent to ΔEFC. Ans.

(c) $D\hat{A}C = G\hat{O}C = y$

$D\hat{C}A = G\hat{C}O = 90° - y$ (common angle)

$A\hat{D}C = O\hat{G}C = 90°$

$\therefore$ ΔADC is similar to ΔOGC. Proved.

(d) Quadrilateral $AOGD$ is a trapezium. Ans.

(e) (i) Radius OG is perpendicular to chord DC.

$\therefore$ $DG = GC$

Area of ΔOGC : Area of ΔADC

$= (GC)^2 : (DC)^2$

$= (1)^2 : (2)^2$

$= 1 : 4$ Ans.

(ii) Area of ΔOGC : Area of quadrilateral $ABCD$

$=$ Area of ΔOGC : 2(Area of ΔADC)

$= 1 : 2(4)$

$= 1 : 8$ Ans.

40 (J2018/P1/Q14)

An irregular polygon has 22 sides.

(a) Calculate the sum of all its interior angles. [2]

(b) Two of the angles in the polygon are each 170°.
The remaining 20 angles are equal to each other.
Calculate the size of one of the 20 equal angles. [2]

Thinking Process

(a) Apply, sum of interior angles = $(n-2)180$.

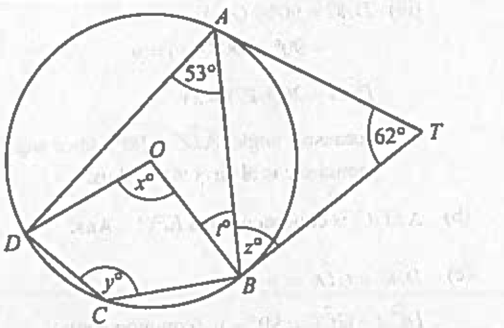
(b) Add up all the angles and equate it to 3600.

Solution

(a) Sum of interior angles $= (22-2)180$
$= 3600$ **Ans.**

(b) Let $x°$ be one of the 20 equal angles.
$\therefore\ 170° + 170° + 20x° = 3600$
$\Rightarrow\ 20x° = 3600 - 340°$
$x° = \dfrac{3600-340°}{20} = 163°$ **Ans.**

41 (J2018/P1/Q22)



The diagram shows a circle, centre O, that passes through A, B, C and D.
The tangents at A and B meet at T.
$A\hat{T}B = 62°$ and $D\hat{A}B = 53°$.

(a) Find x. [1]
(b) Find y. [1]
(c) Find z. [1]
(d) Find t. [1]

Thinking Process

(a) Apply, ∠ at centre is twice ∠ at the circumference).

(b) Note that $ABCD$ is a cyclic quadrilateral.

(c) Note that $TA = TB$ (tangents from external source)

(d) Note that $\angle OBT$ is a right angle.

Solution

(a) $x° = 2 \times 53°$ (∠ at centre is 2 × ∠ at circumference)
$= 106°$ **Ans.**

(b) $ABCD$ is a cyclic quadrilateral.
$y° + 53° = 180°$ (opp. ∠s of a cyclic quad. are supplementary)
$y° = 180° - 53°$
$= 127°$ **Ans.**

(c) ATB is an isosceles triangle.
$\therefore\ \angle TAB = \angle TBA = z°$
$z° + z° + 62° = 180°$ (∠ sum of a Δ)
$2z° = 180° - 62°$
$z° = \dfrac{180°-62°}{2} = 59°$ **Ans.**

(d) $\angle OBT = 90°$ (radius ⊥ tangent)
$\therefore\ t° = 90° - z°$
$= 90° - 59° = 31°$ **Ans.**

42 (N2018/P1/Q12)

Each interior angle of a regular polygon is 175°.
Find the number of sides in the polygon. [2]

Thinking Process

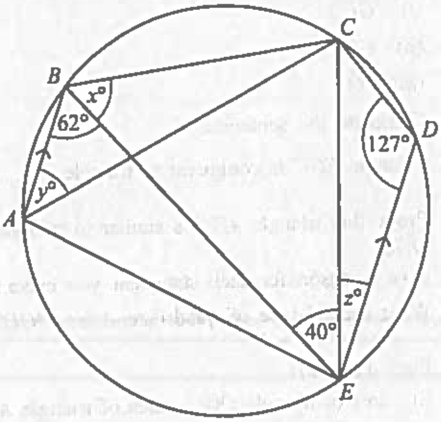
Find the exterior ∠ of the polygon. Apply, Number of sides $= \dfrac{360°}{1 \text{ ext. } \angle}$.

Solution

Exterior angle $= 180° - 175° = 5°$

$\therefore$ Number of sides $= \dfrac{360°}{5°} = 72$ **Ans.**

43 (N2018/P1/Q18)



In the diagram, A, B, C, D and E lie on the circle. AB is parallel to ED.
$A\hat{B}E = 62°$, $C\hat{D}E = 127°$ and $B\hat{E}C = 40°$.

(a) Find x. [1]

(b) Find y. [1]

(c) Find z. [1]

Thinking Process

(a) Note that $BCDE$ is a cyclic quadrilateral.

(b) $\angle BAC$ and $\angle BEC$ are angles in the same segment.

(c) $\angle ABE = \angle DEB$ (alternate angles).

Solution

(a) $BCDE$ is a cyclic quadrilateral,

$x° + 127° = 180°$ (opp. ∠s of a cyclic quad. are supplementary)

$x° = 180° - 127°$

$= 53°$ **Ans.**

(b) $\angle BAC = \angle BEC$ (∠s in the same segment)

$\therefore\ y° = 40°$ **Ans.**

(c) AB is parallel to ED.

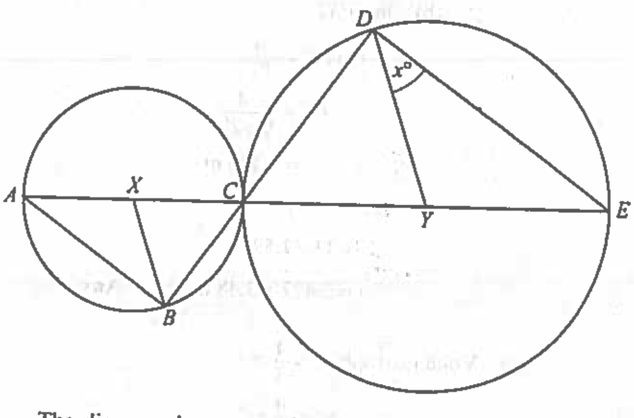
$\Rightarrow\ \angle ABE = \angle DEB$ (alternate ∠s)

$\therefore\ 62° = 40° + z°$

$z° = 62° - 40°$

$= 22°$ **Ans.**

44 *(N2018/P2/Q10)*



The diagram shows two circles that touch at C. A, B and C are points on the small circle, centre X. C, D and E are points on the large circle, centre Y.

$AXCYE$ and BCD are straight lines and $Y\hat{D}E = x°$.

(a) Prove that triangle BCX is similar to triangle DCY. Give a reason for each statement you make. [3]

(b) Find, in terms of x,

(i) $D\hat{C}Y$, [1]

(ii) $B\hat{X}A$. [1]

(c) Given that $BC = 3.5$ cm, $CX = 3.2$ cm and $CD = 5.6$ cm, find the length of AE. [3]

Thinking Process

(a) Prove that two angles of one triangle are equal to two angles of another triangle.

(b) (i) Note that ΔCDY is isosceles.

$\angle CDE = 90°$ (rt. ∠ in semi-circle).

(ii) $\angle BXA$ is the exterior angle of the isosceles triangle BCX.

(c) Apply rule of similarity: ratio of corresponding lengths is equal.

Solution

(a) $B\hat{C}X = D\hat{C}Y$ (vert. opp. ∠s)

ΔBCX and ΔDCY are isosceles triangles

$\therefore\ B\hat{C}X = C\hat{B}X$ and $D\hat{C}Y = C\hat{D}Y$

$\Rightarrow\ C\hat{B}X = C\hat{D}Y$

Since two corresponding angles are equal.

$\therefore\ \Delta BCX$ is similar to ΔDCY **Proved.**

(b) (i) $C\hat{D}E = 90°$ (right ∠ in semi-circle)

$\Rightarrow\ C\hat{D}Y = 90° - x°$

$D\hat{C}Y = C\hat{D}Y$ (ΔCDY is isosceles)

$\therefore\ D\hat{C}Y = 90° - x°$ **Ans.**

(ii) $B\hat{C}X = D\hat{C}Y = 90° - x°$

$C\hat{B}X = B\hat{C}X$ (ΔBCX is isosceles)

$\therefore\ C\hat{B}X = 90° - x°$

now,

$B\hat{X}A = C\hat{B}X + B\hat{C}X$ (Ext. ∠ of a Δ = sum of opp. int ∠s)

$\therefore\ B\hat{X}A = (90° - x°) + (90° - x°)$

$= 180° - 2x°$ **Ans.**

(c) ΔBCX is similar to ΔDCY

$\Rightarrow\ \dfrac{CY}{CX} = \dfrac{DC}{BC}$

$CY = \dfrac{DC}{BC} \times CX$

$= \dfrac{5.6}{3.5} \times 3.2 = 5.12$

$AE = AC + CE$

$= 2(CX) + 2(CY)$

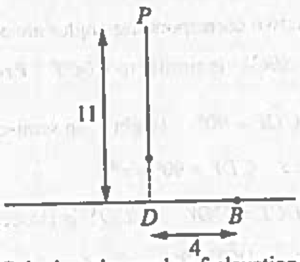
$= 2(3.2) + 2(5.12) = 16.64$ cm. **Ans.**

Topic 14

Trigonometry

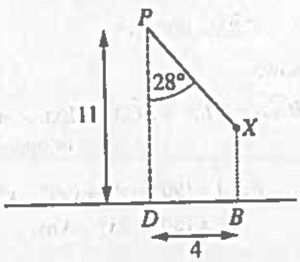
1 (J2009/P2/Q3)

(a) A heavy ball hangs from a point P, 11 m above horizontal ground, by means of a thin wire. The point D is on the ground vertically below P. The point B is on the ground 4 m from D.



(i) Calculate the angle of elevation of P from B. [2]

(ii) The ball swings, with the wire straight, in the vertical plane PDB.



When the ball is at X, directly above B, $D\hat{P}X = 28°$.

Calculate

(a) PX, [2]

(b) XB. [3]

(b) [The volume of a sphere is $\frac{4}{3}\pi r^3$.]

The ball is a sphere of volume 96 cm³. Calculate its radius. [2]

Thinking Process

(a) (i) Find angle PBD.

(ii) (a) To find PX draw a horizontal line from X to PD and apply $\sin\theta = \frac{\text{opp}}{\text{hyp}}$.

(b) Apply $\tan\theta = \frac{\text{opp}}{\text{adj}}$ to the triangle formed in part (ii)(a) above.

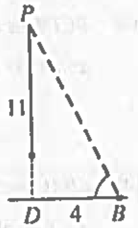
(b) Apply formula of volume of a sphere.

Solution

(a) (i) $\tan P\hat{B}D = \frac{11}{4}$

$P\hat{B}D = 70.0°$

$\therefore$ angle of elevation $= 70°$ **Ans.**

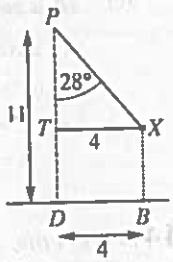


(ii) (a) In ΔPXT

$\sin 28° = \frac{4}{PX}$

$PX = \frac{4}{\sin 28°}$

$= 8.52$ m **Ans.**



(b) In ΔPXT

$\tan 28° = \frac{4}{PT}$

$PT = \frac{4}{\tan 28°}$

$= 7.523$ m

$BX = 11 - PT$

$= 11 - 7.523$

$= 3.477 \approx 3.48$ m (3sf) **Ans.**

(b) Volume of sphere $= \frac{4}{3}\pi r^3$

$96 = \frac{4}{3}\pi r^3$

$r^3 = \frac{96 \times 3}{4\pi}$

$= 22.915$

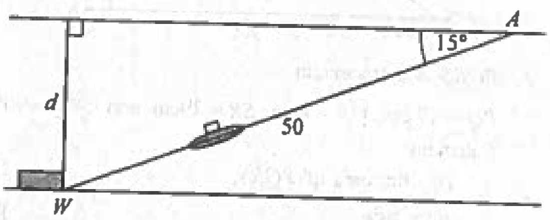
$r = 2.84$

$\therefore$ radius $= 2.84$ cm **Ans.**

2 *(N2009/P2/Q3)*

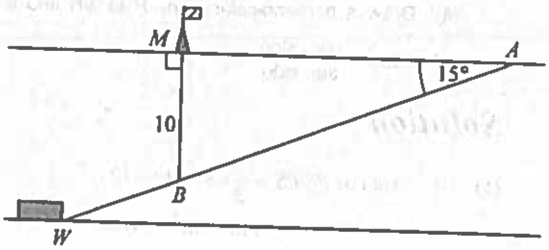
A small submarine dived in a straight line from a point A on the surface to examine an object at the point W on the seabed.

(a)



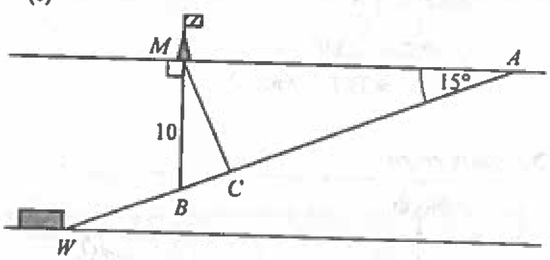
It dived at an angle of 15° to the horizontal and reached W after travelling 50 m.
Calculate the depth, d metres, of the seabed at W. [2]

(b)



A marker is at the point M on the surface.
When at B, the submarine was 10 m vertically below M.
Calculate AB. [3]

(c)



When at C, the submarine was at its nearest point to M.

(i) Find $B\hat{M}C$. [1]

(ii) Calculate CM. [2]

Thinking Process

(a) Apply $\sin\theta = \frac{\text{opp}}{\text{hyp}}$.

(b) Apply $\sin\theta = \frac{\text{opp}}{\text{hyp}}$.

(c) (i) To find angle BMC ∮ find angle MBA.

(ii) Apply $\sin M\hat{B}C = \frac{\text{opp}}{\text{hyp}}$.

Solution

(a) $\sin 15° = \frac{d}{50}$

$d = \sin 15° \times 50$

$= 12.941 \approx 12.9$ m (3sf) **Ans.**

(b) In ΔAMB,

$\sin 15° = \frac{10}{AB}$

$AB = \frac{10}{\sin 15°}$

$= 38.637 \approx 38.6$ m (3sf) **Ans.**

(c) (i) In ΔAMB, $A\hat{M}B = 90°$

$\therefore$ $M\hat{B}A = 90° - 15° = 75°$

Point C is nearest to point M.

$\therefore$ $M\hat{C}B = 90°$

In ΔBMC,

$B\hat{M}C = 90° - M\hat{B}C$

$= 90° - 75°$

$= 15°$ **Ans.**

(ii) In ΔBMC,

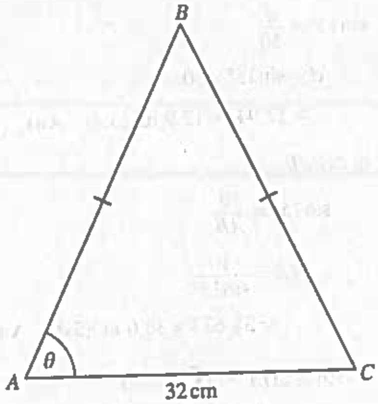
$\sin M\hat{B}C = \frac{CM}{BM}$

$\sin 75° = \frac{CM}{10}$

$CM = \sin 75° \times 10$

$= 9.659 \approx 9.66$ m (3sf) **Ans.**

3 *(J2010/P1/Q23)*



$\sin\theta$	$\frac{15}{17}$
$\cos\theta$	$\frac{8}{17}$
$\tan\theta$	$\frac{15}{8}$

ABC is an isosceles triangle with $AB = BC$ and $AC = 32$ cm.
Using as much information from the table as is necessary, calculate

(a) AB, [2]

(b) the area of triangle ABC. [2]

Thinking Process

(a) Use $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$

(b) Apply, area of $\Delta = \frac{1}{2}ab\sin C$

Solution

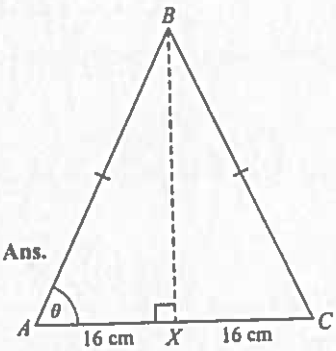
(a) Consider ΔABX

$\cos\theta = \frac{AX}{AB}$

$\frac{8}{17} = \frac{16}{AB}$

$AB = 16 \times \frac{17}{8}$

$= 34$ cm **Ans.**



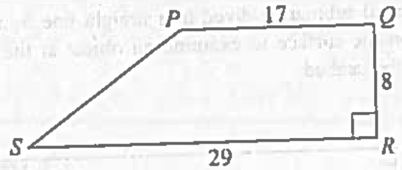
(b) Area of $\Delta ABC = \frac{1}{2}(AB)(AC)\sin\theta$

$= \frac{1}{2}(34)(32)(\frac{15}{17})$

$= 480\text{ cm}^2$ **Ans.**

4 *(J2010/P2/Q7a)*

(a)



$PQRS$ is a trapezium.

$PQ = 17$ cm, $QR = 8$cm, $SR = 29$cm and $S\hat{R}Q = 90°$.

Calculate

(i) the area of $PQRS$, [1]

(ii) $P\hat{S}R$. [2]

Thinking Process

(a) (i) Apply, area of trapezium $= \frac{1}{2}h$(sum of // sides)

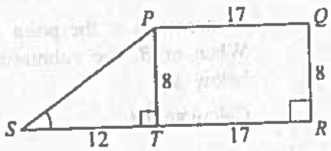
(ii) Draw a perpendicular from P to SR and use $\tan\theta = \frac{\text{opp. side}}{\text{adj. side}}$

Solution

(a) (i) Area of $PQRS = \frac{1}{2} \times 8 \times (29 + 17)$

$= 184\text{ cm}^2$ **Ans.**

(ii) In ΔPST



$PT = 8$ cm,

$ST = 29 - 17$

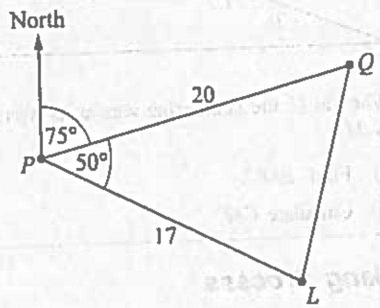
$= 12$ cm.

$\tan P\hat{S}T = \frac{8}{12}$

$\therefore\ P\hat{S}T = 33.69°$

$\approx 33.7°$ **Ans.**

5 *(J2010/P2/Q9)*



The diagram shows two ports, P and Q, and a lighthouse L.

$PQ = 20$ km, $PL = 17$ km, $Q\hat{P}L = 50°$ and the bearing of Q from P is 075°.

(a) Find the bearing of P from L. [1]

(b) Calculate QL. [4]

(c) (i) Calculate $P\hat{L}Q$. [3]

(ii) Hence find the bearing of Q from L. [1]

(d) A boat leaves P and sails in a straight line to Q.

(i) It takes 4 hours and 53 minutes to sail from P to Q.
It arrives at Q at 02 23.
At what time does it leave P? [1]

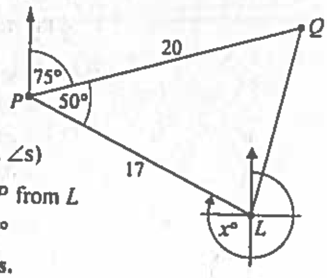
(ii) Calculate the shortest distance between the boat and the lighthouse. [2]

Thinking Process

(a) Draw the North line from L.
(b) Apply cosine rule.
(c) (i) Apply sine rule.
(ii) To find the bearing identify the angle to be found at P.
(d) (i) Count 4 hours, 53 minutes backwards.
(ii) Note that shortest distance is the perpendicular diatance from L to PQ.

Solution

(a)



$x° = 75° + 50°$
$= 125°$ (alt. ∠s)

∴ Bearing of P from L
$= 180° + 125°$
$= 305°$ **Ans.**

(b) Using Cosine Rule

$QL = \sqrt{17^2 + 20^2 - 2(17)(20)\cos 50}$
$= \sqrt{689 - 437.0956}$
$= \sqrt{251.90}$
$= 15.871 \approx 15.9$ km (3 sf) **Ans.**

(c) (i) Using Sine Rule,

$\frac{QL}{\sin Q\hat{P}L} = \frac{PQ}{\sin P\hat{L}Q}$

$\frac{15.871}{\sin 50°} = \frac{20}{\sin P\hat{L}Q}$

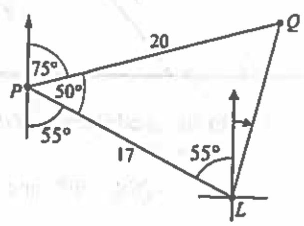
$\sin P\hat{L}Q = \frac{20 \sin 50°}{15.871}$
$= 0.96534$

∴ $P\hat{L}Q = 74.9°$ (1 dp) **Ans.**

(ii)

$\angle PLQ = 74.9°$

Bearing of Q from L
$= 74.9° - 55°$
$= 19.9°$ **Ans.**



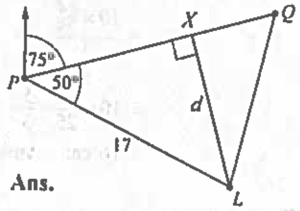
(d) (i) The boat leaves P at $0223 - 0453$
$= 2583 - 0453$
$= 2130$ **Ans.**

(ii) Let d be the shortest distance.

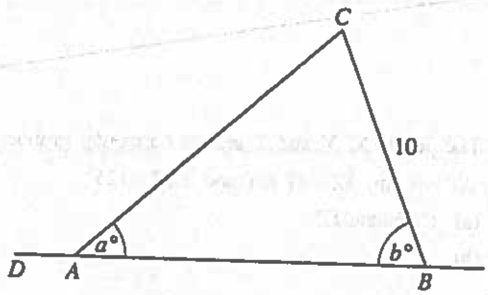
In ΔPLX

$\sin 50° = \frac{d}{17}$

$d = 17 \sin 50°$
$= 13.0$ km **Ans.**



6 *(N2010/P1/Q26)*



$x°$	$a°$	$b°$
$\sin x°$	$\frac{3}{5}$	$\frac{24}{25}$
$\cos x°$	$\frac{4}{5}$	$\frac{7}{25}$
$\tan x°$	$\frac{3}{4}$	$\frac{24}{7}$

In the diagram, DAB is a straight line.
$BC = 10$ cm, $C\hat{A}B = a°$ and $C\hat{B}A = b°$.
Use as much information given in the table as is necessary to answer the following questions.

(a) Write down the value of $\cos D\hat{A}C$. [1]
(b) Calculate AC. [3]

Thinking Process

(a) $\cos(180 - \theta) = -\cos\theta$
(b) Apply Sine Rule.

Solution

(a) $\cos D\hat{A}C = \cos(180° - a°)$
$= -\cos a°$
$= -\frac{4}{5}$ **Ans.**

(b) Using Sine rule,

$\frac{\sin a^\circ}{10} = \frac{\sin b^\circ}{AC}$

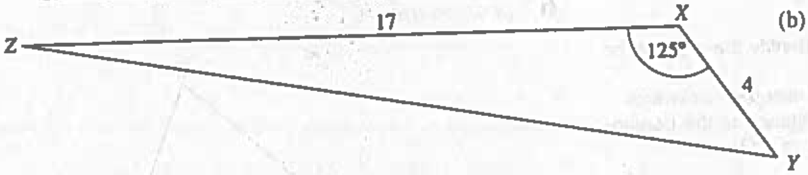
$AC = \frac{10 \sin b^\circ}{\sin a^\circ}$

$= \frac{10 \times \frac{24}{25}}{\frac{3}{5}}$

$= 10 \times \frac{24}{25} \times \frac{5}{3}$

$= 16$ cm **Ans.**

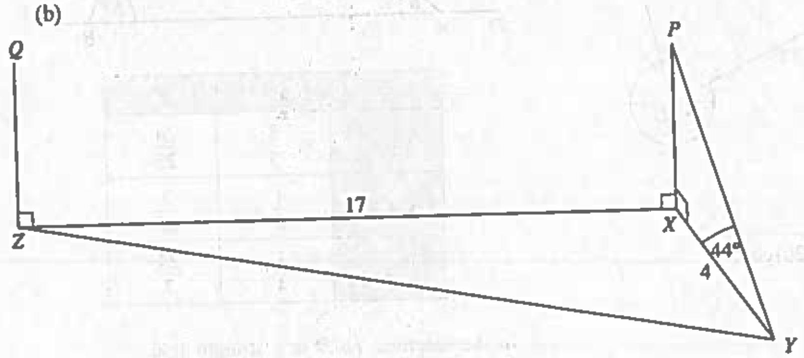
7 *(N2010/P2/Q11)*



The points X, Y and Z are on horizontal ground.
$XY = 4$ km, $XZ = 17$ km and $Y\hat{X}Z = 125°$.

(a) Calculate YZ [4]

(b)



The points P and Q are the same height vertically above X and Z respectively.

(i) When an aircraft was at P, its angle of elevation from Y was 44°.
Calculate PX.
Give your answer in metres, correct to the nearest 100 metres. [3]

(ii) The aircraft took 2 minutes 54 seconds to fly from P to Q.

(a) The aircraft reached its destination 39 minutes 6 seconds after flying over Q.
The flight ended at 15 03.
At what time did the aircraft fly over P? [2]

(b) Calculate the average speed of the aircraft as it flew from P to Q.
Give your answer in kilometres per hour. [3]

Thinking Process

(a) Apply Cosine rule.

(b) (i) Apply $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$.

(ii) (a) Add 2 minutes 54 seconds with 39 minutes 6 seconds and subtract it from 1503.

(b) Average speed = $\frac{\text{total distance}}{\text{total time}}$

Solution

(a) Using cosine rule.

$YZ = \sqrt{(17)^2 + (4)^2 - 2(17)(4)\cos 125°}$

$= \sqrt{305 + 78}$

$= 19.57 \approx 19.6$ km (3 sf) **Ans.**

(b) (i) $\tan 44° = \frac{PX}{4}$

$PX = 4 \tan 44°$

$= 3.863$ km

$= 3863$ m

$\therefore$ $PX = 3900$ m to the nearest 100 m. **Ans.**

(ii) (a) 2 min. 54 seconds
+39 min. 6 seconds
= 41 min. 60 seconds
= 42 minutes

$\therefore$ The aircraft flew over P at.
$1503 - 0042$
$= 1463 - 0042$
$= 1421$ **Ans.**

(b) Time to fly from P to Q
= 2 minutes 54 seconds
= 174 seconds
= 0.04833 hours.

$\therefore$ Average speed

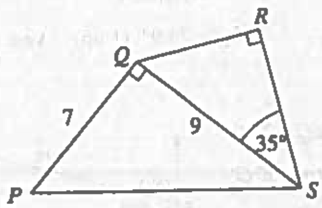
$= \frac{17}{0.04833}$

$= 351.748$

≈ 352 km/h **Ans.**

8 *(N2010/P2/Q4)*

(a)



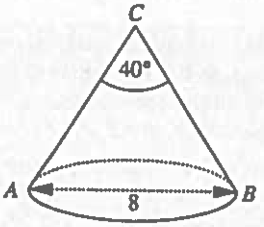
In the quadrilateral $PQRS$, $PQ = 7$cm and $QS = 9$cm.
$P\hat{Q}S = Q\hat{R}S = 90°$ and $Q\hat{S}R = 35°$.

Calculate

(i) $S\hat{P}Q$. [2]

(ii) RS. [2]

(b) **[The area of the curved surface of a cone of radius r and slant height l is πrl]**



The diagram shows a cone ABC.

The diameter $AB = 8$ cm and $A\hat{C}B = 40°$.

Calculate the curved surface area of this cone. [3]

Thinking Process

(a) (i) Apply $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

(ii) Apply $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

(b) Find the slant height l. Draw a perpendicular from C to AB and apply $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$.

Solution

(a) (i) $\tan S\hat{P}Q = \frac{9}{7}$

$S\hat{P}Q = 52.1°$ **Ans.**

(ii) $\cos 35° = \frac{RS}{9}$

$RS = 9\cos 35°$

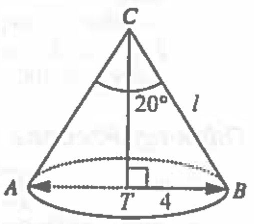
$= 7.37$ cm **Ans.**

(b) Consider ΔCBT

$\sin 20° = \frac{4}{l}$

$l = \frac{4}{\sin 20°}$

$= 11.695$



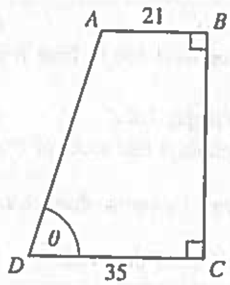
$\therefore$ slant height, $l = 11.695$ cm

Curved surface area $= \pi rl$

$= 3.142 \times 4 \times 11.695$

$= 146.98 \approx 147$ cm^2 **Ans.**

9 *(J2011/P1 Q11)*



$\sin\theta$	$\frac{24}{25}$
$\cos\theta$	$\frac{7}{25}$
$\tan\theta$	$\frac{24}{7}$

$ABCD$ is a trapezium with $AB = 21$ cm and $CD = 35$ cm.

$A\hat{B}C = B\hat{C}D = 90°$ and $A\hat{D}C = \theta$.

Using as much information from the table as is necessary, calculate AD. [2]

Thinking Process

To find AD draw a perpendicular from A to DC.

Solution

In ΔADN

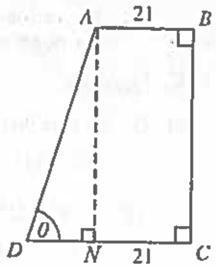
$DN = 35 - 21 = 14$ cm.

$\cos\theta = \frac{DN}{AD}$

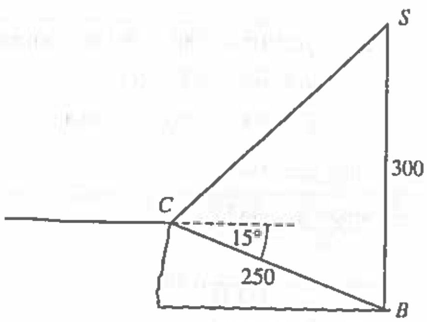
$\frac{7}{25} = \frac{14}{AD}$

$AD = 14 \times \frac{25}{7}$

$= 50$ cm **Ans.**



10 *(J2011/P2 Q10)*



The angle of depression of a buoy, B, from a point, C, on a cliff is 15°.

The distance BC is 250 m.

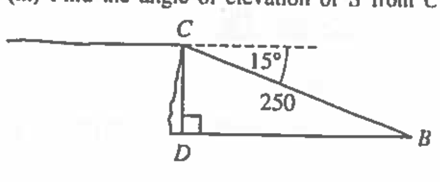
A seagull, S, hovers so that it is vertically above B and $SB = 300$ m.

(a) (i) Find $S\hat{B}C$. [1]

(ii) Find SC. [3]

(iii) Find the angle of elevation of S from C. [3]

(b)



D is a marker at sea level vertically below C and due west of B.

(i) Find DB. [2]

(ii) M is a marker at sea level 200 m from B and $D\hat{B}M = 30°$.
Find the area of triangle DBM. [2]

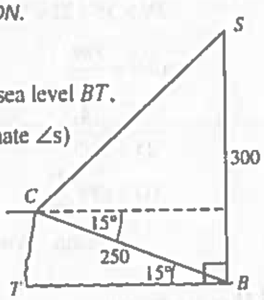
(iii) N is a marker at sea level due south of B and $DN = 450$ m.
A boat sails on a circular course through D, B and N.
Write down the radius of the circle. [1]

Thinking Process

(a) (i) Note that SB is at right angles to the sea level.
(ii) Apply cosine rule.
(iii) Apply sine rule.

(b) (i) Apply $\cos\theta = \frac{adj}{hyp}$.
(ii) Apply area of $\Delta = \frac{1}{2} \times a \times b \times \sin c$.
(iii) The radius of the circle is at the midpoint of the hypotenuse DN.

Solution



(a) (i) SB is at 90° to the sea level BT.

$C\hat{B}T = 15°$ (alternate ∠s)

$\therefore\ S\hat{B}C = 90° - 15°$

$= 75°$ **Ans.**

(ii) In ΔSBC,
using cosine rule,

$SC = \sqrt{(250)^2 + (300)^2 - 2(250)(300)\cos 75°}$

$= \sqrt{152500 - 38822.857}$

$= 337.16 \approx 337$ m (3sf) **Ans.**

(iii) using sine rule,

$\frac{\sin S\hat{C}B}{300} = \frac{\sin 75°}{337.16}$

$\sin S\hat{C}B = \frac{\sin 75°}{337.16} \times 300$

$\sin S\hat{C}B = 0.85947$

$S\hat{C}B = 59.3°$

$\therefore$ angle of elevation of S from C

$= 59.3° - 15° = 44.3°$ **Ans.**

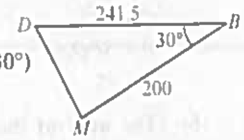
(b) (i) In ΔCBD,

$C\hat{B}D = 15°$ (alternate ∠s)

$\cos C\hat{B}D = \frac{DB}{CB}$

$\cos 15° = \frac{DB}{250}$

$\therefore\ DB = \cos 15° \times 250 = 241.5$ m **Ans.**



(ii) Area of ΔDBM

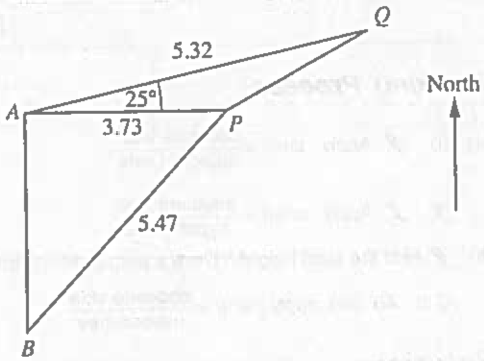
$= \frac{1}{2}(241.5)(200)(\sin 30°)$

$= 12075\ m^2$ **Ans.**

(iii) Radius of circle $= \frac{450}{2} = 225$ m **Ans.**

> In a right angled triangle, if a circle is drawn such that it passes through each of the three vertices of the triangle, then the centre of the circle is always at the mid-point of the triangle's hypotenuse.
> In right ΔDBN, DN is the hypotenuse, therefore the radius of the circle through D, B and N is $\frac{450}{2} = 225$ m.

11 (N2011/P2 Q2)



The diagram shows four points, A, B, P and Q, at sea.
B is due South of A and P is due East of A.
$AP = 3.73$ km, $BP = 5.47$ km, $AQ = 5.32$ km and $P\hat{A}Q = 25°$.

(a) Calculate $A\hat{B}P$. [2]

(b) Calculate PQ. [4]

(c) A boat sailed in a straight line from Q to A.

(i) Find the bearing of A from Q. [1]

(ii) A lighthouse is situated at A.
The top of the lighthouse is 30 m above sea level.
Calculate the angle of depression of the boat from the top of the lighthouse when the boat is 100 m from A. [2]

Thinking Process

(a) Use $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$.

(b) Apply cosine rule.

(c) (i) Draw North-line at Q.
(ii) Draw a right triangle with a base of 100m and height of 30m.

Solution

(a) $\sin A\hat{B}P = \frac{AP}{BP}$

$= \frac{3.73}{5.47}$

$\therefore\ A\hat{B}P = 42.99° \approx 43°$ **Ans.**

(b) using cosine rule,

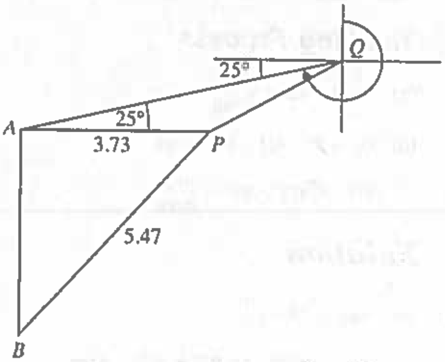
$PQ = \sqrt{(3.73)^2 + (5.32)^2 - 2(3.73)(5.32)\cos 25°}$

$= \sqrt{42.215 - 35.969}$

$= \sqrt{6.246}$

$= 2.499 \approx 2.50$ km **Ans.**

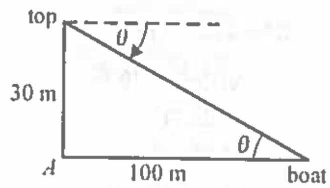
(c) (i)



Bearing of A from $Q = 270° - 25°$

$= 245°$ **Ans.**

(ii)

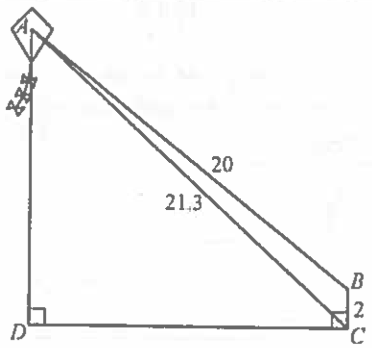


$\tan\theta = \dfrac{30}{100}$

$\theta = 16.7°$

∴ angle of depression from the top of the lighthouse = 16.7° **Ans.**

12 *(J2012 P2 Q8)*



A kite is attached at A to a 20 m length of string and the other end of the string is held at B so that the string is a straight line.

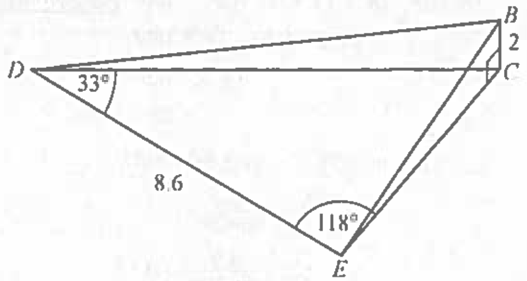
B is 2 m above the ground at C and $AC = 21.3$ m.
D is the point at ground level directly below A such that $A\hat{D}C = B\hat{C}D = 90°$.

(a) Calculate

(i) $A\hat{B}C$, [3]

(ii) AD [3]

(b)



E is another point on the level ground such that $DE = 8.6$ m, $E\hat{D}C = 33°$ and $C\hat{E}D = 118°$.

Calculate

(i) $D\hat{C}E$ [1]

(ii) CE, [3]

(iii) the angle of elevation of B from E. [2]

Thinking Process

(a) (i) ✐ Apply cosine rule.

(ii) Apply sine rule to find angle ACB. Find angle ACD and use $\sin\theta = \dfrac{AD}{AC}$ to find AD.

(b) (i) ✐ Apply sum of angles in a triangle.

(ii) To find CE ✐ apply sine rule.

(iii) Apply $\tan\theta = \dfrac{\text{perp.}}{\text{base}}$

Solution

(a) (i) Using Cosine Rule

$\cos A\hat{B}C = \dfrac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$

$= \dfrac{(20)^2 + (2)^2 - (21.3)^2}{2(20)(2)}$

$= \dfrac{-49.69}{80}$

$= -0.621$

∴ $A\hat{B}C = 128.39° \approx 128.4°$ **Ans.**

(ii) In ΔABC, using Sine Rule.

$\dfrac{\sin A\hat{C}B}{AB} = \dfrac{\sin A\hat{B}C}{AC}$

$\dfrac{\sin A\hat{C}B}{20} = \dfrac{\sin 128.39°}{21.3}$

$\sin A\hat{C}B = \dfrac{20 \sin 128.39°}{21.3}$

$= 0.736$

∴ $A\hat{C}B = 47.4°$

$A\hat{C}D = 90° - 47.4° = 42.6°$

In ΔADC,

$\sin A\hat{C}D = \dfrac{AD}{AC}$

$\sin 42.6° = \dfrac{AD}{21.3}$

$AD = 21.3 \sin 42.6°$

$= 14.4$ m **Ans.**

(b) (i) $D\hat{C}E + C\hat{E}D + E\hat{D}C = 180°$ (∠ sum of a Δ)

$D\hat{C}E + 118° + 33° = 180°$

$D\hat{C}E = 180° - 118° - 33°$

$= 29°$ **Ans.**

(ii) In ΔDCE, using Sine Rule.

$$\frac{CE}{\sin 33°} = \frac{8.6}{\sin 29°}$$

$$CE = \frac{8.6}{\sin 29°} \times \sin 33°$$

$$= \frac{8.6}{0.4848} \times 0.5446$$

$= 9.66 \approx 9.7$ m **Ans.**

(iii) In ΔBCE,

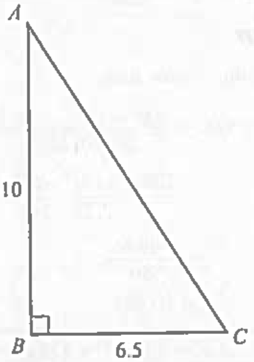
$$\tan B\hat{E}C = \frac{BC}{CE}$$

$$= \frac{2}{9.66}$$

$B\hat{E}C = 11.7°$

∴ angle of elevation of B from E is $11.7°$ **Ans.**

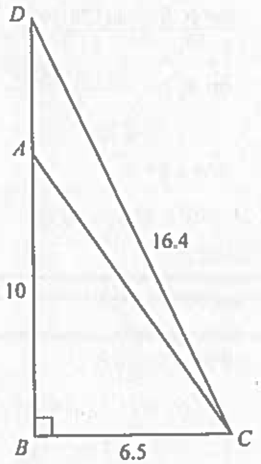
13 (N2012/P2/Q1)



In triangle ABC, $AB = 10$ m, $BC = 6.5$ m and $A\hat{B}C = 90°$.

(a) Find $A\hat{C}B$. [2]

(b)



D is the point on BA produced such that $CD = 16.4$ m.

(i) Find AD.
Give your answer in metres and centimetres, correct to the nearest centimetre. [3]

(ii) Find $D\hat{C}B$. [2]

Thinking Process

(a) Apply $\tan\theta = \frac{\text{opp.}}{\text{adj.}}$

(b) (i) $AD = BD - AB$

(ii) Apply $\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$

Solution

(a) $\tan A\hat{C}B = \frac{10}{6.5}$

∴ $A\hat{C}B = 56.97° \approx 57°$ **Ans.**

(b) (i) In ΔBCD,

$$BD = \sqrt{DC^2 - BC^2}$$

$$= \sqrt{(16.4)^2 - (6.5)^2}$$

$$= \sqrt{226.71}$$

$= 15.056$ m

∴ $AD = BD - AB$

$= 15.056 - 10$

$= 5.056 \approx 5.06$ m

$= 5$ m 6 cm. **Ans.**

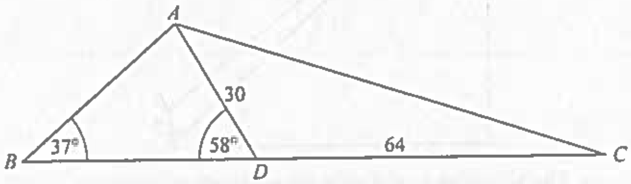
(ii) In ΔBCD,

$$\sin D\hat{C}B = \frac{BD}{DC}$$

$$= \frac{15.056}{16.4}$$

∴ $D\hat{C}B = 66.64° \approx 66.6°$ **Ans.**

14 (N2012/P2/Q9)



A, B, C and D are four points on level ground.
BDC is a straight line.
$AD = 30$ m and $DC = 64$ m.
$A\hat{B}D = 37°$ and $A\hat{D}B = 58°$.

(a) Calculate AB. [3]

(b) Calculate AC. [4]

(c) Calculate the area of triangle ADC. [2]

(d) A vertical tower stands at A.
P is the point on the line BC such that the angle of depression from the top of the tower to the line BC is greatest.
Given that this angle of depression is 34°, calculate the height of the tower. [3]

Thinking Process

(a) Apply sine rule.
(b) Apply cosine rule find angle ADC.
(c) Recall: Area of $\Delta ABC = \frac{1}{2}ab\sin c$.
(d) The greatest angle of depression (or elevation) from the top as seen from BC occurs at a point where the distance from A to BC is shortest.

Solution

(a) In ΔABD, using sine rule,

$$\frac{\sin 37^\circ}{30} = \frac{\sin 58^\circ}{AB}$$

$$AB = \sin 58^\circ \times \frac{30}{\sin 37^\circ}$$

$$= 42.275$$

$$\approx 42.3 \text{ m} \quad \textbf{Ans.}$$

(b) In ΔADC,

$A\hat{D}C = 180^\circ - 58^\circ = 122^\circ$

using cosine rule,

$$AC = \sqrt{30^2 + 64^2 - 2(30)(64)\cos 122^\circ}$$

$$= \sqrt{4996 + 2034.89}$$

$$= \sqrt{7030.89}$$

$$= 83.85 \approx 83.9 \text{ m} \quad \textbf{Ans.}$$

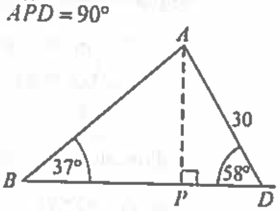
(c) Area of $\Delta ADC = \frac{1}{2}(30)(64)(\sin 122^\circ)$

$$= 814.126$$

$$\approx 814 \text{ m}^2 \quad \textbf{Ans.}$$

(d) For greatest angle of depression, point P is nearest to A, therefore ΔAPD is a right angled triangle with $A\hat{P}D = 90^\circ$

In ΔAPD,



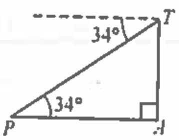
$$\sin 58^\circ = \frac{AP}{30}$$

$$AP = 30 \times \sin 58^\circ$$

$$= 25.44 \text{ m}$$

Let T be the top of the tower

consider ΔTAP



$$\tan T\hat{P}A = \frac{TA}{AP}$$

$$\tan 34^\circ = \frac{TA}{25.44}$$

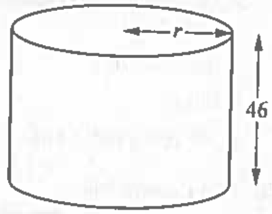
$$TA = 25.44 \times \tan 34^\circ$$

$$= 17.159 \approx 17.2$$

$\therefore$ height of tower = 17.2 m **Ans.**

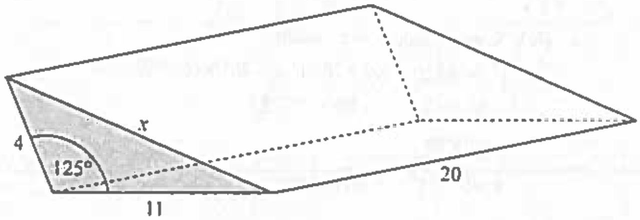
15 (J2013/P2/Q12)

(a)



A cylindrical tank of height 46 cm and radius r cm has a capacity of 70 litres.
Find the radius correct to the nearest centimetre. [3]

(b)



A triangular prism has length 20 cm.
The sides of the shaded cross-section are 4 cm, 11 cm and x cm.
The angle between the sides of length 4 cm and 11 cm is 125°.

(i) Calculate the area of the shaded cross-section. [2]
(ii) Calculate the volume of the prism. [1]
(iii) Calculate x. [4]
(iv) Calculate the surface area of the prism. [2]

Thinking Process

(a) Apply formula for volume of a cylinder.
(b) (i) Apply, area of $\Delta = \frac{1}{2}ab\sin C$
(ii) Volume of prism = base area x height.
(iii) Apply Cosine rule.
(iv) Find areas of all the 5 faces of the prism.

Solution

(a) 1 litre = 1000 cm^3

70 litres = 70000 cm^3

volume of cylinder $= \pi r^2 h$

$$70000 = \pi r^2(46)$$

$$r^2 = \frac{70000}{\pi(46)}$$

$$r^2 = 484.322$$

$$r = 22 \text{ cm} \quad \textbf{Ans.}$$

(b) (i) Shaded area $= \frac{1}{2} \times 4 \times 11 \times \sin 125^\circ$

$$= 18.0213$$

$$\approx 18.02 \text{ cm}^2 \quad \textbf{Ans.}$$

(ii) Volume of the prism

$= $ cross-sectional shaded area $\times$ length of prism

$= 18.0213 \times 20$

$= 360.43$

$\approx 360 \text{ cm}^2$ (3sf) **Ans.**

(iii) Using cosine rule.

$x = \sqrt{(4)^2 + (11)^2 - 2(4)(11)\cos 125°}$

$= \sqrt{137 + 50.475}$

$= \sqrt{187.475}$

$= 13.692 \approx 13.7$ cm (3 sf) **Ans.**

(iv) Surface area of the prism

$= 2(18.02) + (11 \times 20) + (4 \times 20) + (13.692 \times 20)$

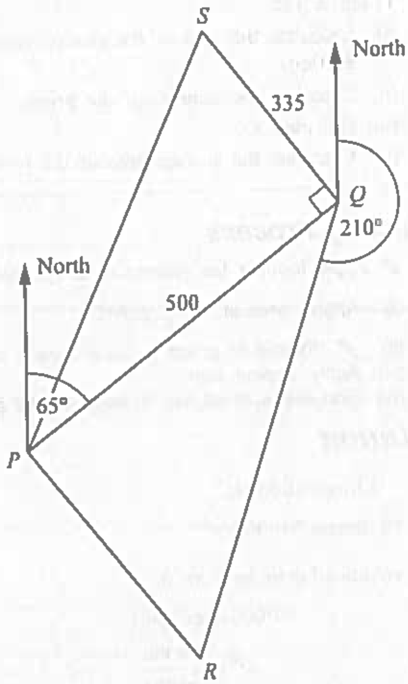
$= 36.04 + 220 + 80 + 273.84$

$= 609.88$

$\approx 610 \text{ cm}^2$ (3 sf) **Ans.**

16 *(J2013/P2/Q6)*

The diagram shows the positions, P, Q, R and S, of four hotels.



The bearing of Q from P is 065° and the bearing of R from Q is 210°.

$PQ = 500$ m, $SQ = 335$ m and $P\hat{Q}S = 90°$.

(a) Calculate $P\hat{Q}R$. [1]

(b) Calculate the shortest distance from P to QR. [2]

(c) Calculate the bearing of S from P. [3]

Thinking Process

(a) Consider the interior angles between parallel Norths.

(b) Draw a perpendicular line from P to QR and calculate the distance.

(c) To find the bearing find angle SPQ.

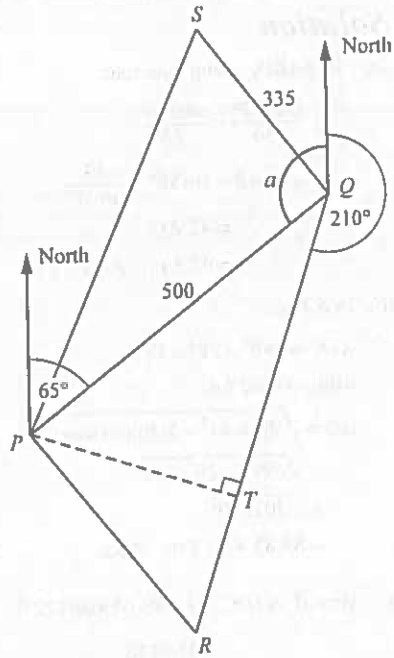
Solution

(a) $\hat{a} = 180° - 65° = 115°$ (int. ∠s between ∥ lines)

$P\hat{Q}R + \hat{a} + 210° = 360°$ (∠ sum around a point)

$\Rightarrow P\hat{Q}R = 360° - 115° - 210°$

$= 35°$ **Ans.**



(b) In ΔPQT,

$\sin PQT = \frac{PT}{PQ}$

$\sin 35° = \frac{PT}{500}$

$PT = \sin 35° \times 500$

$= 286.7882$

≈ 287

$\therefore$ shortest distance from P to $QR = 287$ m **Ans.**

(c) Consider ΔPQS.

$\tan S\hat{P}Q = \frac{SQ}{PQ}$

$\Rightarrow \tan S\hat{P}Q = \frac{335}{500}$

$S\hat{P}Q = 33.82°$

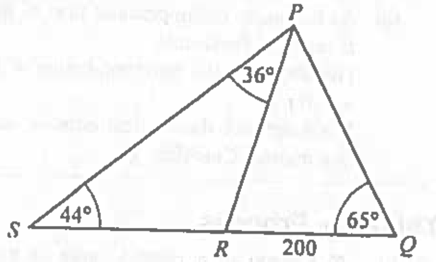
$65° - 33.82° = 31.18°$

$\approx 31.2°$

$\therefore$ bearing of S from $P = 031.2°$ **Ans.**

17 *(N2013/P2/Q10 b)*

(b)



In triangle PQS, $S\hat{Q}P = 65°$ and $Q\hat{S}P = 44°$.
R is the point on QS such that $QR = 200$ m and $R\hat{P}S = 36°$.

(i) In triangle PQR, by using the sine rule, show that $PR = \dfrac{200\sin 65}{\sin 35}$. [2]

(ii) Hence show that $SR = \dfrac{200\sin 65 \sin 36}{\sin 35 \sin 44}$. [2]

(iii) Hence find the length of SR. [1]

(iv) Hence evaluate $\dfrac{\text{area of triangle } SPQ}{\text{area of triangle } PQR}$. [1]

Thinking Process

(b) (i) Find $Q\hat{P}R$. Apply sine rule.

(ii) Apply sine rule on ΔSPR. Substitute the value of PR from (i).

(iii) Solve (b)(ii) for SR.

(iv) Note that two triangles share common height.

Solution

(b) (i) In ΔPQS,

$$44° + 65° + (36° + Q\hat{P}R) = 180°$$
$$145° + Q\hat{P}R = 180°$$
$$Q\hat{P}R = 35°$$

using sine rule,

$$\frac{PR}{\sin 65} = \frac{200}{\sin 35}$$
$$PR = \frac{200}{\sin 35} \times \sin 65$$
$$= \frac{200\sin 65}{\sin 35} \quad \text{Shown.}$$

(ii) Using sine rule to ΔSPR,

$$\frac{SR}{\sin 36} = \frac{PR}{\sin 44}$$
$$SR = PR \times \frac{\sin 36}{\sin 44}$$

from (b)(i), $PR = \dfrac{200\sin 65}{\sin 35}$

$$\Rightarrow \quad SR = \frac{200\sin 65}{\sin 35} \times \frac{\sin 36}{\sin 44}$$
$$\Rightarrow \quad SR = \frac{200\sin 65 \sin 36}{\sin 35 \sin 44} \quad \text{Shown.}$$

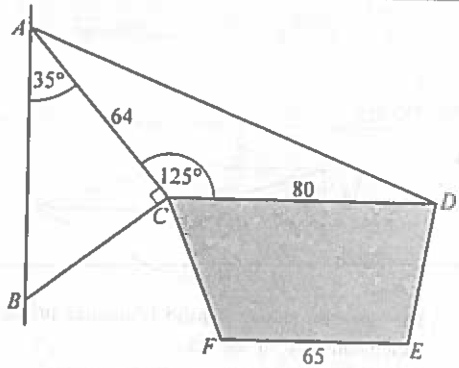
(iii) $SR = \dfrac{200\sin 65 \sin 36}{\sin 35 \sin 44}$

$$= \frac{200(0.9063)(0.5878)}{(0.5736)(0.6947)}$$
$$= \frac{106.545}{0.3985}$$
$$= 267.365 \approx 267 \text{ m (3sf)} \quad \textbf{Ans.}$$

(iv) Let h be the height of P above the line SQ.

$$\frac{\text{area of triangle } SPQ}{\text{area of triangle } PQR} = \frac{\frac{1}{2} \times SQ \times h}{\frac{1}{2} \times RQ \times h}$$
$$= \frac{SQ}{RQ}$$
$$= \frac{267.365 + 200}{200}$$
$$= 2.336$$
$$\approx 2.34 \text{ (3sf)} \quad \textbf{Ans.}$$

18 *(J2014/P2/Q5)*



The diagram shows a framework $ABCD$ supporting a shop sign.
The framework is fixed to a vertical wall AB with CD horizontal.
$AC = 64$ cm and $CD = 80$ cm.
$B\hat{A}C = 35°$, $B\hat{C}A = 90°$ and $A\hat{C}D = 125°$.

(a) Calculate AB. [2]

(b) Calculate AD. [3]

(c) Calculate $A\hat{D}C$. [3]

(d) On the sign $CDEF$, FE is parallel to CD and is 40 cm below it. $FE = 65$ cm.
Calculate the area of the sign $CDEF$. [2]

Thinking Process

(a) Apply, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$.

(b) Apply cosine rule: $a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

(c) Apply sine rule to find $\angle ADC$.

(d) Apply, area of trapezium $= \frac{1}{2}h(\text{sum of // sides})$

Solution

(a) In ΔABC, $\cos 35° = \dfrac{64}{AB}$

$$AB = \frac{64}{\cos 35°}$$

$= 78.129 \approx 78.1$ cm. **Ans.**

(b) In ΔACD, using cosine rule,

$$AD = \sqrt{(64)^2 + (80)^2 - 2(64)(80)\cos 125°}$$

$$AD = \sqrt{10496 + 5873.4227}$$

$= 127.94 \approx 128$ cm (3sf) **Ans.**

(c) In ΔACD, using sine rule.

$$\frac{\sin A\hat{D}C}{AC} = \frac{\sin A\hat{C}D}{AD}$$

$$\Rightarrow \frac{\sin A\hat{D}C}{64} = \frac{\sin 125°}{127.94}$$

$$\Rightarrow \sin A\hat{D}C = \frac{\sin 125°}{127.94} \times 64$$

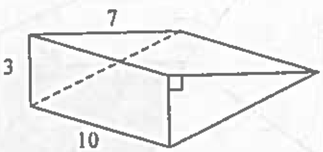
$\therefore$ $A\hat{D}C = 24.19° \approx 24.2°$ (1dp) **Ans.**

(d) Area of trapezium $CDEF = \dfrac{1}{2} \times 40 \times (65 + 80)$

$= 2900 \text{ cm}^2$ **Ans.**

19 *(N2014/P2/Q4)*

(a)

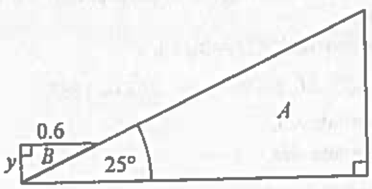


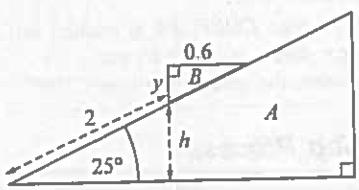
The diagram shows a solid triangular prism. The dimensions are in metres.

(i) Calculate the volume of the prism. [2]

(ii) Calculate the total surface area of the prism. [4]

(b)





The diagrams show the cross-sections of a ramp A and a triangular prism B.

The triangular prism B can move up and down the ramp A.

The ramp is inclined at 25° to the horizontal.

(i) When the prism has moved 2 m up the ramp, it has risen h metres vertically.
Calculate h. [2]

(ii) As it moves, the uppermost face of the prism B remains horizontal.
The length of the horizontal edge of the face is 0.6 m.
The length of the vertical edge of the prism is y metres. Calculate y. [2]

Thinking Process

(a) (i) Volume of a prism = area of the cross-section × length.

(ii) Total surface area of a prism = area of both ends, plus the sum of the areas of all sides.

(b) (i) Apply, $\sin\theta = \dfrac{\text{opp.}}{\text{hyp.}}$

(ii) Use, $\tan\theta = \dfrac{\text{opp.}}{\text{adj.}}$

Solution

(a) (i) Volume of prism = area of cross section × length

$$= \left(\frac{1}{2} \times 7 \times 3\right) \times 10$$

$= \dfrac{210}{2} = 105 \text{ m}^3$ **Ans.**

(ii) Using pythogoras theorem on the cross section triangle.

base width $= \sqrt{(3)^2 + (7)^2} = \sqrt{9 + 49} = \sqrt{58}$ m

Total surface area

$$= 2(\frac{1}{2} \times 3 \times 7) + (7 \times 10) + (3 \times 10) + (\sqrt{58} \times 10)$$

$$= 21 + 70 + 30 + 76.158$$

$= 197.158 \approx 197 \text{ m}^2$ **Ans.**

(b) (i) $\sin 25° = \dfrac{h}{2}$

$$h = 2 \times \sin 25°$$

$= 0.845$ m **Ans.**

(ii) Smallest angle of triangle $B = 25°$ (alt. ∠s)

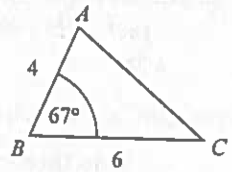
$\therefore$ $\tan 25° = \dfrac{y}{0.6}$

$$y = 0.6 \times \tan 25°$$

$= 0.28$ m **Ans.**

20 *(N2014/P2 Q9)*

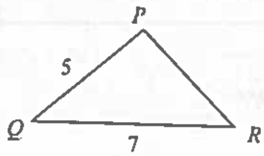
(a)



In triangle ABC, $AB = 4$ m, $BC = 6$ m and $A\hat{B}C = 67°$.

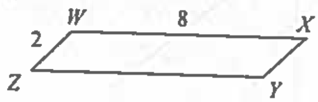
(i) Show that the area of triangle ABC is 11.05 m² correct to 2 decimal places. [1]

(ii)



In triangle PQR, $PQ = 5$ m and $QR = 7$ m.
Area of triangle PQR = Area of triangle ABC.
Find the acute angle PQR. [2]

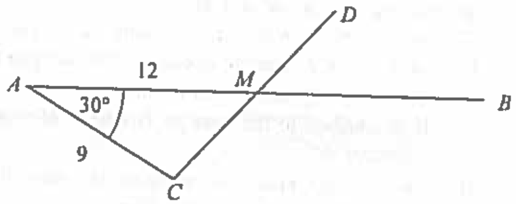
(iii)



In the parallelogram $WXYZ$, $WX = 8$ m and $WZ = 2$ m.
Area of parallelogram $WXYZ$ = Area of triangle ABC.
Find the obtuse angle ZWX. [3]

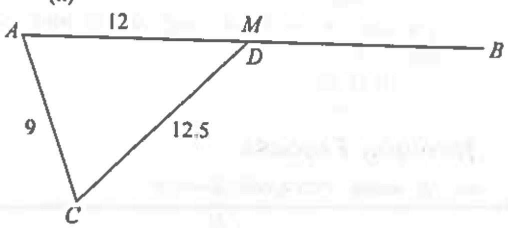
(b) AB, AC and CD are three rods. They can be fixed together in different positions.

(i) $AC = 9$ cm and M is a fixed point on AB such that $AM = 12$ cm.



When $C\hat{A}M = 30°$, calculate CM. [3]

(ii)



In another position, the end D of the rod CD is fixed at the point M.
$CD = 12.5$ cm.

Calculate the increase in $C\hat{A}M$. [3]

Thinking Process

(a) (i) Apply, area of $\Delta = \frac{1}{2}ab\sin C$

(iii) find the height of the parallelogram and use it to find angle ZWX.

(b) (i) Apply Cosine rule to find CM.

(ii) To find the increase in angle find angle CAM
Apply Cosine rule.

Solution

(a) (i) Area of $\Delta ABC = \frac{1}{2}(4)(6)\sin 67°$

$= 11.046 \approx 11.05 \text{ m}^2$ **Shown.**

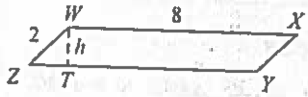
(ii) Area of ΔPQR = area of ΔABC

$\frac{1}{2}(5)(7)\sin P\hat{Q}R = 11.046$

$\sin P\hat{Q}R = 0.6312$

$P\hat{Q}R = 39.14°$ **Ans.**

(iii) Area of parallelogram $WXYZ$ = area of ΔABC



$8 \times h = 11.046$

$h = 1.38$

∴ height of parallelogram = 1.38 m

In ΔZWT

$\cos Z\hat{W}T = \frac{1.38}{2}$

$Z\hat{W}T = 46.37°$

$\approx 46.4°$

∴ $Z\hat{W}X = 46.4° + 90° = 136.4°$ **Ans.**

(b) (i) Using cosine rule,

$CM = \sqrt{(9)^2 + (12)^2 - 2(9)(12)\cos 30°}$

$= \sqrt{225 - 187.061}$

$= \sqrt{37.939}$

$= 6.159 \approx 6.16$ cm (3 sf) **Ans.**

(ii) Using cosine rule,

$\cos C\hat{A}M = \frac{(12)^2 + (9)^2 - (12.5)^2}{2(12)(9)}$

$= \frac{68.75}{216}$

$= 0.3183$

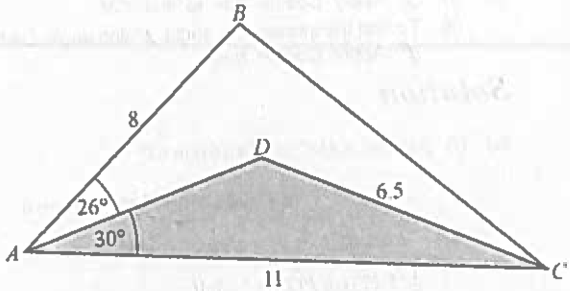
$\Rightarrow C\hat{A}M = 71.44° \approx 71.4°$

∴ increase in $C\hat{A}M = 71.4° - 30°$

$= 41.4°$ **Ans.**

21 (J2015/P2/Q5)

In the diagram, $AB = 8$ cm, $AC = 11$ cm and $DC = 6.5$ cm.
$B\hat{A}D = 26°$ and $D\hat{A}C = 30°$.



(a) Calculate BC. [4]
(b) Calculate the obtuse angle ADC. [3]
(c) Find the percentage of triangle ABC that has been shaded. [4]

Thinking Process

(a) Apply Cosine rule on ΔABC to find BC.
(b) Apply sine rule on ΔADC to find acute $\angle ADC$ then subtract it from 180°.
(c) Express the area of ΔADC as a percentage of area of ΔABC. Use, area of $\Delta = \frac{1}{2}ab\sin C$.

Solution

(a) In ΔABD, using cosine rule.

$$BC = \sqrt{(11)^2 + (8)^2 - 2(11)(8)\cos 56°}$$
$$\Rightarrow BC = \sqrt{121 + 64 - 98.418}$$
$$= \sqrt{86.582}$$
$$= 9.3049 \approx 9.30 \text{ cm (3sf)} \quad \textbf{Ans.}$$

(b) In ΔADC, using sine rule,

$$\frac{\sin A\hat{D}C}{AC} = \frac{\sin D\hat{A}C}{DC}$$
$$\Rightarrow \frac{\sin A\hat{D}C}{11} = \frac{\sin 30°}{6.5}$$
$$\sin A\hat{D}C = \frac{\sin 30°}{6.5} \times 11$$
$$\sin A\hat{D}C = 0.846154$$
$$A\hat{D}C = 57.796°$$
$$\therefore \text{ obtuse angle } ADC = 180° - 57.796°$$
$$= 122.204° \approx 122° \quad \textbf{Ans.}$$

(c) In ΔADC,

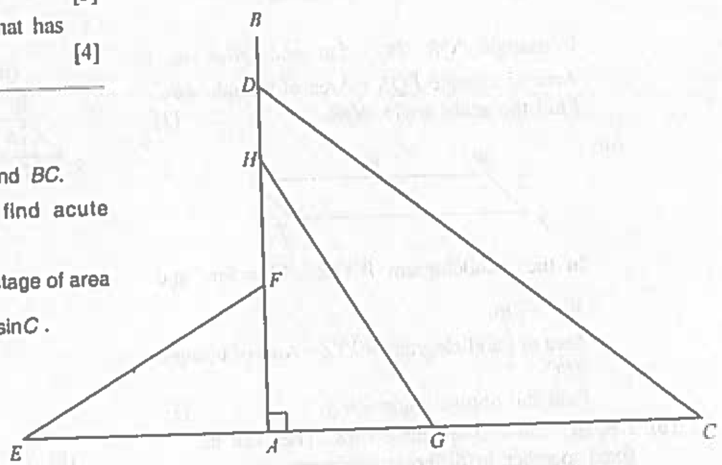
$$A\hat{C}D = 180° - A\hat{D}C - D\hat{A}C \quad (\angle \text{ sum of a } \Delta)$$
$$= 180° - 122° - 30°$$
$$= 28°$$

$$\text{Area of } \Delta ADC = \frac{1}{2}(11)(6.5)\sin 28°$$
$$= 16.784 \text{ cm}^2$$

$$\text{Area of } \Delta ABC = \frac{1}{2}(11)(8)\sin 56°$$
$$= 36.478 \text{ cm}^2$$

$$\text{percentage of } \Delta ABC \text{ shaded} = \frac{16.784}{36.478} \times 100$$
$$= 46\% \quad \textbf{Ans.}$$

22 (N2015/P2/Q5)



The diagram shows a vertical radio mast, AB.
Three of the wires that hold the mast in place are attached to it at F, H and D.
The base A of the mast, and the ends E, G and C of the wires are in a straight line on horizontal ground.

(a) The wire CD has length 65 m.
It is attached to the mast at D where $AD = 40$ m.
Calculate AC. [2]

(b) The wire EF makes an angle of 25° with the horizontal and is of length 30 m.
Calculate AF. [2]

(c) $AH = 35$ m.
The wire HG makes an angle of 30° with the mast AB.
Calculate HG. [3]

Thinking Process

(a) Apply Pythagoras theorem.
(b) Apply $\sin A\hat{E}F = \frac{AF}{EF}$.
(c) Apply $\cos A\hat{H}G = \frac{HA}{HG}$.

Solution

(a) By Pythagoras theorem,

$AC = \sqrt{(65)^2 - (40)^2}$

$= \sqrt{2625}$

$= 51.235 \approx 51.2$ m **Ans.**

(b) In ΔEFA,

$\sin 25° = \frac{AF}{30}$

$AF = 30\sin 25°$

$= 12.6785 \approx 12.7$ m (3sf) **Ans.**

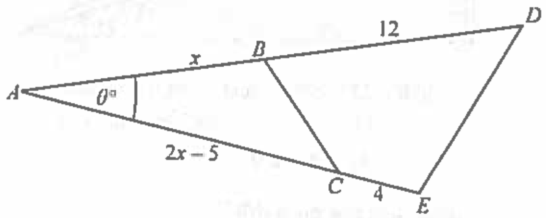
(c) In ΔAGH,

$\cos 30° = \frac{35}{HG}$

$HG = \frac{35}{\cos 30°}$

$= 40.4157 \approx 40.4$ m (3sf) **Ans.**

23 *(N2015 P2 Q7)*



ABD and ACE are straight lines.
$BD = 12$ cm and $CE = 4$ cm.
$AB = x$ cm and $AC = (2x - 5)$ cm.
Angle $BAC = \theta°$.

(a) Show that $\frac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \frac{AB \times AC}{AD \times AE}$. [2]

(b) It is given that $\frac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \frac{1}{3}$.

Using the result from part (a), form an equation in x and show that it simplifies to $2x^2 - 19x + 6 = 0$. [3]

(c) (i) Solve the equation $2x^2 - 19x + 6 = 0$, giving your answers correct to 2 decimal places. [3]

(ii) State, with a reason, which of these solutions does **not** apply to triangle ABC. [1]

(d) Given that $\theta = 25$, calculate BC. [3]

Thinking Process

(a) ✐ Apply, area of $\Delta = \frac{1}{2}ab\sin C$

(c) (i) Solve by quadratic formula.

(ii) Analyse which value of x is not suitable for the sides of the triangle. Give a reason

(d) To find BC ✐ apply cosine rule on ΔABC.

Solution

(a) Area of $\Delta ABC = \frac{1}{2} \times AB \times AC \times \sin\theta$

Area of $\Delta ADE = \frac{1}{2} \times AD \times AE \times \sin\theta$

$\therefore \frac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \frac{\frac{1}{2} \times AB \times AC \times \sin\theta}{\frac{1}{2} \times AD \times AE \times \sin\theta}$

$\Rightarrow \frac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \frac{AB \times AC}{AD \times AE}$ **Shown.**

(b) $\frac{\text{area of triangle } ABC}{\text{area of triangle } ADE} = \frac{1}{3}$

$\Rightarrow \frac{AB \times AC}{AD \times AE} = \frac{1}{3}$

$\Rightarrow \frac{x \times (2x-5)}{(x+12) \times (2x-5+4)} = \frac{1}{3}$

$\Rightarrow \frac{2x^2 - 5x}{(x+12) \times (2x-1)} = \frac{1}{3}$

$\Rightarrow \frac{2x^2 - 5x}{2x^2 + 23x - 12} = \frac{1}{3}$

$\Rightarrow 3(2x^2 - 5x) = 2x^2 + 23x - 12$

$\Rightarrow 6x^2 - 15x = 2x^2 + 23x - 12$

$\Rightarrow 4x^2 - 38x + 12 = 0$

$\Rightarrow 2(2x^2 - 19x + 6) = 0$

$\Rightarrow 2x^2 - 19x + 6 = 0$ **Shown.**

(c) (i) $2x^2 - 19x + 6 = 0$

by quadratic formula.

$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(2)(6)}}{2(2)}$

$= \frac{19 \pm \sqrt{361 - 48}}{4}$

$= \frac{19 \pm \sqrt{313}}{4}$

$\Rightarrow x = \frac{19 + \sqrt{313}}{4}$ or $x = \frac{19 - \sqrt{313}}{4}$

$\therefore x = 9.17$ or 0.33 (2dp) **Ans.**

(ii) For $x = 0.33$

$AC = 2(0.33) - 5 = -4.34$

since length cannot be negative, therefore $x = 0.33$ does not apply to ΔABC.

(d) In ΔABC, when $x = 9.17$

$AB = 9.17$,

$AC = 2(9.17) - 5 = 13.34$

using cosine rule,

$BC = \sqrt{(9.17)^2 + (13.34)^2 - 2(9.17)(13.34)\cos 25°}$

$= \sqrt{84.089 + 177.956 - 221.733}$

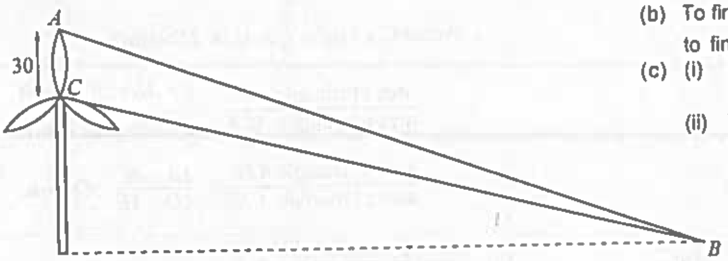
$= \sqrt{40.312}$

$= 6.349$

≈ 6.35 cm (3 sf) **Ans.**

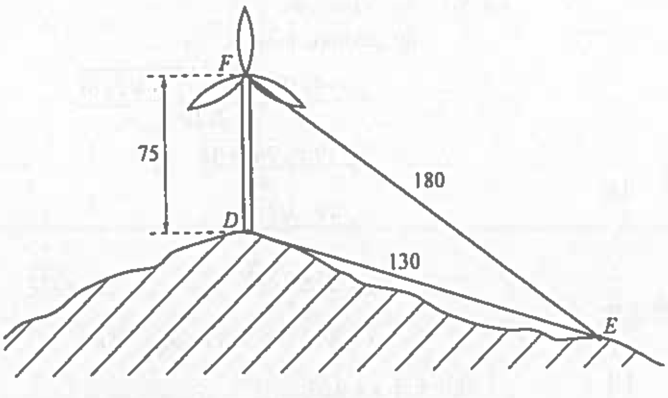
24 (J2016/P2/Q9)

(a)



The diagram shows a vertical wind turbine with blades 30 m long.
The blades are stationary with the point A being the maximum distance possible from the horizontal ground
The point B is such that the angle of elevation of A from B is 34° and the angle of elevation of the centre of the blades, C, from B is 25°.
Calculate the distance AB. [3]

(b) A different wind turbine, shown in the diagram on the next page, has the centre of its blades, F, 75 m from the base of the turbine, D.
Point E is on sloping ground, 180 m from F and 130 m from D.
Calculate the angle of depression of E from F. [4]



(c) P is the point on a blade which is furthest from the centre of the blades.
Each blade is 30 m long.

(i) Calculate the distance travelled by P as the blade completes one revolution. [1]

(ii) The blade completes 15 revolutions per minute.
Calculate the speed of P, giving your answer in kilometres per hour. [2]

(iii) A point Q lies on the straight line between P and the centre of the blades.
Q travels 90 m as the blade completes one revolution.
Calculate PQ. [2]

Thinking Process

(a) To find AB ⇒ apply sine rule on ΔABC.

(b) To find the angle of depression ⇒ apply cosine rule to find $\angle DFE$.

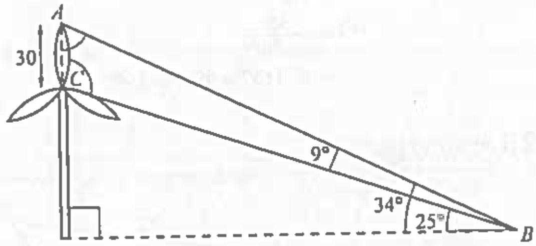
(c) (i) Note that distance travelled by P is the circumference of a circle.

(ii) To find the speed ⇒ find the distance travelled by P in 15 revolutions.

(iii) 90 m is the circumference of the circle made by Q. Therefore, find the radius of the circle and subtract it from 30 m to find PQ.

Solution

(a)



$\angle ACB = 25° + 90°$ (ext. ∠ of a Δ = sum of 2 int. opp ∠s.)
$= 115°$

$\angle ABC = 34° - 25° = 9°$

using sine rule on ΔABC,

$$\frac{AB}{\sin A\hat{C}B} = \frac{AC}{\sin A\hat{B}C}$$

$$\Rightarrow \frac{AB}{\sin 115°} = \frac{30}{\sin 9°}$$

$$\Rightarrow AB = \frac{30}{\sin 9°} \times \sin 115°$$

$= 173.8 \approx 174$ m (3sf) **Ans.**

(b) Using cosine rule.

$$\cos D\hat{F}E = \frac{(75)^2 + (180)^2 - (130)^2}{2(75)(180)}$$

$$= \frac{21125}{27000}$$

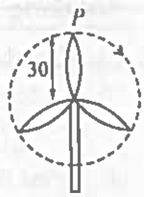
$= 0.7824$

$\therefore \quad D\hat{F}E = 38.519°$

angle of depression of E from F
$= 90° - 38.519°$
$= 51.5°$ **Ans.**

(c) (i) After one revolution, distance travelled by P is the circumference of a circle of radius 30 m.

$\therefore$ distance travelled by P
$= 2\pi(30)$
$= 188.495 \approx 188$ m. **Ans.**



(ii) Distance travelled in 15 revolutions

$= 188.495 \times 15$

$= 2827.425$ m.

$\therefore$ speed of $P = 2827.425$ m/min

$= 2827.425 \times \frac{60}{1000}$

$= 169.6 \approx 170$ km/h. **Ans.**

(iii) Distance travelled by $Q = 2\pi r$

$\Rightarrow \quad 90 = 2\pi r$

$r = \frac{90}{2\pi} = 14.32$

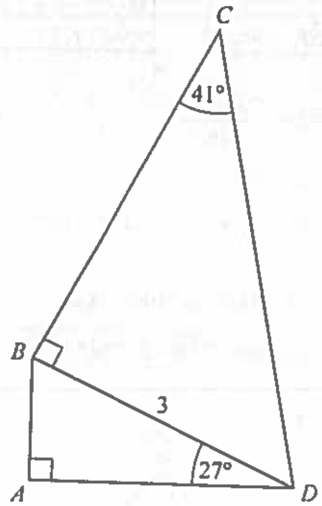
$\therefore$ radius of circle $Q = 14.32$ m

length of $PQ = 30 - 14.32$

$= 15.68 \approx 15.7$ m **Ans.**

25 *(N2016 P2 Q4)*

(a)



In the framework $ABCD$, $BD = 3$ m.

$B\hat{D}A = 27°$, $B\hat{C}D = 41°$. $D\hat{B}C$ and $D\hat{A}B$ are right angles.

(i) Find AD. [2]

(ii) Find CD. [3]

(b) In triangle PQR, $PQ = 3$ m and $QR = 5$ m. The area of triangle $PQR = 6$ m^2.

Find the two possible values of $P\hat{Q}R$. [3]

Thinking Process

(a) (i) Apply, $\cos\theta = \frac{\text{adj.}}{\text{hyp.}}$

(ii) Apply, $\sin\theta = \frac{\text{opp.}}{\text{hyp.}}$

(b) Apply, Area of $\Delta ABC = \frac{1}{2}ab\sin c$

Solution

(a) (i) $\cos 27° = \frac{AD}{3}$

$AD = 3 \times \cos 27°$

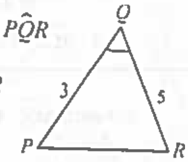
$= 2.67$ m **Ans.**

(ii) $\sin 41° = \frac{3}{CD}$

$CD = \frac{3}{\sin 41°}$

$= 4.57$ m **Ans.**

(b) Area of $\Delta PQR = \frac{1}{2}(3)(5)\sin P\hat{Q}R$



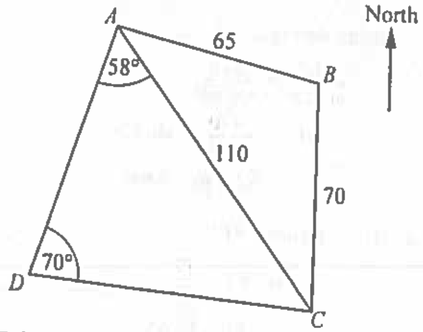
$\Rightarrow \quad 6 = \frac{15}{2}\sin P\hat{Q}R$

$\sin P\hat{Q}R = \frac{12}{15}$

$\Rightarrow \quad P\hat{Q}R = 53.1°$ or $180° - 53.1°$

$\therefore \quad P\hat{Q}R = 53.1°$ or $126.9°$ **Ans.**

26 *(N2016 P2 Q9)*



$ABCD$ is a level playing field.
$AB = 65$ m, $BC = 70$ m and $CA = 110$ m.

$C\hat{D}A = 70°$, $D\hat{A}C = 58°$ and C is due South of B.

(a) Calculate the bearing of A from C. [4]

(b) Calculate AD. [3]

(c) There are two vertical trees, AX and CY, each of height 17 m, one at each end of the path AC.

(i) Calculate the angle of elevation of Y from B. [2]

(ii) A bird flies in a straight line from X to Y. It takes 24 seconds. Calculate the average speed of the bird.
Give your answer in kilometres per hour. [3]

Thinking Process

(a) To find the bearing use cosine rule to find angle ACB.

(b) Apply sine rule to find AD.

(c) (i) Draw a right triangle with base $BC = 70$ m and height $CY = 17$ m. Use $\tan\theta = \frac{\text{perp.}}{\text{base}}$

(ii) Apply, speed $= \frac{\text{distance}}{\text{time}}$. Also note that X and Y are vertically above A and C respectively. Therefore distance $XY = AC$.

Solution

(a) In ΔABC, using cosine rule,

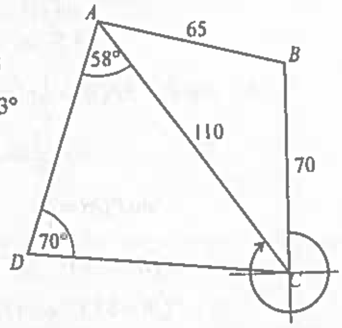
$$\cos A\hat{C}B = \frac{(70)^2 + (110)^2 - (65)^2}{2(70)(110)}$$

$$= \frac{12775}{15400}$$

$$= 0.8295$$

$\Rightarrow A\hat{C}B = 33.953°$

$\approx 34°$



bearing of A from C

$= 360° - 34°$

$= 326°$ **Ans.**

(b) In ΔACD,

$A\hat{C}D = 180° - 70° - 58°$

$= 52°$

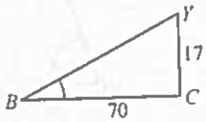
using sine rule.

$$\frac{AD}{\sin 52°} = \frac{110}{\sin 70°}$$

$$AD = \frac{110}{\sin 70°} \times \sin 52°$$

$= 92.2$ m **Ans.**

(c) (i) Consider ΔBCY,



$$\tan C\hat{B}Y = \frac{17}{70}$$

$C\hat{B}Y = 13.65°$

$\therefore$ angle of elevation from $B = 13.7°$ **Ans.**

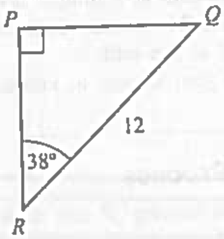
(ii) $XY = AC = 110$ m

$\therefore$ average speed $= \frac{110}{24}$ m/s

$= \frac{110}{24} \times \frac{3600}{1000}$

$= 16.5$ km/h **Ans.**

27 *(J2017/P2/Q6)*



Triangle PQR has a right angle at P, angle $PRQ = 38°$ and $RQ = 12$ cm.

(a) Calculate PQ. [2]

(b) S is a point such that angle PRS is a right angle and $QS = 10$ cm.
Calculate the two possible values of angle QSR. [4]

Thinking Process

(a) apply $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

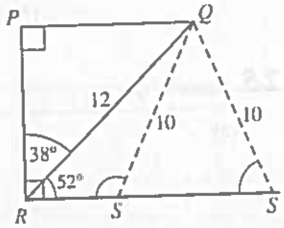
(b) Apply Sine Rule. Note that $\sin\theta = \sin(180 - \theta)$.

Solution

(a) $\sin 38° = \frac{PQ}{12}$

$PQ = 12 \sin 38° = 7.39$ cm **Ans.**

(b) In ΔQSR,



$Q\hat{R}S = 90° - 38°$

$= 52°$

using sine rule.

$$\frac{\sin Q\hat{S}R}{12} = \frac{\sin 52°}{10}$$

$$\sin Q\hat{S}R = \frac{12 \times \sin 52°}{10}$$

$\sin Q\hat{S}R = 0.9456$

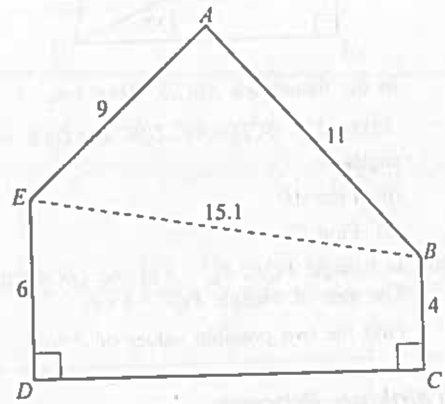
$\Rightarrow Q\hat{S}R = 71.0°$ or $Q\hat{S}R = 180° - 71.0°$

$= 109°$

$\therefore Q\hat{S}R = 71.0°$ or $109°$ **Ans.**

Note that, $\sin\theta = \sin(180 - \theta)$

28 *(J2017/P2/Q8)*



$ABCDE$ is the cross-section of a building.
All the lengths are given in metres.

(a) Calculate DC. [3]

(b) Calculate angle EAB. [3]

(c) Calculate the area of the cross-section. [4]

(d) A model of the building is made using the scale 1 : 50.
What is the area of the cross-section of the model?
Give your answer in square centimetres. [2]

Thinking Process

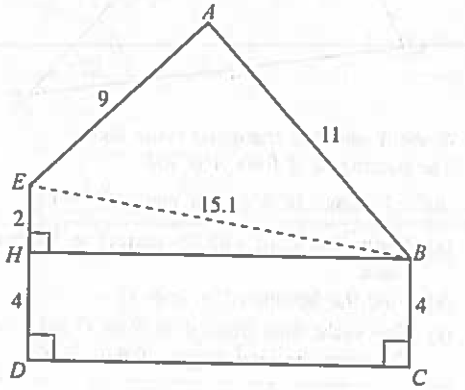
(a) Draw a line parallel to DC at point B. Use the triangle formed to find DC.

(b) Apply cosine rule: $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

(c) Area of the cross-section = area of triangle ABE + area of trapezium $BCDE$.

(d) To find the area in cm^2 express the given scale in terms of area, then calculate the actual area in terms of cm^2 according to given scale.

Solution



(a) In ΔEBH, using pythagoras theorem,

$$HB = \sqrt{(15.1)^2 - (2)^2}$$
$$= \sqrt{224.01} = 14.97$$

$\therefore$ $DC = 14.97$ cm **Ans.**

(b) Using cosine rule,

$$\cos E\hat{A}B = \frac{9^2 + 11^2 - 15.1^2}{2(9)(11)}$$
$$= \frac{-26.01}{198}$$

$\therefore$ $E\hat{A}B = 97.5°$ **Ans.**

(c) Area of $\Delta ABE = \frac{1}{2}(AB)(AE)\sin E\hat{A}B$

$$= \frac{1}{2}(11)(9)\sin 97.548°$$
$$= 49.07 \text{ cm}^2$$

Area of trapezium $BCDE = \frac{1}{2}(14.97)(4+6)$

$$= 74.85 \text{ cm}^2$$

Area of the cross-section

= area of ΔABE + area of trapezium $BCDE$.

$= 49.07 + 74.85$

$= 123.92 \approx 124$ cm^2 **Ans.**

(d) $1 : 50$

$\Rightarrow$ 1 cm : 50 cm

$\Rightarrow$ 1 cm : $\frac{50}{100}$ m

$\Rightarrow$ 1 cm^2 : $(0.5)^2$ m^2

$\Rightarrow$ 1 cm^2 : 0.25 m^2

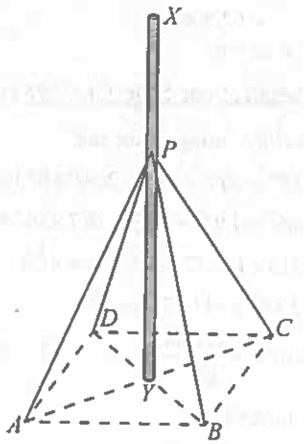
0.25 m^2 represents 1 cm^2

124 m^2 represents $\frac{1}{0.25} \times 124 = 496$ cm^2

$\therefore$ area of cross-section of the model

$= 496$ cm^2 **Ans.**

29 *(N2017 P2 Q11)*



A vertical mast, XY, is positioned on horizontal ground. The mast is supported by four cables attached to the mast at P and to the ground at points A, B, C and D. Y is the centre of the square $ABCD$.

$PY = 7.50$ m.

(a) Given that $AB = 3.65$ m, show that $AY = 2.58$ m correct to 3 significant figures. [3]

(b) Calculate the length of one of the cables used to support the mast. [2]

(c) Calculate $A\hat{P}B$. [3]

(d) The angle of elevation of X from A is 77.0°.

(i) Calculate the height, XY of the mast. [2]

(ii) Calculate the angle of elevation of X from the midpoint of AB. [2]

Thinking Process

(a) Apply Pythagoras theorem to triangle ABC.

(b) Apply Pythagoras theorem to triangle PAY.

(c) Apply cosine rule.

(d) (i) Identify the right-angled triangle XAY. Apply trigonometric ratio to find XY.

(ii) Draw a line from Y to the midpoint of AB and calculate its length. Use trigonometric ratio to find the angle of elevation.

Solution

(a) In ΔABC, using pythagoras theorem.

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (3.65)^2 + (3.65)^2$

$\Rightarrow AC^2 = 26.645$

$\Rightarrow AC = 5.16$

$\therefore AY = \frac{1}{2}(AC)$

$= \frac{1}{2}(5.16) = 2.58$ **Shown.**

(b) In ΔPAY, using pythagoras theorem.

$PA^2 = PY^2 + AY^2$

$= (7.50)^2 + (2.58)^2$

$= 62.906$

$\Rightarrow PA = 7.93$

$\therefore$ length of one cable $PA = 7.93$ m. **Ans.**

(c) In ΔAPB, using cosine rule.

$AB^2 = AP^2 + BP^2 - 2(AP)(BP)\cos A\hat{P}B$

$3.65^2 = 7.93^2 + 7.93^2 - 2(7.93)(7.93)\cos A\hat{P}B$

$13.323 = 125.77 - 125.77\cos A\hat{P}B$

$-112.447 = -125.77\cos A\hat{P}B$

$\cos A\hat{P}B = \frac{112.447}{125.77} \Rightarrow A\hat{P}B = 26.6°$ **Ans.**

(d) (i) In ΔXAY,

$\tan X\hat{A}Y = \frac{XY}{AY}$

$\tan 77° = \frac{XY}{2.58}$

$XY = 2.58 \times \tan 77°$

$= 11.17$

≈ 11.2 m **Ans.**

(ii) From figure, Let Q be the midpoint of AB.

In ΔAQY, $AQ = \frac{1}{2}AB$

$= \frac{1}{2}(3.65) = 1.825$ m

since $ABCD$ is a square,

$\therefore YQ = AQ = 1.825$ m.

now, consider ΔXYQ,

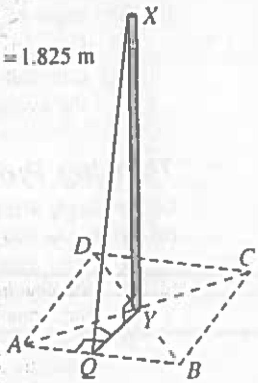
$\tan X\hat{Q}Y = \frac{XY}{YQ}$

$\tan X\hat{Q}Y = \frac{11.17}{1.825}$

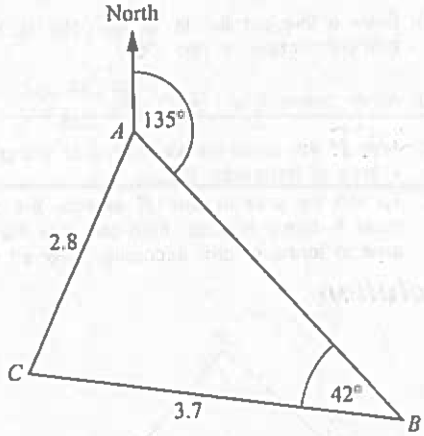
$\Rightarrow X\hat{Q}Y = 80.7°$

$\therefore$ angle of elevation of X from Q

$= 80.7°$ **Ans.**



30 *(J2018/P2/Q7)*



A yacht sails the triangular route shown.
The bearing of B from A is 135°.
$BC = 3.7$ km, $AC = 2.8$ km and $A\hat{B}C = 42°$.

(a) Show that $C\hat{A}B = 62.2°$, correct to 1 decimal place. [3]

(b) Find the bearing of A from C. [2]

(c) The yacht sails from A to B to C to A. Calculate the total length of the route. [4]

Thinking Process

(a) ✎ Apply sine rule to find $\angle CAB$.

(b) To find the bearing ✎ draw north at C.

(c) To find the total length of the route ✎ find $\angle ACB$. Then use sine rule to find AB.

Solution

(a) In ΔABC, using sine rule,

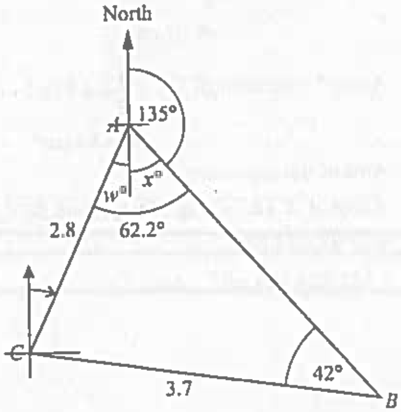
$\frac{\sin C\hat{A}B}{3.7} = \frac{\sin 42°}{2.8}$

$\Rightarrow \sin C\hat{A}B = \frac{\sin 42°}{2.8} \times 3.7$

$\Rightarrow \sin C\hat{A}B = 0.8842$

$\therefore C\hat{A}B = 62.15° \approx 62.2°$ **Shown.**

(b)



$x° + 135° = 180°$ (∠s on a straight line)

$x° = 180° - 135° = 45°$

$w° = 62.2° - x°$

$= 62.2° - 45° = 17.2°$

∴ Bearing of A from C = 017.2° (alt ∠s) **Ans.**

(c) $A\hat{C}B = 180° - 42° - 62.2°$ (∠ sum of a Δ)

$= 75.8°$

Using Sine rule,

$$\frac{AB}{\sin A\hat{C}B} = \frac{AC}{\sin A\hat{B}C}$$

$$\frac{AB}{\sin 75.8°} = \frac{2.8}{\sin 42°}$$

$$AB = \frac{2.8}{\sin 42°} \times \sin 75.8° = 4.06 \text{ km}$$

∴ length of the route = 4.06 + 3.7 + 2.8

= 10.56 ≈ 10.6 km **Ans.**

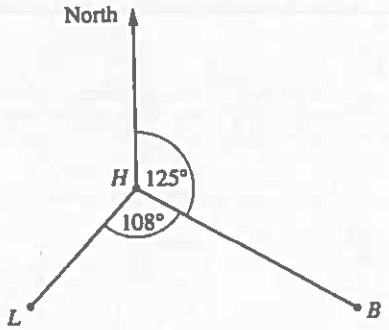
Topic 15

Bearings

1 *(N2008/P1/Q18)*

The diagram shows the positions of a harbour, H, and a lighthouse, L.

A boat is anchored at B where $L\hat{H}B = 108°$.



(a) Given that the bearing of B from H is 125°, find the bearing of
(i) L from H, [1]
(ii) H from B. [1]

(b) At 7 30 a.m. the boat set sail in a straight line from B to H at an average speed of 25km/h. Given that $BH = 70$km, find the time at which the boat reaches the harbour. [2]

Thinking Process

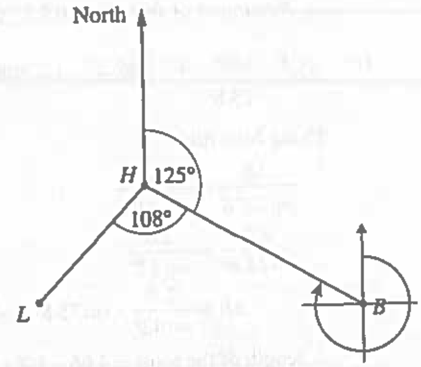
(a) (ii) Draw north at B and find the required bearing.

(b) Recall the formula: speed $= \dfrac{\text{distance}}{\text{time}}$.

Solution

(a) (i) Bearing of L from $H = 125° + 108°$
$= 233°$ **Ans.**

(ii)



Bearing of H from $B = 180° + 125°$
$= 305°$ **Ans.**

(b) Time $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{70}{25} = 2.8$ hrs
$= 2$ h $+ (0.8 \times 60)$ min $= 2$h 48 min.

0730 + 2h 48 min = 0978 = 1018

∴ the boat reaches harbour at 10.18 a.m. **Ans.**

2 *(N2012/P1/Q8)*

A ship travelled from P to Q.
It unloaded its cargo at Q and then returned to P.
The bearing of Q from P is 075°.

(a) Find the bearing of P from Q. [1]

(b) The ship left P at 21 40 and returned to P at 05 33 the following day.
Find the length of time, in hours and minutes, between leaving P and returning to P. [1]

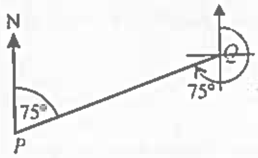
Thinking Process

(a) With given information, draw a line PQ. To find the bearing ∲ draw north at Q

(b) To find the length of time ∲ first subtract 2140 from 2400, then add 5 hours 33 minutes to it.

Solution

(a) Bearing of P from Q
$= 180° + 75°$
$= 255°$ **Ans.**



(b)
$$\begin{array}{r} 24\ 00 \\ -21\ 40 \\ \hline 02\ 20 \end{array}$$

the ship took 2 hours 20 minutes untill midnight

02 20 + 05 33 = 07 53

∴ Length of time is 7 hours 53 minutes. **Ans.**

3 *(J2011/P1/Q16)*

The scale drawing shows three towns, A, B and C.
The scale of the drawing is 1 cm to 25 km.

(a) Measure the bearing of A from C. [1]

(b) Find the bearing of C from A. [1]

(c) Find the actual distance, in kilometres, from B to C. [1]

Thinking Process

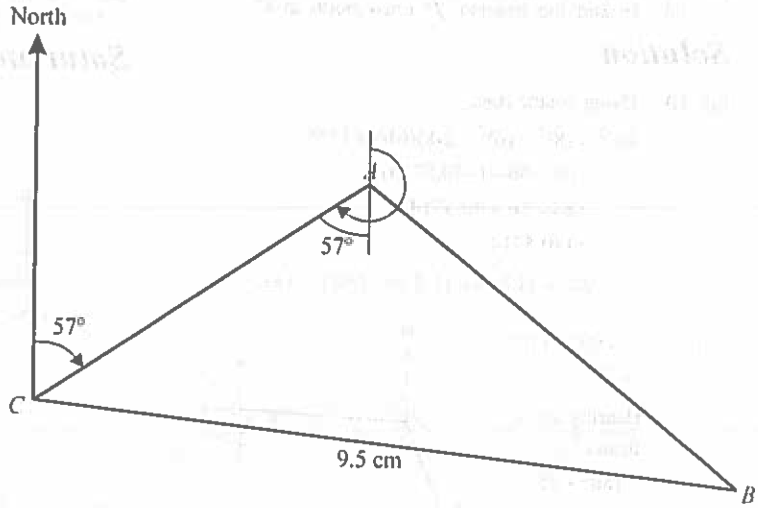
(a) Use a protractor to measure the bearing.
(b) Draw a north-line from A.
(c) Measure BC and multiply by 25.

Solution

(a) Bearing of A from C
$= 057°$ **Ans.**

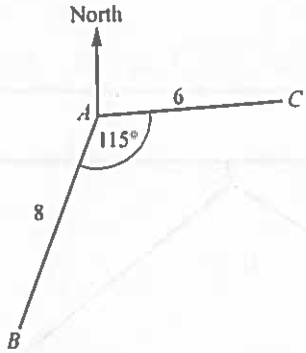
(b) Bearing of C from A
$= 180° + 57° = 237°$ **Ans.**

(c) $BC = 9.5$ cm
given scale: 1 cm = 25 km
$\therefore \; BC = 9.5 \times 25$
$= 237.5$ km **Ans.**



4 *(N2013/P2/Q10 a)*

(a)



Two boats sail from A. One boat sails to B, and the other boat sails to C.

$AB = 8$ km, $AC = 6$ km and $B\hat{A}C = 115°$.

(i) Calculate the distance, BC, between the boats. [4]

(ii) The bearing of B from A is 200°. Find the bearing of A from C. [2]

Thinking Process

(a) (i) Apply cosine rule.

(ii) To find the bearing draw north at C.

Solution

(a) (i) Using cosine rule,

$BC^2 = (8)^2 + (6)^2 - 2(8)(6)\cos 115°$

$= 64 + 36 - (-40.5714)$

$= 64 + 36 + 40.5714$

$= 140.5714$

$\therefore \quad BC = 11.856 \approx 11.9$ km (3sf) **Ans.**

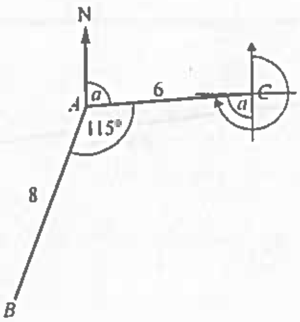
(ii) $\angle a = 200° - 115°$

$= 85°$

$\therefore$ Bearing of A from C

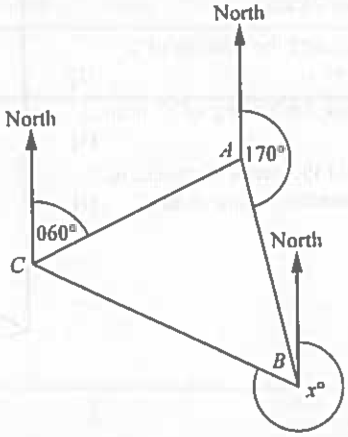
$= 180° + 85°$

$= 265°$ **Ans.**



5 *(J2016/P1/Q17)*

In the diagram, the bearing of B from A is 170°.
The bearing of A from C is 060°.
The bearing of C from B is $x°$.

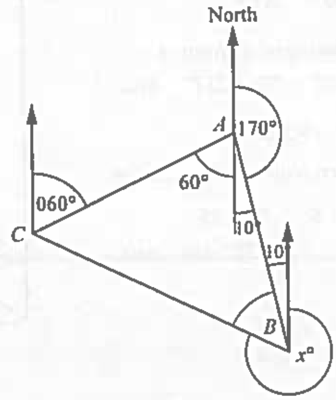


Given that triangle ABC is isosceles, find the three possible values of x. [3]

Thinking Process

Triangle ABC is isosceles when $AB = AC$, or $CA = CB$ or $BC = BA$. Consider each case separately to find three possible values of x.

Solution



Option 1: ΔABC is isosceles with $AB = AC$

$\therefore \quad \angle ACB = \angle ABC$

from figure, $\angle CAB = 60° + 10° = 70°$

$\angle ABC = \dfrac{180° - 70°}{2} = 55°$ (base $\angle$ of isosceles Δ)

$\therefore \quad x = 360° - (55° + 10°)$

$= 360° - 65°$

$= 295°$ **Ans.**

Option 2: ΔABC is isosceles with $CA = CB$

$\therefore \quad \angle CBA = \angle CAB = 70°$

$\therefore \quad x = 360° - (70° + 10°)$

$= 360° - 80°$

$= 280°$ **Ans.**

Option 3: ΔABC is isosceles with $BA = BC$

$\therefore \angle BAC = \angle BCA = 70°$

$\angle ABC + 70° + 70° = 180°$ ($\angle$ sum of a Δ)

$\angle ABC + 140° = 180°$

$\angle ABC = 40°$

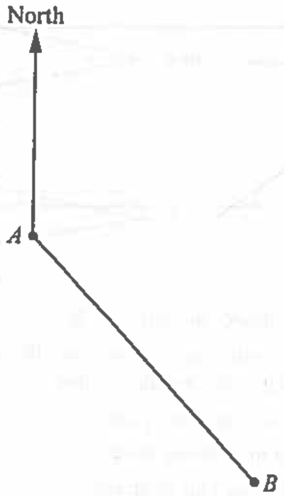
$\therefore x = 360° - (40° + 10°)$

$= 360° - 50°$

$= 310°$ **Ans.**

6 *(J2017/P1/Q5)*

The diagram shows the position of two villages A and B.



(a) Measure the bearing of B from A. [1]

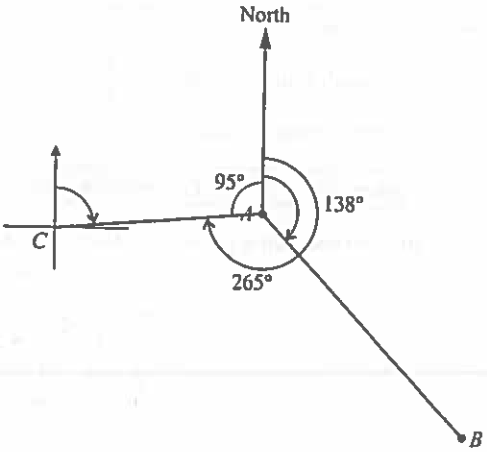
(b) The bearing of village C from A is 265°.
Work out the bearing of A from C. [1]

Thinking Process

(a) Measure the angle by using protractor.

(b) Draw North at C and use north lines as parallel lines to find the required bearing.

Solution



(a) Bearing of B from $A = 138°$ **Ans.**

(b) Bearing of C from $A = 265°$

$360° - 265° = 95°$ ($\angle$s around pt. A)

$\therefore$ Bearing of A from C

$= 180° - 95°$ (int. $\angle$s between $\parallel$ lines)

$= 85°$ **Ans.**

Topic 16

Probability

1 (J2007/P2/Q5 b,c)

(b) Emma chose one word, at random, from the 25 words. Find the probability that this word had

(i) 5 or 6 letters, [1]

(ii) fewer than 9 letters. [1]

(c) Peter chose one word, at random, from the 25 words. He then chose a second word, at random, from the remaining words.
Expressing each answer as a fraction in its lowest terms, find the probability that

(i) both words had 6 letters, [1]

(ii) one word had 2 letters and the other had 4 letters. [2]

Thinking Process

(b) (i) $P(A \text{ or } B) = P(A) + P(B)$.

(ii) Note that number of letters in all the words are less than 9.

(c) Note that Peter choses second word from the remaining words, i.e. from 24 words.

Solution

(b) (i) $P(5 \text{ letters}) = \frac{5}{25} = \frac{1}{5}$.

$P(6 \text{ letters}) = \frac{4}{25}$

$\therefore\ P(5 \text{ or } 6 \text{ letters}) = \frac{1}{5} + \frac{4}{25} = \frac{9}{25}$ Ans

(ii) Number of letters in all the 25 words is fewer (or less) than 9.

$\therefore\ P(\text{fewer than } 9) = 1$ Ans

(c) (i) $P(\text{both words had 6 letters}) = \left(\frac{4}{25}\right) \times \left(\frac{3}{24}\right)$

$= \frac{1}{50}$ Ans

(ii)

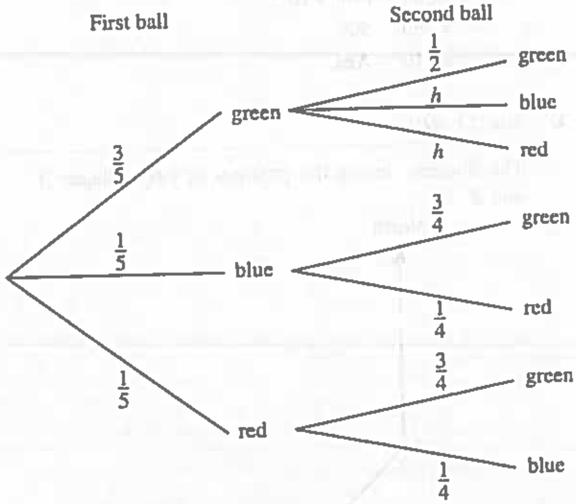
P(one word had 2 letters and the other had 4 letters)

$= (P(2 \text{ letters}) \times P(4 \text{ letters})) + (P(4 \text{ letters}) \times P(2 \text{ letters}))$

$= \left(\frac{2}{25}\right) \times \left(\frac{5}{24}\right) + \left(\frac{5}{25}\right) \times \left(\frac{2}{24}\right) = \frac{1}{60} + \frac{1}{60} = \frac{1}{30}$ Ans

2 (N2007/P1/Q21)

A bag contains 1 red, 1 blue and 3 green balls.
Two balls are taken from the bag, at random, without replacement.
The tree diagram that represents these events is drawn below.



(a) Write down the value of h. [1]

(b) Expressing each answer in its simplest form, calculate the probability that

(i) both balls are green, [1]

(ii) both balls are blue, [1]

(iii) neither ball is green. [1]

Thinking Process

(a) Note that the balls are not being replaced.

(b) (i) Use tree diagram.

(ii) You cannot take out 2 blue balls as there is only one blue ball in the bag.

(iii) Consider all possible combinations of blue & red.

Solution

(a) $h = \frac{1}{4}$ Ans

(b) (i) $P(\text{both balls are green}) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$ Ans

(ii) $P(\text{both balls are blue}) = 0$ Ans

Note that there is only one blue ball

(iii) $P(\text{neither ball is green}) = [P(\text{blue}) \times P(\text{red})] + [P(\text{red}) \times P(\text{blue})]$

$= \left(\frac{1}{5} \times \frac{1}{4}\right) + \left(\frac{1}{5} \times \frac{1}{4}\right)$

$= \frac{1}{20} + \frac{1}{20} = \frac{1}{10}$ Ans

3 *(J2008/P2/Q5 b)*

(b) Tina has two fair, normal 6-sided dice. One is red and the other is blue.
She throws both of them once.
You may find it helpful to draw a possibility diagram to answer the following questions.
Find, as a fraction in its lowest terms, the probability that

(i) the red die shows a 2 and the blue die does not show a 2, [1]

(ii) the sum of the two numbers shown is equal to 5, [1]

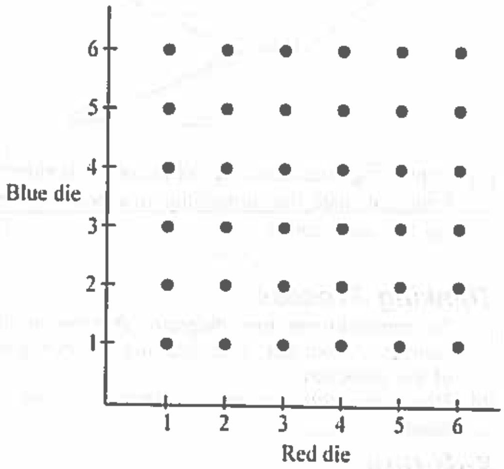
(iii) one die shows a 3 and the other shows an even number. [2]

Thinking Process

(b) Draw a possibility diagram.
(i) Count the number of dots that satisfy the given condition.
(ii) Count the number of dots in which the two numbers add up to 5.
(iii) Count the number of dots in which one die shows 3 and the other an even number.

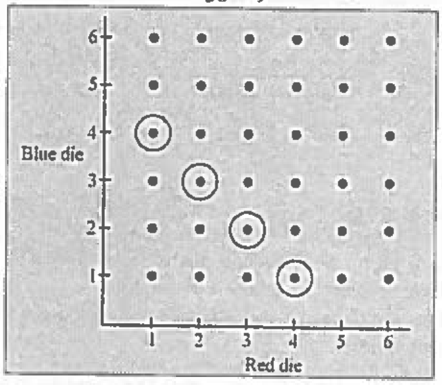
Solution *with* TEACHER's COMMENTS

(b)



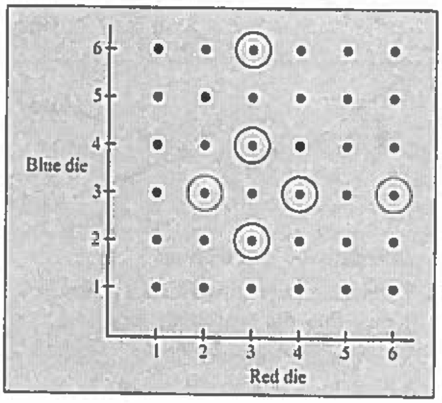
(i) $P(\text{2 on red and blue without 2}) = \frac{5}{36}$ **Ans.**

(ii) $P(\text{sum is 5}) = \frac{4}{36} = \frac{1}{9}$ **Ans.**



(iii) $P(\text{3 on one and even on other}) = \frac{6}{36}$

$= \frac{1}{6}$ **Ans.**



4 *(N2008/P1/Q8)*

A bag contains red, green and yellow pegs.
A peg is taken at random from the bag.
The probability that it is red is 0.35 and the probability that it is green is 0.4.

(a) Find the probability that it is
(i) yellow, [1]
(ii) not red. [1]

(b) Originally there were 16 green pegs in the bags. Find the total number of pegs. [1]

Thinking Process

(a) (i) Subtract the sum of two given probabilities from 1.
(ii) Subtract the probability of red from 1.
(b) Note that 16 green pegs represent a probability of 0.4.

Solution

(a) (i) $P(\text{yellow}) = 1 - (0.35 + 0.4)$
$= 1 - 0.75 = 0.25$ **Ans.**

(ii) $P(\text{not red}) = 1 - 0.35 = 0.65$ **Ans.**

(b) $P(\text{green peg}) = 0.4$

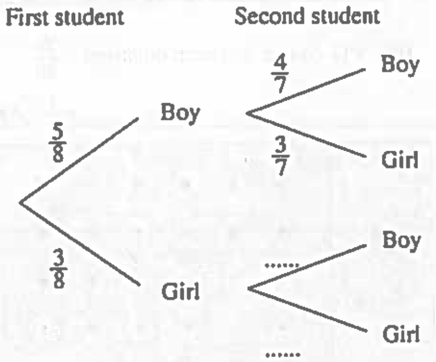
$$\Rightarrow \frac{\text{no. of green pegs}}{\text{total no. of pegs}} = 0.4$$

$$\frac{16}{\text{total no. of pegs}} = 0.4$$

$$\text{total no. of pegs} = \frac{16}{0.4} = 40 \quad \textbf{Ans.}$$

5 *(N2009/P1/Q21)*

In a group of 8 students there are 5 boys and 3 girls. Two students are chosen at random.
The tree diagram shows the possible outcomes and their probabilities.



(a) Complete the tree diagram. [1]

(b) Expressing each answer as a fraction in its lowest terms, find the probability that

(i) two boys are chosen, [1]

(ii) at least one boy is chosen. [2]

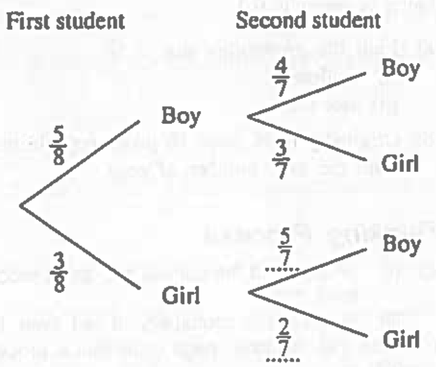
Thinking Process

(b) (i) P(boy) × P(boy).

(ii) 1 − P(Two girls are chosen).

Solution

(a)



(b) (i) $P(\text{two boys are chosen}) = P(\text{boy}) \times P(\text{boy})$

$$= \frac{5}{8} \times \frac{4}{7} = \frac{5}{14} \quad \textbf{Ans.}$$

(ii) $P(\text{at least one boy is chosen})$

$$= 1 - (P(\text{Girl}) \times P(\text{Girl}))$$

$$= 1 - \left(\frac{3}{8} \times \frac{2}{7}\right)$$

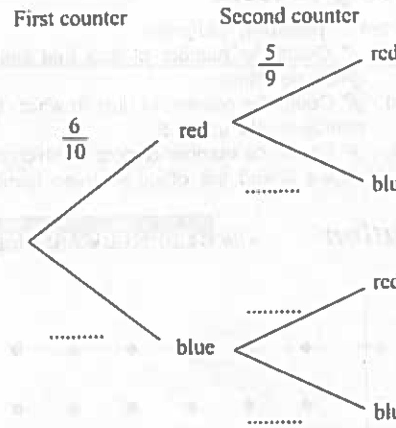
$$= 1 - \frac{3}{28}$$

$$= \frac{25}{28} \quad \textbf{Ans.}$$

6 *(J2010/P1/Q16)*

A bag contains 6 red counters and 4 blue counters. Two counters are taken from the bag at random, without replacement.

(a) Complete the tree diagram below that represents these events.



[1]

(b) Expressing your answer as a fraction in its simplest form, calculate the probability that both counters are the same colour. [2]

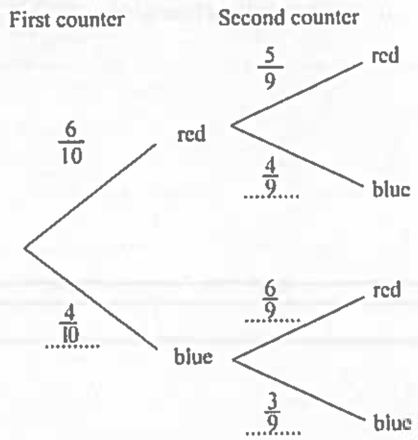
Thinking Process

(a) To complete the tree diagram, consider the number of counters, blue and red at each stage of the selection.

(b) P(both counters are red) + P(both counters are blue).

Solution

(a)



(b) P(both counters are of same colour)

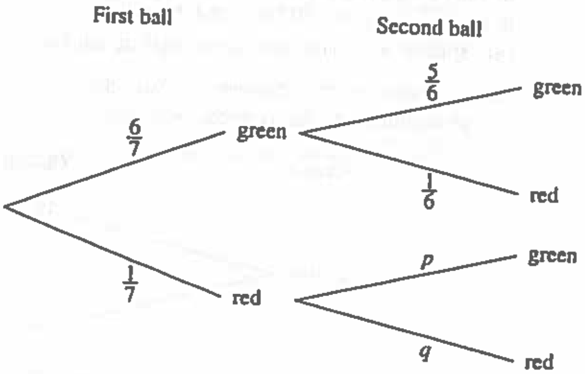
$= P(RR) + P(BB)$

$= \left(\frac{6}{10} \times \frac{5}{9}\right) + \left(\frac{4}{10} \times \frac{3}{9}\right)$

$= \frac{1}{3} + \frac{2}{15} = \frac{7}{15}$ Ans.

7 (N2010 P1 Q19)

A bag contains 7 balls, 6 of which are green and 1 is red.
Two balls are taken from the bag, at random, without replacement.
The tree diagram that represents these events is drawn below.



(a) Find the values of p and q. [1]

(b) Expressing each answer as a fraction in its simplest form, find the probability that

(i) both balls are green, [1]

(ii) the two balls have different colours. [1]

Thinking Process

(a) To find p and q ✐ consider the number of balls at each stage of the selection.

(b) (i) Find P(green) x P(green)

(ii) Find P(1st is green, 2nd is red) + P(1st is red, 2nd is green).

Solution

(a) $p = \frac{6}{6} = 1, \quad q = 0$ Ans.

(b) (i) $P(\text{both balls are green}) = \frac{6}{7} \times \frac{5}{6}$

$= \frac{5}{7}$ Ans.

(ii) P(both balls have different colours)

$= P(GR) + P(RG)$

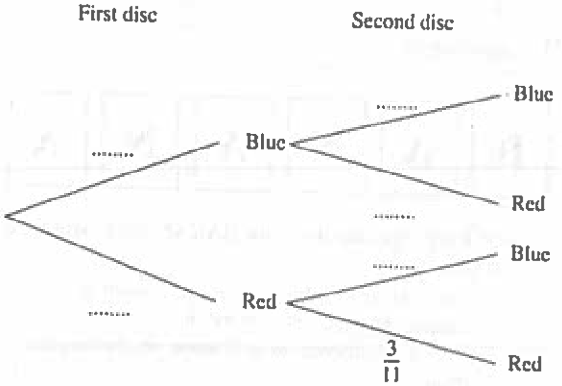
$= \left(\frac{6}{7} \times \frac{1}{6}\right) + \left(\frac{1}{7} \times 1\right)$

$= \frac{1}{7} + \frac{1}{7}$

$= \frac{2}{7}$ Ans.

8 (N2011 P2 Q11 b)

A bag contains 12 discs.
There are 8 blue and 4 red discs.
A disc is picked out at random and not replaced.
A second disc is then picked out at random and not replaced.
The tree diagram below shows the possible outcomes and one of their probabilities.



(i) Complete the tree diagram. [2]

(ii) Expressing each of your answers as a fraction in its lowest terms, calculate the probability that

(a) both discs are red, [1]

(b) at least one disc is blue. [2]

(iii) A third disc is picked out at random. Calculate the probability that all three discs are red. [1]

Thinking Process

(i) To complete the tree diagram ✐ consider the number of discs at each stage of selection.

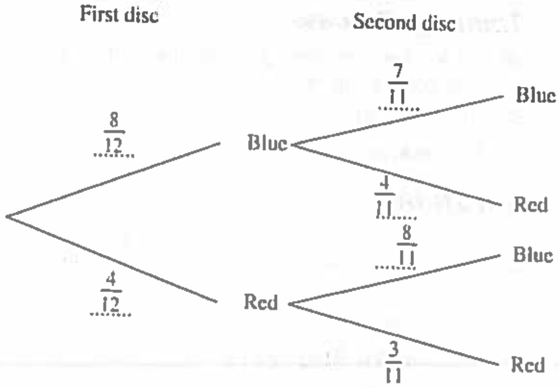
(ii) (a) Find P(red) x P(red).

(b) Use Probability = $1 - P$(red) x P(red).

(iii) Find P(red) x P(red) x P(red).

Solution

(i)



(ii) (a) $P(\text{both discs are red}) = \frac{4}{12} \times \frac{3}{11}$

$= \frac{1}{11}$ Ans.

(b) $P(\text{at least one disc is blue}) = 1 - \frac{1}{11}$

$= \frac{10}{11}$ **Ans.**

(iii) $P(\text{all 3 discs are red}) = \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$

$= \frac{1}{55}$ **Ans.**

9 *(J2011/P2/Q3)*

B	A	N	A	N	A

The letters spelling the word BANANA are written on six tiles.

(a) Find the probability that a tile chosen at random has the letter N on it.
Give your answer as a fraction in its simplest form. [1]

(b) The six tiles are placed in a bag.
Three tiles are chosen at random without replacement.
The first is placed in Position 1, the second in Position 2 and the third in Position 3.

________ ________ ________
Position 1 Position 2 Position 3

(i) Find the probability that the three tiles spell BAN.
Give your answer as a fraction in its simplest form. [2]

(ii) The tiles are now replaced and the process is repeated.
Find the probability that the three tiles spell either ANN or ANA.
Give your answer as a fraction in its simplest form. [2]

Thinking Process

(a) To find the probability, count the number of tiles that contain the letter N.

(b) (i) $P(B) \times P(A) \times P(N)$

(ii) $P(A) \times P(N) \times P(N) + P(A) \times P(N) \times P(A)$

Solution

(a) $P(\text{tile chosen has the letter N}) = \frac{2}{6} = \frac{1}{3}$ **Ans.**

(b) (i) $P(\text{the three tiles spell BAN})$

$= P(B) \times P(A) \times P(N)$

$= \frac{1}{6} \times \frac{3}{5} \times \frac{2}{4}$

$= \frac{1}{20}$ **Ans.**

(ii) $P(\text{the three tiles spell ANN or ANA})$

$= \left(P(A) \times P(N) \times P(N)\right)$

$+ \left(P(A) \times P(N) \times P(A)\right)$

$= \left(\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4}\right) + \left(\frac{3}{6} \times \frac{2}{5} \times \frac{2}{4}\right)$

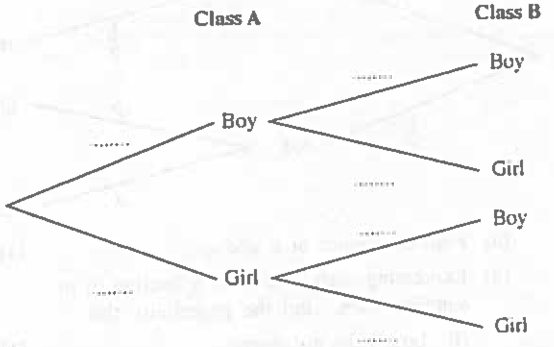
$= \frac{1}{20} + \frac{1}{10}$

$= \frac{3}{20}$ **Ans.**

10 *(J2012/P1/Q21)*

In class A there are 10 boys and 15 girls.
In class B there are 20 boys and 10 girls.
One student is picked from each class at random.

(a) Complete the tree diagram to show the probabilities of the possible outcomes.



[2]

(b) Find the probability that one student is a boy and one is a girl.
Express your answer as a fraction in its lowest terms. [2]

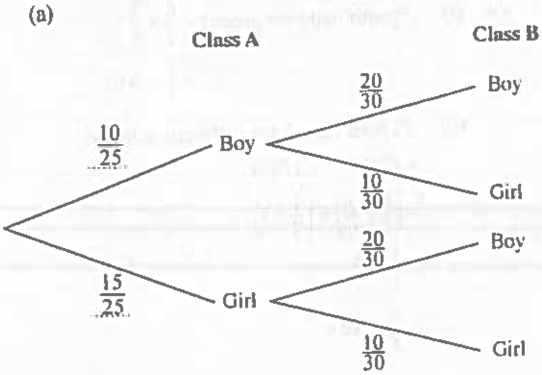
Thinking Process

(a) Consider the number of boys and girls at each stage of the selection.

(b) P(one boy and one girl) = P(boy, girl) + P(girl, boy).

Solution

(a)



(b) P(one boy and one girl)

$= P(\text{boy} \times \text{girl}) + P(\text{girl} \times \text{boy})$

$= \left(\frac{10}{25} \times \frac{10}{30}\right) + \left(\frac{15}{25} \times \frac{20}{30}\right)$

$= \frac{2}{15} + \frac{2}{5}$

$= \frac{8}{15}$ **Ans.**

11 *(N2012 P1 Q16)*

2	3	4
Card A	Card B	Card C

Three cards, A, B and C are marked with the numbers 2, 3 and 4 respectively.
One card is chosen, at random.
A second card is then chosen, at random, from the remaining two cards.
The sum of the numbers on the two chosen cards is calculated.

(a) What is the probability that the sum is 3? [1]

(b) Complete the table to show all the possible outcomes.
You may not need all the columns. [1]

First card	A								
Second card	B								
Sum	5								

(c) What is the probability that the sum is 7? [1]

Thinking Process

(a) ✎ No such probability.
(c) To find the required probability ✎ count the number of outcomes from the table that give a sum of 7.

Solution

(a) $P(\text{sum is } 3) = 0$ **Ans.**

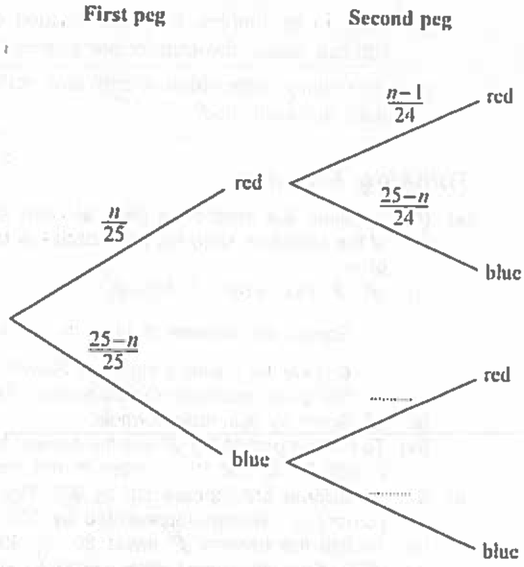
(b)

First card	A	A	B	B	C	C	
Second card	B	C	A	C	A	B	
Sum	5	6	5	7	6	7	

(c) $P(\text{sum is } 7) = \frac{2}{6} = \frac{1}{3}$ **Ans.**

12 *(J2013 P2 Q10)*

(a) A bag contains red and blue pegs.
Altogether there are 25 pegs of which n are red.
Rashid picks two pegs without replacement.
The tree diagram shows the possible outcomes and their probabilities.



(i) Complete the tree diagram. [2]

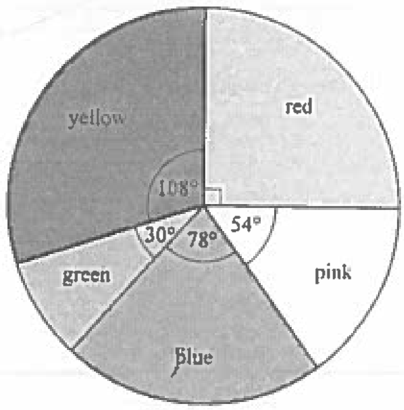
(ii) (a) Write an expression, as a single fraction in terms of n, for the probability that Rashid picks a red peg then a blue peg in that order. [1]

(b) The probability that Rashid picks a red peg then a blue peg in that order is $\frac{1}{p}$.
Given that the number of red pegs, n, satisfies the equation $n^2 - 25n + 150 = 0$, find p. [2]

(iii) Solve $n^2 - 25n + 150 = 0$, to find the possible values of n. [2]

(iv) Given that at the start there are more blue pegs than red pegs in the bag, find the probability that Rashid picks two red pegs. [2]

(b) Each member of a group of children was asked their favourite colour.
The pie chart represents the results.



(i) The number of children whose favourite colour is red is 75.
Find the number of children in the group. [1]

(ii) Find, in its simplest form, the fraction of children whose favourite colour is green. [1]

(iii) How many more children answered yellow than answered blue? [1]

Thinking Process

(a) (i) Consider the number of pegs at each stage of the selection. write the probabilities in terms of n.

(ii) (a) Find $P(\text{red}) \times P(\text{blue})$

(b) Equate the answer of (ii)(a) to $\frac{1}{p}$ and simplify the resulting equation. Solve it with the given equation simultaneously for p.

(iii) Solve by quadratic formula.

(iv) To find the probability use the answer found in part (iii) to find the number of red pegs.

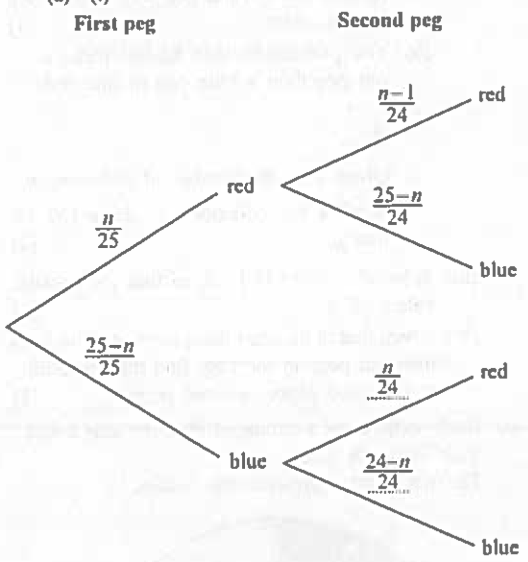
(b) (i) 75 children are represented by 90°. Find the number of children represented by 360°.

(ii) To find the fraction divide 30° by 360°.

(iii) Find the difference between angle for yellow and angle for blue.

Solution

(a) (i)



(ii) (a) $P(\text{red peg then a blue peg})$

$$= \left(\frac{n}{25}\right) \times \left(\frac{25-n}{24}\right)$$

$$= \frac{25n - n^2}{600} \quad \textbf{Ans.}$$

(b) Given that,

$$P(\text{a red peg then a blue peg}) = \frac{1}{p}$$

$$\Rightarrow \quad \frac{25n - n^2}{600} = \frac{1}{p}$$

$$\Rightarrow \quad 25n - n^2 = \frac{600}{p}$$

$$\Rightarrow \quad -(n^2 - 25n) = \frac{600}{p}$$

$$\Rightarrow \quad n^2 - 25n = -\frac{600}{p} \quad \cdots\cdots\cdots(1)$$

given equation is,

$$n^2 - 25n + 150 = 0$$

$$\Rightarrow \quad n^2 - 25n = -150 \quad \cdots\cdots\cdots(2)$$

from equations (1) and (2),

$$-150 = -\frac{600}{p}$$

$$150p = 600 \quad \Rightarrow \quad p = 4 \quad \textbf{Ans.}$$

(iii) $n^2 - 25n + 150 = 0$

by quadratic formula,

$$n = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(1)(150)}}{2(1)}$$

$$= \frac{25 \pm \sqrt{625 - 600}}{2}$$

$$= \frac{25 \pm \sqrt{25}}{2}$$

$$= \frac{25 \pm 5}{2}$$

$$= \frac{25+5}{2} \quad \text{or} \quad = \frac{25-5}{2}$$

$$= 15 \quad \text{or} \quad = 10$$

$$\therefore \quad n = 15 \text{ or } 10 \quad \textbf{Ans.}$$

(iv) Since there are more blue pegs than red, thus from part (iii), we deduce that,

number of red pegs = 10, and

number of blue pegs = 15

$$\therefore \quad P(\text{two red pegs}) = \frac{10}{25} \times \frac{9}{24}$$

$$= \frac{3}{20} \quad \textbf{Ans.}$$

(b) (i) 90° represents —— 75 children

$$360° \text{ represents} \text{ —— } \frac{75}{90} \times 360 = 300 \text{ children}$$

$\therefore$ No. of children in the group = 300 **Ans.**

(ii) Fraction of children whose favourite colour

$$\text{is green} = \frac{30°}{360°}$$

$$= \frac{1}{12} \quad \textbf{Ans.}$$

(iii) $108° - 78° = 30°$

30° represents more children who answered yellow.

360° represents — 300 children

30° represents — $\frac{300}{360} \times 30 = 25$ children

∴ 25 more children answered yellow than answered blue **Ans.**

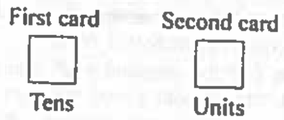
13 *(N2013/P1/Q21)*

2	3	3	4	4	4

The numbers 2, 3, 3, 4, 4, 4 are written on six cards. Two cards are chosen, at random, without replacement, to form a 2-digit number.

The first card chosen shows the number of Tens.

The second card chosen shows the number of Units.



Expressing each answer in its simplest form, find the probability that the two cards show

(a) a number greater than 20, [1]

(b) the number 33, [1]

(c) the number 43 or the number 32. [2]

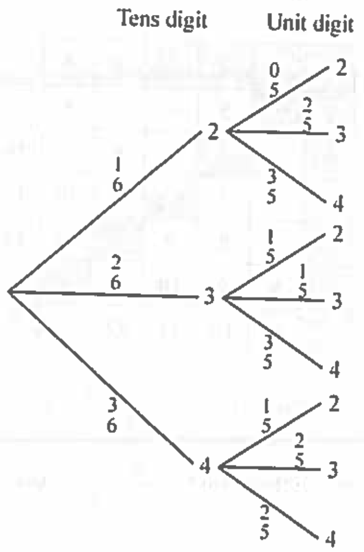
Thinking Process

(a) Note that all possible outcomes will show a number greater than 20.

(b) Find $P(3) \times P(3)$.

(c) $P(A \text{ or } B) = P(A) + P(B)$.

Solution



(a) $P(\text{a number greater than } 20) = 1$ **Ans.**

(b) $P(\text{the number } 33) = \frac{2}{6} \times \frac{1}{5}$

$= \frac{2}{30} = \frac{1}{15}$ **Ans.**

(c) $P(\text{the number 43 or 32}) = \left(\frac{3}{6} \times \frac{2}{5}\right) + \left(\frac{2}{6} \times \frac{1}{5}\right)$

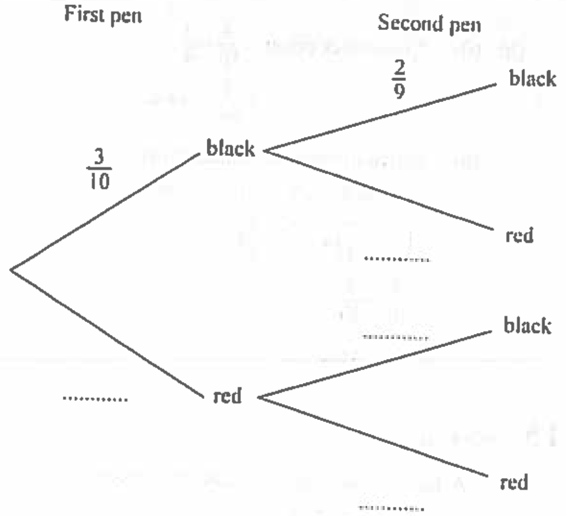
$= \frac{6}{30} + \frac{2}{30}$

$= \frac{8}{30} = \frac{4}{15}$ **Ans.**

14 *(J2014/P1/Q21)*

Luis has 3 black pens and 7 red pens in a case. He takes two pens from the case at random without replacement.

(a) Complete the tree diagram to show the possible outcomes and their probabilities.



[1]

(b) Find, as a fraction in its lowest terms, the probability that

(i) Luis takes two black pens, [1]

(ii) Luis takes two different coloured pens. [2]

Thinking Process

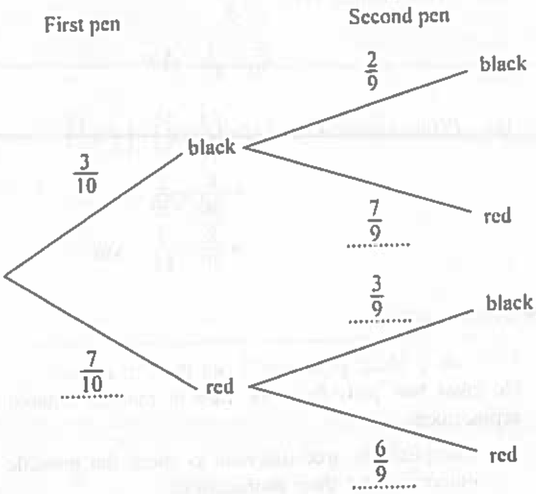
(a) To complete the tree diagram consider the number of pens at each stage of the selection.

(b) (i) Find $P(\text{black}) \times P(\text{black})$

(ii) Find $P(\text{black, red}) + P(\text{red, black})$.

Solution

(a)



(b) (i) $P(\text{two black pens}) = \frac{3}{10} \times \frac{2}{9}$

$= \frac{1}{15}$ **Ans.**

(ii) $P(\text{two different coloured pens})$

$= P(\text{black, red}) + P(\text{red, black})$

$= \left(\frac{3}{10} \times \frac{7}{9}\right) + \left(\frac{7}{10} \times \frac{3}{9}\right)$

$= \frac{7}{30} + \frac{7}{30}$

$= \frac{7}{15}$ **Ans.**

15 *(J2014/P2/Q4)*

A bag contains six identical balls numbered 2, 3, 4, 5, 6 and 7.

(a) A ball is taken from the bag at random. Find, as a fraction in its lowest terms, the probability that the number on the ball is

(i) a multiple of 3, [1]

(ii) prime. [1]

(b) All six balls are replaced in the bag. Two balls are taken from the bag, one after the other, without replacement. The numbers on the two balls are added together.

(i) Complete this possibility diagram to show all the outcomes.

+	2	3	4	5	6	7
2		5	6	7	8	9
3						
4						
5						
6						
7						

[1]

(ii) Find the probability that the sum of the numbers is

(a) odd, [1]

(b) less than 8. [1]

Thinking Process

(a) (i) To find the probability ✎ count the balls numbered with multiples of 3.

(ii) To find the probability ✎ count the balls numbered with prime numbers.

(b) (i) To complete the diagram ✎ add the row heading with the column heading for each box.

(ii) (a) To find the probability ✎ count the odd numbers in the diagram.

(b) To find the probability ✎ count the numbers in the possibility diagram that give a sum of less than 8.

Solution

(a) (i) Multiples of 3 are: 3, 6

$P(\text{multiple of } 3) = \frac{2}{6} = \frac{1}{3}$ **Ans.**

(ii) Prime numbers are: 2, 3, 5, 7

$P(\text{prime number}) = \frac{4}{6} = \frac{2}{3}$ **Ans.**

(b) (i)

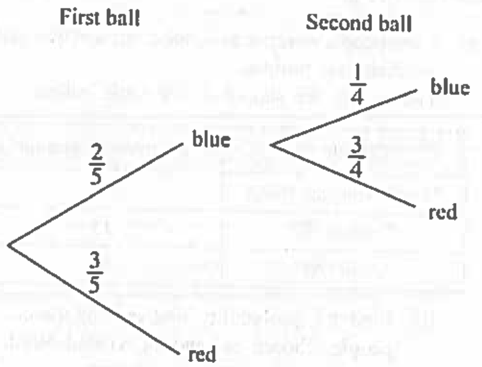
+	2	3	4	5	6	7
2		5	6	7	8	9
3	5		7	8	9	10
4	6	7		9	10	11
5	7	8	9		11	12
6	8	9	10	11		13
7	9	10	11	12	13	

(ii) (a) $P(\text{sum is odd}) = \frac{18}{30} = \frac{3}{5}$ **Ans.**

(b) $P(\text{less than } 8) = \frac{8}{30} = \frac{4}{15}$ **Ans.**

16 (N2014/P1/Q21)

A bag contains 5 balls, 2 of which are blue and 3 are red.
One ball is taken, at random, from the bag.
If it is red it is put back into the bag.
If it is blue it is not put back into the bag.
A second ball is taken, at random, from the bag.
Part of the tree diagram that represents these outcomes is drawn below.



(a) Complete the tree diagram. [1]

(b) Expressing each answer as a fraction in its simplest form, find the probability that

(i) both balls taken are blue, [1]

(ii) the second ball taken is blue. [2]

Thinking Process

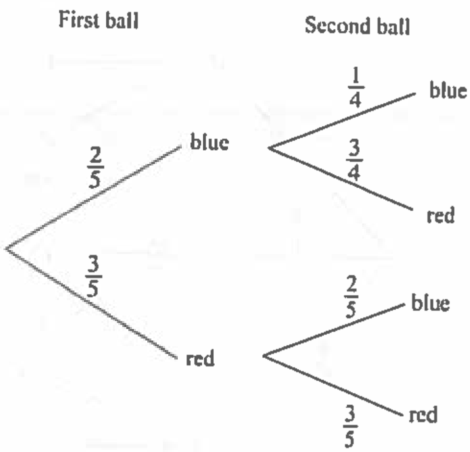
(a) To complete the tree diagram consider the number of red and blue balls at each stage of the selection.

(b) (i) Find $P(\text{blue}) \times P(\text{blue})$.

(ii) Find $P(\text{blue, blue}) + P(\text{red, blue})$

Solution

(a)



(b) (i) $P(\text{both balls are blue}) = \frac{2}{5} \times \frac{1}{4}$

$= \frac{2}{20} = \frac{1}{10}$ Ans.

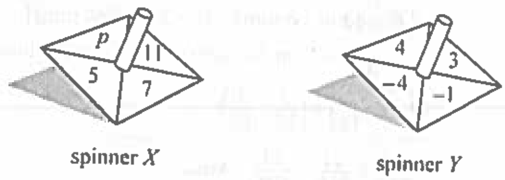
(ii) $P(\text{second ball is blue}) = P(BB) + P(RB)$

$= \left(\frac{2}{5} \times \frac{1}{4}\right) + \left(\frac{3}{5} \times \frac{2}{5}\right)$

$= \frac{1}{10} + \frac{6}{25}$

$= \frac{17}{50}$ Ans.

17 (N2014/P2 Q3)



In a game, when it is Mary's turn, she spins each of these fair spinners once.
Mary's score for the turn is worked out using the formula $xm + yn$, where x is the number on spinner X and y is the number on spinner Y.
The possibility space diagram shows Mary's possible scores.

y (number on spinner Y) \ x (number on spinner X)	5	7	11	p
−4	37	47	67	97
−1	28	38	58	88
3	16	26	46	76
4	13	23	43	73

(a) Find the probability that Mary's score is less than 15. [1]

(b) Calculate the probability that on two consecutive turns, Mary scores less than 40 on one and more than 75 on the other. [3]

(c) The diagram shows 7 on spinner X and −1 on spinner Y.
Using the formula, the score for this turn is $7m - n = 38$.

(i) Using the table, find $7m + 3n$. [1]

(ii) Hence find m and n. [2]

(d) Find p. [2]

Thinking Process

(a) Count the no. of scores that are less than 15.

(b) Find $P(\text{less than 40 in 1st turn, more than 75 in 2nd turn}) + P(\text{more than 75 in 1st turn, less than 40 in 2nd turn})$.

(c) (i) From the table, find the score that corresponds to 7 on spinner X and 3 on spinner Y.

(ii) Solve the two equations found in (c) and (c)(i) simultaneously for m and n.

(d) Using the table and the given formula, form an equation in terms of p, m and n. Solve the equation for p.

Solution

(a) $P(\text{score less than } 15) = \frac{1}{16}$ **Ans.**

(b) $P(\text{score} < 40 \text{ on one and} > 75 \text{ on the other turn})$

$= [P(< 40 \text{ in 1st turn}) \times P(> 75 \text{ in 2nd turn})]$
$+ [P(> 75 \text{ in 1st turn}) \times P(< 40 \text{ in 2nd turn})]$

$= \left(\frac{7}{16} \times \frac{3}{16}\right) + \left(\frac{3}{16} \times \frac{7}{16}\right)$

$= \frac{21}{256} + \frac{21}{256} = \frac{21}{128}$ **Ans.**

(c) (i) $7m + 3n = 26$ **Ans.**

(ii) $7m - n = 38 \Rightarrow n = 7m - 38$ ········ (1)

$7m + 3n = 26$ ········ (2)

substitute (1) into (2).

$7m + 3(7m - 38) = 26$

$7m + 21m - 114 = 26$

$28m = 140$

$m = 5$

substitute $m = 5$ into (1)

$n = 7(5) - 38 = -3$

$\therefore$ $m = 5$, $n = -3$ **Ans.**

(d) Using p and -4 from the table, the given formula becomes,

$pm - 4n = 97$

substitute values of m and n.

$p(5) - 4(-3) = 97$

$5p + 12 = 97$

$5p = 85$

$p = 17$ **Ans.**

18 *(J2015 P1 Q4)*

A bag contains red counters, blue counters and yellow counters.
There are 60 counters in the bag.
The probability that a counter taken at random from the bag is red is $\frac{2}{5}$.
The probability that a counter taken at random from the bag is blue is $\frac{5}{12}$.
How many yellow counters are in the bag? [2]

Thinking Process

To find the number of yellow counters ⇒ find the total number of red and blue counters.

Solution

Number of red counters in the bag $= \frac{2}{5} \times 60 = 24$

Number of blue counters in the bag $= \frac{5}{12} \times 60 = 25$

$\therefore$ number of yellow counters $= 60 - (24 + 25)$
$= 11$ **Ans.**

19 *(J2015 P2 Q11 a)*

(a) Some people were asked which continent they visited on their last holiday.
The results are shown in the table below.

Continent	Number of people
North America (NA)	7
Europe (E)	15
Asia (A)	8

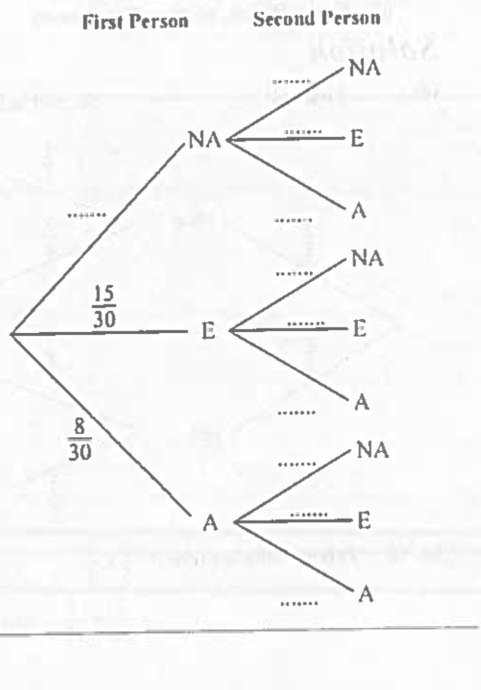
(i) Find the probability that one of these people, chosen at random, visited North America. [1]

(ii) Find the probability that one of these people, chosen at random, did **not** go to Asia. Give your answer as a fraction in its lowest terms. [1]

(iii) Two of these people are chosen at random. The tree diagram opposite shows the possible outcomes and some of their probabilities.

(a) Complete the tree diagram. [2]

(b) What is the probability that the two people went to the same continent? [2]



Thinking Process

(a) (ii) To find the probability ✐ add the number of people who did not go to Asia.

(iii) (a) To complete the tree diagram ✐ consider the number of people at each stage of the selection.

(b) Find $P(\text{NA, NA}) + P(\text{E, E}) + P(\text{A, A})$

Solution

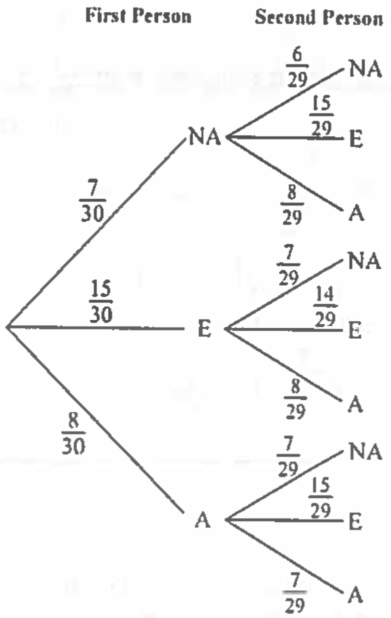
(a) (i) P(one of the people chosen visited NA)

$= \frac{7}{30}$ Ans.

(ii) P(one of the people chosen did not go to Asia)

$= \frac{22}{30} = \frac{11}{15}$ Ans.

(iii) (a)



(b) P(two people went to the same continent)

$= P(\text{NA, NA}) + P(\text{E, E}) + P(\text{A, A})$

$= \left(\frac{7}{30} \times \frac{6}{29}\right) + \left(\frac{15}{30} \times \frac{14}{29}\right) + \left(\frac{8}{30} \times \frac{7}{29}\right)$

$= \frac{7}{145} + \frac{7}{29} + \frac{28}{435}$

$= \frac{154}{435}$ Ans.

20 *(N2015/P1/Q23)*

A fair 4-sided spinner is numbered 1, 2, 3 and 4.

(a) Anil spins it once.
He gets his score by doubling the number obtained.
Complete the table to show the probabilities of his scores.

Score	2	4	6	8
Probability				

[1]

(b) Billie spins it twice. She gets her score by adding the numbers obtained.

(i) Complete the possibility diagram.

First spin (columns), Second spin (rows)

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4				

[1]

(ii) Complete the table showing the probabilities for some of Billie's scores.

Score	> 2	> 4	> 6	> 8
Probability	$\frac{15}{16}$			

[1]

(c) Find the probability that Billie scores more than Anil. [2]

Thinking Process

(a) Note that the probability for each outcome is $\frac{1}{4}$.

(b) (i) To complete the possibility diagram ✐ add the row heading with the column heading for each box.

(ii) Use the possibility diagram in (b) (i) to complete the table.

(c) Find $P(\text{Anil } 2, \text{Billie} > 2) + P(\text{Anil } 4, \text{Billie} > 4) + P(\text{Anil } 6, \text{Billie} > 6) + P(\text{Anil } 8, \text{Billie} > 8)$

Solution

(a)

Score	2	4	6	8
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

(b) (i)

First spin (columns), Second spin (rows)

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

(ii)

Score	> 2	> 4	> 6	> 8
Probability	$\frac{15}{16}$	$\frac{10}{16}$	$\frac{3}{16}$	0

(c) P(Billie scores more than Anil)

$= P$(Anil scores 2, Billie scores > 2)
$+ P$(Anil scores 4, Billie scores > 4)
$+ P$(Anil scores 6, Billie scores > 6)
$+ P$(Anil scores 8, Billie scores > 8)

$$= \left(\frac{1}{4}\times\frac{15}{16}\right)+\left(\frac{1}{4}\times\frac{10}{16}\right)+\left(\frac{1}{4}\times\frac{3}{16}\right)+\left(\frac{1}{4}\times 0\right)$$

$$= \frac{15}{64}+\frac{10}{64}+\frac{3}{64}$$

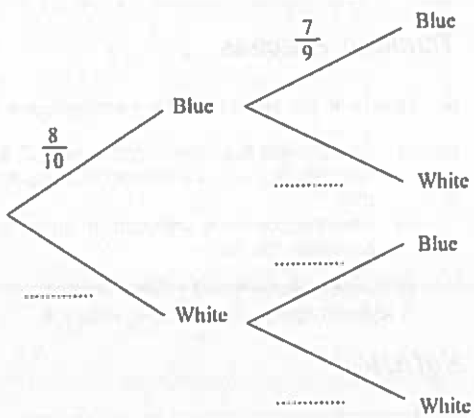
$$= \frac{28}{64} = \frac{7}{16} \quad \text{Ans.}$$

21 *(J2016/P1/Q20)*

A bag contains 10 counters of which 8 are blue and 2 are white.
Two counters are taken from the bag at random without replacement.

(a) Complete the tree diagram to show the possible outcomes and their probabilities.

First counter Second counter



[1]

(b) Find, as a fraction, the probability that

(i) both counters are blue, [1]

(ii) one counter is blue and the other is white. [2]

Thinking Process

(a) To complete the tree diagram while consider the number of white and blue counters at each stage of the selection.

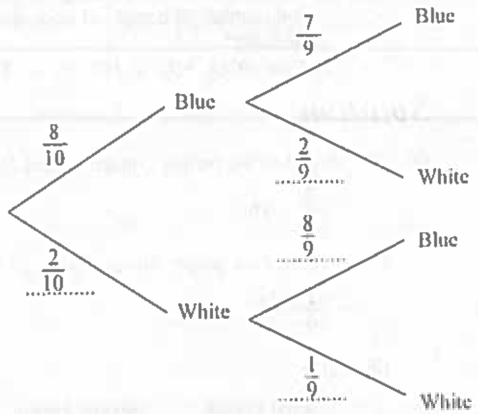
(b) (i) Find P(blue) × P(blue).

(ii) Find P(blue, white) + P(white, blue)

Solution

(a)

First counter Second counter



(b) (i) P(both counters are blue) $= \frac{8}{10}\times\frac{7}{9}$

$$= \frac{56}{90} = \frac{28}{45} \quad \text{Ans.}$$

(ii) P(one is blue and the other is white)

$= P(BW) + P(WB)$

$$= \left(\frac{8}{10}\times\frac{2}{9}\right)+\left(\frac{2}{10}\times\frac{8}{9}\right)$$

$$= \frac{16}{90}+\frac{16}{90}$$

$$= \frac{32}{90} = \frac{16}{45} \quad \text{Ans.}$$

22 *(N2016/P1/Q18)*

[1] [2] [3] [4]

Four cards are marked with the numbers 1, 2, 3 and 4.
One card is chosen at random.
A second card is then chosen, at random, from the remaining three cards.
The sum of the numbers on the two chosen cards is calculated.

(a) Complete the table to show the possible outcomes.

		First card			
		1	2	3	4
Second card	1	▨			
	2		▨		
	3			▨	
	4				▨

[1]

(b) What is the probability that the sum is less than 2? [1]

(c) What is the probability that the sum is greater than 5? [1]

Thinking Process

(a) To complete the table ✐ add the row heading with the column heading for each box.

(b) ✐ No such probability.

(c) To find the probability ✐ count the numbers in the table that give a sum of greater than 5.

Solution

(a)

Second card \ First card	1	2	3	4
1		3	4	5
2	3		5	6
3	4	5		7
4	5	6	7	

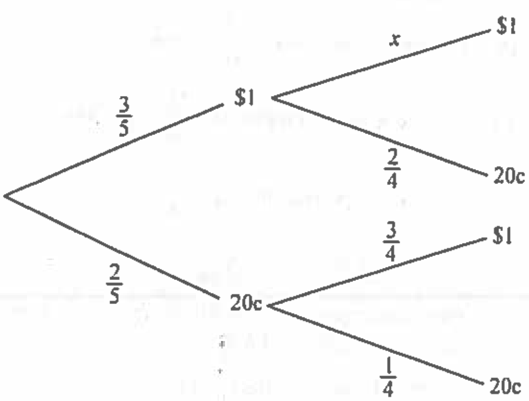
(b) $P(\text{sum is less than } 2) = 0$ **Ans.**

(c) $P(\text{sum is greater than } 5) = \frac{4}{12}$

$= \frac{1}{3}$ **Ans.**

23 *(N2016/P2/Q11 b)*

(b) Amira has three \$1 coins and two 20c coins in her purse.
She picks out coins at random, one after the other.
The coins are not replaced.
The tree diagram shows the possible outcomes and their probabilities when picking out two coins.



(i) Find x. [1]

(ii) Find the probability that the total value of the two coins picked out is 40 cents. [1]

(iii) Find the probability that the total value of the two coins picked out is \$1.20. [2]

(iv) At a car park, the charge is \$1.40.
Amira picks out three coins, one after the other.
Find the probability that the total value of the three coins is \$1.40. [2]

Thinking Process

(b) (i) To find x ✐ consider the number of \$1 coins at the second stage of the selection.

(iv) Consider all the possible outcomes in which the three coins picked up add up to \$1.40.

Solution

(b) (i) $x = \frac{2}{4} = \frac{1}{2}$ **Ans.**

(ii) $P(\text{total value is } 40\text{c}) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$ **Ans.**

(iii) $P(\text{total value is } \$1.20) = \left(\frac{3}{5} \times \frac{2}{4}\right) + \left(\frac{2}{5} \times \frac{3}{4}\right)$

$= \frac{6}{20} + \frac{6}{20} = \frac{3}{5}$ **Ans.**

(iv) $P(\text{total value is } \$1.40)$

$= P(\$1, 20\text{c}, 20\text{c}) + P(20\text{c}, \$1, 20\text{c}) + P(20\text{c}, 20\text{c}, \$1)$

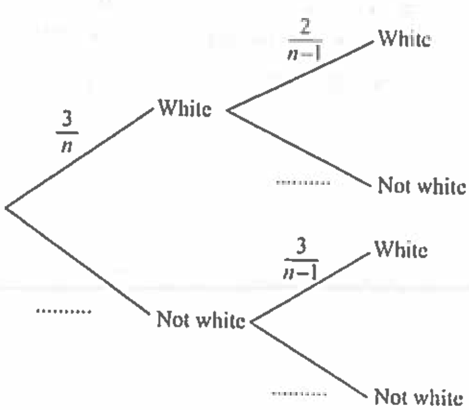
$= \left(\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}\right) + \left(\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}\right)$

$= \frac{6}{60} + \frac{6}{60} + \frac{6}{60} = \frac{3}{10}$ **Ans.**

24 *(J2017/P1/Q24)*

A bag contains n balls.
3 of the balls are white.
Two balls are taken from the bag, at random, without replacement.

(a) Complete the tree diagram.



[2]

(b) The probability that both balls are white is $\frac{1}{15}$.

Show that $n^2 - n - 90 = 0$. [2]

(c) Find the value of n. [2]

Thinking Process

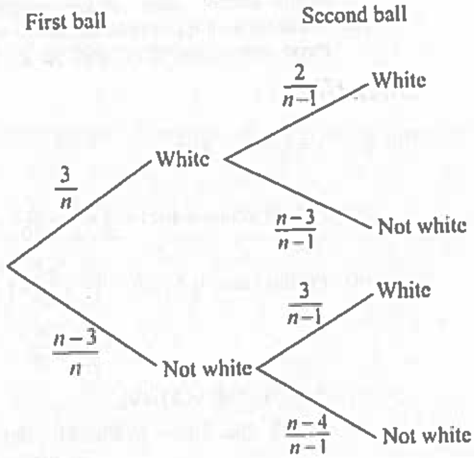
(a) To complete the tree diagram ✎ consider the number of balls at each stage of the selection.

(b) Find $P(\text{white}) \times P(\text{white})$

(c) Factorise and solve for n.

Solution

(a)



(b) $P(\text{both balls are white}) = \frac{3}{n} \times \frac{2}{n-1}$

$\Rightarrow \quad \frac{1}{15} = \frac{6}{n(n-1)}$

$n(n-1) = 90$

$n^2 - n - 90 = 0$ **Shown.**

(c) $n^2 - n - 90 = 0$

$\Rightarrow n^2 + 9n - 10n - 90 = 0$

$\Rightarrow n(n+9) - 10(n+9) = 0$

$\Rightarrow (n+9)(n-10) = 0$

$\Rightarrow n + 9 = 0$ or $n - 10 = 0$

$n = -9$ (reject) $\quad n = 10$

$\therefore \quad n = 10$ **Ans.**

25 *(J2017 P2 Q3)*

Rowena spins two fair spinners, each numbered 1 to 4.

Her score is the value when the numbers on the two spinners are multiplied together.

The table shows some of Rowena's possible scores.

×	1	2	3	4
1	1	2	3	4
2	2	4		
3				
4				

(a) Complete the table of possible scores. [2]

(b) Find the probability that Rowena's score is less than 4. [1]

(c) Find the probability that Rowena's score is an even number.

Give your answer as a fraction in its lowest terms. [2]

(d) Phoebe says that Rowena's score is more likely to be a square number than a factor of 6.

Is she correct? Show your working. [2]

Thinking Process

(a) To complete the table ✎ multiply the row headings with the column headings.

(b) Count the number of scores from the table that are less than 4.

(c) Count the number of scores from the table that are even numbers.

(d) Compare, $P(\text{score is a perfect square})$ with $P(\text{score is a factor of 6})$.

Solution

(a)

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

(b) $P(\text{score is less than 4}) = \frac{5}{16}$ **Ans.**

(c) $P(\text{score is an even number}) = \frac{12}{16} = \frac{3}{4}$ **Ans.**

(d) $P(\text{score is a perfect square}) = \frac{6}{16}$

$P(\text{score is a factor of 6}) = \frac{7}{16}$

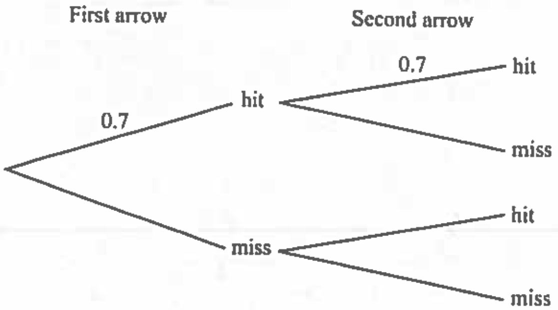
From above probabilities, Rowena's score is more likely to be a factor of 6.

$\therefore$ Phoebe is not correct. **Ans.**

26 *(N2017/P1/Q19)*

Each time an archer fires an arrow, the probability that she hits the target is 0.7 .
She fires two arrows.

(a) Complete the tree diagram.



[1]

(b) Find the probability that

(i) she hits the target twice, [1]

(ii) she hits the target exactly once. [1]

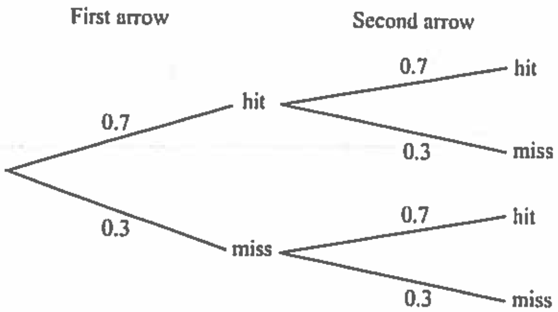
Thinking Process

(b) (i) Find *P*(hit) x *P*(hit)

(ii) Find *P*(hit, miss) + *P*(miss, hit).

Solution

(a)



(b) (i) $P(\text{she hits the target twice}) = 0.7 \times 0.7$

$= 0.49$ **Ans.**

(ii) $P(\text{she hits the target once})$

$= P(\text{hit}) \times P(\text{miss}) + P(\text{miss}) \times P(\text{hit})$

$= (0.7 \times 0.3) + (0.3 \times 0.7)$

$= 0.21 + 0.21 = 0.42$ **Ans.**

27 *(J2018/P1/Q12)*

A dice is thrown 400 times.
The results are shown in the table.

Number thrown	1	2	3	4	5	6
Frequency	65	80	70	75	50	60

(a) Find the relative frequency of throwing the number 2. [1]

(b) Imran throws the dice 1000 times.
How many times would you expect the number 2 to be thrown? [1]

Thinking Process

(a) To find the relative frequency, divide the frequency of 80 by the total number of students.

(b) ✐ Multiply the answer to part (a) by 1000.

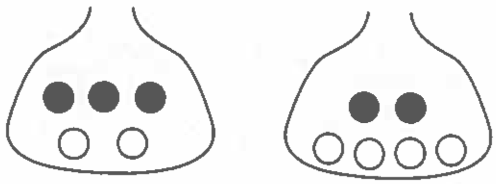
Solution

(a) Relative frequency $= \frac{80}{400} = \frac{1}{5}$ **Ans.**

(b) Number of times 2 is expected to be thrown

$= \frac{1}{5} \times 1000 = 200$ **Ans.**

28 *(N2018/P1/Q17)*

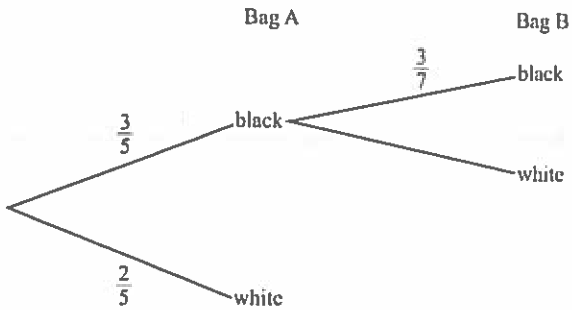


Bag A Bag B

Bag A contains 3 black and 2 white beads.
Bag B contains 2 black and 4 white beads.

A bead is chosen, at random, from Bag A and placed in Bag B.
A bead is then chosen, at random, from Bag B.

(a) Complete the tree diagram.



[2]

(b) Find the probability that a black bead is taken from Bag B. [2]

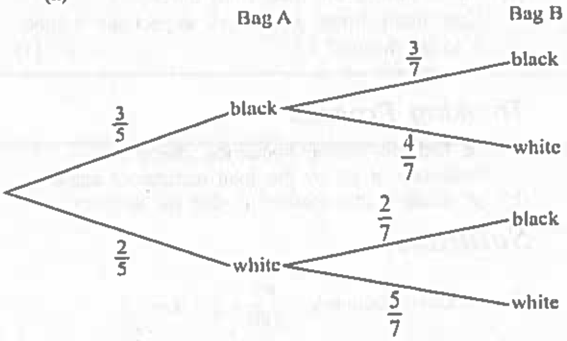
Thinking Process

(a) To complete the tree diagram ✐ consider the number of black and white beads at each stage of the selection.

(b) Find *P*(black, black) + *P*(white, black)

Solution

(a)



(b) $P(\text{black bead from Bag B}) = P(BB) + P(WB)$

$$= \left(\frac{3}{5}\times\frac{3}{7}\right)+\left(\frac{2}{5}\times\frac{2}{7}\right)$$

$$= \frac{9}{35}+\frac{4}{35}=\frac{13}{35} \quad \text{Ans.}$$

Topic 17

Transformations

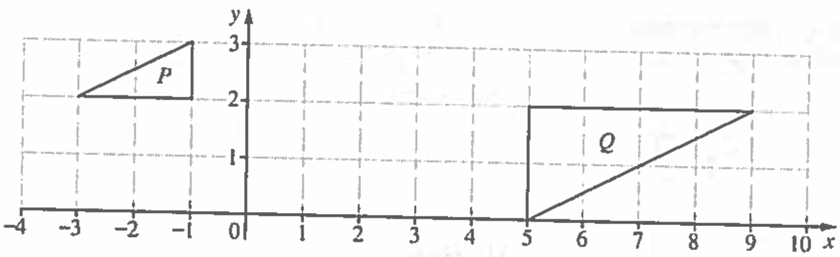
1 (J2009 P2 Q11)

(a) $A = \begin{pmatrix} 0 & 3 \\ -1 & x \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}$

(i) Express $2A - 3B$ in terms of x. [2]

(ii) Given that $A = B^{-1}$, find the value of x. [2]

(b)



The diagram shows the triangles P and Q.

(i) The enlargement E maps triangle P onto triangle Q.
For this enlargement,

(a) write down the scale factor, [1]

(b) find the coordinates of the centre of enlargement. [2]

(ii) The single transformation T is represented by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.
Describe T completely. [2]

(iii) L is the point (k, 2).
T maps L onto (8, 2).

(a) Find the value of k. [1]

(b) Find the coordinates of ET(L). [2]

Thinking Process

(a) (i) Perform the required calculation.

(ii) Apply $B^{-1} = \frac{1}{\det(B)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(b) (i) (a) Divide the length of the image by the corresponding length of the object.

(b) Join the corresponding points to get the centre of enlargement.

(ii) Note that T represents a shear.

(iii) (b) Multiply T with L and then perform enlargement.

Solution

(a) (i) $2A - 3B = 2\begin{pmatrix} 0 & 3 \\ -1 & x \end{pmatrix} - 3\begin{pmatrix} 1 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 6 \\ -2 & 2x \end{pmatrix} - \begin{pmatrix} 3 & -3 \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} -3 & 9 \\ -3 & 2x \end{pmatrix}$ **Ans.**

(ii) $\text{Det. } B = (1 \times 0) - (-1 \times \frac{1}{3}) = \frac{1}{3}$

$A = B^{-1}$

$\begin{pmatrix} 0 & 3 \\ -1 & x \end{pmatrix} = \frac{1}{\frac{1}{3}}\begin{pmatrix} 0 & 1 \\ -\frac{1}{3} & 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 3 \\ -1 & x \end{pmatrix} = 3\begin{pmatrix} 0 & 1 \\ -\frac{1}{3} & 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 3 \\ -1 & x \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ -1 & 3 \end{pmatrix}$

$\therefore$ $x = 3$ **Ans.**

(b) (i) (a) Scale factor = –2 **Ans.**

(b) Centre of enlargement = (1, 2) **Ans.**

(ii) T is a shear along x-axis with shear factor 2. The invariant line is x-axis.

(iii) (a) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} k \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

$\Rightarrow k + 4 = 8$

$k = 4$ **Ans.**

(b) T maps L onto (8, 2)

$\therefore$ $T(L) = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

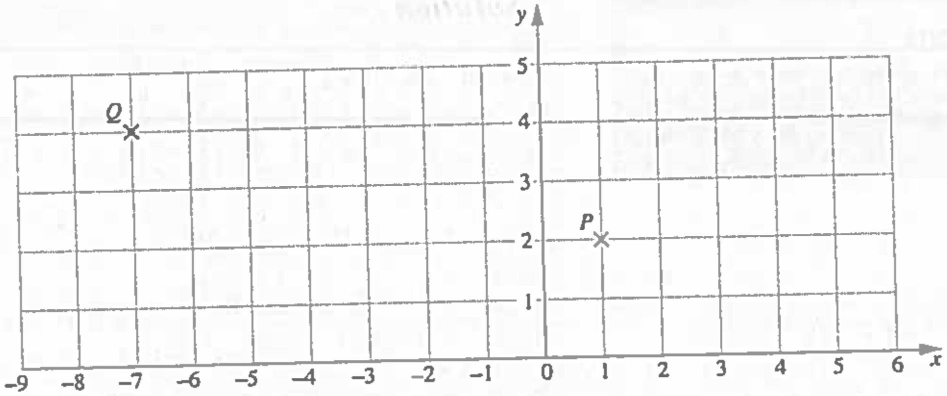
Performing enlargement E on (8, 2) with centre (1, 2) and scale factor –2,

$ET(L) = \begin{pmatrix} -13 \\ 2 \end{pmatrix}$

$\therefore$ coordinates of ET(L) = (–13, 2) **Ans.**

2 *(N2009 P1 Q25)*

(a)

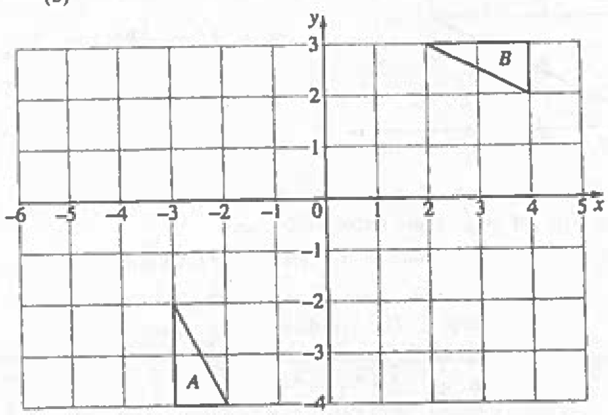


The grid above shows the points $P(1, 2)$ and $Q(-7, 4)$.

(i) P can be mapped onto Q by a translation. Write down its column vector. [1]

(ii) P can also be mapped onto Q by an enlargement, centre (5, 1). Write down its scale factor. [1]

(b)



The diagram shows triangles A and B.

(i) Describe fully the single transformation that maps triangle A onto triangle B. [1]

(ii) Triangle A can also be mapped onto triangle B by a reflection in the line $x = -1$ followed by a rotation. Write down the centre of this rotation. [2]

Thinking Process

(a) (i) Observe how far the point P has moved in the x and y directions to be translated to Q.

(ii) From the given centre, find the ratio of image distance to that of object distance.

(b) (i) Observe that it is a reflection.

(ii) Reflect triangle A. Find the centre of rotation by constructing 2 sets of perpendicular bisectors.

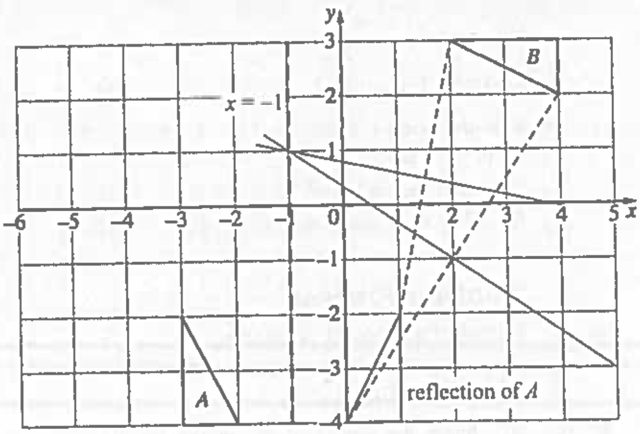
Solution

(a) (i) Column vector of translation $= \begin{pmatrix} -8 \\ 2 \end{pmatrix}$ **Ans.**

(ii) Scale factor of enlargement = 3 **Ans.**

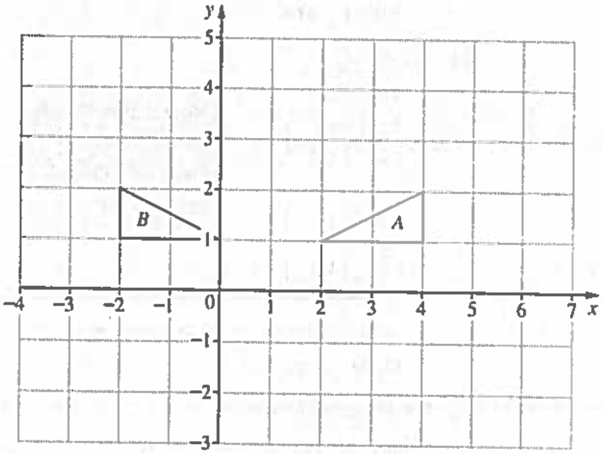
(b) (i) ΔA is mapped onto ΔB by a reflection along the line $y = -x$ **Ans.**

(ii) Centre of rotation is (–1, 1) **Ans.**



3 *(J2009/P1/Q20)*

The diagram shows triangles A and B.



(a) The translation $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ maps ΔA onto ΔC.

On the diagram, draw and label ΔC. [1]

(b) The rotation 90° clockwise, centre (2, 0), maps ΔA onto ΔD.

On the diagram, draw and label ΔD. [2]

(c) Describe fully the single transformation which maps ΔA onto ΔB. [2]

Thinking Process

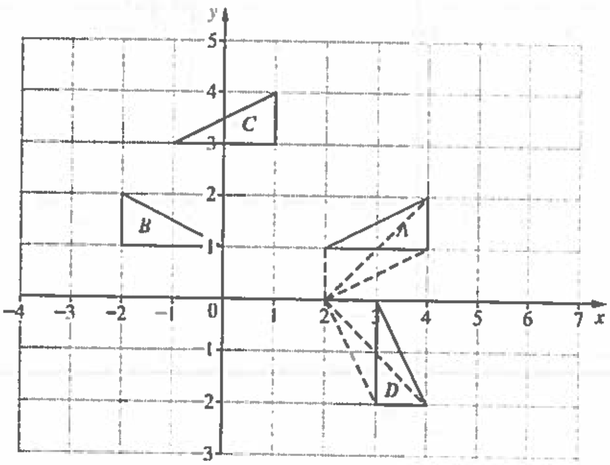
(a) Every point of ΔA moves 3 units to the left and 2 units up.

(b) Rotate ΔA about (2, 0).

(c) Identify the transformation.

Solution

(a) & (b)

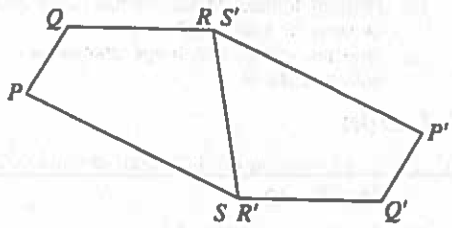


(c) A reflection along the line $x = 1$. **Ans.**

4 *(N2010/P2/Q10)*

(a) $PQRS$ and $P'Q'R'S'$ are congruent quadrilaterals.
R is the same point as S'.
S is the same point as R'.

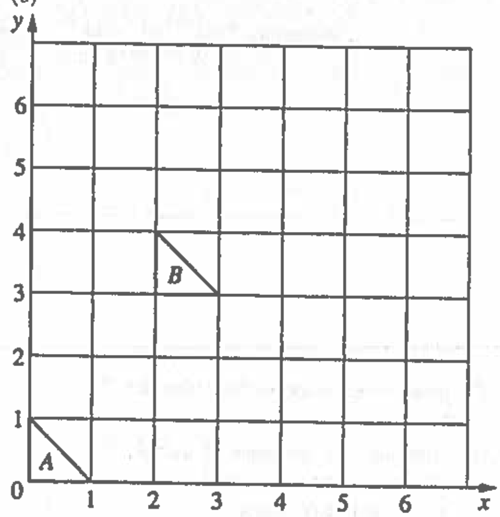
A single transformation maps P onto P', Q onto Q', R onto R' and S onto S'.



(i) Describe fully this transformation. [3]

(ii) Write down two facts connecting PQ and $Q'P'$. [1]

(b)



The diagram shows triangle A and triangle B.

(i) A translation, T, maps triangle A onto triangle B.
State the column vector representing this translation. [1]

(ii) The transformation, S, that maps triangle A onto triangle C is represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

Find the vertices of triangle C. [2]

(iii) Given that $TS(A) = D$, find the vertices of triangle D. [1]

(iv) Triangle E has vertices (0,0), (2,0) and (0,3).
It is given that $(0,0) \to (0,0)$, $(1,0) \to (2,0)$ and $(0,1) \to (0,3)$.
Find the matrix that represents this transformation. [2]

Thinking Process

(a) (i) Recognise that it is a rotation.
(ii) Compare PQ and $Q'P'$.

(b) (i) Consider the transformation of corresponding vertices of triangle A and triangle B.
(ii) Multiply the matrix by each pair of coordinates of triangle A.
(iii) Perform translation on triangle C to find the vertices of triangle D.
(iv) Find the matrix that maps triangle A onto triangle E.

Solution

(a) (i) It is a rotation of 180° about the mid-point of line SR. **Ans.**

(ii) 1. PQ is parallel to $Q'P'$.
2. Length of PQ is equal to length of $Q'P'$.

(b) (i) Point (1, 0) of ΔA is mapped onto point (3, 3) of ΔB

$\therefore$ Translation, $T = \begin{pmatrix}3\\3\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$ **Ans.**

(ii) $\begin{pmatrix}2 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}0 & 2 & 0\\0 & 0 & 1\end{pmatrix}$

$\therefore$ vertices of ΔC are (0, 0), (2, 0) and (0, 1) **Ans.**

(iii) $TS(A) = D$

$\Rightarrow T(C) = D$

$\Rightarrow \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$

$\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$

$\begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$

Note that in (b) (ii), transformation S maps ΔA onto ΔC $\Rightarrow S(A) = C$

$\therefore$ vertices of ΔD are (2, 3), (4, 3) and (2, 4) **Ans.**

(iv) Let M be the matrix.

Taking two sets of coordinates.

$M\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \begin{pmatrix}2 & 0\\0 & 3\end{pmatrix}$

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ is an identity matrix.

$\therefore$ matrix, $M = \begin{pmatrix}2 & 0\\0 & 3\end{pmatrix}$ **Ans.**

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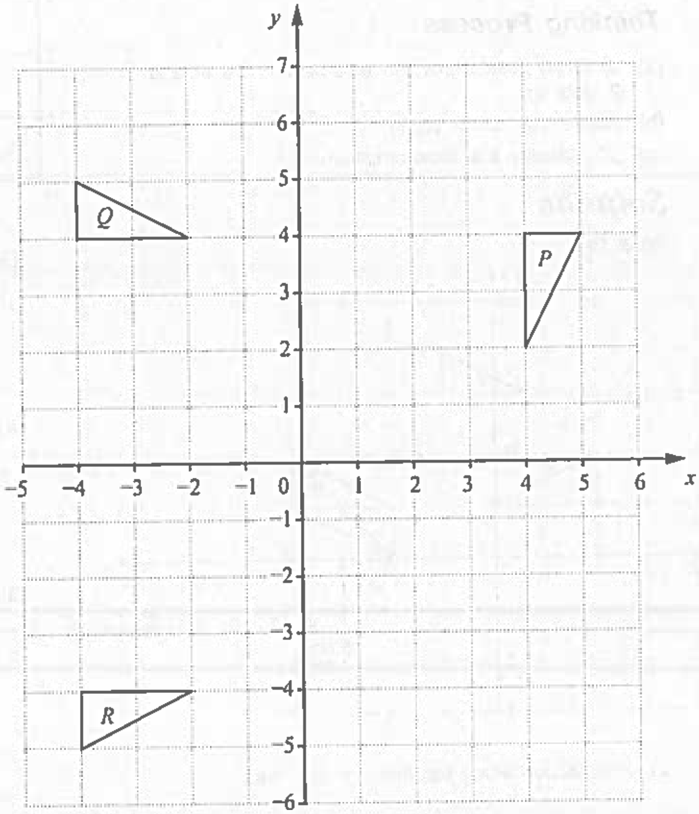
5 *(J2010/P1 Q24)*

The diagram below shows three triangles, P, Q and R.

(a) Triangle T is the image of triangle P under an enlargement with centre (5, 2) and scale factor 2.
Draw and label triangle T on the diagram. [2]

(b) Describe fully the **single** transformation that maps triangle P onto triangle Q. [2]

(c) Find the matrix representing the transformation that maps triangle Q onto triangle R. [1]



Thinking Process

(a) Enlarge triangle P according to given information.

(b) Take note that it is a rotation. & find the center of rotation.

(c) Note that triangle Q is mapped onto triangle R through a reflection along x-axis.

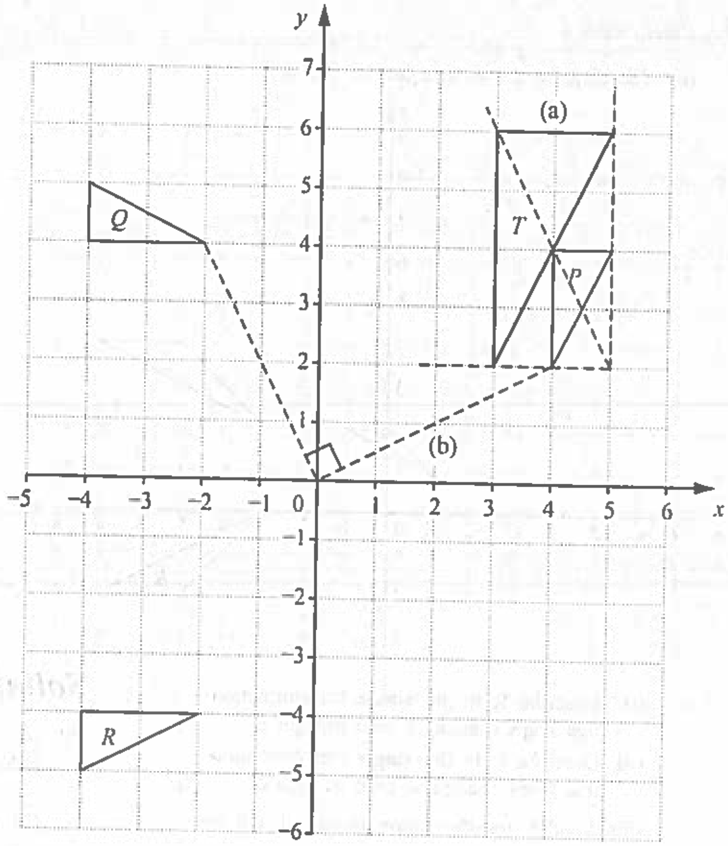
Solution

(a) Refer to graph.

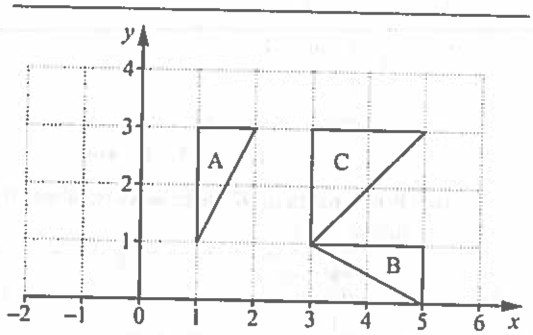
(b) ΔP is mapped onto ΔQ by a rotation of 90° anticlockwise about the origin. **Ans.**

(c) ΔQ is mapped onto ΔR by a reflection along x-axis.

∴ The matrix is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **Ans.**



6 *(N2011/P1/Q27)*



The diagram shows triangles A, B and C.

(a) Triangle A is mapped onto triangle B by an **anticlockwise** rotation.

(i) Write down the angle of rotation. [1]

(ii) Find the coordinates of the centre of rotation. [1]

(b) Triangle A is mapped onto triangle C by a stretch.

(i) Write down the scale factor. [1]

(ii) Write down the equation of the invariant line. [1]

Thinking Process

(a) (i) Recognise that the rotation is anti-clockwise.

(ii) ✎ Draw perpendicular bisectors.
✎ Find intersection of perpendicular bisectors.

(b) (i) To find stretch factor ✎ find the increase in length of a stretched side.

(ii) To find the invariant line ✎ find the line that would fit the scale factor.

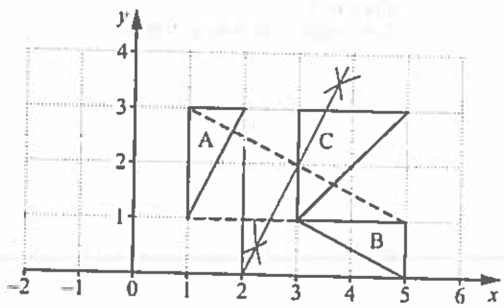
Solution

(a) (i) 270° **Ans.**

(ii) Centre is at (2, 0) **Ans.**

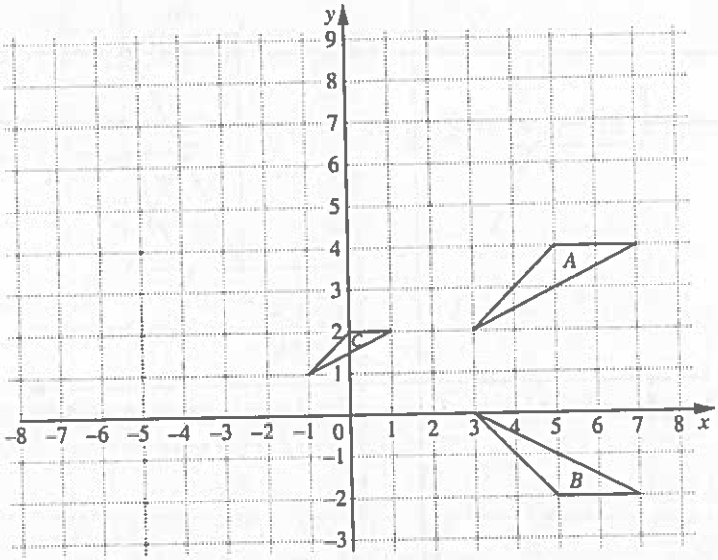
(b) (i) Scale factor, $k = 2$ **Ans.**

(ii) Equation of invariant line: $x = -1$ **Ans.**



7 (J2011 P2 Q8)

(b) The diagram shows triangles A, B and C.



(i) Describe fully the single transformation that maps triangle A onto triangle B. [2]

(ii) Describe fully the single transformation that maps triangle A onto triangle C. [2]

(iii) Another transformation is represented by the matrix $\mathbf{P}$, where $\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

This transformation maps triangle A onto triangle D.
Find the vertices of triangle D. [2]

(iv) Describe fully the single transformation represented by the matrix $\mathbf{P}$. [2]

Thinking Process

(b) (i) Note that it is a reflection.
(ii) It is an enlargement. Find the scale factor and the centre.
(iii) Pre-multiply matrix P by each pair of coordinates of triangle A.
(iv) Recognise that it is a rotation.

Solution

(b) (i) ΔA is mapped onto ΔB by a reflection along the line $y = 1$. **Ans.**

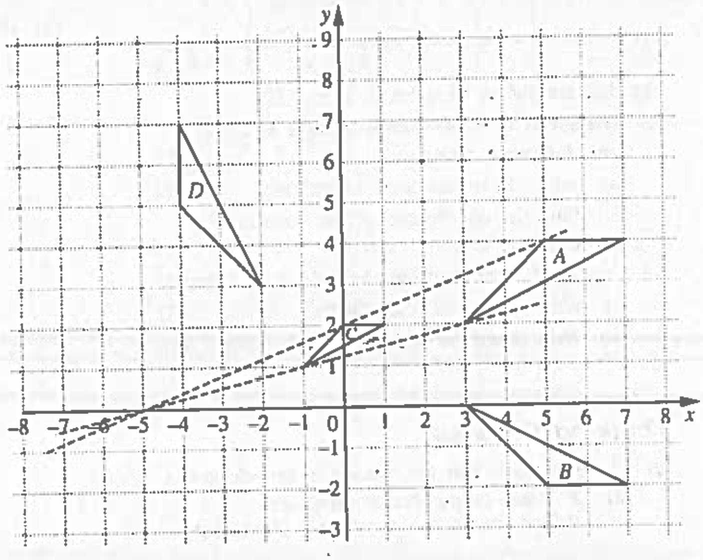
(ii) It is an enlargement, scale factor $\frac{1}{2}$, centre (–5, 0). **Ans.**

(iii) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 5 & 7 \\ 2 & 4 & 4 \end{pmatrix}$

$= \begin{pmatrix} -2 & -4 & -4 \\ 3 & 5 & 7 \end{pmatrix}$

∴ vertices of ΔD are,
(–2; 3), (–4, 5), (–4, 7) **Ans.**

(iv) P is a rotation, 90° anticlockwise about the origin. **Ans.**



8 *(N2011/P2 Q5)*

(a) $\begin{pmatrix}3 & -1 & 0\\ 1 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\ 11\\ y\end{pmatrix}=\begin{pmatrix}4\\ 9\end{pmatrix}$.

Find x and y. [2]

(b) (i) The transformation A is represented by the matrix $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$. Find, in terms of a, b, c and d as appropriate,

(a) the image of (1, 0) under the transformation A, [1]

(b) the image of (0, 1) under the transformation A. [1]

(ii) The transformation B maps (1, 0) onto (1, 3) and (0, 1) onto (–3, –2). Write down the matrix that represents transformation B. [1]

(iii) Describe fully the transformation given by the matrix $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. [2]

Thinking Process

(a) Perform matrix multiplication and solve for x and y.

(b) (i) (a) Pre-multiply $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ by $\begin{pmatrix}1\\ 0\end{pmatrix}$.

(b) Pre-multiply $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ by $\begin{pmatrix}0\\ 1\end{pmatrix}$.

(ii) To write the matrix ✐ examine the answers to part (b) (i).

(iii) ✐ Note that it is a reflection along x-axis.

Solution *with* TEACHER's COMMENTS

(a) $\begin{pmatrix}3 & -1 & 0\\ 1 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\ 11\\ y\end{pmatrix}=\begin{pmatrix}4\\ 9\end{pmatrix}$

$\begin{pmatrix}3x-11+0\\ x+0+y\end{pmatrix}=\begin{pmatrix}4\\ 9\end{pmatrix}$

$3x-11=4$	$x+y=9$
$3x=15$	$5+y=9$
$x=5$	$y=4$

$\therefore\quad x=5,\ y=4$ **Ans.**

(b) (i) (a) $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}a\\ c\end{pmatrix}$

$\therefore$ image of $(1, 0)=(a, c)$ **Ans.**

(b) $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}b\\ d\end{pmatrix}$

$\therefore$ image of $(0, 1)=(b, d)$ **Ans.**

(ii) $B=\begin{pmatrix}1 & -3\\ 3 & -2\end{pmatrix}$ **Ans.**

> To write matrix B, observe the results in part (b) (i)
> $(1, 0)\to(a, c)$, $(0, 1)\to(b, d)$

(iii) It is a reflection along x-axis. **Ans.**

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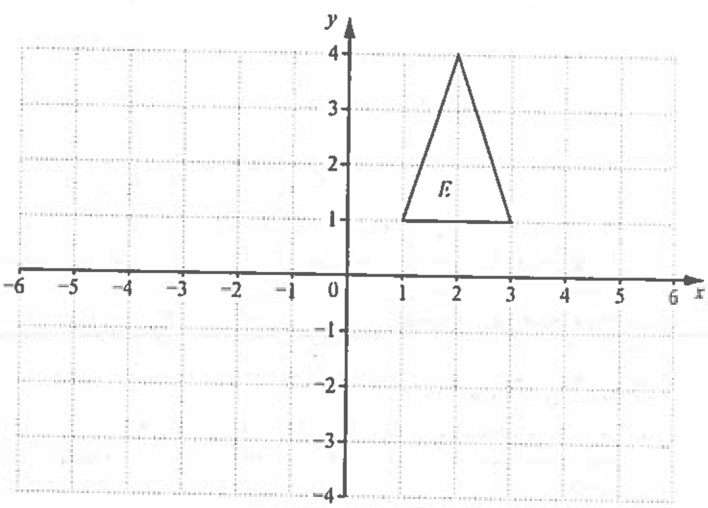
9 *(J2012/P2 Q9 c)*

(c) (i) Triangle E is mapped onto triangle F by a reflection in the line $y=-x$.
Draw and label triangle F. [2]

(ii) The transformation that maps triangle E onto triangle G is represented by the matrix $\begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix}$. Draw and label triangle G. [2]

Thinking Process

(c) (ii) To find triangle G ✐ multiply the matrix by each pair of coordinates of triangle E. Draw triangle G on the grid.



Solution

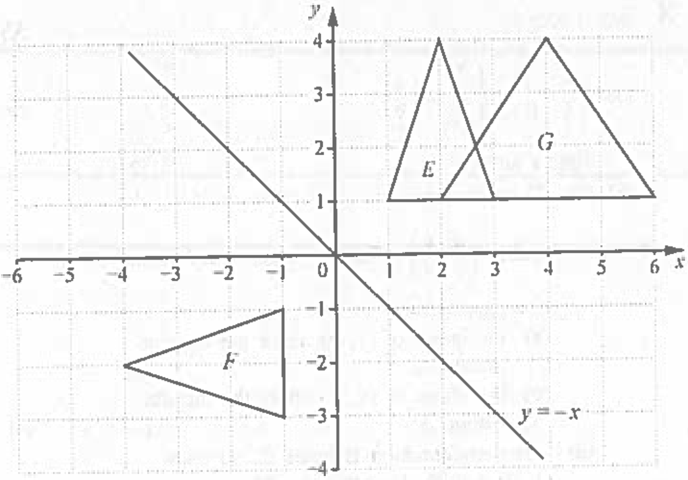
(c) (i) Refer to graph.

(ii) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 1 & 1 & 4 \end{pmatrix}=\begin{pmatrix} 2 & 6 & 4 \\ 1 & 1 & 4 \end{pmatrix}$

$\therefore$ vertices of ΔG are

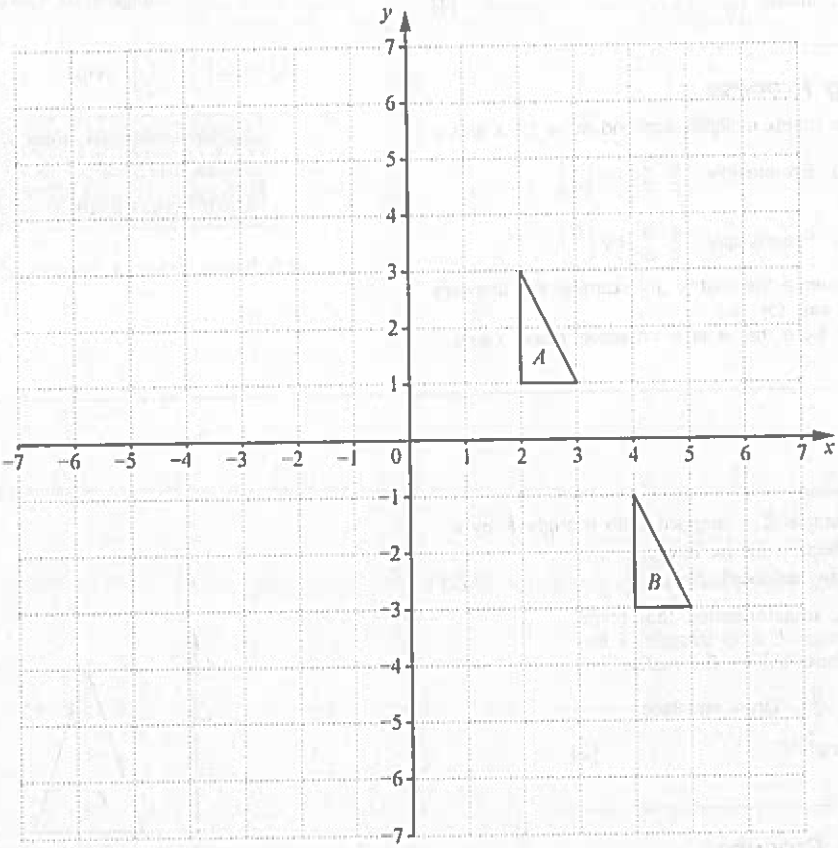
(2, 1), (6, 1) and (4, 4).

Refer to graph for ΔG.



10 *(J2012/P1/Q13)*

The diagram shows two triangles, A and B.



(a) Write down the vector that represents the translation that maps triangle A onto triangle B. [1]

(b) Triangle C is an enlargement of triangle A with centre (5, 3) and scale factor 3.
Draw and label triangle C. [2]

Thinking Process

(a) ΔA is mapped onto ΔB if each point of ΔA is moved 2 units to the right and 4 units down.

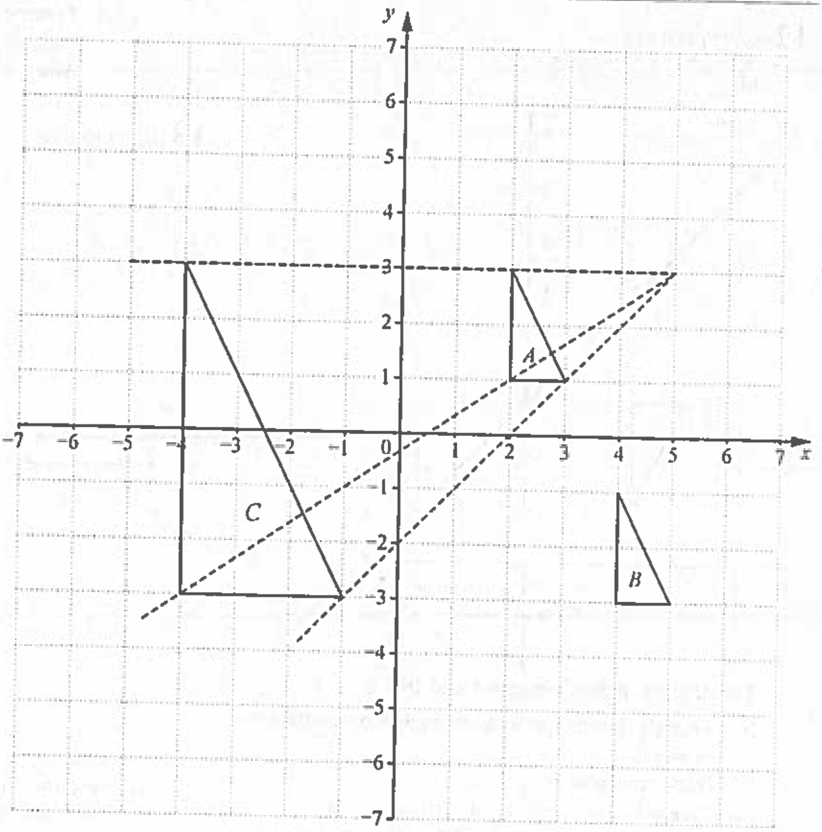
(b) Join each vertex of ΔA to the centre (5, 3) and extend the length to 3 times its original length from the centre.

Solution

(a) Translation vector

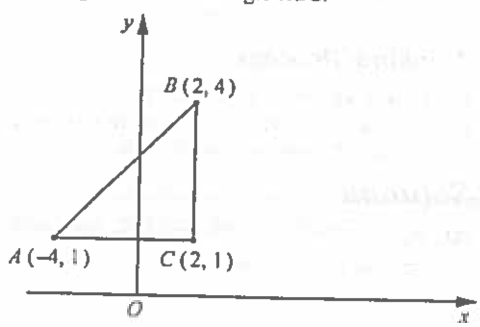
$= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ **Ans.**

(b) Refer to graph.



11 *(N2012/P1/Q22)*

The diagram shows triangle *ABC*.



Triangle *ABC* is translated by $\begin{pmatrix} 9 \\ 2 \end{pmatrix}$ onto triangle *A'B'C'*.

(a) Find the coordinates of *C'*. [1]

(b) What special type of quadrilateral is *BCC'B'*? [1]

(c) Find the area of quadrilateral *BCC'B'*. [2]

Thinking Process

(a) Point C moves 9 units to the right and 2 units up.

(b) Identify the quadrilateral.

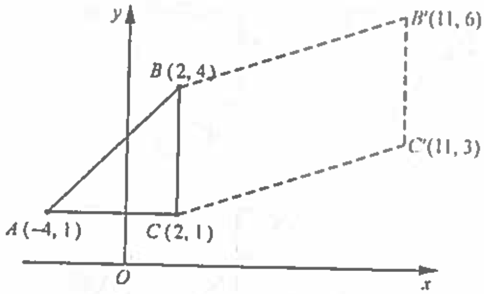
(c) Area of parallelogram = base x height.

Solution

(a) $\begin{pmatrix} 9 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$ $\therefore$ *C'* is (11, 3) **Ans.**

(b) For *B'*, $\begin{pmatrix} 9 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 11 \\ 6 \end{pmatrix}$

$\Rightarrow$ *B'* is (11, 6).



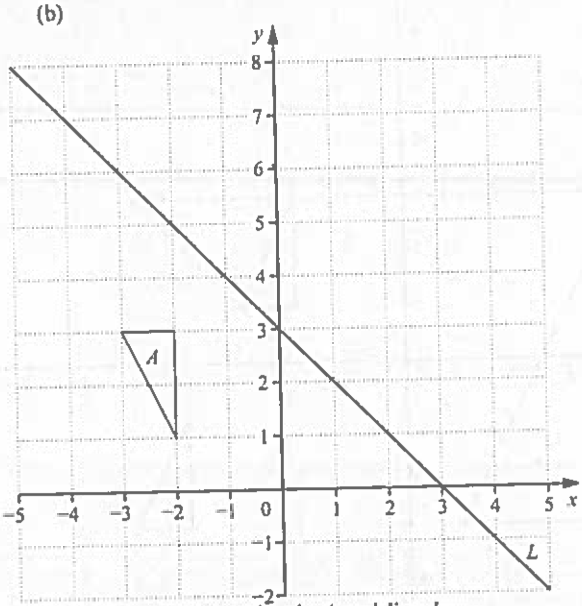
$\therefore$ *BCC'B'* is a parallelogram. **Ans.**

(c) Area of $BCC'B' = b \times h$

$= 9 \times 3 = 27 \text{ units}^2$ **Ans.**

12 *(N2012 P2 Q11 b)*

(b)



The diagram shows triangle A and line L.

(i) Triangle A is mapped onto triangle B by a reflection in line L.
Draw and label triangle B. [2]

(ii) Triangle A is mapped onto triangle C by an anticlockwise rotation of 90°, centre (0, 3).
Draw and label triangle C. [2]

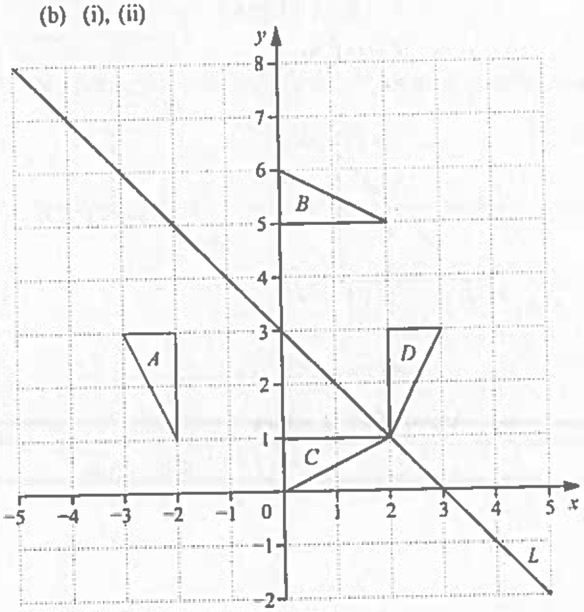
(iii) Triangle C is mapped onto triangle D by a reflection in line L. Describe the single transformation that maps triangle B onto triangle D. [3]

Thinking Process

(b) (iii) Note that it is a clockwise rotation.

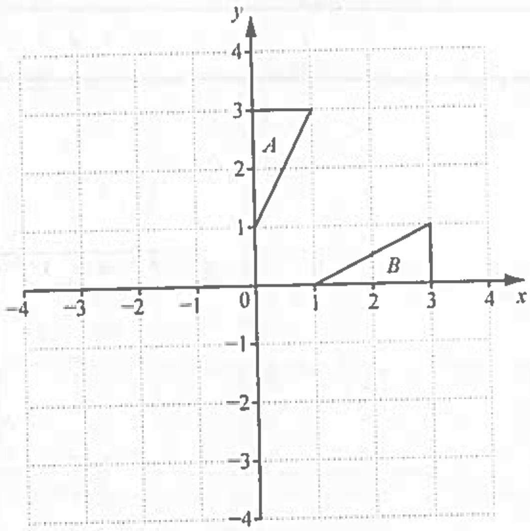
Solution

(b) (i), (ii)



(iii) Triangle B is mapped onto triangle D by a rotation of 90° clockwise with center at (0, 3).
Ans.

13 *(N2013 P1 Q20)*



The diagram shows triangles A and B.

(a) Describe fully the single transformation that maps triangle A onto triangle B. [2]

(b) Triangle A is mapped onto triangle C by the transformation **T**.
T is a rotation, centre the origin, through 270° clockwise.

(i) On the diagram, draw triangle C. [1]

(ii) Find the matrix that represents **T**. [1]

Thinking Process

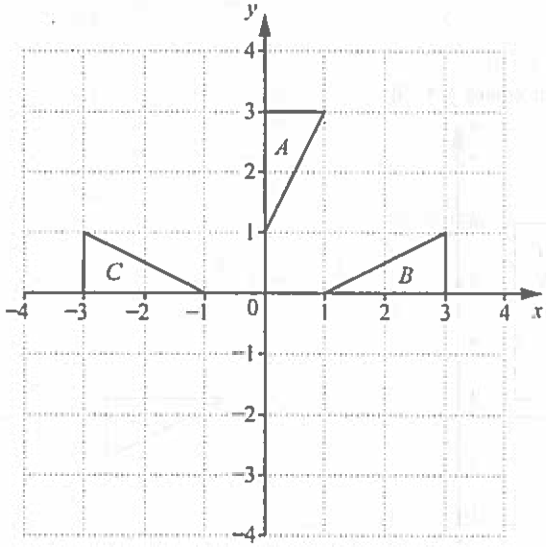
(a) Recognise that it is a reflection.

(b) (ii) Take note that 270° clockwise rotation is same as 90° anti-clockwise rotation.

Solution

(a) ΔA is mapped onto ΔB by a reflection along the line $y = x$. **Ans.**

(b) (i)

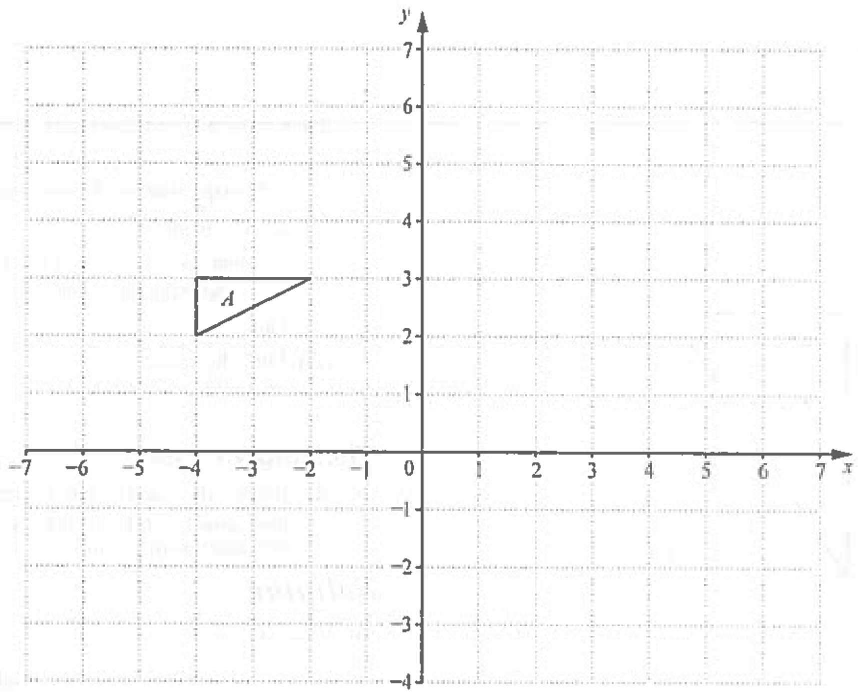


(ii) 270° clockwise rotation about origin is same as 90° anticlockwise rotation.

∴ the matrix is: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **Ans.**

14 *(J2013/P1 Q6)*

The diagram shows triangle A.



(a) Reflect triangle A in the line $x = 1$.
Label the image B. [1]

(b) Rotate triangle A through 90° clockwise about the point (−1, 3).
Label the image C. [1]

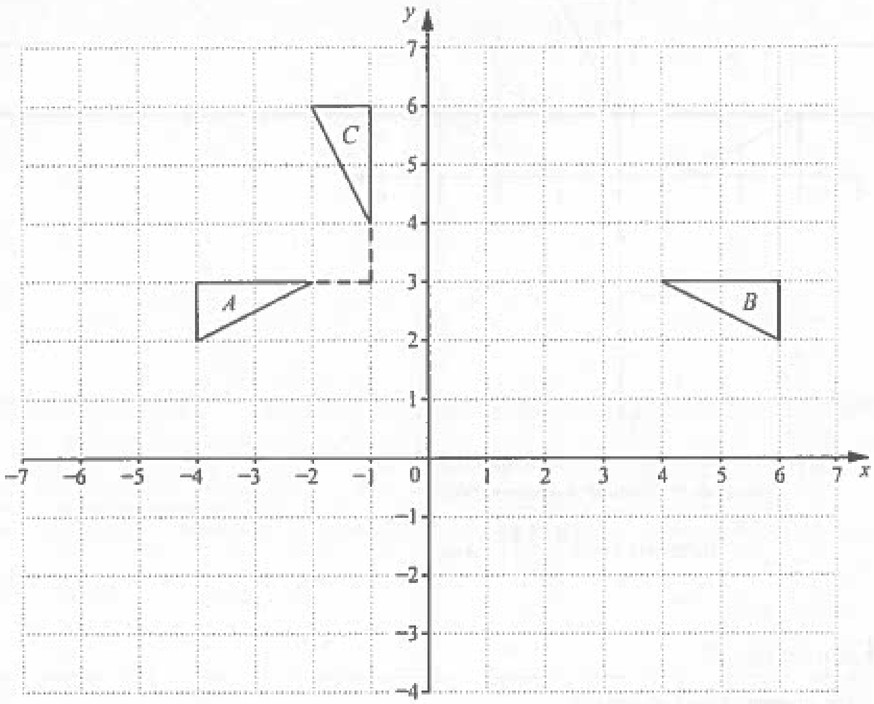
Thinking Process

(a) Draw ΔB such that it is a reflection of ΔA along $x = 1$.

(b) Draw ΔC such that it is the rotation of ΔA about centre $(-1, 3)$.

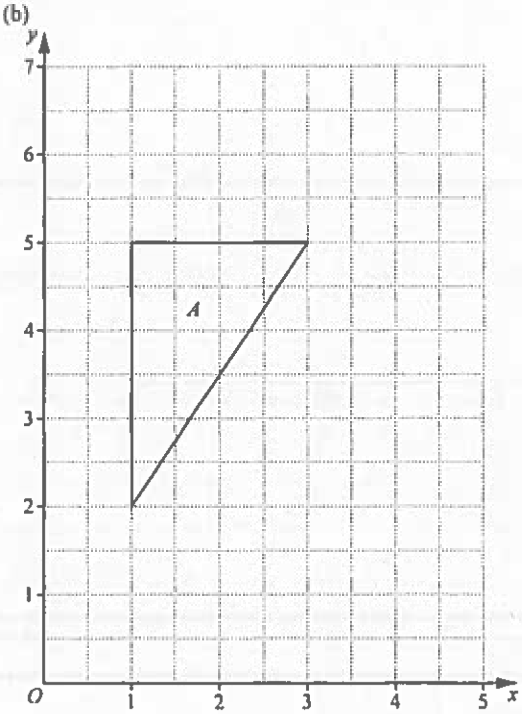
Solution

(a) & (b)



15 *(N2013/P2/Q12)*

(b)



Triangle A has vertices (1, 2), (1, 5) and (3, 5).

(i) An enlargement, centre (1, 2), scale factor 1.5, maps triangle A onto triangle B.
Draw triangle B. [2]

(ii) An enlargement, centre (1, 2), scale factor −0.5, maps triangle A onto triangle C.
Draw triangle C. [2]

(iii) Find the ratio
area of triangle C : area of triangle B. [1]

Thinking Process

(b) (iii) Since ΔB and ΔC are similar, the ratio of their areas = ratio of the squares of corresponding lengths.

Solution

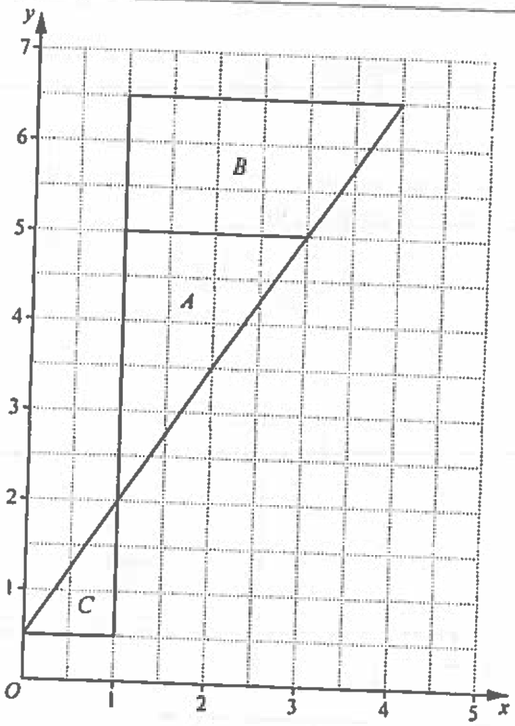
(b) (i) & (ii) Refer to next page.

(iii) area of triangle C : area of triangle B

$(2)^2 : (6)^2$

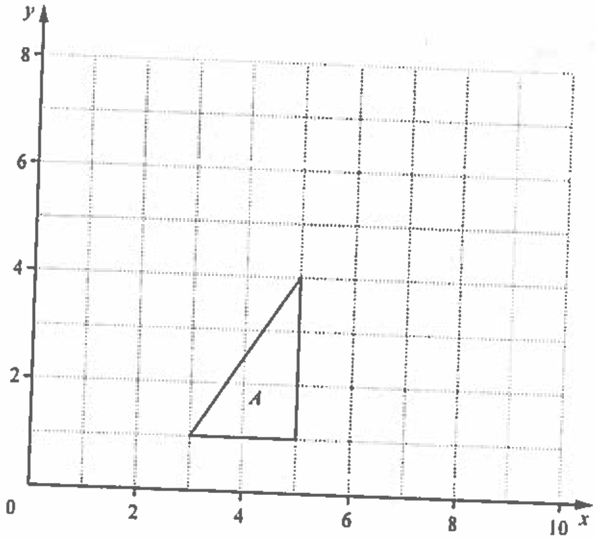
$4 : 36$

$1 : 9$ **Ans.**



16 *(N2014/P2 Q7b)*

(b)



Triangle A has vertices (3, 1), (5, 1) and (5, 4).

The transformation S_1 is represented by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

S_1 maps triangle A onto triangle B.

(i) Draw and label triangle B. [2]

(ii) The transformation S_2 is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

Find the matrix that represents the combined transformation S_2S_1. [2]

(iii) The combined transformation S_2S_1 maps triangle A onto triangle C.
Find the matrix which represents the transformation that maps triangle C onto triangle A. [2]

Thinking Process

(b) (i) Pre-multiply the matrix S_1 by each pair of coordinates of triangle A.

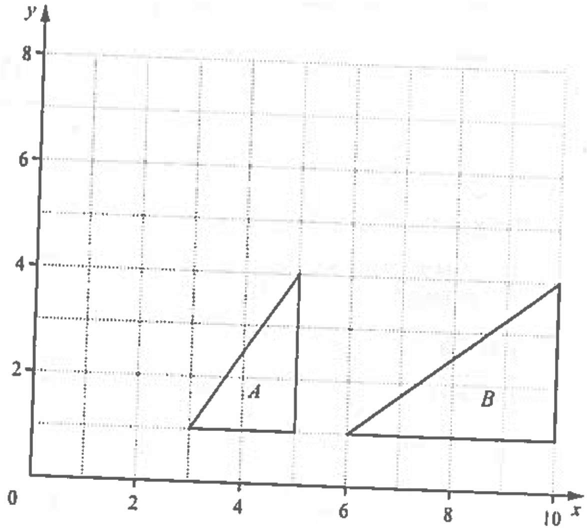
(ii) Multiply matrix S_2 by S_1.

(iii) Apply inverse transformation find $(S_2S_1)^{-1}$.

Solution

(b) (i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 5 & 5 \\ 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 10 & 10 \\ 1 & 1 & 4 \end{pmatrix}$

$\therefore$ the vertices of ΔB are (6, 1), (10, 1) and (10, 4).



(ii) $S_2S_1 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}$ **Ans.**

(iii) The matrix that maps ΔC onto ΔA is $(S_2S_1)^{-1}$

Determinant of $S_2S_1 = 2 - 0 = 2$

$\therefore \quad (S_2S_1)^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ -2 & 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{2} & 0 \\ -1 & 1 \end{pmatrix}$ **Ans.**

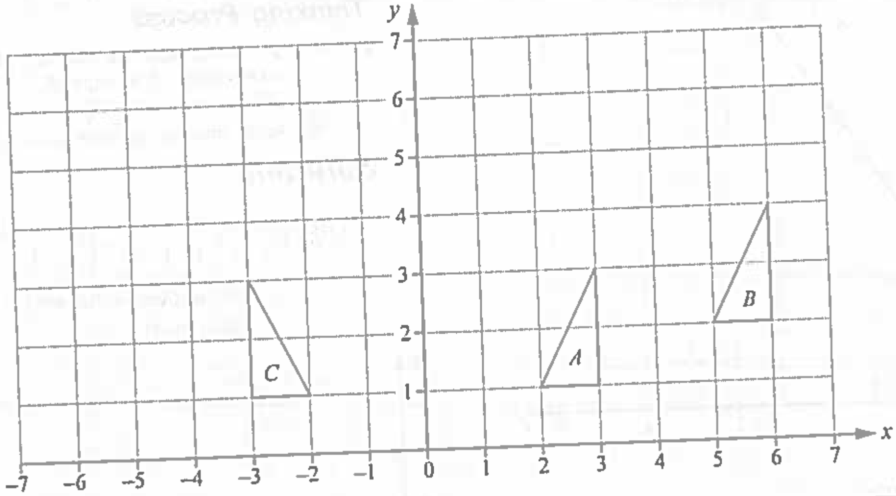
17 (J2014/P1/Q17)

The diagram shows triangles A, B and C.

(a) Triangle A can be mapped onto triangle B by a translation. Write down the column vector for the translation. [1]

(b) Find the matrix representing the transformation that maps triangle A onto triangle C. [1]

(c) Triangle A is mapped onto triangle D by an enlargement, scale factor 2, centre (5, 0). Draw and label triangle D. [2]



Thinking Process

(a) Note that every point of ΔA moves 3 units to the right and 1 unit up.

(b) Identify the line of reflection.

Solution

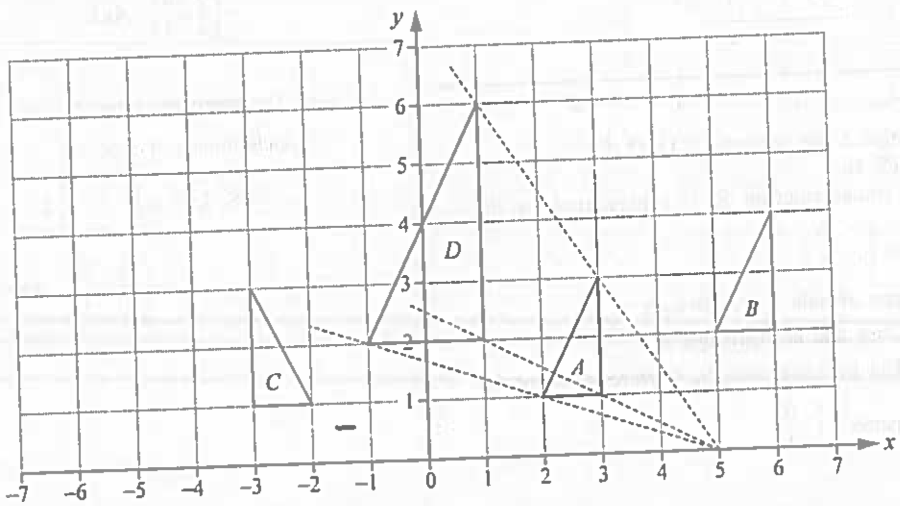
(a) Point (2, 1) of ΔA is mapped onto point (5, 2) of ΔB.

$\therefore$ column vector $= \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **Ans.**

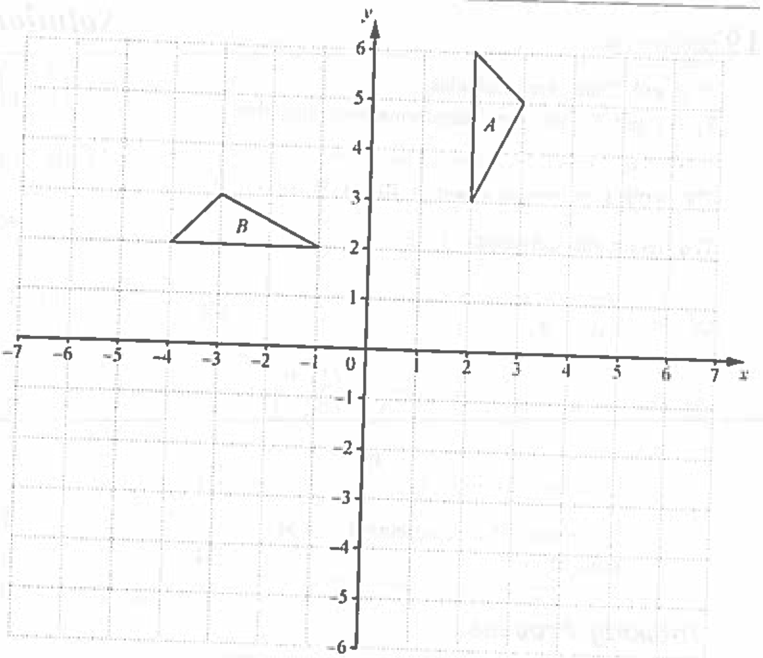
(b) ΔA is mapped onto ΔC by a reflection along y-axis.

$\therefore$ matrix is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **Ans.**

(c)



18 (J2015/P2 Q10 a)



(a) (i) Describe fully the single transformation that maps triangle A onto triangle B. [2]

(ii) Triangle B is mapped onto triangle C by a translation, vector $\begin{pmatrix}-2\\-3\end{pmatrix}$.

Draw and label triangle C. [2]

(iii) Triangle A is mapped onto triangle D by a reflection in the line $y = x$.

Draw and label triangle D. [2]

(iv) Triangle E is geometrically similar to triangle A and its longest side is 12 cm.

Calculate the area of triangle E. [2]

Thinking Process

(a) (i) Recognise that it is an anticlockwise rotation.

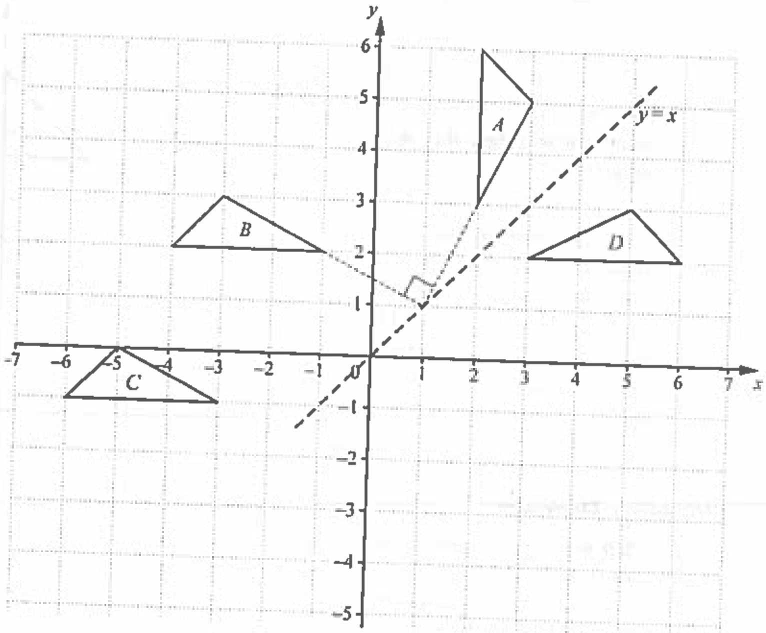
(ii) Triangle B moves 2 units to the left and 3 units down.

(iv) Apply rule of similarity: $\dfrac{\text{Area}_1}{\text{Area}_2} = \left(\dfrac{\text{height}_1}{\text{height}_2}\right)^2$

Solution

(a) (i) Triangle A is mapped onto triangle B by a rotation of 90° anti-clockwise with center at (1, 1). **Ans.**

(ii), (iii) Refer to graph.



(iv) Area of $\Delta A = \frac{1}{2} \times 3 \times 1$

$= \frac{3}{2}$ cm^2

Applying rule of similarity:

$\dfrac{\text{area of } \Delta E}{\text{area of } \Delta A} = \left(\dfrac{12}{3}\right)^2$

$\dfrac{\text{area of } \Delta E}{\frac{3}{2}} = 16$

area of $\Delta E = 16 \times \frac{3}{2}$

$= 24$ cm^2

19 (N2015/P1/Q26)

A, B and C are three triangles.
T_1, T_2 and T_3 are three transformations such that
$T_1(A) = B$, $T_2(A) = C$ and $T_3(C) = B$.
The vertices of triangle A are (1, 0), (0, 1) and (1, 3).

The matrix that represents T_1 is $\begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$.

(a) Find $\begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}$. [2]

(b) The matrix that represents T_2 is $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find the inverse of $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. [1]

(ii) The matrix that represents T_3 is **M**. Find **M**. [2]

Thinking Process

(a) Multiply respective row by respective column.

(b) (i) Recall: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{\text{Det. } A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

where Det. $A = ad - bc$

(ii) To find **M** ✐ use the given information and the answer to part (b) (i).

Solution

(a) $\begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 8 \\ 0 & 1 & 3 \end{pmatrix}$ **Ans**

(b) (i) Determinant $= (2 \times 1) - 0 = 2$

inverse $= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$ **Ans.**

(ii) $T_1(A) = B$.

$\therefore$ from part (a). $B = \begin{pmatrix} 2 & 2 & 8 \\ 0 & 1 & 3 \end{pmatrix}$

$T_2(A) = C$

$\Rightarrow C = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$

$T_3(C) = B \Rightarrow \mathbf{M}(C) = B$

taking two sets of corresponding coordinates from triangles B and C.

$\mathbf{M}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow \mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}^{-1}$

$\Rightarrow \mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow \mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ **Ans.**

20 (N2015/P2/Q11 b)

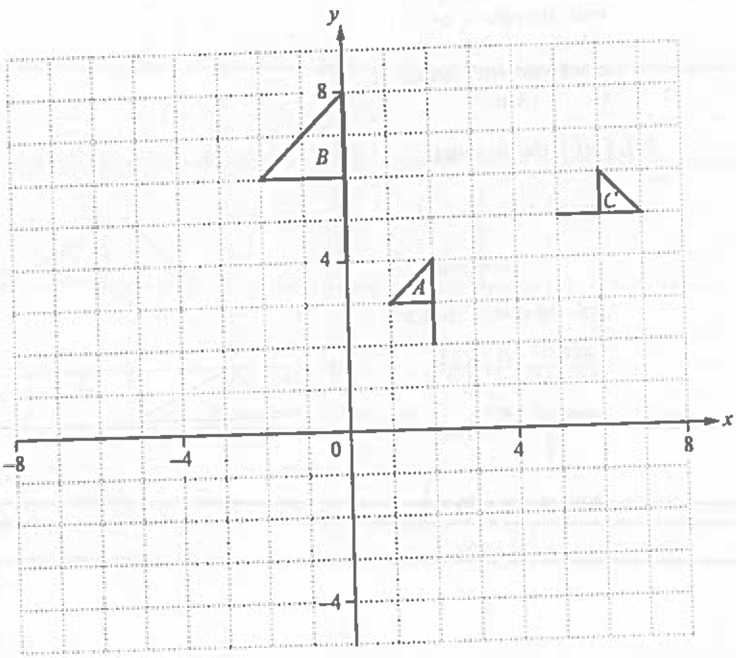
(b) (i) Flag A is mapped onto flag T by the translation $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$.

Draw, and label, flag T. [1]

(ii) Describe fully the enlargement that will map flag A onto flag B. [2]

(iii) Find the centre of the rotation that will map flag A onto flag C. [1]

(iv) Rotate flag B through 45° anticlockwise about the origin. Label the image R. [2]



Thinking Process

(b) (i) Shift the flag A 3 units to the left and 6 units down.

(ii) To describe the enlargement ✐ Find the scale factor and center of enlargement.

(iii) To find the centre of rotation draw the perpendicular bisectors of the lines joining the corresponding vertices of the two triangles.

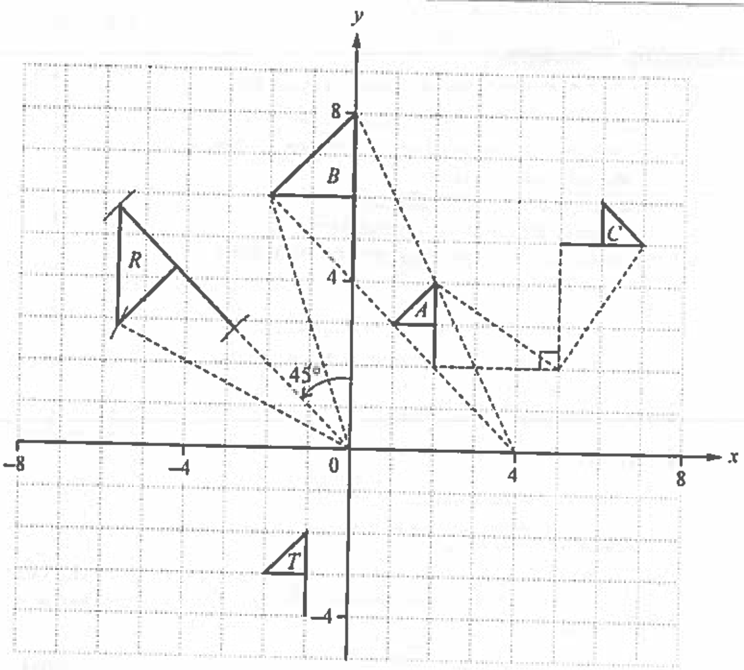
Solution

(b) (i) Refer to graph.

(ii) Flag A is mapped onto flag B by an enlargement with centre (4, 0) and scale factor 2.

(iii) Centre of rotation is (5, 2) Ans.

(iv) Refer to graph.



21 *(J2016/P2/Q10)*

Triangles A, B, C and D are drawn on a centimetre square grid.

(a) The perimeter of triangle A is $(a+\sqrt{b})$ cm, where a and b are integers. Find a and b. [2]

(b) Triangle A is mapped onto triangle B by the translation T. Write down the column vector that represents T. [1]

(c) Describe fully the single transformation that maps triangle B onto triangle C. [2]

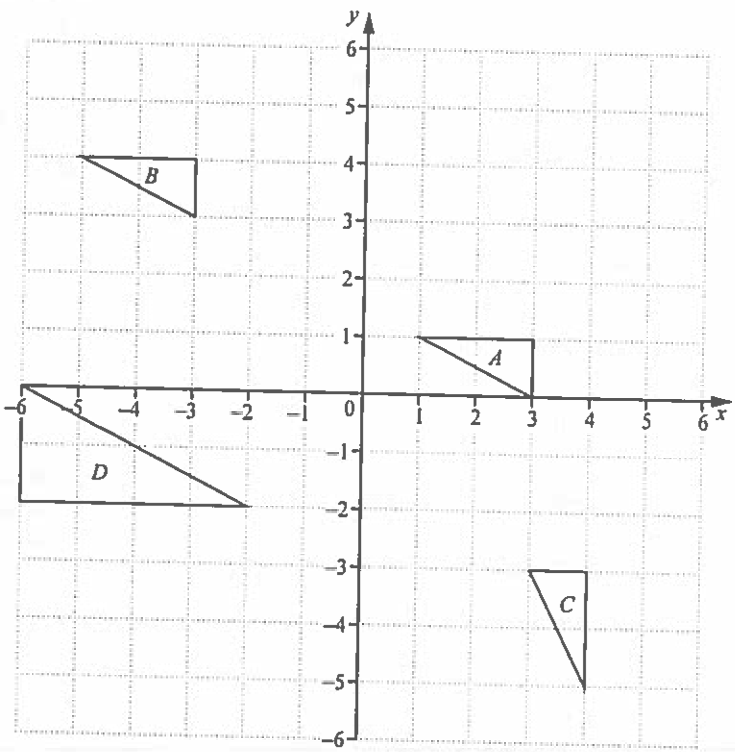
(d) Describe fully the single transformation that maps triangle B onto triangle D. [3]

(e) Write down the matrix that represents the transformation which maps triangle D onto triangle A. [1]

(f) The transformation V is a reflection in the line $y=0$. The transformation W is a rotation 90° clockwise about (0, 0). The single transformation X is equivalent to the transformation V followed by the transformation W.

(i) The point (g, h) is mapped onto the point P by the transformation X. Find the coordinates of P. [1]

(ii) Describe fully the single transformation X. [2]



Thinking Process

(a) To find the perimeter ✎ apply Pythagoras theorem to find the slant height of ΔA.

(b) ✎ Note that every point of ΔA moves 6 units to the left and 3 unit up.

(c) Recognise that it is a reflection.

(d) Recognise that it is an enlargement.

(e) Observe that it is an enlargement, with scale factor $-\frac{1}{2}$.

(f) (i) First find the matrix of transformation X, then multiply the matrix by object (g, h).

(ii) Observe that the matrix of transformation X represents reflection.

Solution

(a) Using pythagoras theorem.

hypotenuse of $\Delta A = \sqrt{(1)^2 + (2)^2}$

$= \sqrt{5}$

perimeter of $\Delta A = 1 + 2 + \sqrt{5}$

$= (3 + \sqrt{5})$ cm

$\therefore\ a = 3,\ b = 5$ **Ans.**

(b) Point (1, 1) of ΔA is mapped onto point (−5, 4) of ΔB.

$\therefore$ column vector $= \begin{pmatrix} -5 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 3 \end{pmatrix}$ **Ans.**

(c) ΔB is mapped onto ΔC by a reflection along the line $y = x$. **Ans.**

(d) ΔB is mapped onto ΔD by an enlargement, centre (−4, 2) with scale factor −2. **Ans.**

(e) ΔD is mapped onto ΔA by an enlargement, centre (0, 0) with scale factor $-\frac{1}{2}$.

$\therefore$ matrix is: $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$ **Ans.**

(f) (i) The matrix of transformation V is. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The matrix of transformation W is. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\therefore$ The matrix of transformation X is.

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

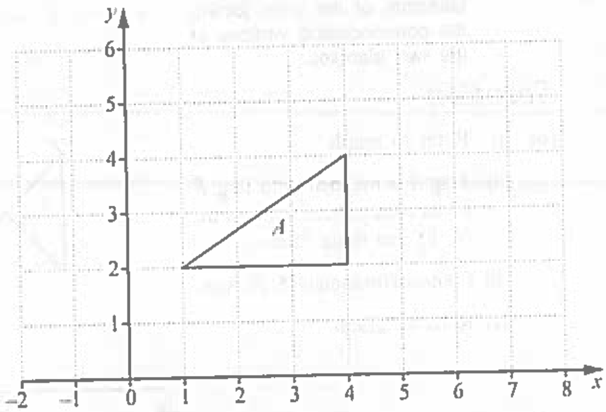
transformation X maps (g, h) onto P

$\Rightarrow \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} g \\ h \end{pmatrix} = \begin{pmatrix} -h \\ -g \end{pmatrix}$

$\therefore$ coordinates of P are $(-h, -g)$ **Ans.**

(ii) Transformation X is a reflection in the line $y = -x$ **Ans.**

22 *(N2016 P1 Q7)*

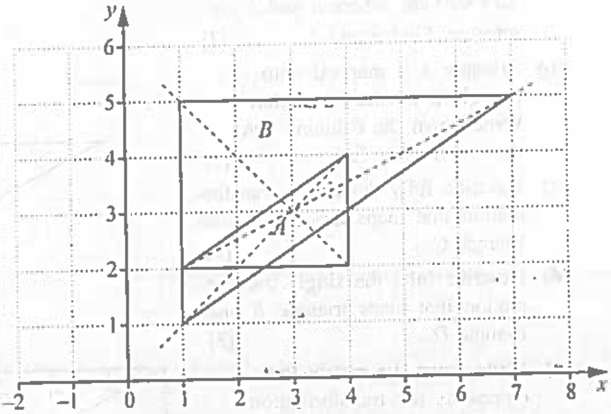


The diagram shows triangle A.
Triangle A is mapped onto triangle B by an enlargement.
The enlargement has centre (3, 3) and scale factor −2. Draw and label triangle B. [2]

Thinking Process

Join each vertex of ΔA to the centre (3, 3) and extend the length to 2 times its original length from the centre on the opposite side.

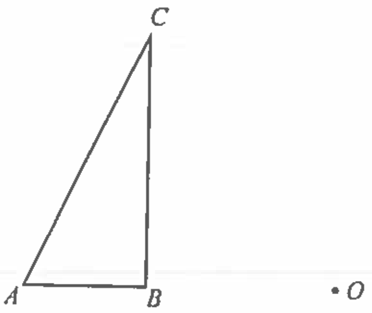
Solution



Note that when scale factor is negative, the image formed is on the opposite side of the centre of enlargement, and it is upside down.

23 *(N2016/P1/Q13)*

Triangle ABC is mapped onto triangle $A'B'C'$ by a rotation, centre O, through 110° clockwise.
Draw and label triangle $A'B'C'$.

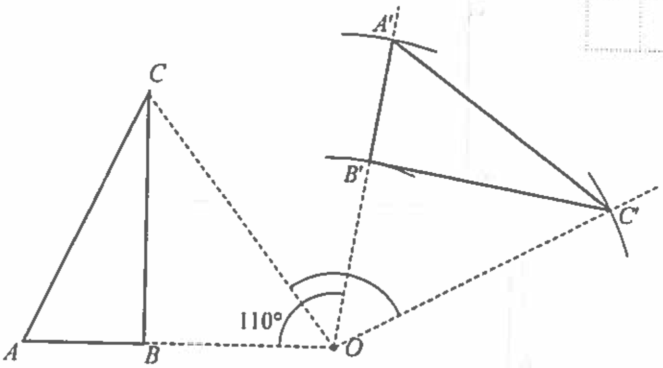


[3]

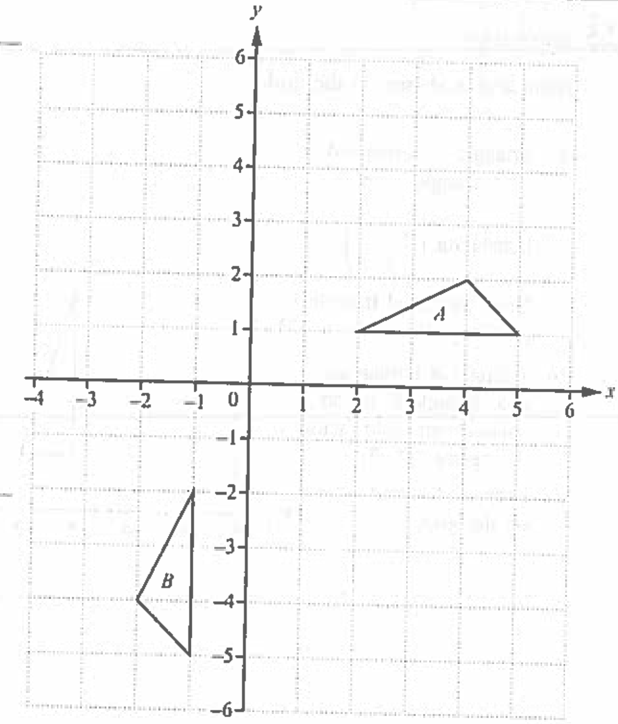
Thinking Process

Using a protractor and compass, rotate each point of the triangle 110° clockwise about O.

Solution



24 *(J2017/P1/Q15)*



The diagram shows triangles A and B.

(a) Describe fully the single transformation that maps triangle A onto triangle B. [2]

(b) Triangle A is mapped onto triangle C by a rotation, 90° anti-clockwise about the origin.
On the diagram, draw triangle C. [2]

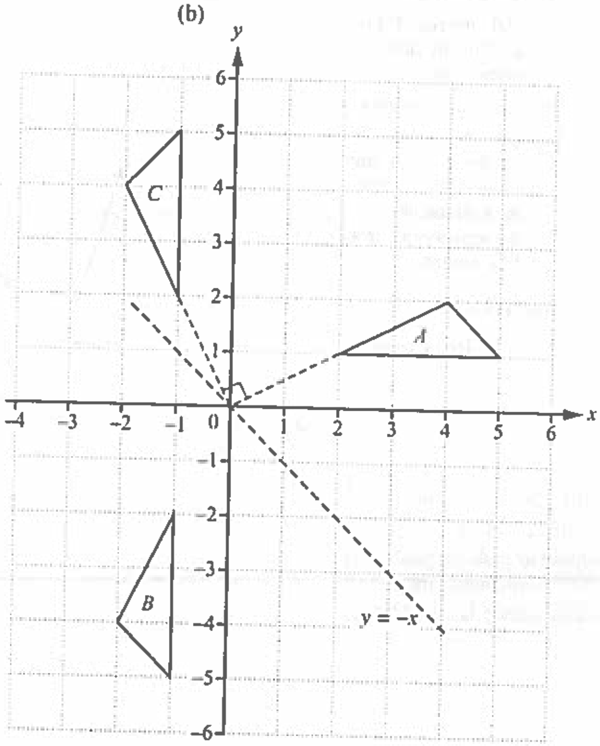
Thinking Process

(a) Recognise that it is a reflection.

Solution

(a) ΔA is mapped onto ΔB by a reflection in the line $y = -x$. Ans.

(b)



25 (N2017/P2/Q4)

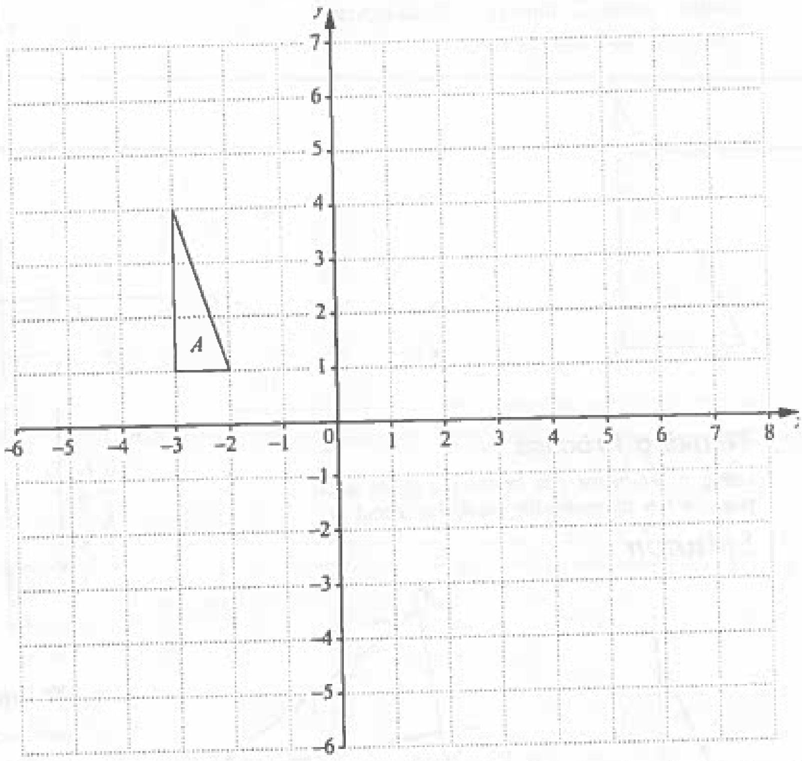
Triangle A is shown on the grid.

(a) Triangle A is mapped onto triangle B by a translation of $\begin{pmatrix} 7 \\ -5 \end{pmatrix}$.

Draw and label triangle B on the grid. [2]

(b) Triangle A is mapped onto triangle C by an enlargement scale factor -2, centre $(-1, 2)$.

Draw and label triangle C on the grid. [2]



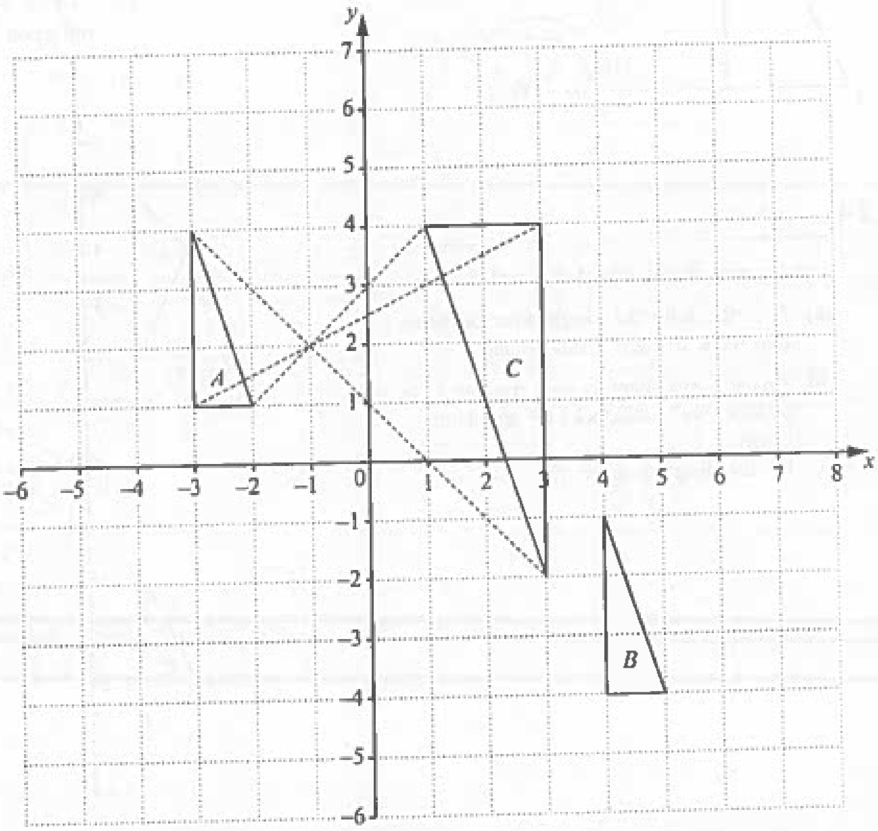
Thinking Process

(a) Every point of ΔA moves 7 units to the left and 5 units down.

(b) Join each vertex of ΔA to the centre $(-1, 2)$ and extend the length to 2 times its original length from the centre.

Solution

(a) & (b) Refer to graph.



(b) Take note that ΔC will appear at the opposite side of centre of enlargement since the scale factor is negative.

26 *(J2018/P1/Q24)*

Triangle A is mapped onto triangle B by a translation, followed by an enlargement with centre (10, −4).

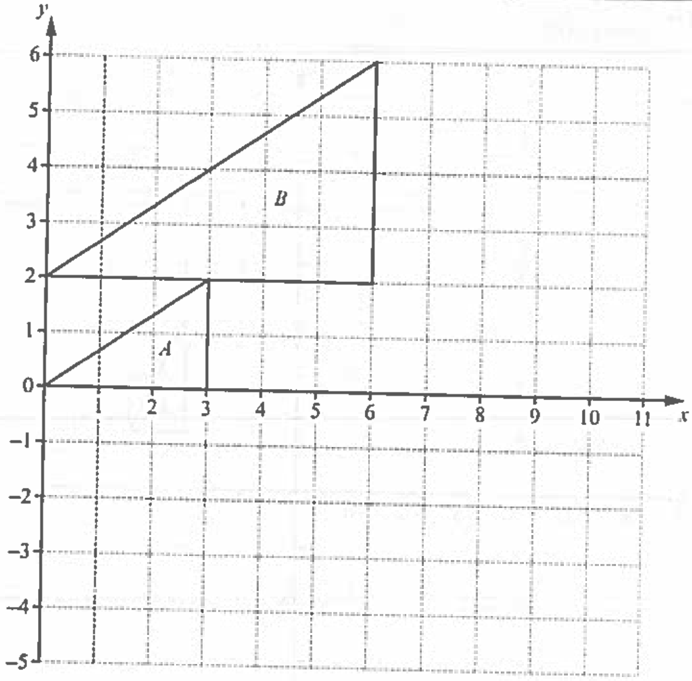
The translation maps triangle A onto triangle C.

The enlargement maps triangle C onto triangle B.

(a) Write down the scale factor of the enlargement. [1]

(b) Draw triangle C on the grid. [2]

(c) Find the column vector that represents the translation that maps triangle A onto triangle C. [1]



Thinking Process

(a) Divide the length of the image by the corresponding length of the object.

(b) Use the given centre of enlargement and the scale factor to locate triangle C.

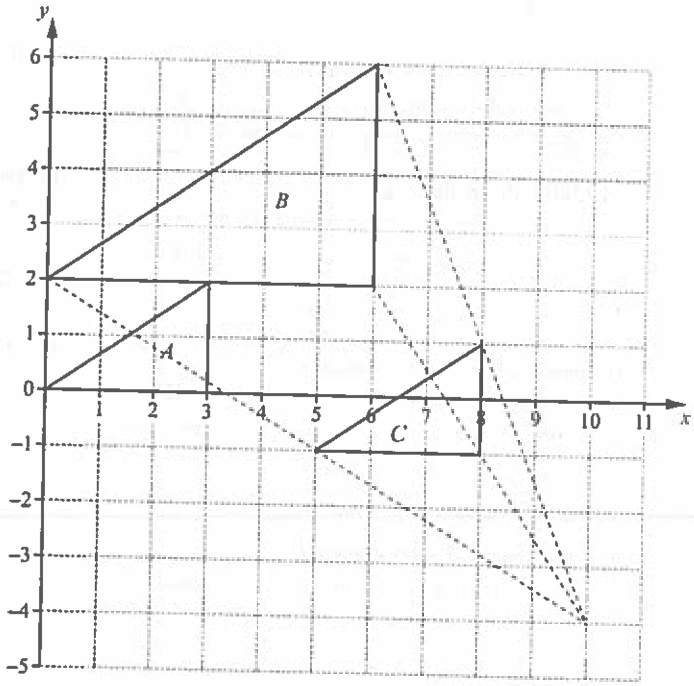
(c) Observe how far each point of triangle A has moved in the x and y directions to be translated to triangle C.

Solution

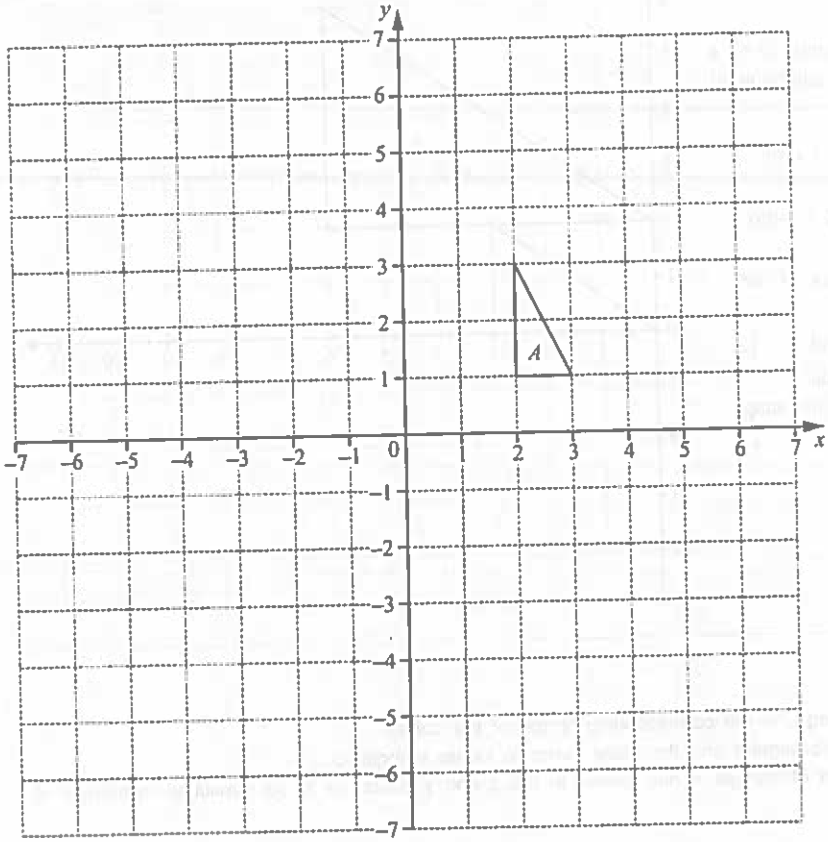
(a) Scale factor of the enlargement = 2 **Ans.**

(b) Refer to graph.

(c) Column vector of translation = $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ **Ans.**



27 *(N2018/P2/Q7)*



Triangle A is drawn on the grid.

(a) Transformation P is represented by the matrix

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}.$$

P maps triangle A onto triangle B.

(i) Draw and label triangle B. [2]

(ii) Describe fully the single transformation P. [2]

(iii) Write down the ratio area of triangle A : area of triangle B. [1]

(b) Transformation Q is represented by the matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Q maps triangle B onto triangle C.

Draw and label triangle C. [2]

(c) Transformation Y is represented by the matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}.$$

Y maps triangle A onto triangle D.

Find the matrix that represents the transformation that maps triangle D onto triangle A. [2]

Thinking Process

(a) (i) Multiply the matrix P by each pair of coordinate of triangle A.

(ii) Note that triangle A is mapped onto triangle B by an enlargement.

(iii) $\dfrac{\text{Area of image}}{\text{Area of object}} = (\text{scale factor})^2$

(b) Multiply the matrix Q by each pair of coordinate of triangle B.

(c) Find the inverse of matrix Y.

Solution

(a) (i) $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -4 & -6 & -4 \\ -2 & -2 & -6 \end{pmatrix}$

$\therefore$ The vertices of ΔB are $(-4, -2)$, $(-6, -2)$ and $(-4, -6)$.

(ii) The transformation P is an enlargement of scale factor -2 with centre of enlargement being the origin.

(iii) $\dfrac{\text{Area of } \Delta A}{\text{Area of } \Delta B} = \left(-\dfrac{1}{2}\right)^2$

$= \dfrac{1}{4}$

$\therefore$ area of ΔA : area of ΔB

$= 1:4$ **Ans.**

(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -4 & -6 & -4 \\ -2 & -2 & -6 \end{pmatrix}$

$= \begin{pmatrix} -4 & -6 & -4 \\ 2 & 2 & 6 \end{pmatrix}$

$\therefore$ The vertices of ΔC are $(-4, 2)$, $(-6, 2)$ and $(-4, 6)$.

(c) The matrix that maps ΔD onto ΔA is the inverse of matrix Y

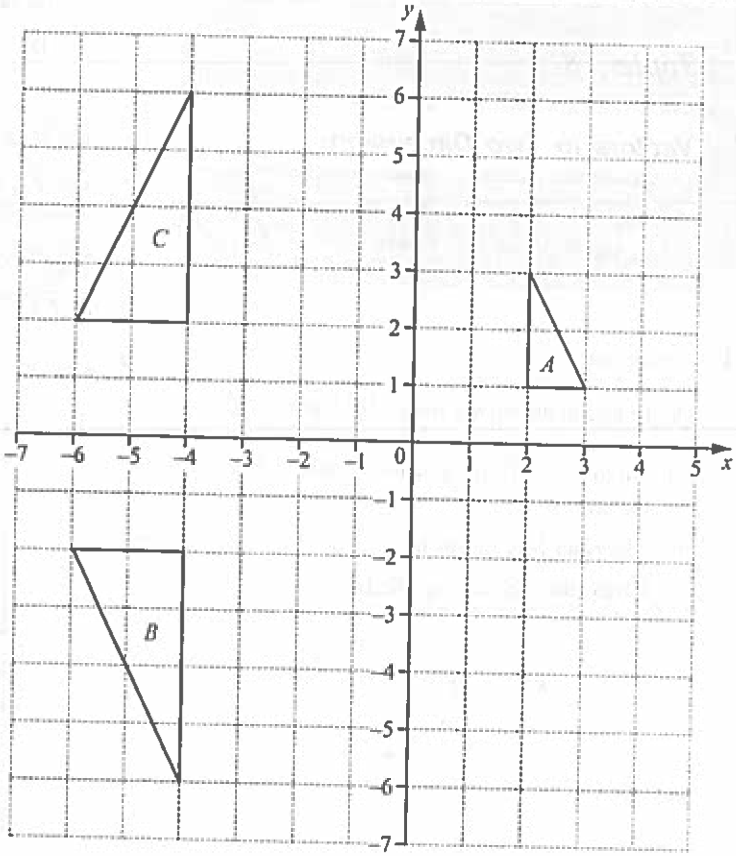
$Y = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

Det. $Y = 3 - 0 = 3$

$Y^{-1} = \dfrac{1}{3}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

$\therefore$ The matrix that maps ΔD onto ΔA

$= \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ **Ans.**



Topic 18

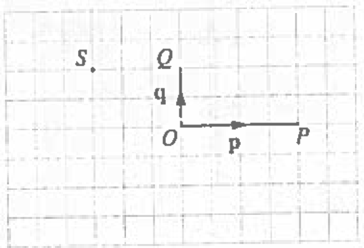
Vectors in Two Dimensions

1 (J2007/P1/Q8)

On the grid in the answer space, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

(a) Given that $\overrightarrow{OR} = \mathbf{p} - \mathbf{q}$, mark the point R clearly on the grid. [1]

(b) The point S is shown on the grid.
Given that $\overrightarrow{OS} = \mathbf{q} + h\mathbf{p}$, find h. [1]



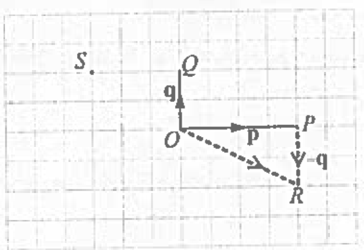
Thinking Process

(a) To find R, add the vector $-\mathbf{q}$ with $\mathbf{p}$.

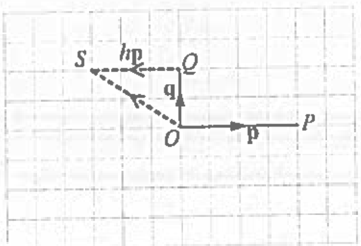
(b) Look for a value of the constant h such that $\overrightarrow{QS} = h\mathbf{p}$.

Solution with TEACHER's COMMENTS

(a)



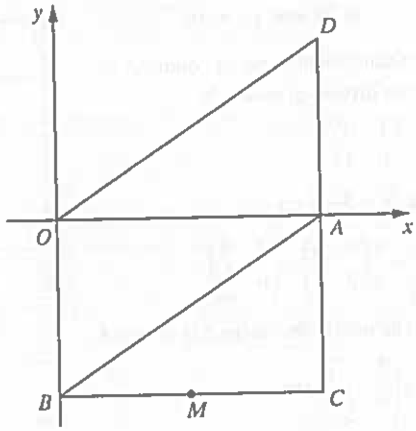
(b)



By drawing the figure as above, we see that $\overrightarrow{QS} = -\frac{3}{4}\mathbf{p}$. Therefore the value of h is $-\frac{3}{4}$. Ans.

> You must observe that $\overrightarrow{OP}$ is parallel to $\overrightarrow{QS}$. $\overrightarrow{OP}$ has a magnitude of 4 units whereas $\overrightarrow{QS}$ has a magnitude 3 units and its direction is also opposite to $\overrightarrow{OP}$. Therefore h is negative and 3/4th of $\overrightarrow{OP}$

2 (J2007 P2 Q6)



In the diagram, A is the point (6, 0) and B is the point (0, –4).

$OACB$ is a rectangle and M is the midpoint of BC.

$\overrightarrow{CA} = \overrightarrow{AD}$

(a) Describe, fully, the symmetry of quadrilateral $ODAB$. [2]

(b) Express as column vectors

(i) $\overrightarrow{CD}$ [1]

(ii) $\overrightarrow{OC}$ [1]

(iii) $\overrightarrow{DO}$ [1]

(c) What type of triangle is OCD? [1]

(d) The transformation P maps the rectangle $OBCA$ onto the quadrilateral $OBAD$.
It also maps M onto M'.

(i) Write down the coordinates of M'. [1]

(ii) Identify the transformation P. [1]

Thinking Process

(a) To describe the symmetry note that $\overrightarrow{CA} = \overrightarrow{AD}$ and $OACB$ is a rectangle.

(b) (i) To find $\overrightarrow{CD}$ write $\overrightarrow{CD}$ as $\overrightarrow{CA} + \overrightarrow{AD}$.

(ii) To find $\overrightarrow{OC}$ write $\overrightarrow{OC}$ as $\overrightarrow{OB} + \overrightarrow{BC}$.

(iii) To find $\overrightarrow{DO}$ write $\overrightarrow{DO}$ as $\overrightarrow{DC} + \overrightarrow{CO}$.

(c) Consider the answers of (b) (ii) & (iii).
(d) (i) Note that M' becomes the mid-point of BA after transformation.

Solution with TEACHER's COMMENTS

(a) $OACB$ is a rectangle

$\therefore \quad BO = CA$

also $CA = AD$ (given)

$\Rightarrow \quad BO = AD$

$\Rightarrow \quad ODAB$ is a parallelogram

$\therefore \quad ODAB$ has a rotational symmetry of order 2, about centre (3, 0) **Ans**

> The centre of rotation of a parallelogram is at the mid-point of its diagonals.

(b) Given that: $B(0, -4)$ and $A(6, 0)$

$\therefore \quad \vec{OB} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$ and $\vec{OA} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

(i) $\vec{CD} = \vec{CA} + \vec{AD}$

$= 2\vec{CA}$ (as $\vec{CA} = \vec{AD}$ (given))

$= 2\begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}$ **Ans**

(ii) $\vec{OC} = \vec{OB} + \vec{BC}$

$= \begin{pmatrix} 0 \\ -4 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ **Ans**

(iii) $\vec{DO} = \vec{DC} + \vec{CO}$

$= -\vec{CD} + (-\vec{OC})$

$= -\begin{pmatrix} 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ -8+4 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$ **Ans**

(c) From the results of part (b) (ii) and (iii), we see that $|OC| = |OD|$

$\therefore \quad OCD$ is an isosceles triangle **Ans**

> $|OC| = \sqrt{6^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52}$
> $|DO| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52}$

(d) (i) BC is mapped onto BA. Therefore M' is the mid-point of BA.

mid-point of $BA = \left(\frac{0+6}{2}, \frac{-4+0}{2}\right)$

$= (3, -2)$ **Ans**

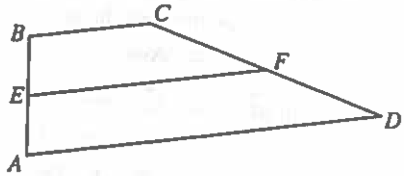
(ii) Shear along y-axis **Ans.**

3 (N2007 P2 Q9 a,b)

(a) Given that

$\vec{PQ} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $\vec{QR} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{RS} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$, find $\vec{PS}$. [1]

(b)



In the diagram

$\vec{AB} = 2\mathbf{b}$, $\vec{AD} = 3\mathbf{a}$ and $\vec{DF} = \mathbf{b} - \mathbf{a}$.

E is the midpoint of AB and F is the midpoint of DC.

(i) Express, as simply as possible, in terms of **a** and/or **b**,

(a) $\vec{EA}$, [1]

(b) $\vec{DC}$, [1]

(c) $\vec{EF}$, [1]

(d) $\vec{BC}$. [1]

(ii) (a) Give the special name of the quadrilateral $ABCD$. Give your reason. [2]

(b) Find the ratio $|\vec{BC}| : |\vec{EF}| : |\vec{AD}|$. [1]

Thinking Process

(a) $\vec{PS} = \vec{PQ} + \vec{QR} + \vec{RS}$.

(b) (i) (a) E is the midpoint of AB. Note that the direction of $\vec{EA}$ is opposite to $\vec{AB}$.

(b) $\vec{DC} = \vec{DF} + \vec{FC}$. F is the midpoint of DC.

(c) $\vec{EF} = \vec{EA} + \vec{AD} + \vec{DF}$.

(d) $\vec{BC} = \vec{BA} + \vec{AD} + \vec{DC}$.

(ii) (a) Note that $BC \parallel AD$.

(b) Compare the given vectors and find the ratio of their magnitudes.

Solution

(a) $\vec{PS} = \vec{PQ} + \vec{QR} + \vec{RS}$

$= \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -2 \end{pmatrix}$ **Ans**

(b) (i) (a) $\vec{EA} = -\mathbf{b}$ Ans

(b) $\vec{DC} = \vec{DF} + \vec{FC}$

$= 2\vec{DF} = 2(\mathbf{b} - \mathbf{a})$ Ans

(c) $\vec{EF} = \vec{EA} + \vec{AD} + \vec{DF}$

$= -\mathbf{b} + 3\mathbf{a} + (\mathbf{b} - \mathbf{a})$

$= 2\mathbf{a}$ Ans

(d) $\vec{BC} = \vec{BA} + \vec{AD} + \vec{DC}$

$= -2\mathbf{b} + 3\mathbf{a} + 2(\mathbf{b} - \mathbf{a})$

$= -2\mathbf{b} + 3\mathbf{a} + 2\mathbf{b} - 2\mathbf{a}$

$= \mathbf{a}$ Ans

(ii) (a) $\vec{AD} = 3\mathbf{a}$ and $\vec{BC} = \mathbf{a}$

$\Rightarrow AD \parallel BC$

$\therefore$ $ABCD$ is a trapezium Ans

(b) $\vec{BC} = \mathbf{a}$

$\vec{EF} = 2\mathbf{a}$

$\vec{AD} = 3\mathbf{a}$

$\therefore \left|\vec{BC}\right| : \left|\vec{EF}\right| : \left|\vec{AD}\right|$

$= 1 : 2 : 3$ Ans

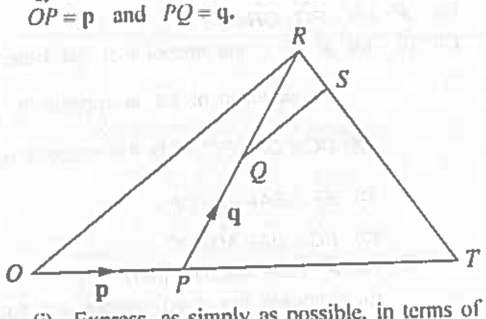
4 (J2008/P2/Q11 b)

(b) In the diagram,

$OT = 3OP$, $RS = \frac{1}{6}RT$ and

Q is the midpoint of PR.

$\vec{OP} = \mathbf{p}$ and $\vec{PQ} = \mathbf{q}$.



(i) Express, as simply as possible, in terms of p and q,

(a) $\vec{OR}$, [1]

(b) $\vec{RT}$. [1]

(c) $\vec{QS}$. [2]

(ii) Write down the value of $\frac{QS}{OR}$. [1]

Thinking Process

(b) (i) (a) ✐ Note that Q is the midpoint of PR.

(b) To find $\vec{RT}$ ✐ write $\vec{RT} = \vec{RO} + \vec{OT}$

(c) To find $\vec{QS}$ ✐ write $\vec{QS} = \vec{QR} + \vec{RS}$

(ii) Substitute the values of QS and OR and find the ratio.

Solution

(b) (i) (a) Q is the midpoint of PR.

$\Rightarrow \vec{PR} = 2\vec{PQ} = 2\mathbf{q}$

$\therefore \vec{OR} = \vec{OP} + \vec{PR}$

$= \mathbf{p} + 2\mathbf{q}$ Ans.

(b) Given that, $OT = 3OP$

$\Rightarrow \vec{OT} = 3\mathbf{p}$

$\therefore \vec{RT} = \vec{RO} + \vec{OT}$

$= -\vec{OR} + \vec{OT}$

$= -(\mathbf{p} + 2\mathbf{q}) + 3\mathbf{p}$

$= 2\mathbf{p} - 2\mathbf{q}$ Ans.

(c) Given that, $RS = \frac{1}{6}RT$

$\Rightarrow \vec{RS} = \frac{1}{6}(2\mathbf{p} - 2\mathbf{q})$

$\vec{QS} = \vec{QR} + \vec{RS}$

$= \mathbf{q} + \frac{1}{6}(2\mathbf{p} - 2\mathbf{q})$

$= \mathbf{q} + \frac{1}{3}\mathbf{p} - \frac{1}{3}\mathbf{q}$

$= \frac{1}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$

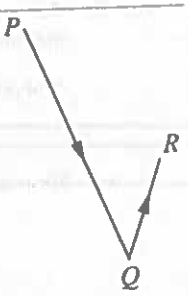
$= \frac{1}{3}(\mathbf{p} + 2\mathbf{q})$ Ans.

(ii) $\frac{\vec{QS}}{\vec{OR}} = \frac{\frac{1}{3}(\mathbf{p} + 2\mathbf{q})}{\mathbf{p} + 2\mathbf{q}} = \frac{1}{3}$

$\therefore \frac{QS}{OR} = \frac{1}{3}$ Ans.

5 (N2008/P2/Q11 a)

(a) $\vec{PQ} = \begin{pmatrix} 12 \\ -35 \end{pmatrix}$ and $\vec{QR} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$.



(i) Find

(a) $\left|\vec{PQ}\right|$. [1]

(b) $\vec{PR}$. [1]

(ii) Given that T is the midpoint of QR, find $\overrightarrow{PT}$. [2]

(iii) $PQRS$ is a parallelogram.
The coordinates of R are (6, 16).
Find the coordinates of S. [2]

Thinking Process

(a) (i) (a) Use formula: $\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2+b^2}$

(b) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

(ii) Apply head to tail on triangle PQT. Note that $\overrightarrow{QT} = \frac{1}{2}\overrightarrow{QR}$

(iii) Use the concept of position vectors and find point P first.

Solution

(a) (i) (a) $\left|\overrightarrow{PQ}\right| = \sqrt{(12)^2 + (-35)^2}$

$= \sqrt{1369} = 37$ units **Ans.**

(b) $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= \begin{pmatrix} 12 \\ -35 \end{pmatrix} + \begin{pmatrix} 4 \\ 14 \end{pmatrix} = \begin{pmatrix} 16 \\ -21 \end{pmatrix}$ **Ans.**

(ii) $\overrightarrow{QT} = \frac{1}{2}\overrightarrow{QR} = \frac{1}{2}\begin{pmatrix} 4 \\ 14 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

$\overrightarrow{PT} = \overrightarrow{PQ} + \overrightarrow{QT}$

$= \begin{pmatrix} 12 \\ -35 \end{pmatrix} + \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 14 \\ -28 \end{pmatrix}$ **Ans.**

(iii) As $PQRS$ is a parallelogram,

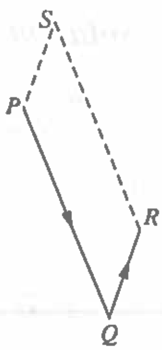
$\overrightarrow{PS} = \overrightarrow{QR} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$, and $\overrightarrow{SR} = \overrightarrow{PQ} = \begin{pmatrix} 12 \\ -35 \end{pmatrix}$

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$\overrightarrow{OP} = \overrightarrow{OR} - \overrightarrow{PR}$

$\overrightarrow{OP} = \begin{pmatrix} 6 \\ 16 \end{pmatrix} - \begin{pmatrix} 16 \\ -21 \end{pmatrix}$

$= \begin{pmatrix} -10 \\ 37 \end{pmatrix}$



$\overrightarrow{PS} = \overrightarrow{OS} - \overrightarrow{OP}$

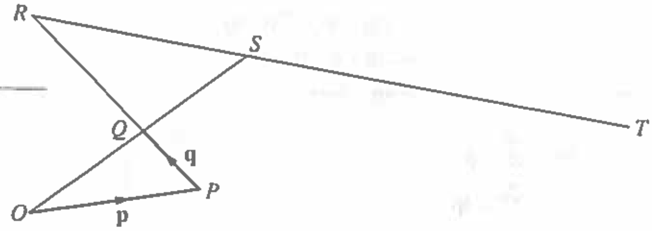
$\overrightarrow{OS} = \overrightarrow{PS} + \overrightarrow{OP}$

$= \begin{pmatrix} 4 \\ 14 \end{pmatrix} + \begin{pmatrix} -10 \\ 37 \end{pmatrix} = \begin{pmatrix} -6 \\ 51 \end{pmatrix}$

$\therefore$ coordinates of S are $(-6, 51)$ **Ans.**

6 *(J2009 P2 Q9 b)*

(b) In the diagram,



$\overrightarrow{OQ} = \overrightarrow{QS}$, $\overrightarrow{QR} = 2\overrightarrow{PQ}$ and $\overrightarrow{ST} = 2\overrightarrow{RS}$.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{PQ} = \mathbf{q}$.

(i) Express, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{q}$,

(a) $\overrightarrow{OQ}$, [1]

(b) $\overrightarrow{RS}$, [1]

(c) $\overrightarrow{OS}$, [1]

(d) $\overrightarrow{OT}$. [1]

(ii) Hence write down two facts about O, P and T. [2]

Thinking Process

(b) (i) (a) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$.

(b) $\overrightarrow{RS} = \overrightarrow{RQ} + \overrightarrow{QS}$.

(c) Observe that $\overrightarrow{OQ} = \overrightarrow{QS}$.

(d) $\overrightarrow{OT} = \overrightarrow{OS} + \overrightarrow{ST}$.

(ii) Express $\overrightarrow{OP}$ as a ratio of $\overrightarrow{OT}$.

Solution

(b) (i) (a) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= \mathbf{p} + \mathbf{q}$ **Ans.**

(b) Given that,

$\overrightarrow{QR} = 2\overrightarrow{PQ} = 2\mathbf{q}$

$\overrightarrow{OQ} = \overrightarrow{QS} = \mathbf{p} + \mathbf{q}$

$\therefore$ $\overrightarrow{RS} = \overrightarrow{RQ} + \overrightarrow{QS}$

$= -2\mathbf{q} + \mathbf{p} + \mathbf{q} = \mathbf{p} - \mathbf{q}$ **Ans.**

(c) Given that, $\overrightarrow{OQ} = \overrightarrow{QS}$

$\therefore$ $\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{QS}$

$= 2\overrightarrow{OQ}$

$= 2(\mathbf{p} + \mathbf{q})$ **Ans.**

(d) Given that, $\vec{ST} = 2\vec{RS} = 2(\mathbf{p} - \mathbf{q})$

$\therefore \quad \vec{OT} = \vec{OS} + \vec{ST}$

$= 2(\mathbf{p} + \mathbf{q}) + 2(\mathbf{p} - \mathbf{q})$

$= 2(\mathbf{p} + \mathbf{q} + \mathbf{p} - \mathbf{q})$

$= 4\mathbf{p}$ **Ans.**

(ii) $\vec{OP} = \mathbf{p}$

$\vec{OT} = 4\mathbf{p}$

$\Rightarrow \quad \vec{OT} = 4\vec{OP}$

This implies

1. O, P, and T are collinear.
2. OT is 4 times OP.

7 *(N2009/P1/Q15)*

$\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$

(a) Express $\mathbf{a} + 2\mathbf{b}$ as a column vector. [1]

(b) (i) Find $|\mathbf{a}|$. [1]

(ii) Given that $\dfrac{|\mathbf{b}|}{|\mathbf{a}|} = \sqrt{n}$, where n is an integer, find the value of n. [1]

Thinking Process

(a) Multiply column vector **b** by 2 and add it to vector **a**.

(b) (i) Apply formula: $\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2 + b^2}$.

(ii) Substitute the values of $|\mathbf{a}|$ & $|\mathbf{b}|$ in the given equation and solve for n.

Solution

(a) $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ 14 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 10 \end{pmatrix}$ **Ans.**

(b) (i) $|\mathbf{a}| = \sqrt{(3)^2 + (-4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25} = 5$ **Ans.**

(ii) $|\mathbf{b}| = \sqrt{(-1)^2 + (7)^2} = \sqrt{1 + 49} = \sqrt{50}$

$\dfrac{|\mathbf{b}|}{|\mathbf{a}|} = \sqrt{n}$

$\Rightarrow \quad \dfrac{\sqrt{50}}{5} = \sqrt{n}$

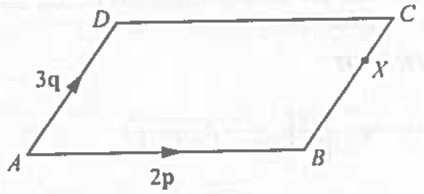
squaring both sides,

$\left(\dfrac{\sqrt{50}}{5}\right)^2 = \left(\sqrt{n}\right)^2$

$\dfrac{50}{25} = n$

$n = 2$ **Ans.**

8 *(J2010/P1/Q17)*



$ABCD$ is a parallelogram.

X is the point on BC such that $BX : XC = 2 : 1$.

$\vec{AB} = 2\mathbf{p}$ and $\vec{AD} = 3\mathbf{q}$.

Find, in terms of **p** and **q**,

(a) $\vec{AC}$, [1]

(b) $\vec{AX}$, [1]

(c) $\vec{XD}$. [1]

Thinking Process

(a) $\vec{AC} = \vec{AB} + \vec{BC}$

(b) Use $\vec{AX} = \vec{AB} + \vec{BX}$. Note that $\vec{BX} = \frac{2}{3}\vec{BC}$.

(c) $\vec{XD} = \vec{XC} + \vec{CD}$

Solution

(a) $\vec{AC} = \vec{AB} + \vec{BC}$

$= 2\mathbf{p} + 3\mathbf{q}$ **Ans.**

(b) $\vec{BX} : \vec{XC} = 2 : 1. \quad \Rightarrow \quad \vec{BX} = \frac{2}{3}\vec{BC}$

$\vec{AX} = \vec{AB} + \vec{BX}$

$\Rightarrow \quad \vec{AX} = \vec{AB} + \frac{2}{3}\vec{BC}$

$= 2\mathbf{p} + \frac{2}{3}(3\mathbf{q})$

$= 2\mathbf{p} + 2\mathbf{q}$ **Ans.**

(c) $\vec{XC} = \frac{1}{3}\vec{BC}$

$\vec{XD} = \vec{XC} + \vec{CD}$

$\Rightarrow \vec{XD} = \frac{1}{3}\vec{BC} + \vec{CD}$

$= \frac{1}{3}(3\mathbf{q}) + (-2\mathbf{p})$

$= \mathbf{q} - 2\mathbf{p}$ **Ans.**

9 *(N2010/P1/Q9)*

$\vec{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

(a) Find $|\vec{AB}|$. [1]

(b) A is the point (0,2).

(i) The equation of the line AB may be written $3y + 4x = k$.
Find the value of k. [1]

(ii) Find the coordinates of the midpoint of AB. [1]

Thinking Process

(a) Find the magnitude of AB by formula:

if $\vec{AB} = \begin{pmatrix} a \\ b \end{pmatrix}$ then $|\vec{AB}| = \sqrt{a^2 + b^2}$

(b) (i) Substitute point A into the given equation and solve for k.

(ii) Find $\vec{OB}$ using $\vec{AB} = \vec{OB} - \vec{OA}$. Apply the midpoint formula i.e. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Solution

(a) $|\vec{AB}| = \sqrt{(3)^2 + (-4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$ units **Ans.**

(b) (i) $3y + 4x = k$

Substituting point A into the equation,

$3(2) + 4(0) = k$

$k = 6$ **Ans.**

(ii) $\vec{AB} = \vec{OB} - \vec{OA}$

$\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \vec{OB} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$\vec{OB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

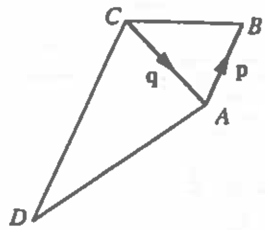
$\therefore$ point B is $(3, -2)$

midpoint of $AB = \left(\frac{0+3}{2}, \frac{2+(-2)}{2}\right)$

$= (1.5, 0)$ **Ans.**

10 *(N2010/P1/Q24)*

In the diagram, $\vec{AB} = \mathbf{p}$, $\vec{CA} = \mathbf{q}$ and $\vec{DC} = 3\vec{AB}$.



(a) Express $\vec{DA}$ in terms of p and q. [1]

(b) E is the point such that $\vec{BE} = k\mathbf{q}$.

(i) Write down the name given to the special quadrilateral $ACBE$. [1]

(ii) Express $\vec{AE}$ in terms of p, q and k. [1]

(iii) Given that D, A and E lie on a straight line, find the value of k. [1]

Thinking Process

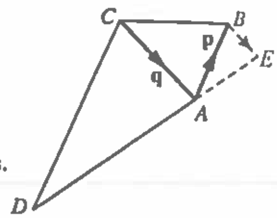
(a) $\vec{DA} = \vec{DC} + \vec{CA}$

(b) (i) To identify the quadrilateral note that $\vec{BE} \parallel \vec{CA}$.

(ii) $\vec{AE} = \vec{AB} + \vec{BE}$

(iii) Write $\vec{DA} = h\vec{AE}$. Find the value of h and then subsequently find k.

Solution



(a) $\vec{DA} = \vec{DC} + \vec{CA}$

$= 3\mathbf{p} + \mathbf{q}$ **Ans.**

(b) (i) trapezium. **Ans.**

(ii) $\vec{AE} = \vec{AB} + \vec{BE}$

$= \mathbf{p} + k\mathbf{q}$ **Ans.**

(iii) Let $\vec{DA} = h\vec{AE}$

$\Rightarrow$ $3p + q = h(p + kq)$

$3p + q = hp + hkq$

comparing coefficient of p, $3 = h$

comparing coefficient of q, $1 = hk$

$1 = 3k$

$k = \frac{1}{3}$ Ans.

> Note that,
> If A, B, and C are collinear, then
> $\vec{AB} = h\vec{BC}$,
> where h is a real number.

11 (J2011/P1/Q5)

$c = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $d = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

(a) Calculate $2c - d$. [1]

(b) Calculate $|d|$. [1]

Thinking Process

(a) Perform the required calculation.

(b) Use $|a| = \sqrt{x^2 + y^2}$, where $a = \begin{pmatrix} x \\ y \end{pmatrix}$

Solution

(a) $2c - d = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 10 \end{pmatrix}$ Ans.

(b) $|d| = \sqrt{(8)^2 + (-6)^2}$

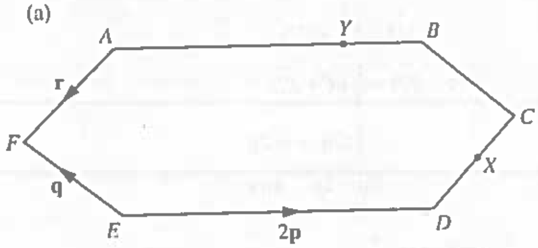
$= \sqrt{64 + 36}$

$= \sqrt{100}$

$= 10$ Ans.

12 (J2011/P2/Q7)

(a)



In the diagram, $ABCDEF$ is a hexagon with rotational symmetry of order 2.

$\vec{ED} = 2p$, $\vec{EF} = q$ and $\vec{AF} = r$.

X is the midpoint of CD and Y is the point on AB such that $AY : YB$ is $3 : 1$.

(i) How many lines of symmetry does $ABCDEF$ have? [1]

(ii) Express, as simply as possible, in terms of one or more of the vectors p, q and r,

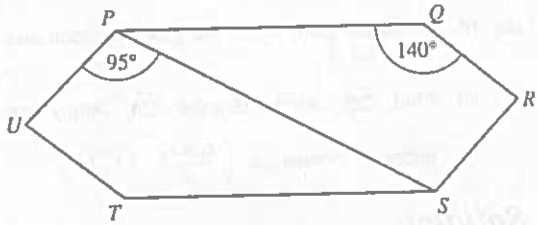
(a) $\vec{EA}$. [1]

(b) $\vec{FC}$. [1]

(c) $\vec{FY}$. [1]

(d) $\vec{YX}$. [1]

(b)



$PQRSTU$ is a similar hexagon to $ABCDEF$.

$U\hat{P}S = 95°$ and $P\hat{Q}R = 140°$.

Find

(i) $Q\hat{P}S$. [1]

(ii) $P\hat{S}R$. [1]

(iii) $P\hat{U}T$. [1]

Thinking Process

(a) (i) Understand the definition of line symmetry.

(ii) (a) $\vec{EA} = \vec{EF} + \vec{FA}$

(b) $\vec{FC} = \vec{FE} + \vec{ED} + \vec{DC}$

(c) $\vec{FY} = \vec{FA} + \vec{AY}$. Note that $\frac{AY}{YB} = \frac{3}{1}$

(d) Find $\vec{YB}$, $\vec{CX}$ and then perform vector addition.

(b) (i) Note that $\angle UPQ = \angle PQR$.

(ii) $\angle PSR = \angle SPU$ (alternate $\angle$s).

(iii) Apply, sum of angles in a quadrilateral.

Solution

(a) (i) Lines of symmetry = 2 **Ans.**

(ii) (a) $\vec{EA} = \vec{EF} + \vec{FA}$

$= \mathbf{q} - \mathbf{r}$ **Ans.**

(b) $\vec{CD} = \vec{AF} = \mathbf{r}$

$\vec{FC} = \vec{FE} + \vec{ED} + \vec{DC}$

$= -\mathbf{q} + 2\mathbf{p} - \mathbf{r}$ **Ans.**

(c) $\dfrac{\vec{AY}}{\vec{YB}} = \dfrac{3}{1}$

$\Rightarrow \dfrac{\vec{AY}}{\vec{AB}} = \dfrac{3}{4} \Rightarrow \vec{AY} = \dfrac{3}{4}\vec{AB}$

$\vec{FY} = \vec{FA} + \vec{AY}$

$= -\mathbf{r} + \dfrac{3}{4}\vec{AB}$

$= -\mathbf{r} + \dfrac{3}{4}(2\mathbf{p})$

$= -\mathbf{r} + \dfrac{3}{2}\mathbf{p}$ **Ans.**

(d) $\vec{YB} = \dfrac{1}{4}\vec{AB}$, $\vec{BC} = \vec{FE}$, $\vec{CX} = \dfrac{1}{2}\vec{CD}$

$\vec{YX} = \vec{YB} + \vec{BC} + \vec{CX}$

$= \dfrac{1}{4}\vec{AB} + \vec{FE} + \dfrac{1}{2}\vec{CD}$

$= \dfrac{1}{4}(2\mathbf{p}) + (-\mathbf{q}) + \dfrac{1}{2}(\mathbf{r})$

$= \dfrac{1}{2}\mathbf{p} - \mathbf{q} + \dfrac{1}{2}\mathbf{r}$ **Ans.**

(b) (i) $U\hat{P}Q = P\hat{Q}R = 140°$ (interior $\angle$s)

$Q\hat{P}S = 140° - 95° = 45°$ **Ans.**

(ii) $P\hat{S}R = S\hat{P}U = 95°$ (alternate $\angle$s) **Ans.**

(iii) $T\hat{S}P = Q\hat{P}S = 45°$ (alternate $\angle$s)

In quadrilateral $PSTU$,

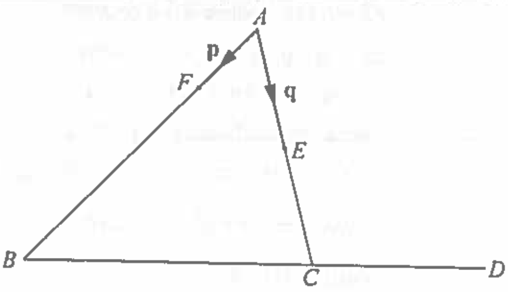
$U\hat{P}S + P\hat{U}T + U\hat{T}S + T\hat{S}P = 360°$

$95° + P\hat{U}T + 140° + 45° = 360°$

$P\hat{U}T = 360° - 280°$

$= 80°$ **Ans.**

13 (N2011/P1/Q28)



In the diagram, F is the point on AB where $AF = \frac{1}{4}AB$. E is the midpoint of AC.

$\vec{AF} = \mathbf{p}$ and $\vec{AE} = \mathbf{q}$.

(a) Express, in terms of $\mathbf{p}$ and $\mathbf{q}$.

(i) $\vec{FE}$. [1]

(ii) $\vec{BC}$. [1]

(b) D is the point on BC produced such that $BD = kBC$.

(i) Express $\vec{FD}$ in terms of k, $\mathbf{p}$ and $\mathbf{q}$. [1]

(ii) Given that F, E and D are collinear, find the value of k. [2]

Thinking Process

(a) (i) $\vec{FE} = \vec{FA} + \vec{AE}$

(ii) $\vec{BC} = \vec{BA} + \vec{AC}$

(b) (i) $\vec{FD} = \vec{FB} + \vec{BD}$

(ii) Express $\vec{FE}$ as $h\vec{FD}$ since F, E, and D are collinear. Find the value of h and subsequently find k.

Solution

(a) (i) $\vec{FE} = \vec{FA} + \vec{AE}$

$= \mathbf{q} - \mathbf{p}$ **Ans.**

(ii) $\vec{AF} = \dfrac{1}{4}\vec{AB} \Rightarrow \vec{AB} = 4\vec{AF} = 4\mathbf{p}$

$\vec{AC} = 2\vec{AE} = 2\mathbf{q}$

$\therefore \vec{BC} = \vec{BA} + \vec{AC}$

$= -4\mathbf{p} + 2\mathbf{q}$ **Ans.**

(b) (i) $\vec{FD} = \vec{FB} + \vec{BD}$

$= 3\mathbf{p} + k\vec{BC}$

$= 3\mathbf{p} + k(-4\mathbf{p} + 2\mathbf{q})$

$= 3\mathbf{p} - 4k\mathbf{p} + 2k\mathbf{q}$

$= \mathbf{p}(3 - 4k) + 2k\mathbf{q}$ **Ans.**

(ii) Since F, E and D are collinear.

$\vec{FE} = h\vec{FD}$, where h is a constant.

$\Rightarrow \quad \mathbf{q} - \mathbf{p} = h(\mathbf{p}(3-4k) + 2k\mathbf{q})$

$-\mathbf{p} + \mathbf{q} = h\mathbf{p}(3-4k) + 2hk\mathbf{q}$

comparing coefficients of p and q.

$-1 = h(3-4k) \Rightarrow -1 = 3h - 4hk$ ....(1)

$1 = 2hk \Rightarrow k = \frac{1}{2h}$ ....(2)

substitute (2) into (1)

$-1 = 3h - 4h(\frac{1}{2h})$

$-1 = 3h - 2$

$3h = 1$

$h = \frac{1}{3}$

substitute $h = \frac{1}{3}$ into (2)

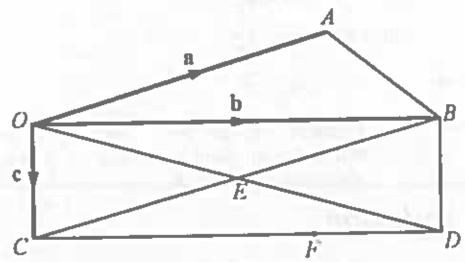
$k = \frac{1}{2(\frac{1}{3})} \Rightarrow k = \frac{3}{2}$ or 1.5 **Ans.**

14 *(J2012/P2/Q7)*

OAB is a triangle and $OBDC$ is a rectangle where OD and BC intersect at E.

F is the point on CD such that $CF = \frac{3}{4}CD$.

$\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$.



(a) Express, as simply as possible, in terms of one or more of the vectors a, b and c,

(i) $\vec{AB}$. [1]

(ii) $\vec{OE}$. [1]

(iii) $\vec{EF}$. [2]

(b) G is the point on AB such that $\vec{OG} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$.

(i) Express $\vec{AG}$ in terms of a and b. Give your answer as simply as possible. [1]

(ii) Find $AG : GB$. [1]

(iii) Express $\vec{FG}$ in terms of a, b and c. Give your answer as simply as possible. [2]

Thinking Process

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$

(ii) $\vec{OE} = \frac{1}{2}\vec{OD}$ ✐ find $\vec{OD}$

(iii) $\vec{EF} = \vec{EO} + \vec{OC} + \vec{CF}$

(b) (i) $\vec{AG} = \vec{OG} - \vec{OA}$

(ii) To find the ratio ✐ express AG and BG in terms of AB.

(iii) To find FG ✐ perform vector addition.

Solution

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \mathbf{b} - \mathbf{a}$ **Ans.**

(ii) $\vec{OE} = \frac{1}{2}\vec{OD}$

$= \frac{1}{2}(\vec{OC} + \vec{CD})$

$= \frac{1}{2}(\mathbf{c} + \mathbf{b})$ **Ans.**

(iii) $\vec{EF} = \vec{EO} + \vec{OC} + \vec{CF}$

$= \vec{EO} + \vec{OC} + \frac{3}{4}\vec{CD}$

$= -\frac{1}{2}(\mathbf{c} + \mathbf{b}) + \mathbf{c} + \frac{3}{4}\mathbf{b}$

$= -\frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{b} + \mathbf{c} + \frac{3}{4}\mathbf{b}$

$= \frac{1}{4}\mathbf{b} + \frac{1}{2}\mathbf{c}$ **Ans.**

(b) (i) $\vec{AG} = \vec{OG} - \vec{OA}$

$= \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} - \mathbf{a}$

$= \frac{2}{5}\mathbf{b} - \frac{2}{5}\mathbf{a}$

$= \frac{2}{5}(\mathbf{b} - \mathbf{a})$ **Ans.**

(ii) $\vec{AG} = \frac{2}{5}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{5}\vec{AB}$

$\Rightarrow \vec{GB} = \frac{3}{5}\vec{AB}$

$\therefore AG : GB = 2 : 3$ **Ans.**

(iii) $\vec{FG} = \vec{FC} + \vec{CO} + \vec{OA} + \vec{AG}$

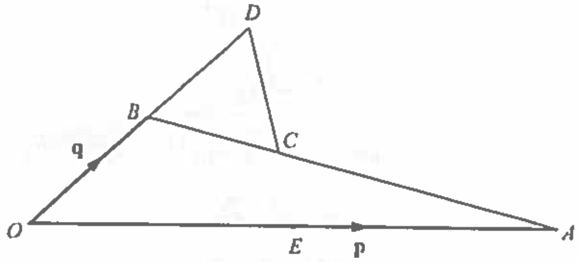
$= -\frac{3}{4}\vec{CD} + \vec{CO} + \vec{OA} + \vec{AG}$

$= -\frac{3}{4}\mathbf{b} - \mathbf{c} + \mathbf{a} + \frac{2}{5}\mathbf{b} - \frac{2}{5}\mathbf{a}$

$= \frac{3}{5}\mathbf{a} - \frac{7}{20}\mathbf{b} - \mathbf{c}$ **Ans.**

15 *(N2012 P2 Q11 a)*

(a)



B is the midpoint of OD and E is the midpoint of OA.
C is the point on AB such that $AC : CB = 2 : 1$.
$\vec{OA} = \mathbf{p}$ and $\vec{OB} = \mathbf{q}$

(i) Find, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(a) $\vec{AB}$, [1]

(b) $\vec{CD}$, [1]

(c) $\vec{ED}$. [1]

(ii) Use your answers to parts (i)(b) and (i)(c) to make two statements about the points E, C and D. [2]

Thinking Process

(a) (i) (a) $\vec{AB} = \vec{OB} - \vec{OA}$.

(b) $\vec{CD} = \vec{CB} + \vec{BD}$ ✎ find $\vec{CB}$ and $\vec{BD}$.

(c) $\vec{ED} = \vec{OD} - \vec{OE}$.

(ii) Express $\vec{CD}$ as a ratio of $\vec{ED}$.

Solution

(a) (i) (a) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \mathbf{q} - \mathbf{p}$ **Ans.**

(b) Given that, $AC : CB = 2 : 1$

$\Rightarrow \vec{CB} = \frac{1}{3}\vec{AB}$

B is the mid-point of OD, $\therefore \vec{BD} = \mathbf{q}$

$\vec{CD} = \vec{CB} + \vec{BD}$

$= \frac{1}{3}\vec{AB} + \mathbf{q}$

$= \frac{1}{3}(\mathbf{q} - \mathbf{p}) + \mathbf{q}$

$= \frac{1}{3}\mathbf{q} - \frac{1}{3}\mathbf{p} + \mathbf{q}$

$= \frac{4}{3}\mathbf{q} - \frac{1}{3}\mathbf{p}$

$= \frac{1}{3}(4\mathbf{q} - \mathbf{p})$ **Ans.**

(c) $\vec{ED} = \vec{OD} - \vec{OE}$

$= 2\vec{OB} - \frac{1}{2}\vec{OA}$

$= 2\mathbf{q} - \frac{1}{2}\mathbf{p}$ **Ans.**

(ii) $\vec{CD} = \frac{1}{3}(4\mathbf{q} - \mathbf{p})$

$= \frac{2}{3}(2\mathbf{q} - \frac{1}{2}\mathbf{p})$

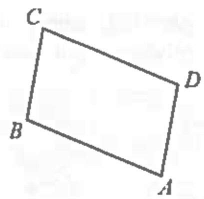
$= \frac{2}{3}\vec{ED}$

this implies that,

1. E, C and D are collinear
2. CD is $\frac{2}{3}$rd of ED.

16 *(J2013 P2 Q9 a)*

(a) $ABCD$ is a parallelogram.



$\vec{AB} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ and $\vec{BC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

(i) Find $\vec{BD}$. [1]

(ii) Calculate $|\vec{AC}|$. [2]

(iii) The parallelogram $ABCD$ is mapped onto the parallelogram $PBQR$.

$\vec{PB} = \begin{pmatrix} -12 \\ 6 \end{pmatrix}$ and $\vec{BQ} = \begin{pmatrix} 3 \\ 12 \end{pmatrix}$.

(a) Describe fully the single transformation that maps the parallelogram $ABCD$ onto the parallelogram $PBQR$. [2]

(b) S is the midpoint of PQ.

Find $\vec{SR}$. [2]

Thinking Process

(a) (i) Note that $\vec{BC} = \vec{AD}$ since $ABCD$ is a parallelogram.

(ii) Use formula: $\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2 + b^2}$ ✎ find $\vec{AC}$.

(iii) (a) To describe the transformation ✎ take note that $\vec{PB} \parallel \vec{AB}$, $\vec{BQ} \parallel \vec{BC}$ and both parallelograms share a common point B.

(b) Apply vector addition: $\vec{SR} = \vec{SP} + \vec{PR}$

Solution with TEACHER's COMMENTs

(a) (i) As $ABCD$ is a parallelogram.

$\Rightarrow \vec{AD} = \vec{BC} = \begin{pmatrix}1\\4\end{pmatrix}$

$\therefore \vec{BD} = \vec{BA} + \vec{AD}$

$= -\vec{AB} + \vec{AD}$

$= \begin{pmatrix}4\\-2\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix}$ **Ans.**

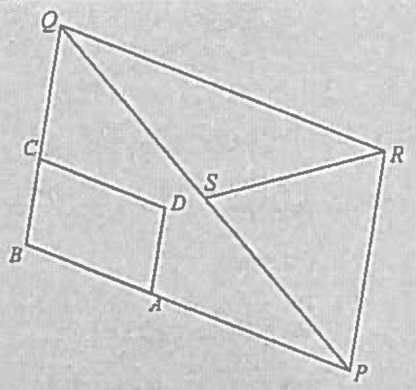
(ii) $\vec{AC} = \vec{AB} + \vec{BC}$

$= \begin{pmatrix}-4\\2\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}-3\\6\end{pmatrix}$

$\therefore \left|\vec{AC}\right| = \sqrt{(-3)^2 + (6)^2}$

$= \sqrt{45} = 6.71$ units. **Ans.**

(iii) (a) $ABCD$ is mapped onto $PBQR$ by an enlargement of scale factor 3, centre B.



$\vec{PB} = \begin{pmatrix}-12\\6\end{pmatrix} = 3\begin{pmatrix}-4\\2\end{pmatrix} \Rightarrow \vec{PB} = 3\vec{AB} \Rightarrow \vec{PB} \parallel \vec{AB}$

similarly,

$\vec{BQ} = \begin{pmatrix}3\\12\end{pmatrix} = 3\begin{pmatrix}1\\4\end{pmatrix} \Rightarrow \vec{BQ} = 3\vec{BC} \Rightarrow \vec{BQ} \parallel \vec{BC}$

Therefore, $PBQP$ is an enlargement of scale factor 3 with centre being at the common point B.

(b) $\vec{PQ} = \vec{PB} + \vec{BQ}$

$= \begin{pmatrix}-12\\6\end{pmatrix} + \begin{pmatrix}3\\12\end{pmatrix} = \begin{pmatrix}-9\\18\end{pmatrix}$

$\Rightarrow \vec{PS} = \frac{1}{2}\begin{pmatrix}-9\\18\end{pmatrix} = \begin{pmatrix}-4.5\\9\end{pmatrix}$

also, $\vec{PR} = \vec{BQ} = \begin{pmatrix}3\\12\end{pmatrix}$ $(\because BQ \parallel PR)$

$\therefore \vec{SR} = \vec{SP} + \vec{PR}$

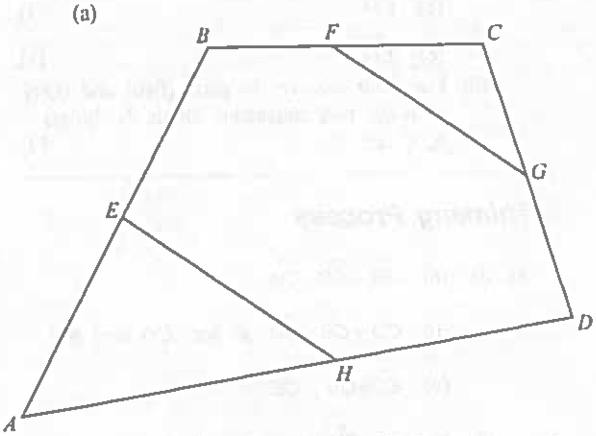
$\Rightarrow \vec{SR} = -\vec{PS} + \vec{PR}$

$= -\begin{pmatrix}-4.5\\9\end{pmatrix} + \begin{pmatrix}3\\12\end{pmatrix}$

$= \begin{pmatrix}4.5\\-9\end{pmatrix} + \begin{pmatrix}3\\12\end{pmatrix} = \begin{pmatrix}7.5\\3\end{pmatrix}$ **Ans.**

17 *(N2013 P2 Q12 a)*

(a)



(i) $\vec{AD} = \begin{pmatrix}6\\1\end{pmatrix}$. Calculate $\left|\vec{AD}\right|$ [1]

(ii) $\vec{AE} = \begin{pmatrix}1\\2\end{pmatrix}$

H is the midpoint of AD. Find $\vec{EH}$. [2]

(iii) $\vec{BF} = \begin{pmatrix}1.5\\0\end{pmatrix}$ $\vec{CG} = \begin{pmatrix}0.5\\-1.5\end{pmatrix}$

F is the midpoint of BC. Find $\vec{FG}$. [1]

(iv) Use your answers to parts (ii) and (iii) to complete the following statement.

The lines EH and FG are .... and .... [1]

(v) Given that E is the midpoint of AB, show that G is the midpoint of CD. [2]

Thinking Process

(a) (i) ✎ If $\vec{AB} = \begin{pmatrix}a\\b\end{pmatrix}$ then $\left|\vec{AB}\right| = \sqrt{a^2 + b^2}$.

(ii) $\vec{EH} = \vec{EA} + \vec{AH}$

(iii) $\vec{FG} = \vec{FC} + \vec{CG}$

(iv) ✐ Compare $\vec{EH}$ and $\vec{FG}$.

(v) Find GD. If $GD = CG$, then G is the midpoint of CD.

Solution

(a) (i) $\left|\vec{AD}\right| = \sqrt{(6)^2 + (1)^2}$

$= \sqrt{37} = 6.08$ **Ans.**

(ii) $\vec{EH} = \vec{EA} + \vec{AH}$

$\vec{EH} = -\vec{AE} + \frac{1}{2}\vec{AD}$

$\vec{EH} = -\begin{pmatrix}1\\2\end{pmatrix} + \frac{1}{2}\begin{pmatrix}6\\1\end{pmatrix}$

$= \begin{pmatrix}-1\\-2\end{pmatrix} + \begin{pmatrix}3\\\frac{1}{2}\end{pmatrix} = \begin{pmatrix}2\\-1.5\end{pmatrix}$ **Ans.**

(iii) F is the midpoint of BC

$\Rightarrow \vec{BF} = \vec{FC} = \begin{pmatrix}1.5\\0\end{pmatrix}$

$\vec{FG} = \vec{FC} + \vec{CG}$

$= \begin{pmatrix}1.5\\0\end{pmatrix} + \begin{pmatrix}0.5\\-1.5\end{pmatrix}$

$= \begin{pmatrix}2\\-1.5\end{pmatrix}$ **Ans.**

(iv) The lines EH and FG are parallel and equal.

(v) $\vec{BC} = 2\vec{BF}$

$= 2\begin{pmatrix}1.5\\0\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix}$

$\vec{AB} = 2\vec{AE}$

$= 2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$

now, $\vec{CD} = \vec{CB} + \vec{BA} + \vec{AD}$

$= \begin{pmatrix}-3\\0\end{pmatrix} + \begin{pmatrix}-2\\-4\end{pmatrix} + \begin{pmatrix}6\\1\end{pmatrix}$

$= \begin{pmatrix}1\\-3\end{pmatrix}$

$\vec{GD} = \vec{CD} - \vec{CG}$

$= \begin{pmatrix}1\\-3\end{pmatrix} - \begin{pmatrix}0.5\\-1.5\end{pmatrix}$

$= \begin{pmatrix}0.5\\-1.5\end{pmatrix}$

as $\vec{GD} = \vec{CG}$

$\therefore$ G is the midpoint of CD **Shown.**

18 *(J2014 P2 Q8)*

(a) In this question you may use the grid below to help you.

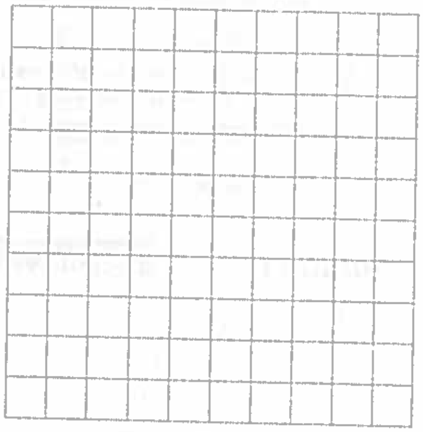
The point P has position vector $\begin{pmatrix}4\\2\end{pmatrix}$ and the point Q has position vector $\begin{pmatrix}8\\-3\end{pmatrix}$.

(i) Find $\vec{PQ}$. [1]

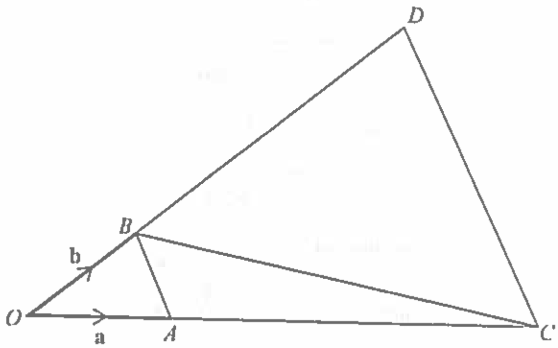
(ii) Find $\left|\vec{PQ}\right|$. [1]

(iii) Find the equation of the line PQ. [2]

(iv) Given that Q is the midpoint of the line PR, find the coordinates of R. [2]



(b)



In the diagram triangles OAB and OCD are similar.

$\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{BC} = 4\mathbf{a} - \mathbf{b}$.

(i) Express, as simply as possible, in terms of **a** and/or **b**

(a) $\vec{AB}$. [1]

(b) $\vec{AC}$. [1]

(c) $\vec{CD}$. [2]

(ii) Find, in its simplest form, the ratio

(a) perimeter of triangle OAB : perimeter of triangle OCD, [1]

(b) area of triangle OAB : area of trapezium $ABDC$. [1]

Thinking Process

(a) (i) $\vec{PQ} = \vec{OQ} - \vec{OP}$

(ii) if $\vec{AB} = \begin{pmatrix} a \\ b \end{pmatrix}$ then $\left|\vec{AB}\right| = \sqrt{a^2 + b^2}$

(iii) To find equation of line PQ ✐ use $y = mx + c$ where m is the gradient and c is the y-intercept.

(iv) Let R be (x, y). Find the midpoint of PQ and equate it to the midpoint of QR.

(b) (i) (a) $\vec{AB} = \vec{OB} - \vec{OA}$

(b) $\vec{AC} = \vec{AB} - \vec{BC}$

(c) ✐ Apply similarity concept and express $\vec{CD}$ as a ratio of $\vec{AB}$

(ii) (a) To find the ratio ✐ consider the ratio of the linear dimensions of two triangles.

(b) Apply concept of area of similar triangles. $\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2$

Solution with TEACHER's COMMENTS

(a) (i) $\vec{PQ} = \vec{OQ} - \vec{OP}$

$= \begin{pmatrix} 8 \\ -3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ **Ans.**

(ii) $\left|\vec{PQ}\right| = \sqrt{(4)^2 + (-5)^2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$ units **Ans.**

(iii) We have. $P(4, 2)$ and $Q(8, -3)$

gradient of $PQ = \frac{-3-2}{8-4} = -\frac{5}{4}$

equation of PQ is: $y = -\frac{5}{4}x + c$

using $P(4, 2)$, $2 = -\frac{5}{4}(4) + c \Rightarrow c = 7$

$\therefore \quad y = -\frac{5}{4}x + 7$ **Ans.**

(iv) Let the coordinates of R be (x, y)

midpoint of $PR = \left(\frac{4+x}{2}, \frac{2+y}{2}\right)$

since Q is the midpoint of PR.

$\Rightarrow \left(\frac{4+x}{2}, \frac{2+y}{2}\right) = (8, -3)$

$\Rightarrow \frac{4+x}{2} = 8 \quad , \quad \frac{2+y}{2} = -3$

$4 + x = 16 \quad , \quad 2 + y = -6$

$x = 12 \quad , \quad y = -8$

$\therefore$ R is $(12, -8)$ **Ans.**

(b) (i) (a) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \mathbf{b} - \mathbf{a}$ **Ans.**

(b) $\vec{AC} = \vec{AB} + \vec{BC}$

$= \mathbf{b} - \mathbf{a} + 4\mathbf{a} - \mathbf{b}$

$= 3\mathbf{a}$ **Ans.**

(c) $\vec{OC} = \vec{OA} + \vec{AC}$

$= \mathbf{a} + 3\mathbf{a} = 4\mathbf{a}$

$\Rightarrow \frac{OA}{OC} = \frac{1}{4}$

since ΔOAB and ΔOCD are similar

$\therefore \quad \frac{OB}{OD} = \frac{AB}{CD} = \frac{OA}{OC} = \frac{1}{4}$

$\Rightarrow \vec{CD} = 4\vec{AB}$

$= 4(\mathbf{b} - \mathbf{a})$ **Ans.**

(ii) (a) perimeter of ΔOAB : perimeter of ΔOCD

$=$ 1 : 4 **Ans.**

> Since ΔOAB and ΔOCD are similar, the ratio of their perimeters is the ratio of their corresponding sides.

(b) $\frac{\text{area of } \Delta OAB}{\text{area of } \Delta OCD} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

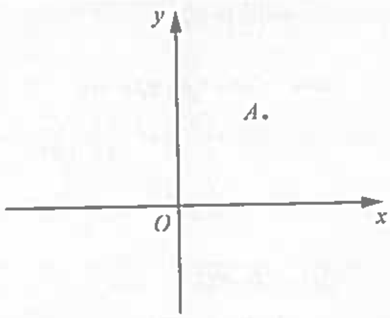
therefore,

area of ΔOAB : area of trapezium $ABDC$

$=$ 1 : 15 **Ans.**

> Note that area of ΔOAB is 1 unit. Area of ΔOGD is 16 units. Therefore area of trapezium $ABDC$ is $16 - 1 = 15$ units.

19 *(N2014 P1 Q16)*



A is the point $(5, 5)$ $\quad \vec{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(a) AB is mapped onto CD by a reflection in the y-axis.

Find $\vec{CD}$. [1]

(b) AB is mapped onto AE by a rotation, centre A, through an angle of 90° clockwise.

Find $\vec{AE}$. [1]

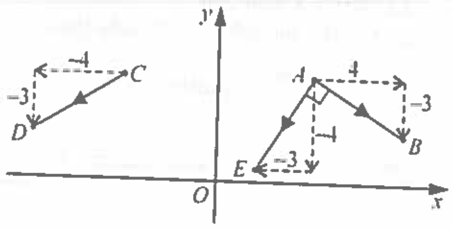
(c) Find $|\vec{AB}|$. [1]

Thinking Process

(a) Draw $\vec{AB}$ on the grid and reflect it in the y-axis.

(c) If $\vec{AB} = \begin{pmatrix} a \\ b \end{pmatrix}$ then $|\vec{AB}| = \sqrt{a^2 + b^2}$

Solution



(a) $\vec{CD} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ **Ans.**

(b) $\vec{AE} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ **Ans.**

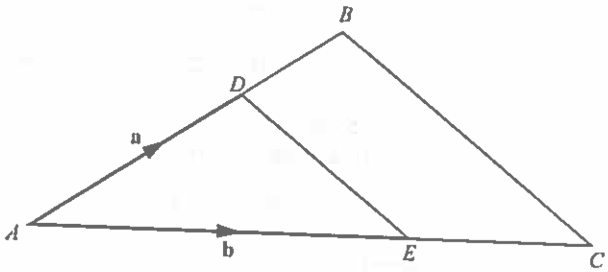
(c) $|\vec{AB}| = \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25} = 5$ **Ans.**

20 *(N2014/P2 Q7a)*

(a)



In the triangle ABC, D divides AB in the ratio $3:2$, and E divides AC in the ratio $3:2$.

$\vec{AD} = \mathbf{a}$ and $\vec{AE} = \mathbf{b}$.

(i) Show, using vectors, that DE is parallel to BC. [3]

(ii) Find the ratio Area of triangle ADE : Area of triangle ABC. [2]

Thinking Process

(a) (i) Using vector subtraction, prove that $\vec{DE} = k\,\vec{BC}$.

Note that $\vec{AB} = \frac{3}{5}\vec{AD}$ and $\vec{AE} = \frac{3}{5}\vec{AC}$.

(ii) Note that ΔADE is similar to ΔABC.

Solution

(a) (i) Given that, $AD:DB = 3:2$

$\Rightarrow \vec{AD}:\vec{AB} = 3:5 \Rightarrow \vec{AD} = \frac{3}{5}\vec{AB}$

also given, $AE:EC = 3:2$

$\Rightarrow \vec{AE}:\vec{AC} = 3:5 \Rightarrow \vec{AE} = \frac{3}{5}\vec{AC}$

now, $\vec{DE} = \vec{AE} - \vec{AD}$

$= \frac{3}{5}\vec{AC} - \frac{3}{5}\vec{AB}$

$= \frac{3}{5}(\vec{AC} - \vec{AB})$

$= \frac{3}{5}\vec{BC}$

since $\vec{DE}$ is $\frac{3}{5}$ times of $\vec{BC}$

∴ DE is parallel to BC **Proved.**

(ii) DE is parallel to BC

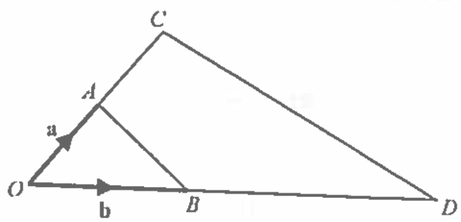
$\Rightarrow$ ΔADE and ΔABC are similar

∴ Area of ΔADE : Area of ΔABC

$3^2 : 5^2$

$9 : 25$ **Ans.**

21 *(J2015 P1 Q25)*



In the diagram, A is the midpoint of OC and B is the point on OD where $OB = \frac{1}{3}OD$.

$\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of $\mathbf{a}$ and $\mathbf{b}$

(i) $\vec{AB}$. [1]

(ii) $\vec{CD}$. [1]

(b) E is the point on CD where $CE : ED = 1 : 2$.

(i) Express $\vec{BE}$, as simply as possible, in terms of a and/or b. [2]

(ii) What special type of quadrilateral is $ABEC$? [1]

Thinking Process

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$

(ii) $\vec{CD} = \vec{OD} - \vec{OC}$

(b) (i) Use $\vec{BE} = \vec{BD} - \vec{DE}$. Note that $\vec{ED} = \frac{2}{3}\vec{CD}$.

(ii) Note that AC and BE are parallel.

Solution

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$

$= \mathbf{b} - \mathbf{a}$ **Ans.**

(ii) Given that, $\vec{OC} = 2\vec{OA}$

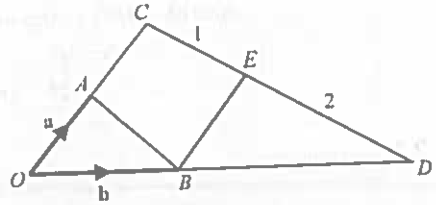
also, $\vec{OB} = \frac{1}{3}\vec{OD} \Rightarrow \vec{OD} = 3\vec{OB}$

$\therefore \vec{CD} = \vec{OD} - \vec{OC}$

$= 3\vec{OB} - 2\vec{OA}$

$= 3\mathbf{b} - 2\mathbf{a}$ **Ans.**

(b) (i)



$\vec{CE} : \vec{ED} = 1 : 2 \Rightarrow \vec{ED} = \frac{2}{3}\vec{CD}$

$\therefore \vec{BE} = \vec{BD} + \vec{DE}$

$= \vec{BD} - \vec{ED}$

$= 2\mathbf{b} - \frac{2}{3}\vec{CD}$

$= 2\mathbf{b} - \frac{2}{3}(3\mathbf{b} - 2\mathbf{a})$

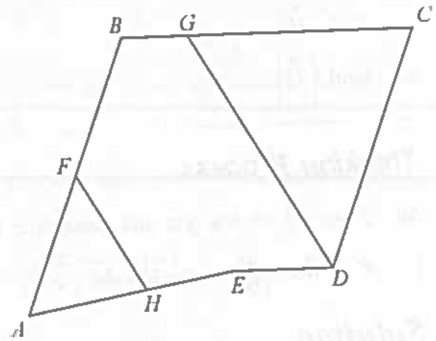
$= 2\mathbf{b} - 2\mathbf{b} + \frac{4}{3}\mathbf{a}$

$= \frac{4}{3}\mathbf{a}$ **Ans.**

(ii) $ABEC$ is a trapezium. **Ans.**

22 (N2015 P2 Q11 a)

(a)



$ABCDE$ is a pentagon.
AFB, AHE and BGC are straight lines.

(i) $\vec{AE} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$. Calculate $|\vec{AE}|$ [1]

(ii) H is the midpoint of AE, and $\vec{FH} = \begin{pmatrix} 2 \\ -3.5 \end{pmatrix}$.

Find $\vec{AF}$. [2]

(iii) G divides BC in the ratio $1 : 2$.

$\vec{BG} = \begin{pmatrix} 2.5 \\ 0 \end{pmatrix}$ and $\vec{CD} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}$.

(a) Find $\vec{GD}$. [1]

(b) Explain why GD is parallel to FH. [1]

(iv) B is the point (3, 10).
Find the coordinates of D. [1]

Thinking Process

(a) (i) If $\vec{AB} = \begin{pmatrix} a \\ b \end{pmatrix}$ then $|\vec{AB}| = \sqrt{a^2 + b^2}$

(ii) Apply $\vec{AF} = \vec{AH} + \vec{HF}$

(iii) (a) Apply $\vec{GD} = \vec{GC} + \vec{CD}$ Use the given ratio to find $\vec{GC}$.

(b) Express GD as kFH.

(iv) To find D write $\vec{BD} = \vec{OD} - \vec{OB}$ and calculate $\vec{OD}$.

Solution

(a) (i) $|\vec{AE}| = \sqrt{(6)^2 + (1)^2}$

$= \sqrt{37}$ units **Ans.**

(ii) $\vec{AF} = \vec{AH} + \vec{HF}$

$= \frac{1}{2}(\vec{AE}) - \vec{FH}$

$= \frac{1}{2}\begin{pmatrix}6\\1\end{pmatrix} - \begin{pmatrix}2\\-3.5\end{pmatrix}$

$= \begin{pmatrix}3\\0.5\end{pmatrix} - \begin{pmatrix}2\\-3.5\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix}$ **Ans.**

(iii) (a) Given that, $\frac{BG}{GC} = \frac{1}{2} \Rightarrow GC = 2BG$

$\vec{GD} = \vec{GC} + \vec{CD}$

$= 2(\vec{BG}) + \vec{CD}$

$= 2\begin{pmatrix}2.5\\0\end{pmatrix} + \begin{pmatrix}-1\\-7\end{pmatrix}$

$= \begin{pmatrix}5\\0\end{pmatrix} + \begin{pmatrix}-1\\-7\end{pmatrix}$

$= \begin{pmatrix}4\\-7\end{pmatrix}$ **Ans.**

(b) $\vec{GD} = \begin{pmatrix}4\\-7\end{pmatrix}$

$= 2\begin{pmatrix}2\\-3.5\end{pmatrix} = 2\vec{FH}$

since $\vec{GD} = 2\vec{FH}$

∴ GD is parallel to FH.

(iv) $\vec{BD} = \vec{BG} + \vec{GD}$

$\vec{OD} - \vec{OB} = \begin{pmatrix}2.5\\0\end{pmatrix} + \begin{pmatrix}4\\-7\end{pmatrix}$

$\vec{OD} - \begin{pmatrix}3\\10\end{pmatrix} = \begin{pmatrix}6.5\\-7\end{pmatrix}$

$\vec{OD} = \begin{pmatrix}6.5\\-7\end{pmatrix} + \begin{pmatrix}3\\10\end{pmatrix}$

$= \begin{pmatrix}9.5\\3\end{pmatrix}$

∴ coordinates of D are (9.5, 3) **Ans.**

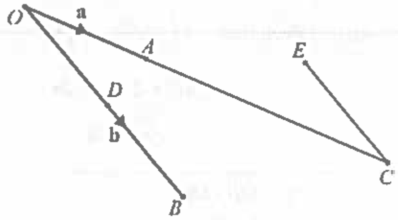
23 *(J2016 P2 Q2)*

(a) $\vec{JK} = \begin{pmatrix}2\\5\end{pmatrix}$ $\vec{KL} = \begin{pmatrix}4\\-2\end{pmatrix}$ $\vec{LM} = \begin{pmatrix}-1\\3\end{pmatrix}$

(i) Find $\vec{JM}$. [1]

(ii) Calculate $|\vec{KL}|$. [2]

(b)



In the diagram, $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.
C is the point such that OAC is a straight line and $AC = 2OA$.
D is the midpoint of OB.

E is the point such that $\vec{EC} = \vec{OD}$.

(i) Express, as simply as possible, in terms of a and b.

(a) $\vec{AD}$. [1]

(b) $\vec{EB}$. [1]

(ii) Find $|\vec{EB}| : |\vec{AD}|$. [1]

Thinking Process

(a) (i) $\vec{JM} = \vec{JK} + \vec{KL} + \vec{LM}$

(ii) if $\vec{AB} = \begin{pmatrix}a\\b\end{pmatrix}$ then $|\vec{AB}| = \sqrt{a^2 + b^2}$

(b) (i) (a) $\vec{AD} = \vec{OD} - \vec{OA}$

(b) Apply, $\vec{EB} = \vec{EC} + \vec{CB}$

(ii) To find the ratio h calculate the magnitudes of $\vec{EB}$ and $\vec{AD}$.

Solution

(a) (i) $\vec{JM} = \vec{JK} + \vec{KL} + \vec{LM}$

$= \begin{pmatrix}2\\5\end{pmatrix} + \begin{pmatrix}4\\-2\end{pmatrix} + \begin{pmatrix}-1\\3\end{pmatrix}$

$= \begin{pmatrix}5\\6\end{pmatrix}$ **Ans.**

(ii) $|\vec{KL}| = \sqrt{(4)^2 + (-2)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$ units **Ans.**

(b) (i) (a) $\vec{AD} = \vec{OD} - \vec{OA}$

$= \frac{1}{2}\mathbf{b} - \mathbf{a}$ **Ans.**

(b) Given that, $\vec{EC} = \vec{OD} = \frac{1}{2}b$

$\vec{AC} = 2\vec{OA} = 2a$

$\therefore \quad \vec{OC} = 3a$

$\vec{CB} = \vec{OB} - \vec{OC}$

$= b - 3a$

now, $\vec{EB} = \vec{EC} + \vec{CB}$

$= \frac{1}{2}b + b - 3a$

$= \frac{3}{2}b - 3a$ **Ans.**

(ii) $\left|\vec{EB}\right| = \sqrt{(-3)^2 + (\frac{3}{2})^2}$

$= \sqrt{9 + \frac{9}{4}} = \sqrt{\frac{45}{4}}$

$\left|\vec{AD}\right| = \sqrt{(-1)^2 + (\frac{1}{2})^2}$

$= \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}}$

now, $\left|\vec{EB}\right| : \left|\vec{AD}\right|$

$\sqrt{\frac{45}{4}} : \sqrt{\frac{5}{4}}$

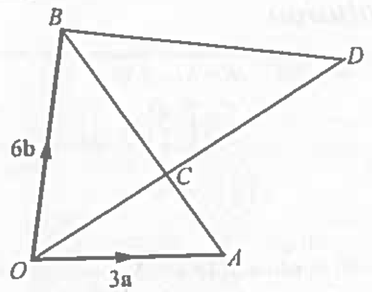
$\sqrt{45} : \sqrt{5}$

$\sqrt{9} : \sqrt{1}$

$3 : 1$ **Ans.**

24 *(N2016 P2 Q10 a)*

(a)



ACB and *OCD* are straight lines.
$AC : CB = 1 : 2$.

$\vec{OA} = 3a$ and $\vec{OB} = 6b$.

(i) Express $\vec{AB}$ in terms of a and b. [1]

(ii) Express $\vec{AC}$ in terms of a and b. [1]

(iii) $\vec{BD} = 5a - b$.
Showing your working clearly, find $OC : CD$. [4]

Thinking Process

(a) (ii) $\vec{AC} = \frac{1}{3}\vec{AB}$

(iii) To find the ratio ~~of~~ express $\vec{OC}$ and $\vec{CD}$ in terms of a and b.

Solution with TEACHER'S COMMENTS

(a) (i) $\vec{AB} = \vec{OB} - \vec{OA}$

$= 6b - 3a = 3(2b - a)$ **Ans.**

(ii) Given that, $AC : CB = 1 : 2$

$\Rightarrow \quad \vec{AC} : \vec{AB} = 1 : 3$

$\Rightarrow \quad \vec{AC} = \frac{1}{3}\vec{AB}$

$= \frac{1}{3} \times 3(2b - a) = 2b - a$ **Ans.**

(iii) $\vec{CB} = \frac{2}{3}\vec{AB}$

$= \frac{2}{3} \times 3(2b - a) = 2(2b - a)$

$\vec{CD} = \vec{CB} + \vec{BD}$

$= 2(2b - a) + 5a - b$

$= 4b - 2a + 5a - b$

$= 3a + 3b$

$= 3(a + b)$

$\therefore \quad \frac{1}{3}\vec{CD} = a + b$

$\vec{OC} = \vec{OA} + \vec{AC}$

$= 3a + 2b - a$

$= 2a + 2b$

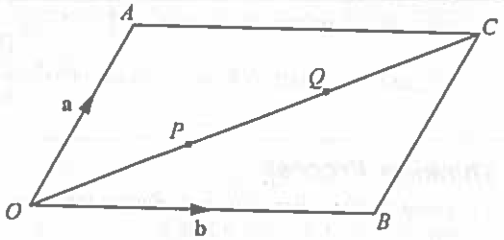
$= 2(a + b)$

$= 2(\frac{1}{3}\vec{CD}) = \frac{2}{3}\vec{CD}$

$\Rightarrow \quad \vec{OC} = \frac{2}{3}\vec{CD}$

$\therefore \quad OC : CD = 2 : 3$ **Ans.**

25 (J2017 P1 Q21)



$OACB$ is a parallelogram.

$\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.

P and Q are points on OC such that $OP = PQ = QC$.

(a) Express, as simply as possible, in terms of a and b,

(i) $\vec{OP}$. [1]

(ii) $\vec{BP}$. [1]

(b) Show that triangles OAQ and CBP are congruent. [2]

Thinking Process

(a) (i) Note that $\vec{OP} = \frac{1}{3}\vec{OC}$.

(ii) Use $\vec{BP} = \vec{OP} - \vec{OB}$.

(b) Observe that triangles are congruent by SAS property.

Solution

(a) (i) $\vec{OP} = \frac{1}{3}\vec{OC}$

$= \frac{1}{3}(\vec{OA} + \vec{AC})$

$= \frac{1}{3}(\mathbf{a} + \mathbf{b})$ **Ans.**

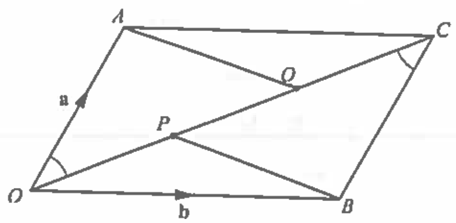
(ii) $\vec{BP} = \vec{OP} - \vec{OB}$

$= \frac{1}{3}(\mathbf{a} + \mathbf{b}) - \mathbf{b}$

$= \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \mathbf{b}$

$= \frac{1}{3}\mathbf{a} - \frac{2}{3}\mathbf{b} = \frac{1}{3}(\mathbf{a} - 2\mathbf{b})$ **Ans.**

(b)



$OACB$ is a parallelogram.

$\therefore\ OA = CB$

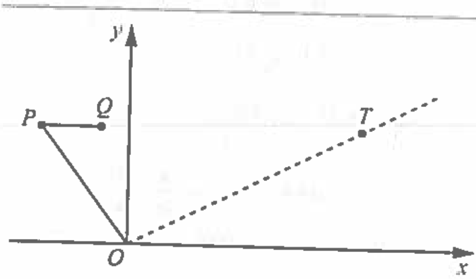
given that, $OP = PQ = QC$

$\therefore\ OQ = CP$

also, $A\hat{O}Q = B\hat{C}P$ (alternate angles)

$\therefore\ \Delta OAQ \equiv \Delta CBP$ (SAS) **Shown.**

26 (N2017 P1 Q27)



In the diagram, $\vec{OP} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ $\vec{PQ} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

(a) Find $|\vec{OP}| + |\vec{PQ}|$. [3]

(b) T is the point where $\vec{PT} = k\vec{PQ}$.

(i) Express $\vec{OT}$ as a column vector in terms of k. [1]

(ii) M is the point such that O, T and M lie on a straight line and $\vec{OM} = \begin{pmatrix} 24 \\ 16 \end{pmatrix}$.

Find the value of k. [2]

Thinking Process

(a) Find the magnitude of OP and PQ by using formula: $\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2 + b^2}$.

(b) (i) Apply vector addition: $\vec{OT} = \vec{OP} + \vec{PT}$

(ii) Write $\vec{OT} = h\vec{OM}$. Find the value of h and then subsequently find k.

Solution

(a) $|\vec{OP}| + |\vec{PQ}|$

$= \sqrt{(-3)^2 + (4)^2} + \sqrt{(2)^2 + (0)^2}$

$= \sqrt{25} + \sqrt{4}$

$= 5 + 2 = 7$ **Ans.**

(b) (i) $\vec{OT} = \vec{OP} + \vec{PT}$

$= \vec{OP} + k\,\vec{PQ}$

$= \begin{pmatrix}-3\\4\end{pmatrix} + k\begin{pmatrix}2\\0\end{pmatrix}$

$= \begin{pmatrix}-3+2k\\4\end{pmatrix}$ **Ans.**

(ii) $\vec{OT} = h\vec{OM}$

$\begin{pmatrix}-3+2k\\4\end{pmatrix} = h\begin{pmatrix}24\\16\end{pmatrix}$

$\begin{pmatrix}-3+2k\\4\end{pmatrix} = \begin{pmatrix}24h\\16h\end{pmatrix}$

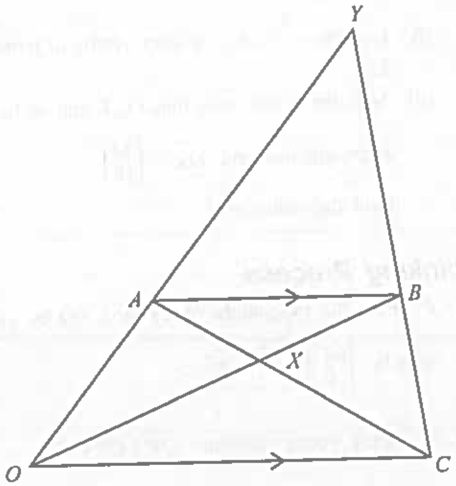
$\Rightarrow\ 4 = 16h\ \Rightarrow\ h = \frac{4}{16} = \frac{1}{4}$

also. $-3+2k = 24h$

$\Rightarrow\ -3+2k = 24(\frac{1}{4})$

$2k = 6+3\ \Rightarrow\ k = \frac{9}{2}$ **Ans.**

27 *(J2018 P2 Q8)*



OYC is a triangle.
A is a point on OY and B is a point on CY.
AB is parallel to OC. AC and OB intersect at X.

(a) Prove that triangle ABX is similar to triangle COX. Give a reason for each statement you make. [3]

(b) $\vec{OA} = 3\mathbf{a}$ and $\vec{OC} = 6\mathbf{c}$ and $CB : BY = 1:2$.
Find, as simply as possible, in terms of a and/or c

(i) $\vec{AB}$. [1]

(ii) $\vec{CY}$. [2]

(c) Find, in its simplest form, the ratio

(i) $OX : XB$. [2]

(ii) area of triangle COX : area of triangle ABX. [1]

(iii) area of triangle AYB : area of trapezium $OABC$. [1]

Thinking Process

(a) Prove that two angles of one triangle are equal to two angles of another triangle.

(b) (i) To find $\vec{AB}$ ∮ consider that ΔAYB is similar to ΔOYC.

(ii) Use $\vec{CY} = \vec{CO} + \vec{OY}$. Note that $\frac{OA}{AY} = \frac{1}{2}$.

(c) (i) To find the ratio ∮ use the rule of similarity i.e. ratio of corresponding lengths is equal.

(ii) Apply concept of area of similar triangles. $\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2$

Solution

(a) $A\hat{B}X = C\hat{O}X$ (alt ∠s. $AB \parallel OC$)

$B\hat{A}X = O\hat{C}X$ (alt ∠s, $AB \parallel DC$)

since two corresponding angles are equal.

∴ ΔABX is similar to ΔCOX. **Proved.**

(b) (i) As $AB \parallel OC$

∴ ΔAYB is similar to ΔOYC

Given that, $\frac{CB}{BY} = \frac{1}{2}$

$\Rightarrow \frac{YB}{YC} = \frac{2}{3}\ \Rightarrow\ YB = \frac{2}{3}YC$

∴ $\vec{AB} = \frac{2}{3}\vec{OC}$

$= \frac{2}{3}(6\mathbf{c}) = 4\mathbf{c}$ **Ans.**

(ii) $\frac{OA}{AY} = \frac{1}{2}\ \Rightarrow\ AY = 2OA$

∴ $\vec{AY} = 2(3\mathbf{a}) = 6\mathbf{a}$

$\Rightarrow\ \vec{OY} = \vec{OA} + \vec{AY}$

$= 3\mathbf{a} + 6\mathbf{a} = 9\mathbf{a}$

now, $\vec{CY} = \vec{CO} + \vec{OY}$

$= -6\mathbf{c} + 9\mathbf{a} = 3(3\mathbf{a} - 2\mathbf{c})$ **Ans.**

(c) (i) ΔABX is similar to ΔCOX

$\Rightarrow \frac{\vec{AB}}{\vec{OC}} = \frac{4\mathbf{c}}{6\mathbf{c}} = \frac{2}{3}$

∴ $\frac{XB}{OX} = \frac{2}{3}\ \Rightarrow\ OX : XB = 3:2$ **Ans.**

(ii) $\dfrac{\text{Area of } \Delta COX}{\text{Area of } \Delta ABX} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

$\therefore$ area of ΔCOX : area of ΔABX
= 9 : 4 **Ans.**

(iii) $\dfrac{\text{Area of } \Delta AYB}{\text{Area of } \Delta OYC} = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$

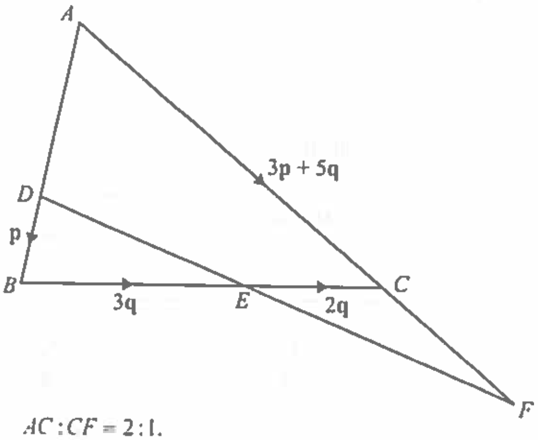
therefore,

area of ΔAYB : area of trapezium $OABC$
= 4 : (9 − 4)
= 4 : 5 **Ans.**

> Note that area of ΔAYB is 4 units. Area of ΔOYC is 9 units. Therefore area of trapezium $OABC$ is $9-4=5$ units.

28 *(N2018 P1 Q25)*

In the diagram, ADB and ACF are straight lines. BC intersects DF at E.



$AC : CF = 2 : 1$.

$\vec{DB} = \mathbf{p}$, $\vec{BE} = 3\mathbf{q}$, $\vec{EC} = 2\mathbf{q}$, and $\vec{AC} = 3\mathbf{p} + 5\mathbf{q}$.

(a) Express $\vec{AB}$ in terms of p. [1]

(b) Express $\vec{CF}$ in terms of p and/or q. [1]

(c) Express $\vec{EF}$ in terms of p and/or q. [1]

(d) $\vec{EF} = k\vec{DE}$. Find k. [2]

Thinking Process

(a) $\vec{AB} = \vec{AC} + \vec{CB}$

(b) Note that $\vec{CF} = \dfrac{1}{2}\vec{AC}$

(c) $\vec{EF} = \vec{EC} + \vec{CF}$

(d) Write $\vec{DE}$ as $\vec{DB} + \vec{BE}$ ✐ Compare $\vec{EF}$ and $\vec{DE}$.

Solution

(a) $\vec{AB} = \vec{AC} + \vec{CB}$
$= 3\mathbf{p} + 5\mathbf{q} + (-2\mathbf{q} - 3\mathbf{q})$
$= 3\mathbf{p} + 5\mathbf{q} - 5\mathbf{q}$
$= 3\mathbf{p}$ **Ans.**

(b) Given that, $AC : CF = 2 : 1$

$\Rightarrow \dfrac{AC}{CF} = \dfrac{2}{1} \Rightarrow CF = \dfrac{1}{2}AC$

$\therefore \vec{CF} = \dfrac{1}{2}\vec{AC}$
$= \dfrac{1}{2}(3\mathbf{p} + 5\mathbf{q})$ **Ans.**

(c) $\vec{EF} = \vec{EC} + \vec{CF}$
$= 2\mathbf{q} + \dfrac{1}{2}(3\mathbf{p} + 5\mathbf{q})$
$= \dfrac{4\mathbf{q} + 3\mathbf{p} + 5\mathbf{q}}{2}$
$= \dfrac{3\mathbf{p} + 9\mathbf{q}}{2} = \dfrac{3}{2}(\mathbf{p} + 3\mathbf{q})$ **Ans.**

(d) $\vec{EF} = k\vec{DE}$

$\Rightarrow \vec{EF} = k(\vec{DB} + \vec{BE})$

$\Rightarrow \dfrac{3}{2}(\mathbf{p} + 3\mathbf{q}) = k(\mathbf{p} + 3\mathbf{q})$

$\Rightarrow k = \dfrac{3}{2}$ **Ans.**

Topic 19

Statistics

1 (J2009 P2 Q10)

Answer THE WHOLE of this question on a sheet of graph paper.

The waiting times of 50 people at a supermarket checkout were recorded.
The results are summarised in the table below.

Time (t minutes)	$1 < t \le 3$	$3 < t \le 4$	$4 < t \le 5$	$5 < t \le 7$	$7 < t \le 9$	$9 < t \le 12$
Number of people	4	10	8	14	8	6

(a) Using a scale of 1 cm to represent 1 minute, draw a horizontal axis for waiting times between 0 and 12 minutes.
Using a scale of 1 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 10 units.
On your axes, draw a histogram to illustrate the distribution of waiting times. [3]

(b) In which class does the upper quartile lie? [1]

(c) Calculate an estimate of the mean waiting time. [3]

(d) One person is chosen, at random, from the 50 people.
Write down the probability that this person waited
(i) less than 1 minute, [1]
(ii) more than 5 minutes. [1]

(e) A second person is now chosen, at random, from the remaining 49 people.
Expressing each answer as a fraction in its lowest terms, calculate the probability that
(i) both people waited more than 5 minutes, [1]
(ii) one person waited more than 5 minutes and the other waited 5 minutes or less. [2]

Thinking Process

(a) Find the width of each interval and hence the frequency density of each interval.

(b) Calculate 75% of total frequency and look for the interval where it lies.

(c) Mean $= \frac{\sum fx}{\sum f}$ & calculate the midpoint of each interval.

(d) (i) Note that no one waited for less than 1 min.
(ii) Add up the number of people who waited for more than 5 minutes and calculate the probability.

(e) (ii) $P(>5 \text{ min}) \times P(<5 \text{ min}) \times 2$

Solution

(a)

Time (t minutes)	Number of people	Width	Frequency density
$1 < t \le 3$	4	2	$\frac{4}{2} = 2$
$3 < t \le 4$	10	1	$\frac{10}{1} = 10$
$4 < t \le 5$	8	1	$\frac{8}{1} = 8$
$5 < t \le 7$	14	2	$\frac{14}{2} = 7$
$7 < t \le 9$	8	2	$\frac{8}{2} = 4$
$9 < t \le 12$	6	3	$\frac{6}{3} = 2$

Refer to histogram on the next page

(b) $\frac{75}{100} \times 50 = 37.5$

$\therefore$ upper quartile lies in class $= 7 < t \le 9$ **Ans.**

(c)

Time (t minutes)	Midpoint, x	Number of people, f	Product, fx
$1 < t \le 3$	2	4	8
$3 < t \le 4$	3.5	10	35
$4 < t \le 5$	4.5	8	36
$5 < t \le 7$	6	14	84
$7 < t \le 9$	8	8	64
$9 < t \le 12$	10.5	6	63
		$\sum f = 50$	$\sum fx = 290$

Mean time $= \frac{\sum fx}{\sum f} = \frac{290}{50} = 5.8$ minutes **Ans.**

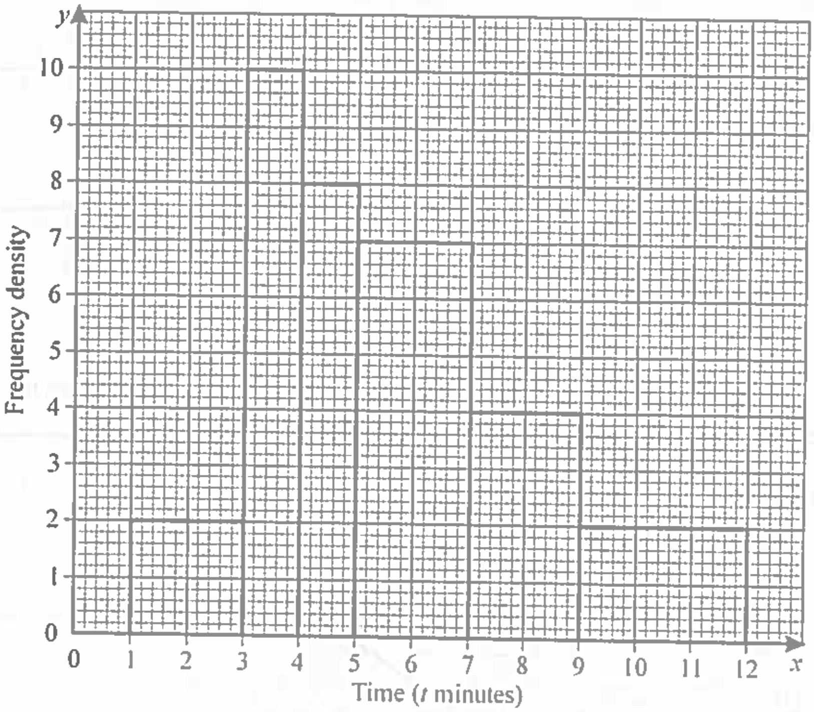
(d) (i) $P(<\text{one minute}) = 0$ **Ans.**

(ii) People waited for more than 5 minutes
$= 14 + 8 + 6 = 28$

$P(\text{more than five minutes}) = \frac{28}{50} = \frac{14}{25}$ **Ans.**

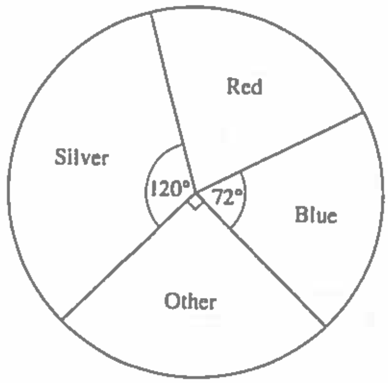
(e) (i) $P(\text{both waited} > 5 \text{ minutes}) = \frac{28}{50} \times \frac{27}{49}$
$= \frac{54}{175}$ **Ans.**

(ii) $P(\text{one waited} > 5 \text{ minutes, other} < 5 \text{ minutes})$
$= \left(\frac{28}{50} \times \frac{22}{49}\right) \times 2$
$= \frac{88}{175}$ **Ans.**



2 *(N2009 P1 Q8)*

The colours of the cars which passed a house were noted. The results are shown in the pie chart below.



There were 12 blue cars. How many cars

(a) passed the house, [1]

(b) were red? [2]

Thinking Process

(a) Note that 360° represents total number of cars.
(b) Find the angle that represents red cars.

Solution

(a) 72° represents —— 12 cars

360° represents —— $\frac{12}{72} \times 360 = 60$ cars

$\therefore$ 60 cars passed the house. **Ans.**

(b) Angle representing red cars

$= 360° - 120° - 90° - 72°$ (∠s around a pt.)

$= 78°$

360° represents —— 60 cars

78° represents —— $\frac{60}{360} \times 78 = 13$ cars

$\therefore$ number of red cars = 13 **Ans.**

3 *(N2009 P1 Q11)*

The table below shows the number of pets owned by 20 families.

Number of pets	0	1	2	3	4	5	6	7
Number of families	2	5	3	2	4	1	1	2

Find

(a) the modal number of pets, [1]

(b) the mean number of pets. [2]

Thinking Process

(a) Look for the number of pets which have the highest frequency.
(b) To find mean divide the total number of pets by total frequency.

Solution

(a) Modal number = 1 **Ans.**

(b) Mean

$$= \frac{0\times 2+1\times 5+2\times 3+3\times 2+4\times 4+5\times 1+6\times 1+7\times 2}{20}$$

$$= \frac{5+6+6+16+5+6+14}{20}$$

$$= \frac{58}{20}$$

$$= \frac{29}{10} = 2.9 \quad \textbf{Ans.}$$

4 *(J2009/P1/Q9)*

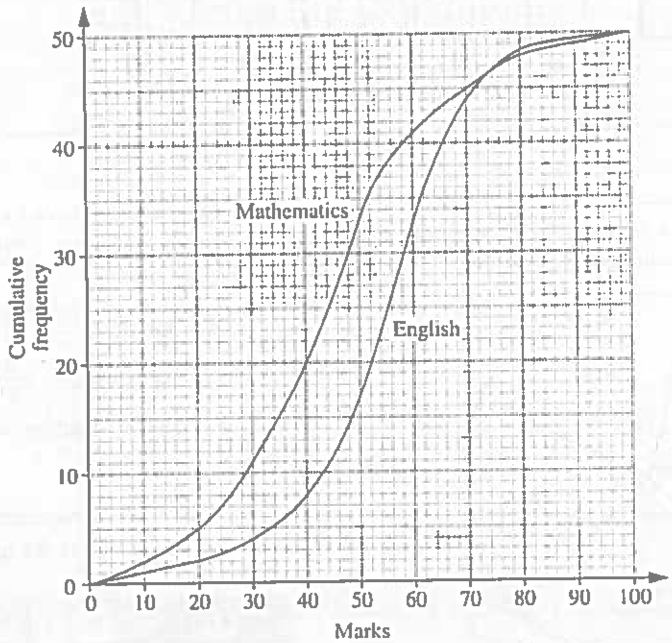
Fifty students each took a Mathematics and an English test. The distributions of their marks are shown in the cumulative frequency graph.

(a) Use the graph

(i) to estimate the median mark in the English test, [1]

(ii) to estimate the 20th percentile mark in the Mathematics test. [1]

(b) State, with a reason, which test the students found more difficult. [1]



Thinking Process

(a) (i) To find median mark ✎ find the marks in the English test when cumulative frequency is 25.

(ii) ✎ Find the marks in Mathematics test when the cumulative freq. is 20% of 50, i.e. at 10.

(b) To find which test was more difficult ✎ consider the marks obtained in both tests for any given percentile.

Solution

(a) (i) median mark = 55 **Ans.**

(ii) 20% of 50

$$= \frac{20}{100}\times 50 = 10$$

∴ when cumulative freq. = 10

marks in Mathematics test = 29 **Ans.**

(b) Mathematics test was more difficult.

Median mark in Mathematics test is 44. Whereas median mark in English test is 55.

5 *(N2009/P2 Q10)*

Answer the whole of this question on a sheet of graph paper.

80 electric light bulbs of brand A were tested to find how long each bulb lasted. The results are summarised in the table below.

Time (t hours)	$t \le 50$	$50 < t \le 100$	$100 < t \le 150$	$150 < t \le 200$	$200 < t \le 250$	$250 < t \le 300$	$300 < t \le 350$	$350 < t \le 400$
Number of bulbs	1	2	6	34	26	8	2	1

(a) Copy and complete the following cumulative frequency table.

Time (t hours)	$t \le 50$	$t \le 100$	$t \le 150$	$t \le 200$	$t \le 250$	$t \le 300$	$t \le 350$	$t \le 400$
Number of bulbs	1	3						80

[1]

(b) Using a horizontal scale of 2 cm to represent 50 hours and a vertical scale of 2 cm to represent 10 bulbs, draw a smooth cumulative frequency curve for these brand A bulbs. [3]

(c) Use your graph to estimate
(i) the median, [1]
(ii) the 10th percentile. [1]

(d) 80 brand B bulbs were also tested and a report on the test gave the following information.
3 bulbs lasted 50 hours or less.
No bulbs lasted more than 350 hours.
The median time was 250 hours.
The upper quartile was 275 hours.
The interquartile range was 75 hours.
On the same axes, draw a smooth cumulative frequency curve for the brand B bulbs. [3]

(e) Use your graph to estimate the number of bulbs that lasted 260 hours or less
(i) for brand A, [1]
(ii) for brand B. [1]

(f) Which brand of bulb is more likely to last longer than 250 hours? Justify your answer. [1]

Thinking Process

(a) Add up the number of bulbs in the first table to fill in the blanks in the second table.
(b) using scales given, draw the cumulative curve.
(c) (i) Find 50% of the total frequency. Mark the value on the graph and read for x-values.
(ii) Find 10% of the total frequency. Mark the value on the graph and read for x-value.
(d) Plot the points using the information given, and draw a cumulative frequency curve on the same graph.
(e) At 260 hrs, read the corresponding y-values of both brands.
(f) Consider the chance that a bulb will last beyond a certain number of hours.

Solution

(a)

Time (t hours)	$t \le 50$	$t \le 100$	$t \le 150$	$t \le 200$	$t \le 250$	$t \le 300$	$t \le 350$	$t \le 400$
Number of bulbs	1	3	9	43	69	77	79	80

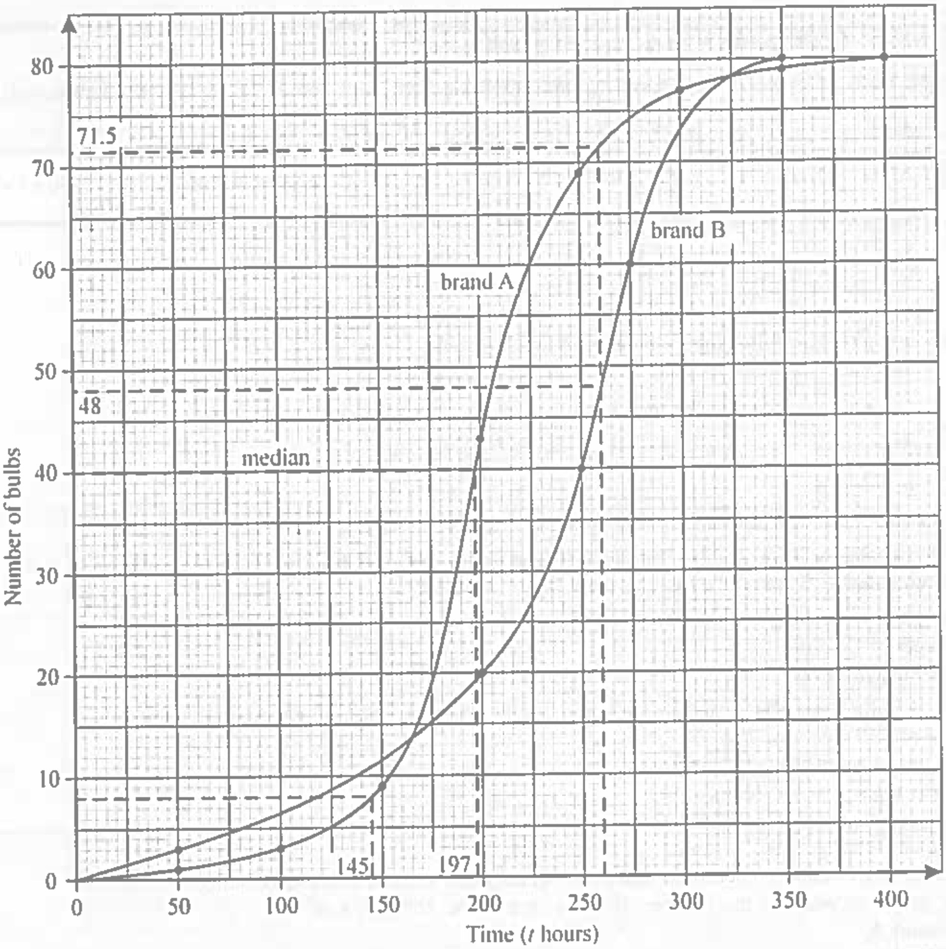
(b) Refer to graph on the next page.

(c) (i) median = 197 hours. **Ans.**
(ii) 10th percentile = 145 hours. Ans.

(d) Refer to graph on the next page.

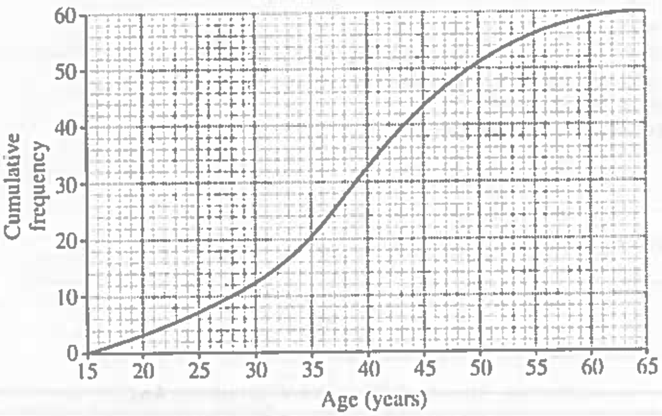
(e) (i) Brand A: $71.5 \approx 72$ bulbs. **Ans.**
(ii) Brand B: 48 bulbs. **Ans.**

(f) Brand B.
From graph, 40 bulbs of brand B will last longer than 250 hours. Whereas only 11 bulbs of brand A will last longer then 250 hours.



6 *(J2010/P1 Q20)*

The graph shows the cumulative frequency curve for the ages of 60 employees.



Use the graph to estimate

(a) the median, [1]

(b) the interquartile range, [2]

(c) the number of employees aged over 50. [1]

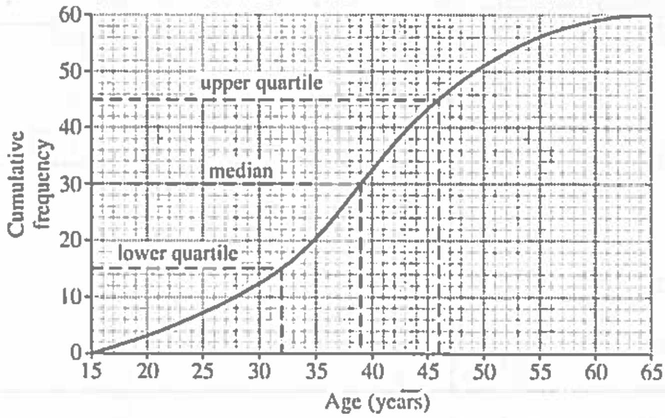
Thinking Process

(a) To find median ⇨ find the age when cumulative frequency = 30.

(b) Find the difference between the upper quartile and the lower quartile.

(c) To find the number of employees aged over 50 ⇨ find the number of employees whose age is equal to 50.

Solution



(a) Median = 39 years. **Ans.**

(b) Upper quartile = 46 years

Lower quartile = 32 years

∴ Interquartile range = 46 − 32 = 14 years. **Ans.**

(c) From graph, the number of employees whose age is equal to 50 years = 51.

∴ number of employees aged over 50 = 60 − 51 = 9 **Ans.**

7 *(J2010 P2 Q11)*

Answer THE WHOLE of this question on a sheet of graph paper.

(a) The time taken by 140 children to run 200 metres was recorded.

The results are summarised in the table below.

Time (t seconds)	$22 \le t < 24$	$24 \le t < 26$	$26 \le t < 31$	$31 \le t < 36$	$36 \le t < 46$
Frequency	12	18	42	28	40

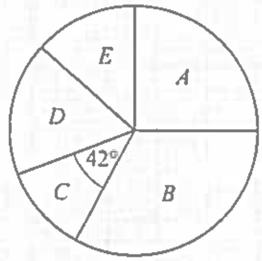
(i) Using a scale of 1 cm to represent 2 seconds, draw a horizontal axis for time from 22 seconds to 46 seconds.
Using a scale of 1 cm to represent 1 unit, draw a vertical axis for frequency density from 0 to 9 units.
On your axes, draw a histogram to represent the information in the table. [3]

(ii) Estimate the number of children who took less than 25 seconds to run 200 metres. [1]

(iii) One child was chosen at random.
Calculate the probability that the time taken by this child was less than 36 seconds.
Express your answer as a fraction in its lowest terms. [1]

(iv) Out of the 30 children who took less than 26 seconds, two were chosen at random.
Calculate the probability that they both took less than 24 seconds. [2]

(b) Some boys were put into five groups, A, B, C, D and E, based on the times they took to run 100 metres.
The pie chart shows the proportion of boys in each group.



Group A	contains $\frac{1}{4}$ of the boys.
Group B	contains 35% of the boys.
Group C	is represented by a sector with an angle of 42°.
Group D	contains 9 boys.

(i) Find the fraction of boys in group C.
Give your answer in its lowest terms. [1]

(ii) Given that the number of boys in group B is 21, find the total number of boys who ran the 100 metres. [2]

(iii) Calculate the number of boys in group E. [2]

Thinking Process

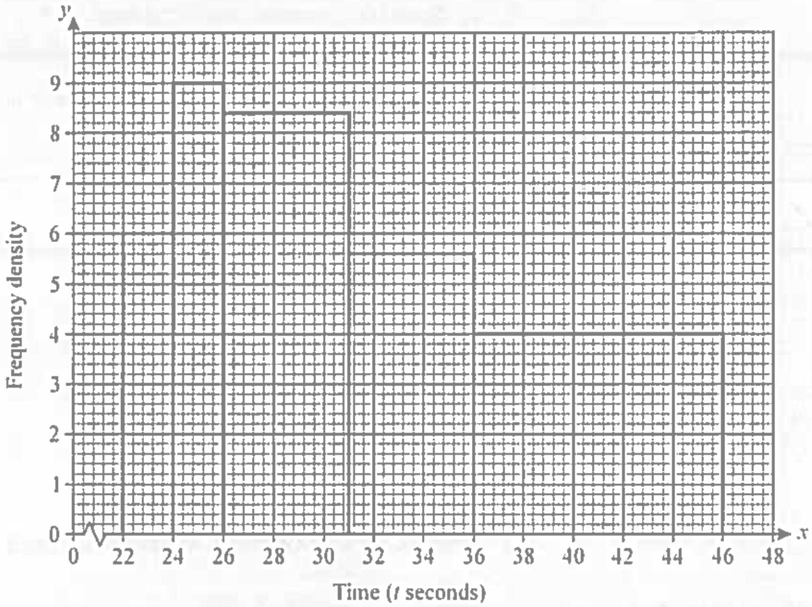
(a) (i) To draw a histogram ⇒ find the frequency density for each range.
(ii) Estimate by referring to the histogram drawn.
(iii) Express number of children who took less than 36 seconds as a fraction of total time.
(iv) ⇒ 2 x P(less than 24 seconds). Note that 2 children are selected out of 30.

(b) (i) ⇒ Express 42° as a fraction of 360°.
(ii) ⇒ 35% — 21. Find 100%.
(iii) To find the number of boys in group E ⇒ calculate the number of boys in each group.

Solution

(a) (i)

Time (t seconds)	Frequency	Width	Frequency density
$22 \le t < 24$	12	2	$\frac{12}{2} = 6$
$24 \le t < 26$	18	2	$\frac{18}{2} = 9$
$26 \le t < 31$	42	5	$\frac{42}{5} = 8.4$
$31 \le t < 36$	28	5	$\frac{28}{5} = 5.6$
$36 \le t < 46$	40	10	$\frac{40}{10} = 4$



(ii) From histogram, we see that frequency density from 24 to 25 seconds = 9

$\Rightarrow$ frequency = f.d × width

$= 9 \times 1 = 9$

$\therefore$ number of children who took less than 25 seconds $\approx 12 + 9 = 21$ **Ans.**

(iii) From the table, number of children who took less than 36 seconds $= 12 + 18 + 42 + 28$

$= 100$

$\therefore$ $P(\text{time} < 36 \text{ seconds}) = \dfrac{100}{140} = \dfrac{5}{7}$ **Ans.**

(iv) $P(\text{time} < 24 \text{ seconds}) = \dfrac{12}{30} \times \dfrac{11}{29}$

$= \dfrac{22}{145}$ **Ans.**

(b) (i) Fraction of boys in group $C = \dfrac{42}{360}$

$= \dfrac{7}{60}$ **Ans.**

(ii) 35% — 21

100% — $\dfrac{21}{35} \times 100 = 60$

$\therefore$ total no. of boys = 60 **Ans.**

(iii) No. of boys in group $A = \dfrac{1}{4} \times 60 = 15$

No. of boys in group $B = 21$

No. of boys in group $C = \dfrac{7}{60} \times 60 = 7$

No. of boys in group $D = 9$

$\therefore$ No. of boys in group E

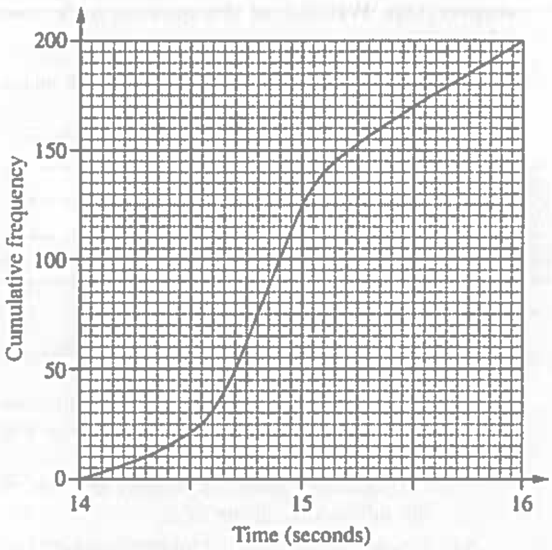
$= 60 - (15 + 21 + 7 + 9)$

$= 60 - 52 = 8$ **Ans.**

8 *(N2010/P1/Q18)*

The times taken for 200 children to run 100m were recorded.

The cumulative frequency curve summarises the results.



Use the curve to find

(a) the lower quartile, [1]

(b) the number of children who took at least 15.5 seconds. [2]

Thinking Process

(a) From graph, find 25% of 200.

(b) Find the number of children who took more than 15.5 seconds.

Solution

(a) 25% of 200

$= \frac{25}{100} \times 200 = 50$

$\therefore$ From graph, lower quartile = 14.7 seconds. **Ans.**

(b) From graph, at 15.5 seconds, number of children = 170

$\therefore$ number of children who took at least 15.5 seconds = 200 − 170 = 30 **Ans.**

9 *(N2010 P2 Q5)*

The table shows the distribution of the masses of 90 apples.

Mass (m grams)	$60 < m \le 80$	$80 < m \le 90$	$90 < m \le 95$	$95 < m \le 100$	$100 < m \le 110$	$110 < m \le 130$
Frequency	10	16	20	21	22	1

(a) In which interval does the median lie? [1]

(b) Calculate an estimate of the mean. [3]

(c) A histogram is drawn to represent this information.

(i) Calculate the frequency density of the interval $90 < m \le 95$. [1]

(ii) The rectangle representing the apples with masses in the interval $80 < m \le 90$ has width 2 cm and height 4 cm.
Find the width and height of the rectangle representing the apples with masses in the interval $90 < m \le 95$. [2]

Thinking Process

(a) Add the frequencies in each class in ascending (or descending) order of class. Look for the interval where the cumulative frequency is 45.

(b) Use mean $= \frac{\sum fx}{\sum f}$ Compute the mid-point (x) of each interval.

(c) (i) Frequency density $= \frac{\text{frequency}}{\text{class width}}$

(ii) Find the frequency densities of respective intervals and use proportionality concept to find the width and height of the said interval.

Solution

(a) Median lie in interval: $90 < m \le 95$ **Ans.**

(b)

Mass (m grams)	Midpoint, x	Freq. f	fx
$60 < m \le 80$	$\frac{60+80}{2} = 70$	10	700
$80 < m \le 90$	$\frac{80+90}{2} = 85$	16	1360
$90 < m \le 95$	$\frac{90+95}{2} = 92.5$	20	1850
$95 < m \le 100$	$\frac{95+100}{2} = 97.5$	21	2047.5
$100 < m \le 110$	$\frac{100+110}{2} = 105$	22	2310
$110 < m \le 130$	$\frac{110+130}{2} = 120$	1	120
		$\sum f = 90$	$\sum fx = 8387.5$

$$\text{mean} = \frac{\sum fx}{\sum f} = \frac{8387.5}{90}$$

$= 93.1944 \approx 93.2$ grams **Ans**

(c) (i) Frequency density $= \frac{20}{5} = 4$ **Ans**

(ii)

Interval	Frequency	Class width	Freq. density
$80 < m \le 90$	16	10	$\frac{16}{10} = 1.6$
$90 < m \le 95$	20	5	$\frac{20}{5} = 4$

Now, class width of 10 represents rectangle width of 2 cm

$\therefore$ class width of 5 will represent rectangle width of 1 cm **Ans.**

freq. density of 1.6 represents rectangle height of 4 cm

$\therefore$ freq. density of 4 will represent a rectangle height of $\frac{4}{1.6} \times 4 = 10$ cm **Ans.**

10 *(J2011 P2 Q12)*

The time taken by each of 320 students taking a Physics test was recorded.
The following table shows a distribution of their times.

Time (m minutes)	$60 < m \le 70$	$70 < m \le 80$	$80 < m \le 90$	$90 < m \le 100$	$100 < m \le 110$	$110 < m \le 120$
Frequency	24	92	104	68	24	8

(a) Complete the cumulative frequency table below.

Time (m minutes)	$m \le 60$	$m \le 70$	$m \le 80$	$m \le 90$	$m \le 100$	$m \le 110$	$m \le 120$
Cumulative frequency	0	24	116				

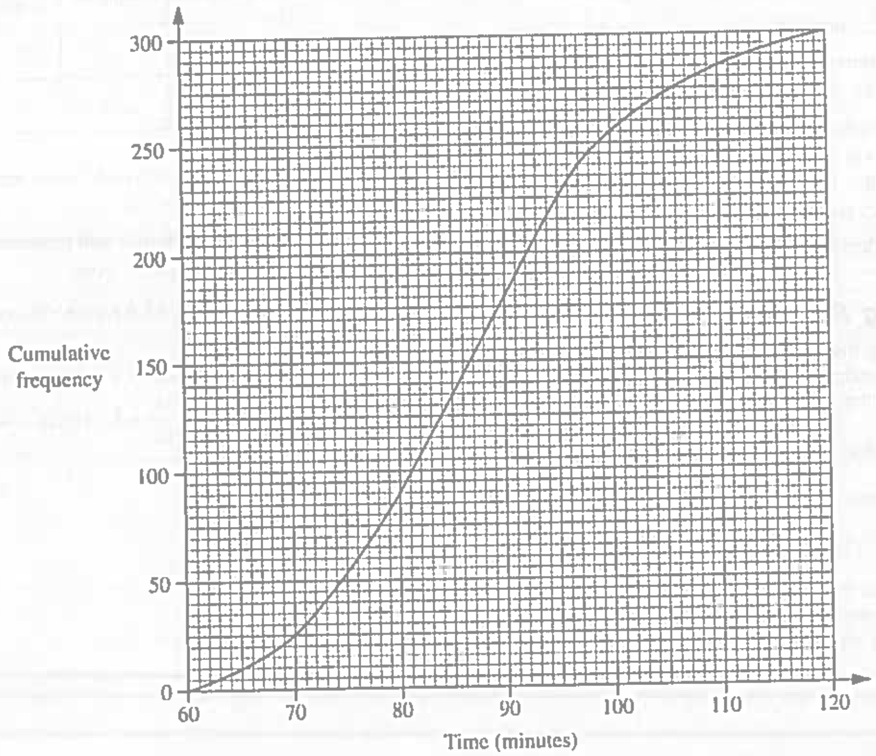
[1]

(b) For this part of the question use the graph paper.

(i) Using a scale of 2 cm to represent 10 minutes, draw a horizontal m-axis for $60 \le m \le 120$.
Using a scale of 1 cm to represent 20 students, draw a vertical axis for cumulative frequencies from 0 to 320.
On your axes, draw a smooth cumulative frequency curve to illustrate the information. [3]

(ii) Use your graph to estimate
(a) the median, [1]
(b) the interquartile range, [2]
(c) the percentage of students who took at least 95 minutes to complete the test. [2]

(iii) A group of 300 students of similar ability took an equivalent test the previous year.
The following graph shows a distribution of their times.
(a) Find the 20th percentile. [1]
(b) Find the percentage of students who took at least 95 minutes to complete the test. [1]
(c) Hence make a comparison between the two tests. [1]



Thinking Process

(a) (i) To complete the table ⇒ sum up the frequencies of the corresponding groups.

(b) (i) Using the given scales, draw the cumulative frequency curve.

(ii) (a) ⇒ Find 50% of total frequency.

(b) Find 25% and 75% of total frequency. Mark the two values on the graph. Read the corresponding x-values and find their difference.

(c) To fnd the percentage ⇒ Find the number of students who took 95 minutes or more to complete the test.

(iii) (a) Find 20% of 300.

(b) Find the number of students who took 95 minutes or more.

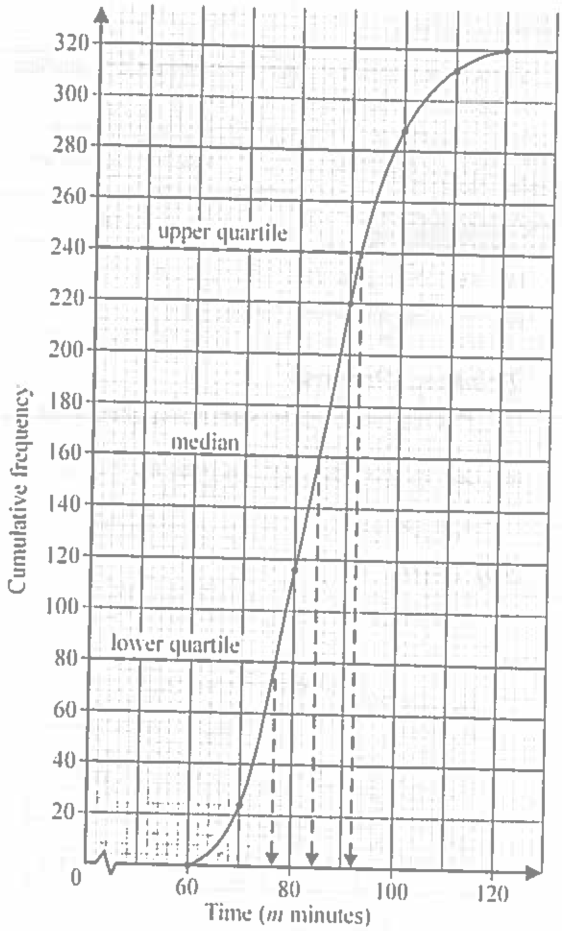
(c) Compare the medians of both graphs.

Solution

(a)

Time (m minutes)	$m \le 60$	$m \le 70$	$m \le 80$	$m \le 90$	$m \le 100$	$m \le 110$	$m \le 120$
Cumulative frequency	0	24	116	220	288	312	320

(b) (i) Refer to graph on the right.



(ii) (a) Median = 84.5 minutes **Ans.**

(b) Upper quartile = 92 minutes
Lower quartile = 76.5 minutes
Interquartile range = 92 − 76.5
= 15.5 minutes

(c) Number of students who took 95 minutes or more = 320 − 264 = 56

$\therefore$ percentage $= \dfrac{56}{320} \times 100 = 17.5\%$ **Ans.**

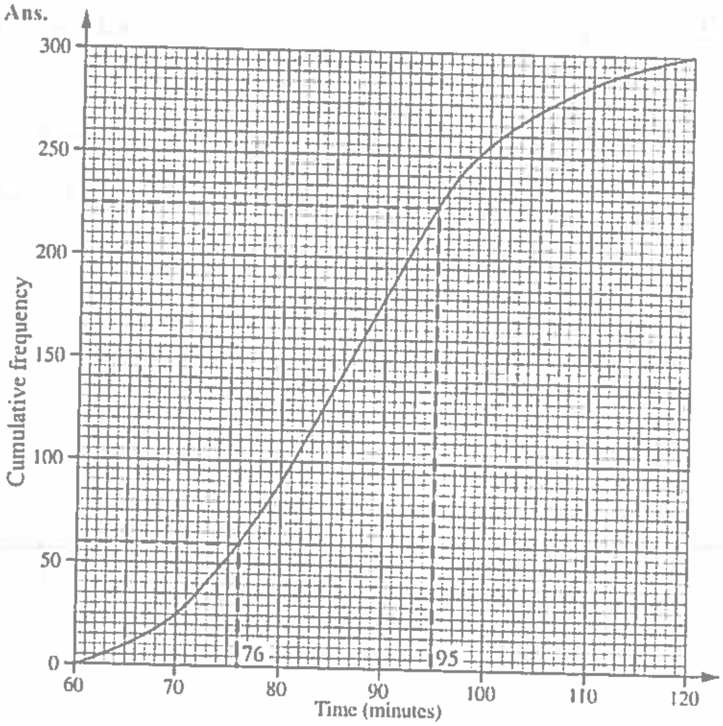
(iii) (a) 20% of 320 $= \dfrac{20}{100} \times 300 = 60$

$\therefore$ from graph, 20th percentile = 76 minutes **Ans.**

(b) Number of students who took 95 minutes or more = 300 − 225 = 75

$\therefore$ percentage $= \dfrac{75}{300} \times 100 = 25\%$ **Ans.**

(c) Comparing the median marks of both graphs, we see that the students in the previous year took longer time.
Hence the test in the previous year was perhaps harder than the present years test.



11 (J2011 P1 Q20)

The table shows the distribution of the number of complete lengths swum by a group of swimmers.

Number of complete lengths (n)	$0 < n \le 20$	$20 < n \le 40$	$40 < n \le 60$	$60 < n \le 80$
Frequency	5	20	10	5

(a) Find the modal class. [1]

(b) Calculate an estimate of the mean. [3]

Thinking Process

(a) Look for the class with highest frequency.

(b) Use mean $= \frac{\sum fx}{\sum f}$ calculate the mid-point of each interval.

Solution

(a) Modal class $= 20 < n \le 40$ **Ans.**

(b)

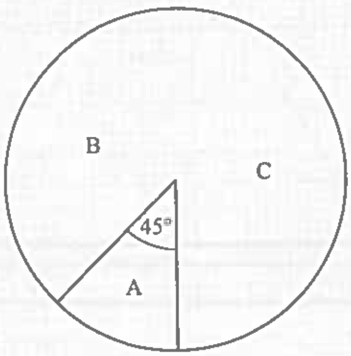
Lengths (n)	Midpoint, x	Frequency, f	fx
$0 < n \le 20$	10	5	50
$20 < n \le 40$	30	20	600
$40 < n \le 60$	50	10	500
$60 < n \le 80$	70	5	350
		$\sum f = 40$	$\sum fx = 1500$

Mean $= \frac{\sum fx}{\sum f} = \frac{1500}{40} = 37.5$ **Ans.**

12 (N2011 P1 Q15)

In a survey, some people were asked which of three songs, labelled A, B and C, they liked best.
The diagram shows part of a pie chart illustrating the results.
The angle of the sector that represents the people who liked C best is 168°.

(a) Complete the pie chart. [1]



(b) Expressing your answer in its lowest terms, find the fraction of people in the survey who liked C best. [1]

(c) Given that 30 people liked A best, calculate the number of people in the survey. [1]

Thinking Process

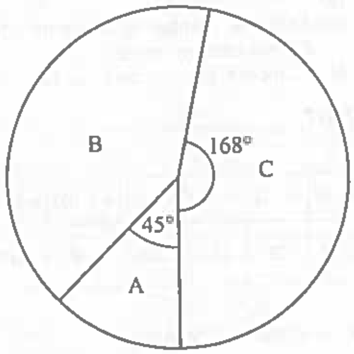
(a) Draw an angle of 168° at the center of the circle with a protractor.

(b) Divide 168° by 360°.

(c) 45° represent 30 people. find what 360° represents.

Solution

(a)



(b) $\frac{168°}{360°} = \frac{7}{15}$ **Ans.**

(c) 45° —— 30 people

360° —— $(\frac{30}{45} \times 360)$ people

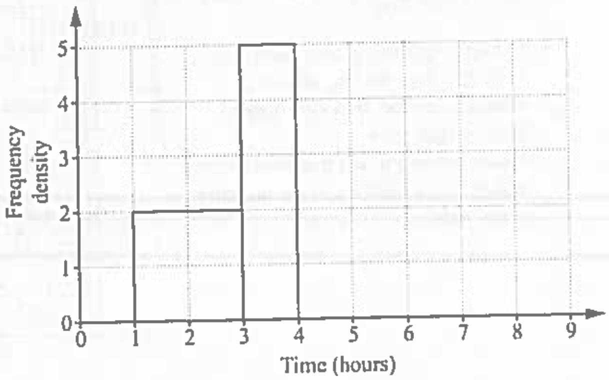
$= 240$ people **Ans.**

13 (N2011 P1 Q16)

The distribution of the lengths of time taken by an engineer to repair some washing machines is given in the table.

Time (t hours)	$1 < t \le 3$	$3 < t \le 4$	$4 < t \le 5$	$5 < t \le 8$
Frequency	k	5	4	3

The histogram represents some of this information.



(a) Find k. [1]

(b) Complete the histogram. [2]

Thinking Process

(a) Use, Frequency = frequency density × class width

(b) To complete the histogram find the frequency density for each range.

Solution

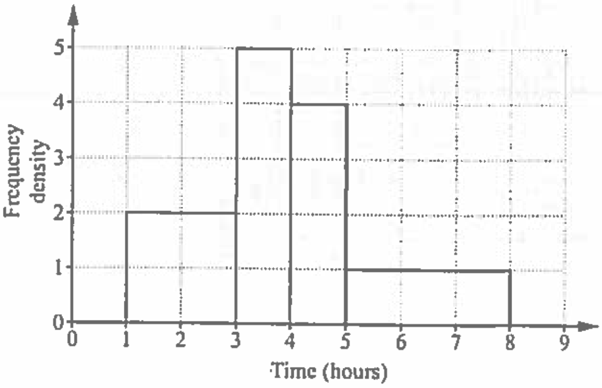
(a) Frequency = frequency density × class width

$k = 2 \times 2$

$= 4$ **Ans.**

(b) For $4 < t \le 5$, freq. density $= \frac{4}{1} = 4$

For $5 < t \le 8$, freq. density $= \frac{3}{3} = 1$



14 *(N2011/P1/Q24)*

The table shows the number of goals scored by 40 football teams during one weekend.

Number of goals	0	1	2	3	4	5	6
Number of teams	16	6	6	6	4	0	2

Find

(a) the mode, [1]

(b) the median, [1]

(c) the mean. [2]

Thinking Process

(a) Mode is the data with the highest frequency.

(b) To find median find the number of goals in the middle position.

(c) Mean $= \frac{\text{total number of goals}}{\text{total number of teams}}$

Solution

(a) Mode = 0 **Ans.**

(b) Median = 1 **Ans.**

(c) Mean

$= \frac{0\times16+1\times6+2\times6+3\times6+4\times4+5\times0+6\times2}{40}$

$= \frac{0+6+12+18+16+0+12}{40}$

$= \frac{64}{40}$

$= \frac{8}{5} = 1\frac{3}{5}$ or 1.6 **Ans.**

15 *(N2011/P2 Q11 a)*

(a) A sports club has 120 members.
The cumulative frequency table for their ages is shown below.

Age (x years)	$x \le 5$	$x \le 15$	$x \le 25$	$x \le 35$	$x \le 45$	$x \le 55$	$x \le 65$
Cumulative frequency	0	12	30	60	96	114	120

(i) On the grid, draw a horizontal x-axis for $0 \le x \le 70$, using a scale of 2 cm to represent 10 years and a vertical axis from 0 to 120, using a scale of 2 cm to represent 20 members.
On your axes draw a smooth cumulative frequency curve to illustrate the information in the table. [3]

(ii) Find the upper quartile age. [1]

(iii) Find the interquartile range of the ages. [1]

(iv) Members who are not more than 15, and members who are over 50, pay reduced fees. Use your graph to find an estimate of the number of members who pay reduced fees. [1]

Thinking Process

(a) (i) Plot the curve with the values given in the table.

(ii) Find 75% of 120

(iii) IQR = UQ − LQ

(iv) Find the number of members who are less than 15 years old and those who are more than 50 years old.

Solution

(a) (i) Refer to graph on the next page.

(ii) Upper quartile = 43 years. **Ans.**

(iii) Interquartile range

= upper quartile − lower quartile

$= 43 - 25 = 18$ years **Ans.**

(iv) From graph,

Number of members below 15 years = 12

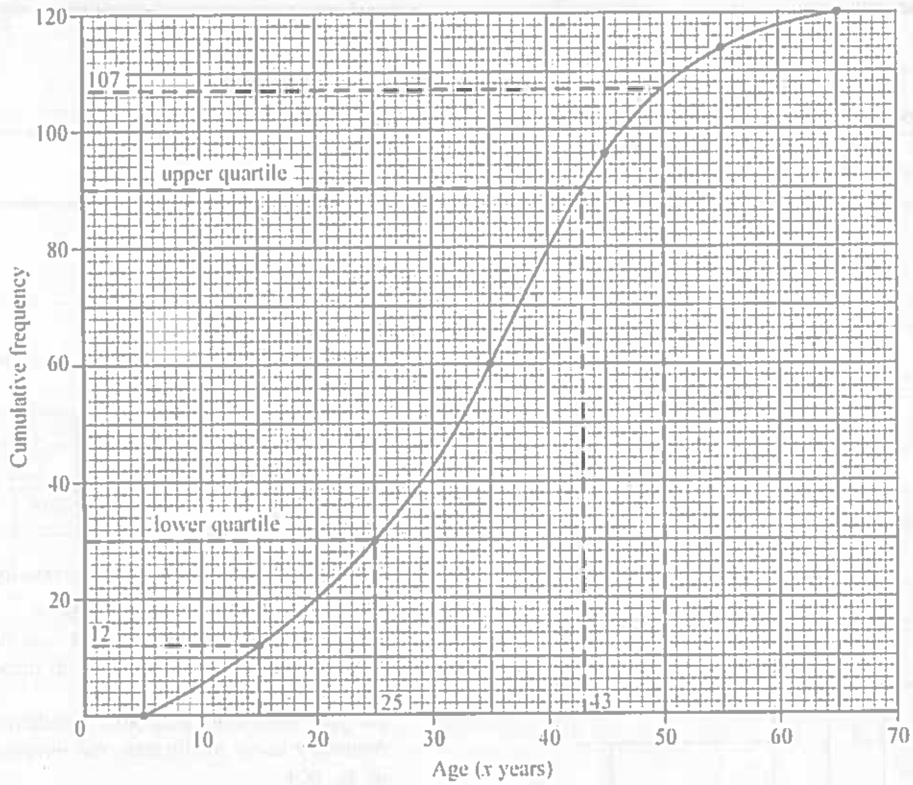
Number of members above 50 years

$= 120 - 107 = 13$

∴ number of members who pay reduced fee

$= 12 + 13 = 25$ **Ans.**

Graph for part (a) (i)

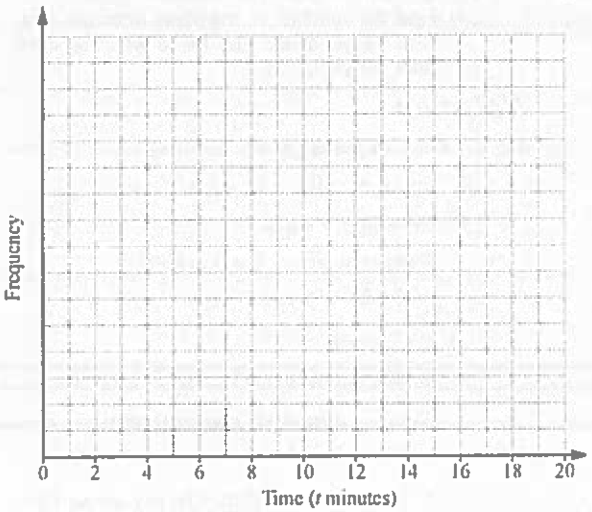


16 *(J2012 P1 Q23)*

The table summarises the times, in minutes, taken by a group of people to complete a puzzle.

Time (t minutes)	$0 < t \le 4$	$4 < t \le 8$	$8 < t \le 12$	$12 < t \le 16$	$16 < t \le 20$
Frequency	4	8	7	4	2

(a) On the grid draw a frequency polygon to represent this information.



[2]

(b) Write down the modal class. [1]

(c) How many people took more than 8 minutes to complete the puzzle? [1]

(d) Imran says:

'The longest time to complete the puzzle was 20 minutes.'

Explain why he may not be correct. [1]

Thinking Process

(a) Draw a frequency polygon by plotting each frequency against the mid-value of the class interval.

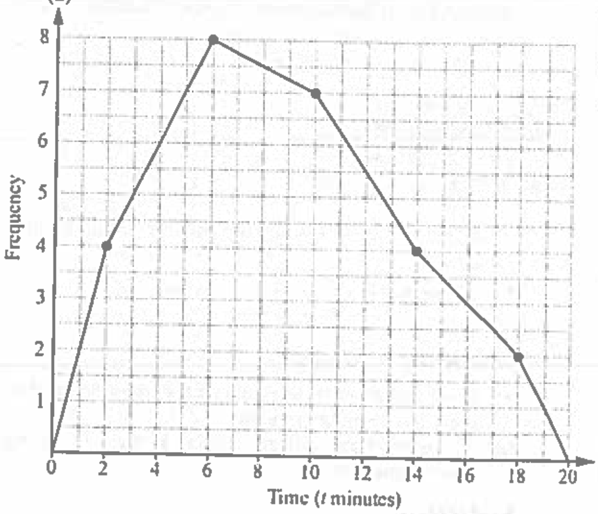
(b) Look for the class with highest frequency.

(c) Use the given table to find the number of people who took more than 8 minutes.

(d) Note that the people in group $16 < t \le 20$ can take any time between 16 to 20 minutes to complete the puzzle.

Solution

(a)



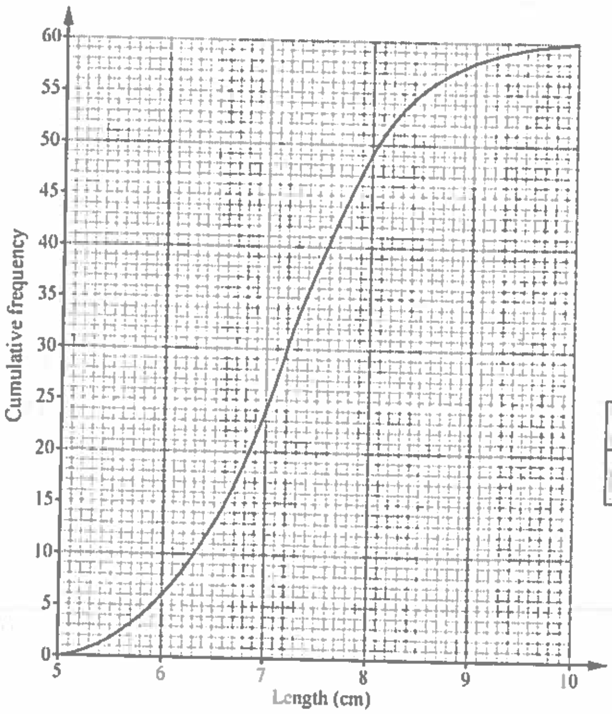
(b) Modal class = $4 < t \le 8$ **Ans.**

(c) Number of people who took more than 8 minutes
$= 7 + 4 + 2 = 13$ **Ans.**

(d) Imran may not be correct as the people in the longest time group $16 < t \le 20$ might take less than 20 minutes to complete the puzzle.

17 *(J2012 P2 Q5)*

(a) The cumulative frequency graph shows the distribution of the lengths of 60 leaves.



(i) Complete the table to show the distribution of the lengths of the leaves.

Length (l cm)	$5 < l \le 6$	$6 < l \le 7$	$7 < l \le 8$	$8 < l \le 9$	$9 < l \le 10$
Frequency	6	18			2

[1]

(ii) Use the graph to estimate the median. [1]

(iii) Use the graph to estimate the interquartile range. [2]

(iv) One of these leaves is chosen at random. Estimate the probability that it has a length of **more than** 7.5 cm. [2]

(b) The distribution of the widths of these leaves is shown in the table below.

Width (w cm)	$3 < w \le 4$	$4 < w \le 5$	$5 < w \le 6$	$6 < w \le 7$	$7 < w \le 8$	$8 < w \le 9$
Frequency	4	15	20	13	5	3

(i) Calculate an estimate of the mean width. [3]

(ii) Calculate the percentage of leaves with a width of more than 6 cm. [2]

Thinking Process

(a) (i) To complete the table ✐ use the relationship between frequency and cumulative frequency.

(ii) ✐ Find 50% of total frequency and read the corresponding value for the length.

(iii) ✐ Read 75% of frequency and 25% of frequency. Subtract.

(iv) From graph, find the number of leaves that have length more than 7.5 cm. Express them as a fraction of total leaves.

(b) (i) Mean $= \dfrac{\sum f x}{\sum f}$ ✐ use the mid-class width from each category in the computation.

(ii) Express the number of leaves that have widths greater than 6 cm as a percentage of total frequency.

Solution

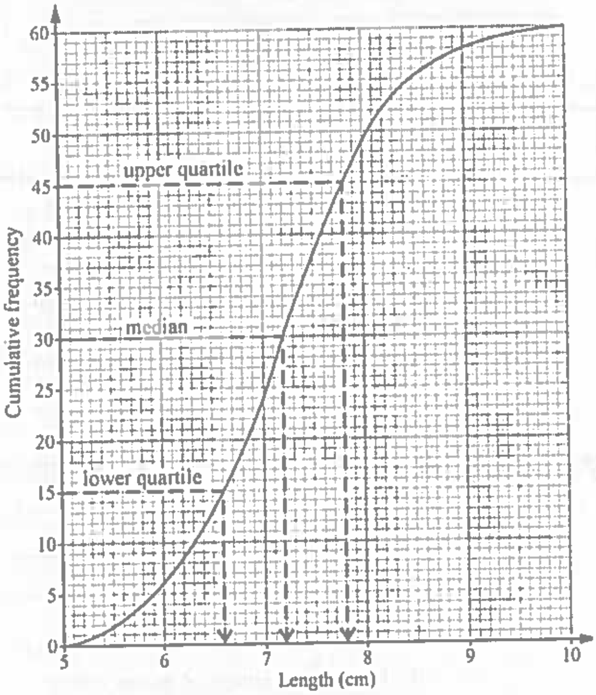
(a) (i) From graph, the cumulative frequencies are:

Length (l cm)	$5 < l \le 6$	$6 < l \le 7$	$7 < l \le 8$	$8 < l \le 9$	$9 < l \le 10$
Cumulative frequency	6	24	49	58	60

$\therefore$ for class $7 < l \le 8$, frequency $= 49 - 24$
$= 25$ **Ans.**

for class $8 < l \le 9$, frequency $= 58 - 49$
$= 9$ **Ans.**

(ii) Median length = 7.2 cm **Ans.**



(iii) Interquartile range
= upper quartile – lower quartile
$= 7.8 - 6.6 = 1.2$ cm **Ans.**

(iv) Number of leaves with length of more than 7.5 cm $= 60 - 38 = 22$

$P(\text{length} > 7.5 \text{ cm}) = \frac{22}{60} = \frac{11}{30}$ **Ans.**

(b) (i)

Width (w cm)	Midpoint (x)	Frequency (f)	fx
$3 < w \le 4$	3.5	4	14
$4 < w \le 5$	4.5	15	67.5
$5 < w \le 6$	5.5	20	110
$6 < w \le 7$	6.5	13	84.5
$7 < w \le 8$	7.5	5	37.5
$8 < w \le 9$	8.5	3	25.5
		$\sum f = 60$	$\sum fx = 339$

Mean width $= \frac{\sum fx}{\sum f}$

$= \frac{339}{60} = 5.65$ cm **Ans.**

(ii) Number of leaves with a width of more than 6 cm $= 13 + 5 + 3 = 21$

$\frac{21}{60} \times 100 = 35\%$ **Ans.**

18 *(N2012/P1/Q9)*

The number of goals scored by some football teams during one weekend was recorded.
The table shows the results.

Number of goals scored	0	1	2	3	4
Number of teams	x	1	5	4	2

(a) If the mode is 0, find the smallest possible value of x. [1]

(b) If the median is 1, find the value of x. [1]

Thinking Process

(a) Since mode is 0, the value of x must be greater then the highest frequency of x.

(b) Remember that the median is the data in the middle position.

Solution

(a) Smallest possible value of $x = 6$ **Ans.**

(b) $x = 11$ **Ans.**

19 *(N2012/P2/Q6)*

The heights of 150 children are measured.
The results are summarised in the table.

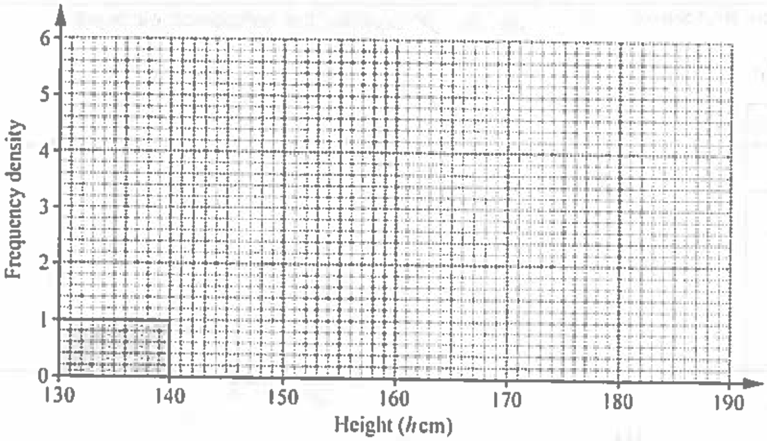
Height (h cm)	$130 < h \le 140$	$140 < h \le 150$	$150 < h \le 155$	$155 < h \le 160$	$160 < h \le 170$	$170 < h \le 190$
Frequency	10	30	20	30	35	25

(a) Calculate an estimate of the mean height. [3]

(b) (i) One child is chosen at random.
Find the probability that this child has a height greater than 160 cm. [1]

(ii) Two children are chosen at random without replacement.
Find the probability that the height of one child is greater than 160 cm and the height of the other is 150 cm or less. [2]

(c) Complete the histogram to represent the information in the table.



[3]

Thinking Process

(a) Mean = $\frac{\sum fx}{\sum f}$

(b) (i) Express the number of children with height greater than 160 cm as a fraction of total number of children.

(ii) 2 × P(more than 160 cm) × P(150 cm or less).

(c) Note that the distribution is of unequal widths. Find the width of each interval and hence frequency density of each interval.

Solution

(a)

height (h cm)	Midpoint (x)	Frequency (f)	fx
$130 \le h < 140$	135	10	1350
$140 \le h < 150$	145	30	4350
$150 \le h < 155$	152.5	20	3050
$155 \le h < 160$	157.5	30	4725
$160 \le h < 170$	165	35	5775
$170 \le h < 190$	180	25	4500
		$\sum f = 150$	$\sum fx = 23750$

Mean height $= \frac{\sum fx}{\sum f}$

$= \frac{23750}{150} = 158.33$

≈ 158 cm **Ans.**

(b) (i) No. of children with height greater than 160 cm = 35 + 25 = 60

$P(\text{ht. of child} > 160 \text{ cm})$

$= \frac{60}{150} = \frac{2}{5}$ **Ans.**

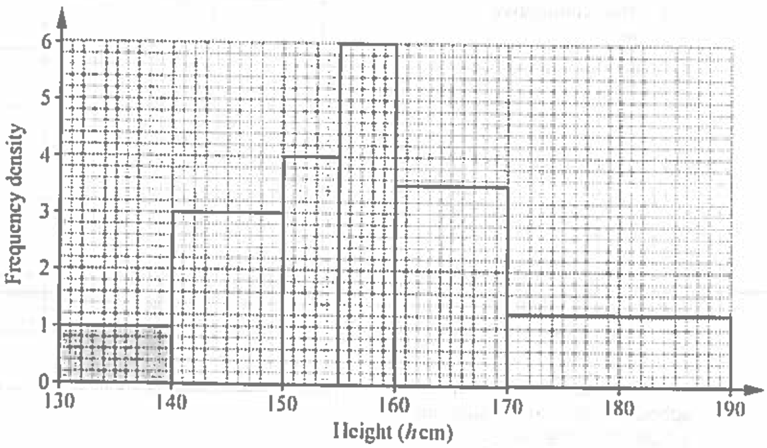
(ii) No. of children with height 150 cm or less $= 10 + 30 = 40$

P(ht. of one child is > 160 cm and ht. of other is 150 cm or less) $= \left(\frac{60}{150} \times \frac{40}{149}\right) + \left(\frac{40}{150} \times \frac{60}{149}\right)$

$= \frac{32}{149}$ **Ans.**

(c)

Height (h cm)	Width	Frequency	Frequency density
$130 \le h < 140$	10	10	$\frac{10}{10} = 1$
$140 \le h < 150$	10	30	$\frac{30}{10} = 3$
$150 \le h < 155$	5	20	$\frac{20}{5} = 4$
$155 \le h < 160$	5	30	$\frac{30}{5} = 6$
$160 \le h < 170$	10	35	$\frac{35}{10} = 3.5$
$170 \le h < 190$	20	25	$\frac{25}{20} = 1.25$



20 *(J2013/P1/Q19)*

(a) Keith records the number of letters he receives each day for 20 days.
His results are shown in the table.

Number of letters	Frequency
0	4
1	6
2	3
3	2
4	1
5	4

(i) Write down the mode. [1]

(ii) Work out the mean. [2]

(b) Over the same 20 days, Emma received a mean of 1.7 letters each day.
How many letters did Emma receive altogether? [1]

Thinking Process

(a) (i) Look for the number of letters with highest frequency.

(ii) Mean = $\frac{\text{total number of letters}}{\text{total frequency}}$.

(b) Total number of letters = mean × total frequency.

Solution

(a) (i) Mode = 1 **Ans.**

(ii) Mean

$$= \frac{(4\times0)+(6\times1)+(3\times2)+(2\times3)+(1\times4)+(4\times5)}{20}$$

$$= \frac{0+6+6+6+4+20}{20}$$

$$= \frac{42}{20}$$

$$= \frac{21}{10} = 2.1 \quad \textbf{Ans.}$$

(b) Number of letters = mean × total frequency

$= 1.7 \times 20$

$= 34$ **Ans.**

21 *(J2013/P1/Q21)*

A group of 80 students took a Physics test.
This table shows the distribution of their marks.

Mark (m)	$0 < m \le 10$	$10 < m \le 20$	$20 < m \le 30$	$30 < m \le 40$	$40 < m \le 50$	$50 < m \le 60$
Frequency	4	12	14	22	18	10

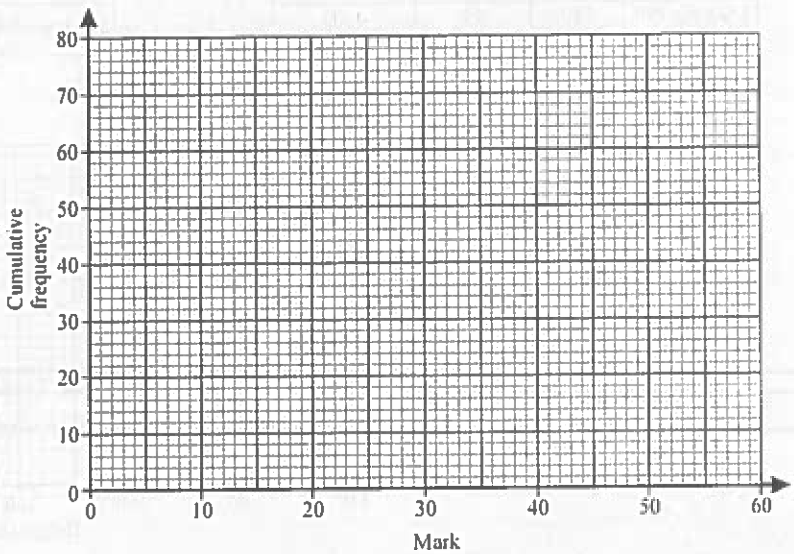
(a) Complete the cumulative frequency table.

Mark (m)	$m \le 10$	$m \le 20$	$m \le 30$	$m \le 40$	$m \le 50$	$m \le 60$
Cumulative frequency						

[1]

(b) Draw a cumulative frequency curve for this information. [2]

(c) The pass mark for the test is 45.
Use your cumulative frequency curve to estimate the number of students who passed. [2]



Thinking Process

(a) Use frequency table to complete the cumulative frequency table.

(b) Using the cumulative frequency table, plot the points and draw a smooth curve.

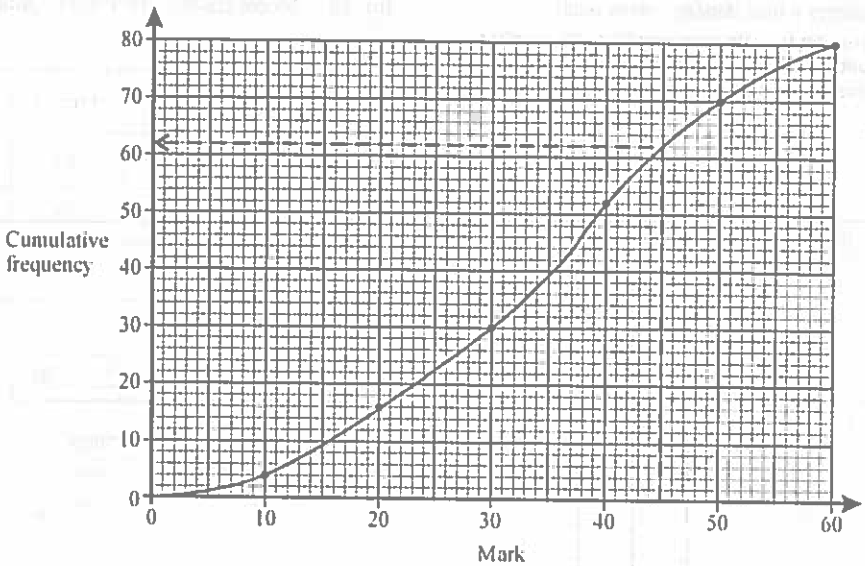
(c) Read from the graph the cumulative frequency that corresponds to 45 marks. Subtract it from total frequency.

Solution

(a)

Mark (m)	$m \le 10$	$m \le 20$	$m \le 30$	$m \le 40$	$m \le 50$	$m \le 60$
Cumulative frequency	4	16	30	52	70	80

(b)



(c) From graph, the number of students who took 45 marks = 62

∴ number of students who passed = 80 − 62 = 18 **Ans.**

22 *(J2013 P2 Q7)*

(a) The distribution of the times spent by 200 customers at a restaurant one evening is shown in the table.

Time (t minutes)	$30 \le t < 60$	$60 \le t < 80$	$80 \le t < 90$	$90 \le t < 100$	$100 \le t < 120$
Frequency	24	p	q	58	28

The diagram on the right shows part of the histogram that represents this data.

(i) Complete the histogram. [1]

(ii) Find p and q. [2]

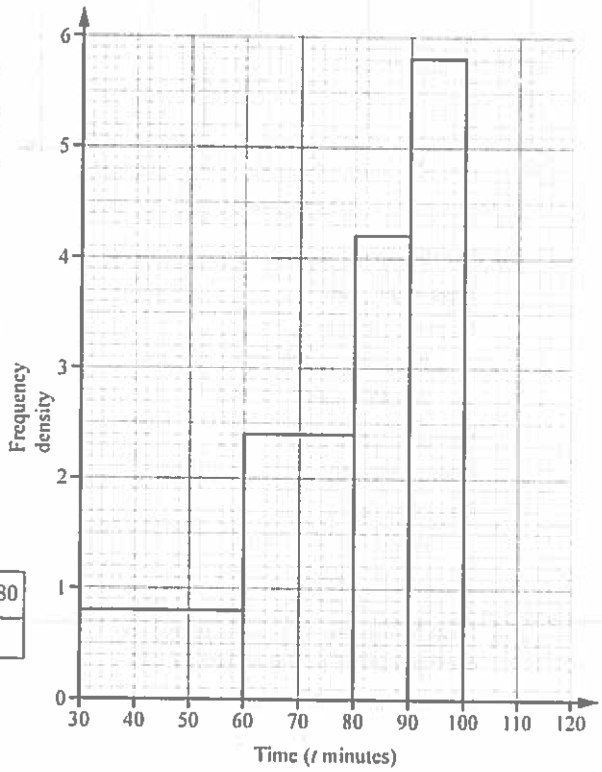
(iii) Estimate the probability that a customer, chosen at random, spent more than 95 minutes in the restaurant. [1]

(b) The table below shows the distribution of the ages of these customers.

Age (y years)	$0 < y \le 20$	$20 < y \le 40$	$40 < y \le 60$	$60 < y \le 80$
Frequency	34	57	85	24

(i) State the modal class. [1]

(ii) Calculate an estimate of the mean age of these customers. [3]



Thinking Process

(a) (i) Find the frequency density of the required interval.

(ii) To find p and q read out the heights and widths of the corresponding columns.

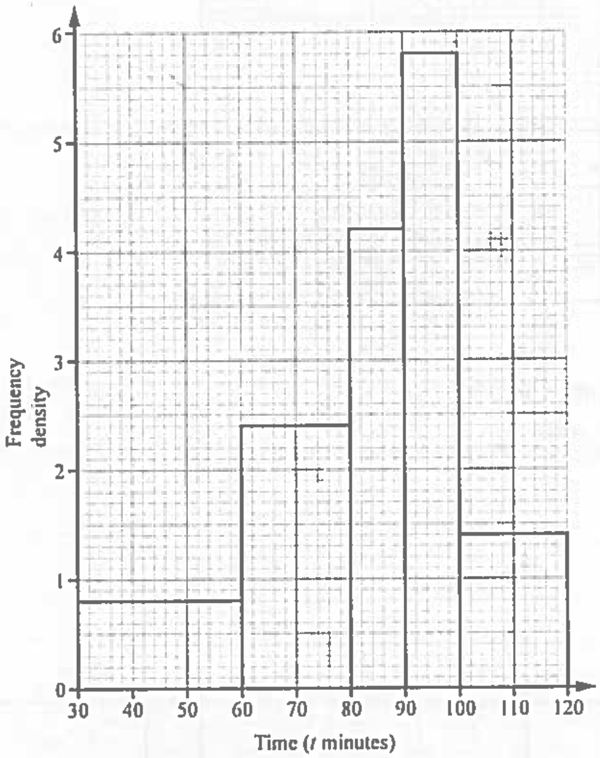
consider the formula:

Frequency = freq. density × class width.

(iii) By referring to histogram, express the number of customers who spent more than 95 minutes as a fraction of total customers.

Solution

(a) (i)



(ii) For the interval $60 \le t < 80$,

class width = 20

frequency density = 2.4

$\therefore$ Frequency = $2.4 \times 20 = 48$

$\Rightarrow$ $p = 48$ **Ans.**

For the interval $80 \le t < 90$,

class width = 10

frequency density = 4.2

$\therefore$ Frequency = $4.2 \times 10 = 42$

$\Rightarrow$ $q = 42$ **Ans.**

(iii) From histogram, we see that frequency density from 95 to 100 minutes = 5.8

$\Rightarrow$ frequency = f.d × width

$= 5.8 \times 5 = 29$

$\therefore$ number of customers who spent more than 95 minutes $\approx 29 + 28 = 57$

P(No. of customers who spent > 95 minutes)

$= \frac{57}{200}$ **Ans.**

(b) (i) Modal classs: $40 < y \le 60$ **Ans.**

(ii)

Age (y years)	Midpoint, x	Freq. f	fx
$0 < y \le 20$	$\frac{0+20}{2} = 10$	34	340
$20 < y \le 40$	$\frac{20+40}{2} = 30$	57	1710
$40 < y \le 60$	$\frac{40+60}{2} = 50$	85	4250
$60 < y \le 80$	$\frac{60+80}{2} = 70$	24	1680
		$\sum f = 200$	$\sum fx = 7980$

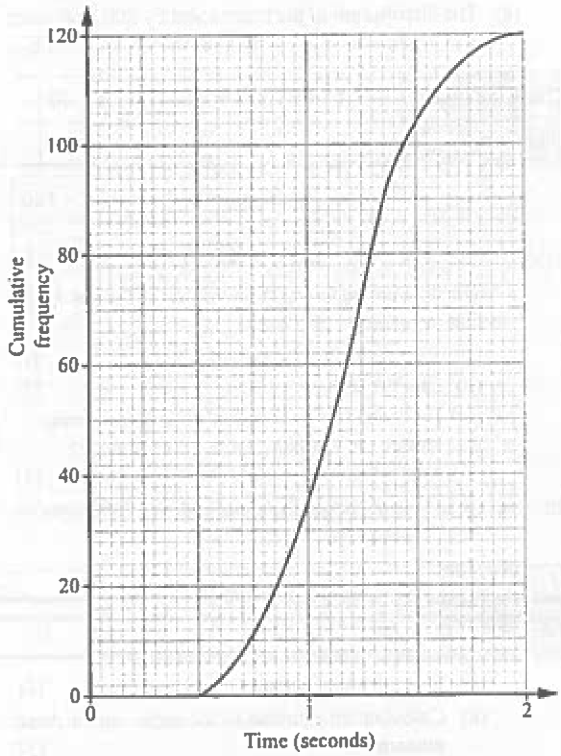
$$\text{mean age} = \frac{\sum fx}{\sum f} = \frac{7980}{200}$$

$= 39.9$ years **Ans**

23 *(N2013/P1/Q14)*

The times taken by each of 120 runners to react to the starting gun were recorded.

The cumulative frequency curve summarises the results.



(a) Find the upper quartile. [1]
(b) Find the 40th percentile. [1]
(c) Find the number of students who took less than 1.5 seconds. [1]

Thinking Process

(a) Find 75% of 120. Read the corresponding value of time.
(b) Find 40% of 120. Read the corresponding value of time.

Solution

(a) 75% of 120

$= \frac{75}{100} \times 120 = 90$

$\therefore$ from graph, upper quartile = 1.35 seconds **Ans.**

(b) 40% of 120

$= \frac{40}{100} \times 120 = 48$

$\therefore$ from graph,

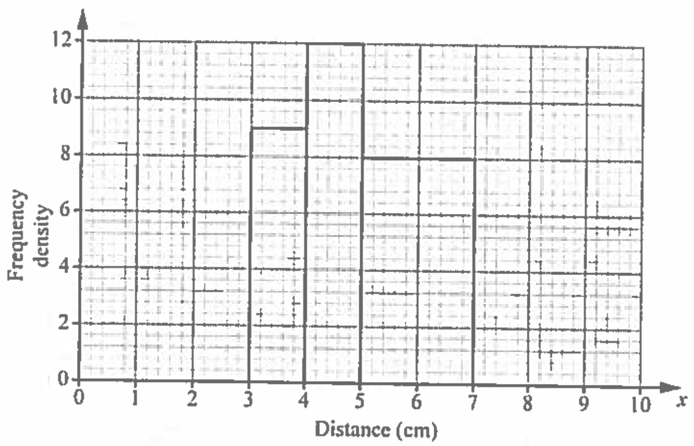
40th percentile = 1.1 seconds **Ans.**

(c) From graph, the number of students who took less than 1.5 seconds = 104 **Ans.**

24 *(N2013/P1 Q18)*

In an experiment with a group of snails, the distance moved in one minute by each snail was recorded.
Some of the results are shown in the table and illustrated in the histogram.

Distance (x centimetres)	$2 < x \le 3$	$3 < x \le 4$	$4 < x \le 5$	$5 < x \le 7$	$7 < x \le 9$
Frequency	6	9	12	P	4



(a) Use the histogram to find the value of p. [1]
(b) Complete the histogram. [2]
(c) One snail is chosen at random.
Find the probability that this snail did not move more than 4 cm. [1]

Thinking Process

(a) To find p consider the formula:

Frequency = freq. density × class width

(b) To complete the histogram find the frequency density of the required intervals.
(c) Find the number of snails that moved less than 4 cm and divide it by total frequency.

Solution

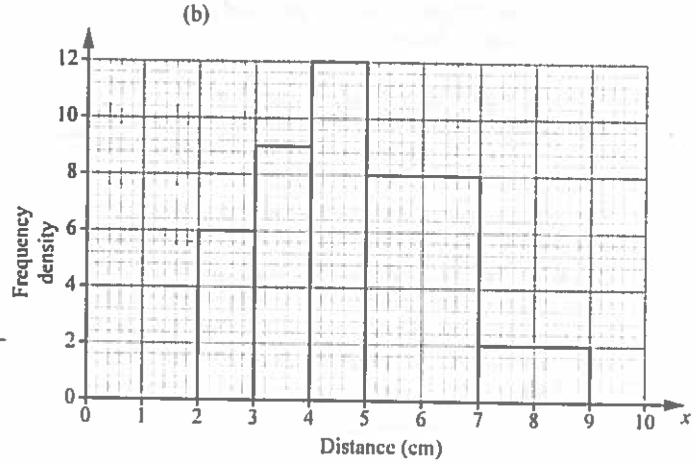
(a) Frequency = frequency density × class width

$= 8 \times 2$

$= 16$

$\therefore \quad p = 16$ **Ans.**

(b)



(c) Total number of snails = $6 + 9 + 12 + 16 + 4 = 47$

No. of snails that did not move more than 4 cm

$= 6 + 9 = 15$

$\therefore \quad P$(the snail did not move more than 4 cm)

$= \frac{15}{47}$ **Ans.**

25 *(N2013 P2 Q2)*

(a) The results of a survey of the number of cars owned by 50 families are given in the table below.

Number of cars	0	1	2	3
Number of families	4	35	6	5

(i) Calculate the mean number of cars per family. [2]

(ii) When the same 50 families were surveyed at a later date, the results were as follows.

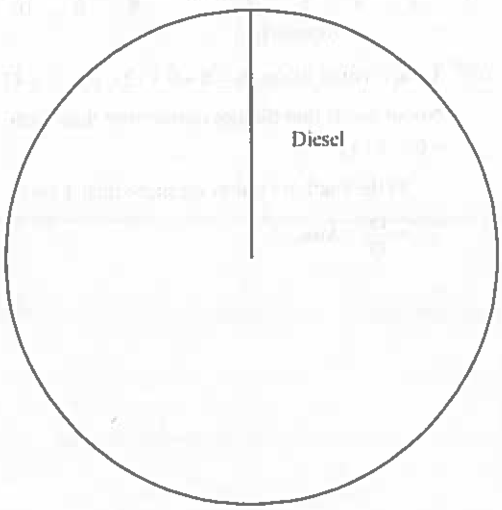
Number of cars	0	1	2	3
Number of families	x	37	y	5

The mean number of cars per family stayed the same as before.
Find x and y. [2]

(b) A service station sells diesel, unleaded and super unleaded fuel.
During one week, 13 500 litres of diesel and 36 000 litres of unleaded were sold.
The total number of litres of fuel sold that week was 54 000.

(i) What fraction of the total number of litres sold was super unleaded?
Give your answer in its lowest terms. [1]

(ii) Complete the pie chart to represent the amounts of fuel sold.



[3]

Thinking Process

(a) (i) To find the mean ⇒ divide the total number of cars by the total frequency.

(ii) Equate total frequency to 50 and form an equation. Equate mean to the answer found in (i). Solve for x and y.

(b) (i) To find fraction ⇒ compute the number of litres of super unleaded fuel.

(ii) To complete the pie chart ⇒ calculate the angle representing each type of fuel.

Solution

(a) (i) Mean $= \dfrac{(0\times4)+(1\times35)+(2\times6)+(3\times5)}{50}$

$= \dfrac{0+35+12+15}{50}$

$= \dfrac{62}{50} = 1.24$ **Ans.**

(ii) Total frequency = 50

$\Rightarrow \quad x+37+y+5=50$

$x+y=8 \quad \cdots\cdots(1)$

mean $= \dfrac{(0\times x)+(1\times37)+(2\times y)+(3\times5)}{50}$

from part (i), mean number of cars = 1.24

$\Rightarrow \quad \dfrac{(0\times x)+(1\times37)+(2\times y)+(3\times5)}{50} = 1.24$

$37+2y+15=62$

$2y=10$

$y=5$

substitute $y=5$ into (1).

$x+5=8 \quad \Rightarrow \quad x=3.$

$\therefore \quad x=3, \ y=5$ **Ans.**

(b) (i) Number of litres of super unleaded fuel sold

$= 54000-13500-36000$

$= 4500$

$\therefore$ fraction of super unleaded sold

$= \dfrac{4500}{54000} = \dfrac{1}{12}$ **Ans.**

(ii) Angle represented by diesel

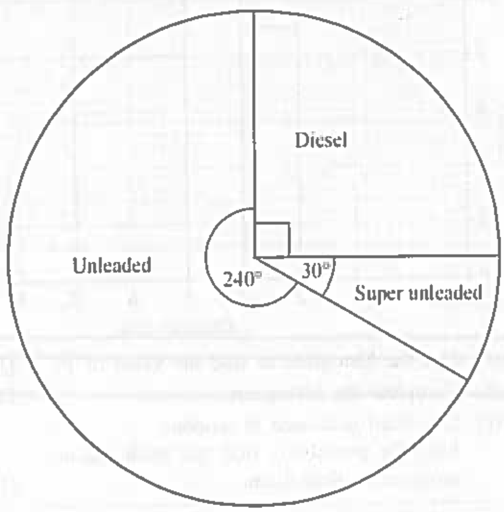
$= \dfrac{13500}{54000}\times360^\circ = 90^\circ$

Angle represented by unleaded

$= \dfrac{36000}{54000}\times360^\circ = 240^\circ$

Angle represented by super unleaded

$= \dfrac{4500}{54000}\times360^\circ = 30^\circ$



26 *(J2014 P1 Q24)*

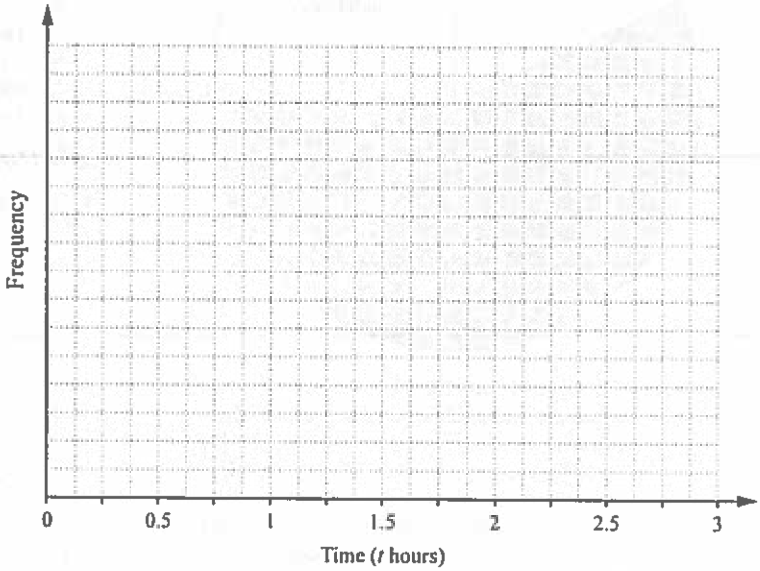
Some students were asked how long they had each spent doing homework the day before. The results are summarised in the table.

Time (t hours)	$0 < t \le 0.5$	$0.5 < t \le 1$	$1 < t \le 1.5$	$1.5 < t \le 2$	$2 < t \le 2.5$	$2.5 < t \le 3$
Girls	0	5	8	6	0	1
Boys	3	3	4	5	3	2

(a) On the grid, draw a frequency polygon to represent this information for the girls and another frequency polygon for the boys. [3]

(b) Write down the modal group for the girls. [1]

(c) Make a comment comparing the distribution of the times spent by the girls with the times spent by the boys. [1]

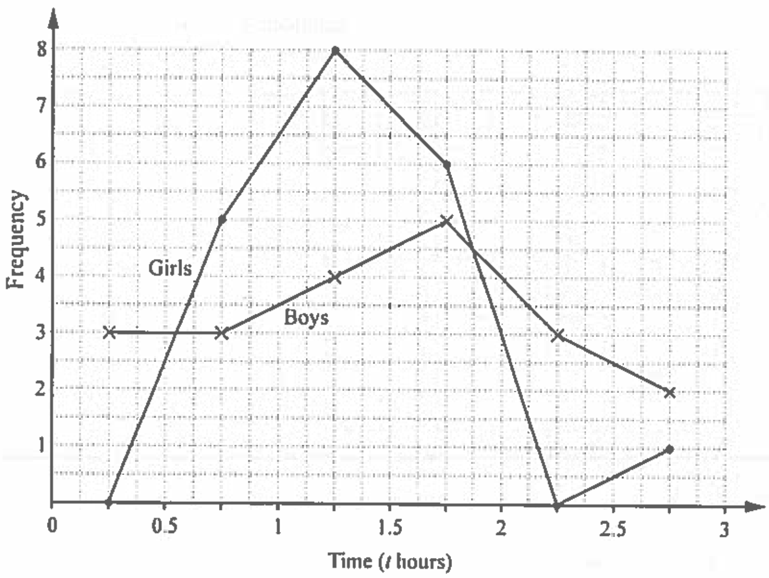


Thinking Process

(a) To draw a frequency polygon ✐ plot each frequency against the mid-value of the class interval.
(b) ✐ Look for the class with highest frequency.
(c) For comparison, consider the modal groups for the two graphs or the range of times spent.

Solution

(a)



(b) $1 < t \le 1.5$ Ans.

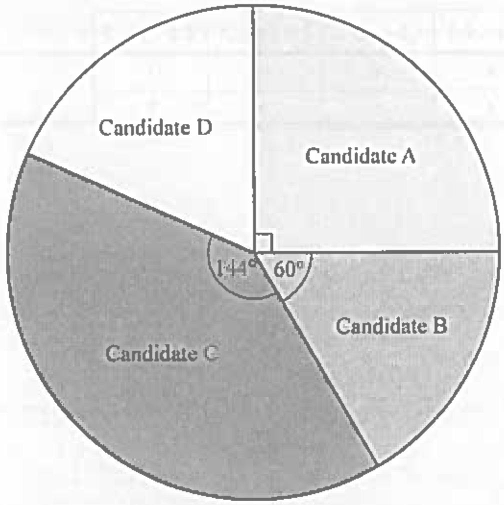
(c) Comparing the modal groups, we see that the modal group for boys i.e. $1.5 < t \le 2$ hours is higher than the modal group for girls. Therefore boys spent longer time doing their homework than girls.

Alternatively:

Most girls spent between 0.75 hours to 2 hours doing their homework, whereas boys times are more evenly spread between 0.25 hours to 3 hours.

27 (J2014 P2 Q7)

(a) The pie chart summarises the results of a local election.



(i) Candidate B received 1600 votes. Work out the total number of people who voted in the election. [2]

(ii) What fraction of the vote did candidate D receive? Give your answer in its lowest terms. [1]

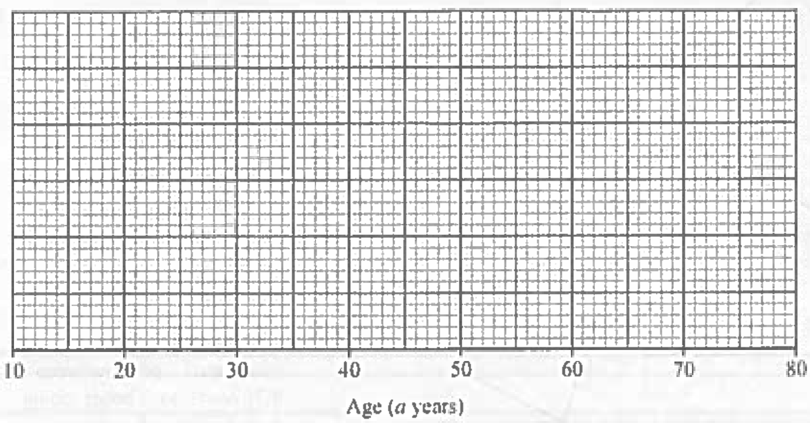
(iii) How many more votes than candidate A did candidate C receive? [2]

(b) The table summarises the ages of the members of a film club.

Age (a years)	$15 \le a < 20$	$20 \le a < 30$	$30 \le a < 40$	$40 \le a < 60$	$60 \le a < 80$
Frequency	12	36	45	33	24

(i) Calculate an estimate of the mean age of the members. [3]

(ii) On the grid below, draw a histogram to represent this data. [3]



(iii) Find an estimate for the number of members of the film club who are over 50. [1]

Thinking Process

(a) (i) Note that 1600 votes are represented by 60°. Find the number of votes represented by 360°.

(ii) To find the fraction ✐ Find the angle represented by Candidate D.

(iii) ✐ Find the number of votes received by candidate A and C.

(b) (i) Use mean $= \frac{\sum fx}{\sum f}$ ✐ Compute the mid-point (x) of each interval.

(ii) To draw histogram ✐ first find the frequency density for each range.

(iii) Estimate by referring to the histogram drawn.

Solution

(a) (i) 60° represents —— 1600 votes

360° represents —— $\frac{1600}{60°} \times 360°$

$= 9600$ votes

∴ total number of people who voted $= 9600$ **Ans.**

(ii) Angle representing candidate D

$= 360° - 144° - 60° - 90°$ (∠s around a pt.)

$= 66°$

fraction of votes that candidate D received

$= \frac{66°}{360°} = \frac{11}{60}$ **Ans.**

(iii) Votes received by candidate $A = \frac{90}{360} \times 9600$

$= 2400$

Votes received by candidate $C = \frac{144}{360} \times 9600$

$= 3840$

difference, $3840 - 2400 = 1440$

∴ candidate C receives 1440 more votes than candidate A **Ans.**

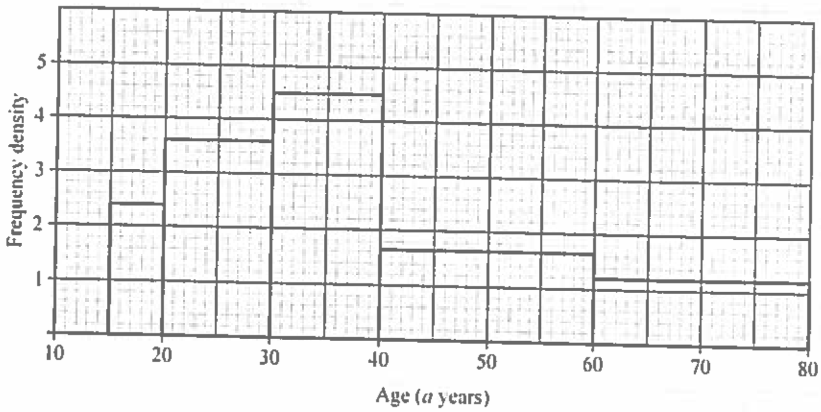
(b) (i)

Age (a years)	Midpoint, x	Frequency f	Product, fx
$15 \le a < 20$	17.5	12	210
$20 \le a < 30$	25	36	900
$30 \le a < 40$	35	45	1575
$40 \le a < 60$	50	33	1650
$60 \le a < 80$	70	24	1680
		$\sum f = 150$	$\sum fx = 6015$

$$\text{mean} = \frac{\sum fx}{\sum f} = \frac{6015}{150}$$
$$= 40.1 \text{ years} \quad \textbf{Ans.}$$

(ii)

Age (a years)	Freq.	Width	Freq. density
$15 \le a < 20$	12	5	$\frac{12}{5} = 2.4$
$20 \le a < 30$	36	10	$\frac{36}{10} = 3.6$
$30 \le a < 40$	45	10	$\frac{45}{10} = 4.5$
$40 \le a < 60$	33	20	$\frac{33}{20} = 1.65$
$60 \le a < 80$	24	20	$\frac{24}{20} = 1.2$



(iii) From histogram, we see that frequency density from 50 to 60 years = 1.65

$\Rightarrow$ frequency = f.d × width
$= 1.65 \times 10$
$= 16.5$

$\therefore$ number of members over 50 years $= 16.5 + 24$
$= 40.5 \approx 41$ **Ans.**

28 *(N2014 P1 Q3)*

In an experiment, a red die and a blue die were thrown 10 times.
Each time, the score on the red die was subtracted from the score on the blue die.
The results are given below.

5 −4 −3 4 0 2 −1 −3 3 −2

For these results, find
(a) the median, [1]
(b) the mean. [2]

Thinking Process

(a) To find median ⇒ write the given numbers in increasing order.
(b) To find mean ⇒ divide the total results by total frequency.

Solution

(a) Writing the values in increasing order, we have.

−4 −3 −3 −2 −1 0 2 3 4 5

middle positions are 5th and 6th

$\therefore$ median $= \frac{-1+0}{2} = -\frac{1}{2}$ **Ans.**

(b) Mean

$= \frac{5+(-4)+(-3)+4+0+2+(-1)+(-3)+3+(-2)}{10}$

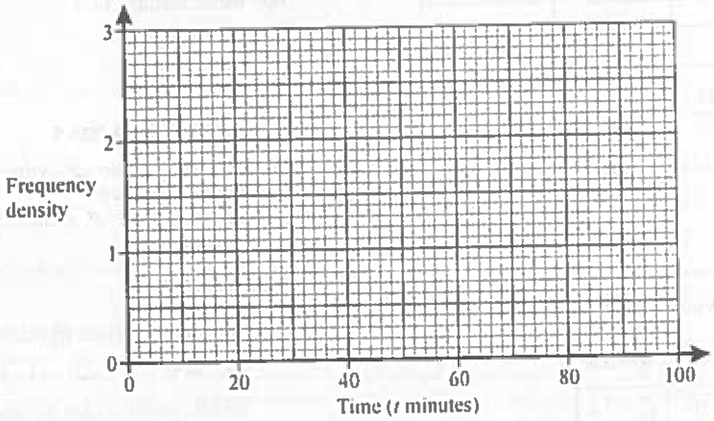
$= \frac{5-4-3+4+0+2-1-3+3-2}{10} = \frac{1}{10}$ **Ans.**

29 (N2014/P2/Q11)

(a) 100 students were each asked how long they spent talking on their mobile phone during one day. The results are summarised in the table.

Time (t minutes)	$0 < t \leq 10$	$10 < t \leq 20$	$20 < t \leq 40$	$40 < t \leq 60$	$60 < t \leq 80$	$80 < t \leq 100$
Frequency	10	30	12	16	20	12

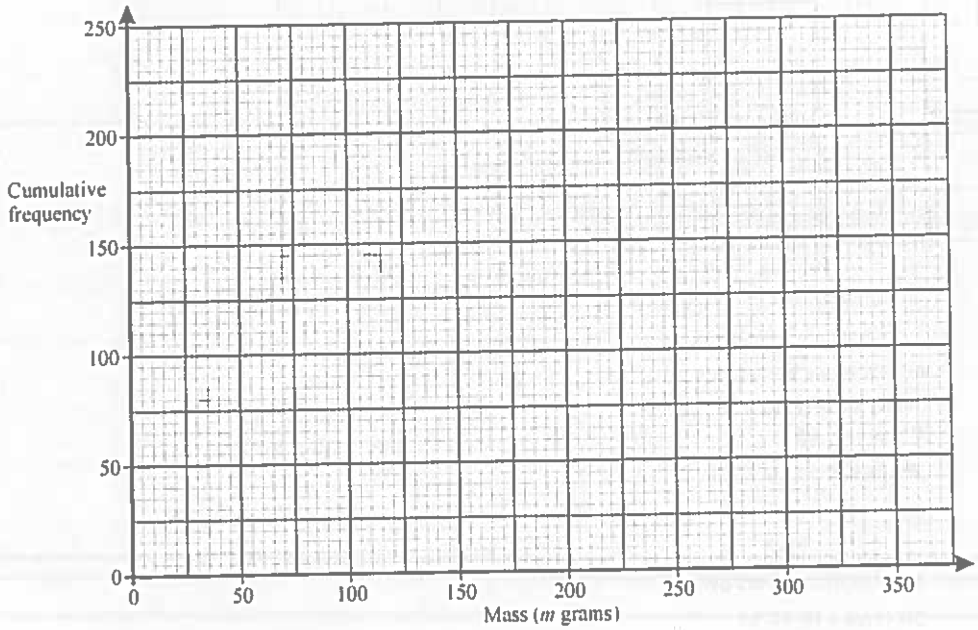
On the axes below, draw a histogram to represent these results.



(b) The masses, in grams, of 240 potatoes were found.
The cumulative frequency table for these results is shown below.

Mass (m grams)	$m \leq 50$	$m \leq 100$	$m \leq 150$	$m \leq 200$	$m \leq 250$	$m \leq 300$	$m \leq 350$
Cumulative frequency	0	4	54	132	204	236	240

(i) Draw a smooth cumulative frequency curve to illustrate this information.



[2]

(ii) (a) Find the median. [1]

(b) Find the inter-quartile range. [2]

(iii) Complete the frequency table below.

Mass (m grams)	$50 < m \le 100$	$100 < m \le 150$	$150 < m \le 200$	$200 < m \le 250$	$250 < m \le 300$	$300 < m \le 350$
Frequency	4					

[1]

(iv) A potato with a mass greater than 250 grams is classed as extra large.
(a) How many of these potatoes are extra large? [1]
(b) Which percentile of the distribution can be used to find this number? [2]

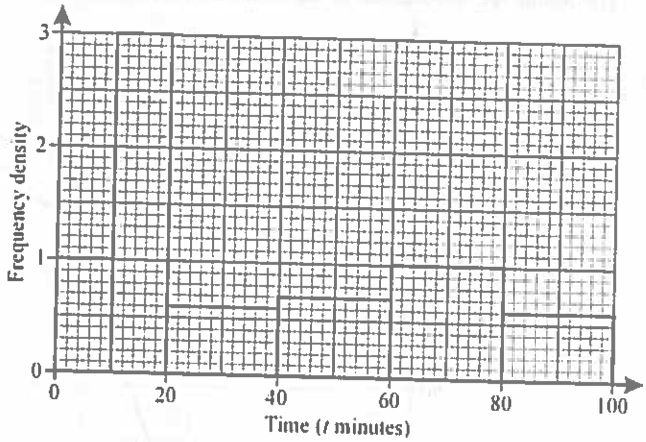
Thinking Process

(a) To draw the histogram ✎ find the frequency density for each interval.
(b) (ii) (a) Find 50% of total frequency and read the corresponding value for the length.
(b) Find 25% and 75% of total frequency. Mark the two values on the graph. Read the corresponding x-values and find their difference.
(iii) Use the relationship between frequency and cumulative frequency. ✎ Subtract the number of potatoes in the first table to fill in the blanks in the second table.
(iv) (a) From graph, find the number of potatoes that corresponds to 250 grams. Subtract the number 240.
(b) ✎ find

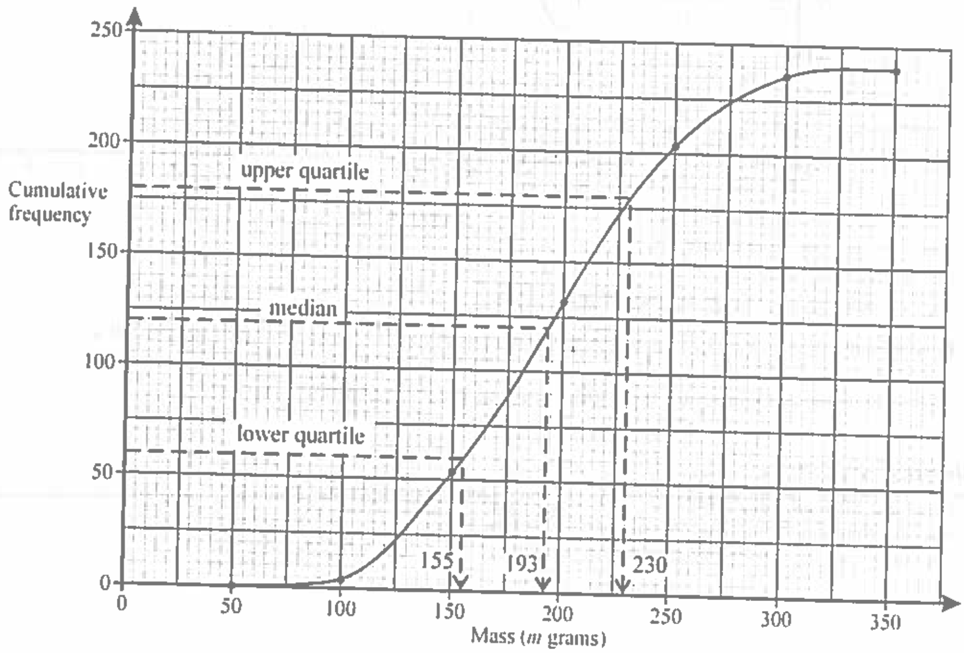
Solution

(a)

Time (t minutes)	Freq.	Width	Freq. density
$0 < t \le 10$	10	10	$\frac{10}{10} = 1$
$10 < t \le 20$	30	10	$\frac{30}{10} = 3$
$20 < t \le 40$	12	20	$\frac{12}{20} = 0.6$
$40 < t \le 60$	16	20	$\frac{16}{20} = 0.8$
$60 < t \le 80$	20	20	$\frac{20}{20} = 1$
$80 < t \le 100$	12	20	$\frac{12}{20} = 0.6$



(b) (i)



(ii) (a) Median length = 193 grams **Ans.**

(b) Interquartile range = upper quartile – lower quartile
$= 230 - 155 = 75$ grams **Ans.**

(iii)

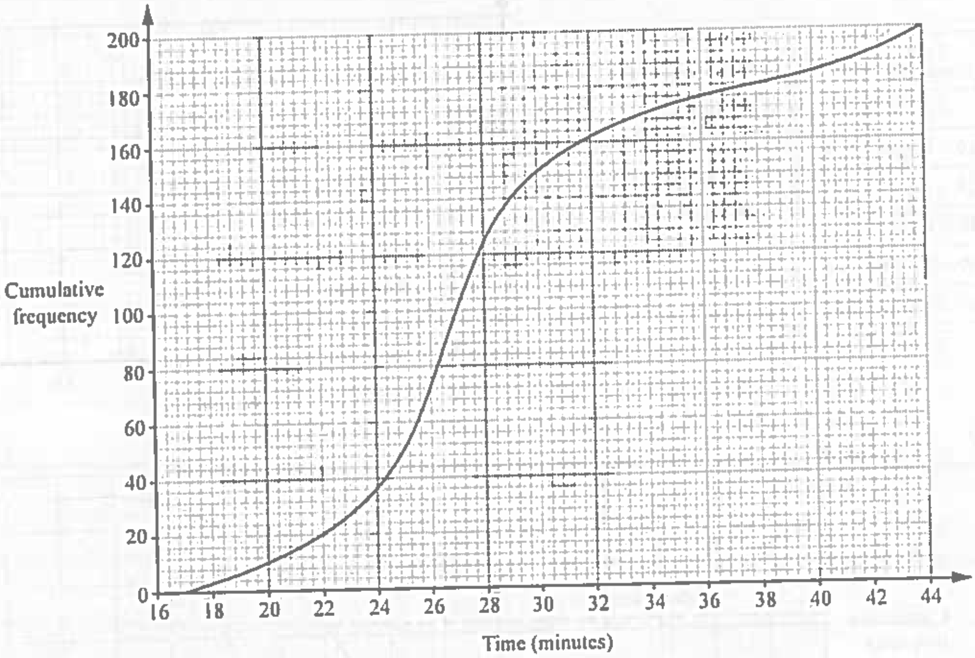
Mass (m grams)	$50 < m \le 100$	$100 < m \le 150$	$150 < m \le 200$	$200 < m \le 250$	$250 < m \le 300$	$300 < m \le 350$
Frequency	4	50	78	72	32	4

(iv) (a) From graph, the number of potatoes with mass 250 grams = 204
$\therefore$ number of potatoes with mass greater than 250 grams $= 240 - 204 = 36$ **Ans.**

(b) $\frac{204}{240} \times 100 = 85\%$
$\therefore$ 85th percentile can be used to find this number. **Ans.**

30 *(J2015/P1/Q20)*

The times taken for 200 people to complete a 5 km race were recorded.
The results are summarised in the cumulative frequency diagram.



(a) Use the diagram to estimate
(i) the median time, [1]
(ii) the interquartile range of the times. [2]

(b) It was found that the recording of the times was inaccurate. The correct times were all one minute more than recorded.
Write down the median and interquartile range of the correct times. [1]

Thinking Process

(a) (i) Find 50% of 200. Mark the value on the graph and read for x-values.
(ii) Interquartile range = upper quartile – lower quartile.
(b) Add one minute in the median time. Interquartile range remains unchanged.

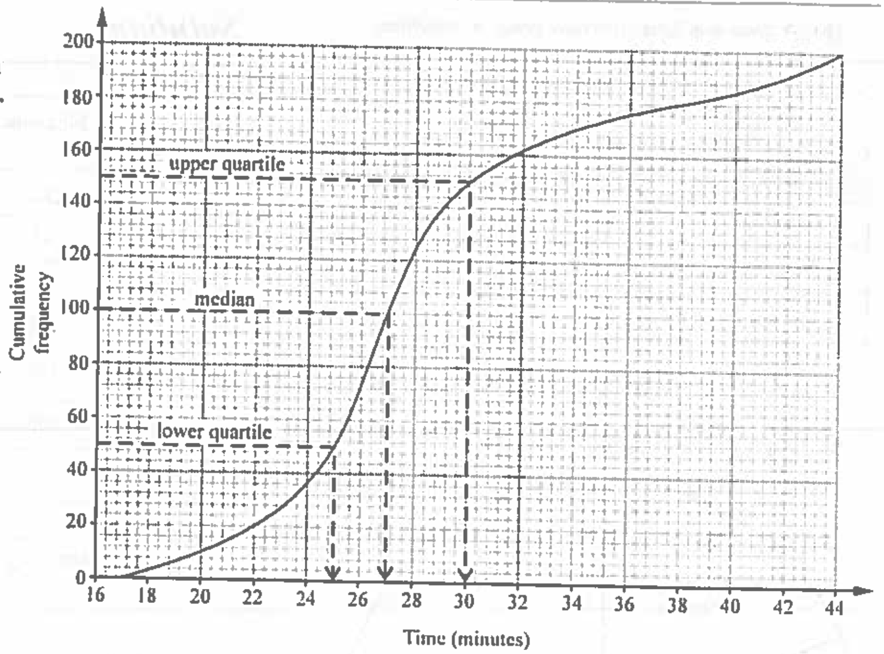
Solution

(a) (i) From graph, median
$= 27$ minutes **Ans.**

(ii) Interquartile range
= upper quartile
− lower quartile
$= 30 - 25$
$= 5$ minutes **Ans.**

(b) Median
$= 28$ minutes. **Ans.**

Interquartile range
$= 5$ minutes. **Ans.**



31 *(J2015/P1/Q12)*

Omar has a pack of number cards.
He picks these five cards.

−2	−4	−2	4	1

(a) Write down the mode of the five numbers. [1]

(b) He takes another card from the pack.

(i) If the mean of the six numbers is −1, what number did he pick? [1]

(ii) If the difference between the highest and lowest of the six numbers is 12, what are the two possible numbers he could have picked? [1]

Thinking Process

(a) Mode is the number that occurs most.

(b) (i) $\text{Mean} = \dfrac{\text{sum of all numbers}}{\text{total frequency}}$

(ii) Look for two numbers such that their difference with the highest and lowest given numbers i.e. 4 and −4 is 12.

Solution

(a) Mode $= -2$ **Ans.**

(b) (i) $\text{Mean} = \dfrac{-2+(-4)+(-2)+4+1+x}{6}$

$-1 = \dfrac{-3+x}{6}$

$-6 = -3 + x$

$x = -3$

∴ the number card he picks is -3 **Ans.**

(ii) Two possible numbers are: 8, and −8 **Ans.**

> The highest and lowest number given on number cards is 4 and −4.
> ∴ $4-(-8)=12$ and $8-(-4)=12$

32 *(J2015/P2/Q3)*

(a) In a survey, 50 students were asked how long they spent exercising during one particular week.
The results are summarised in the table.

Time (t minutes)	Frequency
$0 \le t < 30$	10
$30 \le t < 60$	15
$60 \le t < 90$	11
$90 \le t < 120$	7
$120 \le t < 150$	5
$150 \le t < 180$	2

(i) Calculate an estimate of the mean time each student spent exercising that week. [3]

(ii) During that week, the time Simon spent exercising is shown below.

Tuesday 12.37 p.m. until 1.24 p.m.
Thursday 8.57 a.m. until 9.42 a.m.

Which interval is his time recorded in? [1]

(b) A gym has four different types of machines. Carol is going to draw a pie chart to show how many times the machines are used in one day. She has started to make a table.

Machine	Frequency	Angle of sector
Running	90	120°
Rowing	75	
Cycling	57	
Weights		64°

(i) Complete the table. [2]

(ii) Complete the pie chart.

[1]

Thinking Process

(a) (i) Use mean = $\frac{\sum fx}{\sum f}$ ✎ Compute the mid-point (x) of each interval.

(ii) To find the interval ✎ find the total number of minutes spent during two days.

(b) (i) Note that frequency of 90 is represented by 120°. Find the angle that represents frequency of 75 and 57 respectively. Similarly find the frequency represented by 64°.

(ii) To complete the pie chart ✎ use the angles found in part (b) (i).

Solution

(a) (i)

Time (t minutes)	Midpoint, x	Frequency f	Product, fx
$0 \le t < 30$	15	10	150
$30 \le t < 60$	45	15	675
$60 \le t < 90$	75	11	825
$90 \le t < 120$	105	7	735
$120 \le t < 150$	135	5	675
$150 \le t < 180$	165	2	330
		$\sum f = 50$	$\sum fx = 3390$

Mean time $= \frac{\sum fx}{\sum f} = \frac{3390}{50}$

$= 67.8$ minutes **Ans.**

(ii) Time spent on Tuesday $= 0124 - 1237$
$= 1284 - 1237$
$= 47$ minutes.

Time spent on Thursday $= 0942 - 0857$
$= 45$ minutes.

Total time spent during the week
$= 47 + 45 = 92$ minutes.

$\therefore$ required interval: $90 \le t < 120$ **Ans.**

(b) (i) Freq. of 90 represents —— 120°

Freq. of 75 represents —— $\frac{120°}{90} \times 75 = 100°$

Freq. of 57 represents —— $\frac{120°}{90} \times 57 = 76°$

$\therefore$ angle of sector for rowing = 100° **Ans.**

angle of sector for cycling = 76° **Ans.**

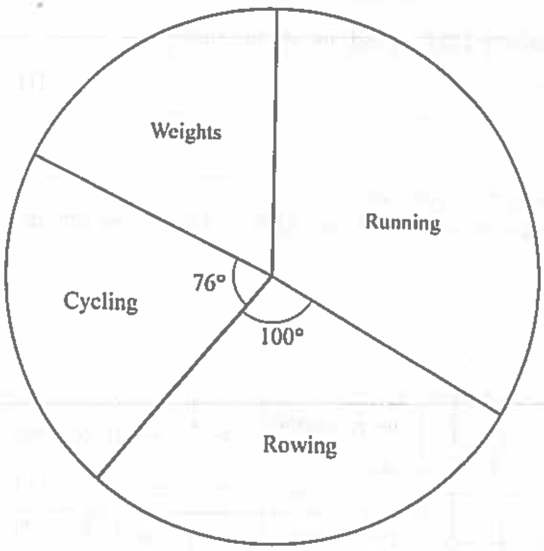
for weights:

Freq. represented by 120° = 90

Freq. represented by 64° $= \frac{90}{120°} \times 64 = 48$

$\therefore$ frequency of weights = 48 **Ans.**

(ii)

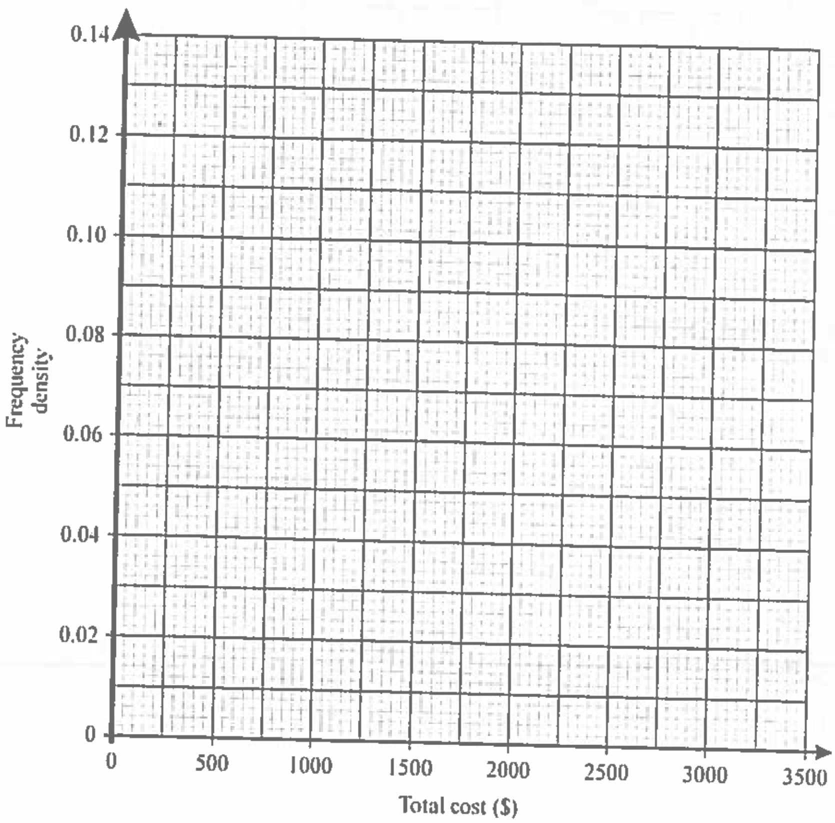


33 *(J2015/P2 Q11 b)*

(b) The table shows the distribution of the total cost per person for holidays in 2014 for another group of people.

Total cost per person ($c)	$0 \le c < 250$	$250 \le c < 500$	$500 \le c < 1000$	$1000 \le c < 2000$	$2000 \le c < 3500$
Frequency	35	20	15	8	6

(i) Draw a histogram to represent this data.



[3]

(ii) Estimate the number of people who spent less than \$700 on holidays in 2014. [2]

(iii) Of the people who spent less than \$250 on holidays in 2014, $\frac{2}{7}$ did not go on holiday.

How many people did not go on holiday in 2014? [1]

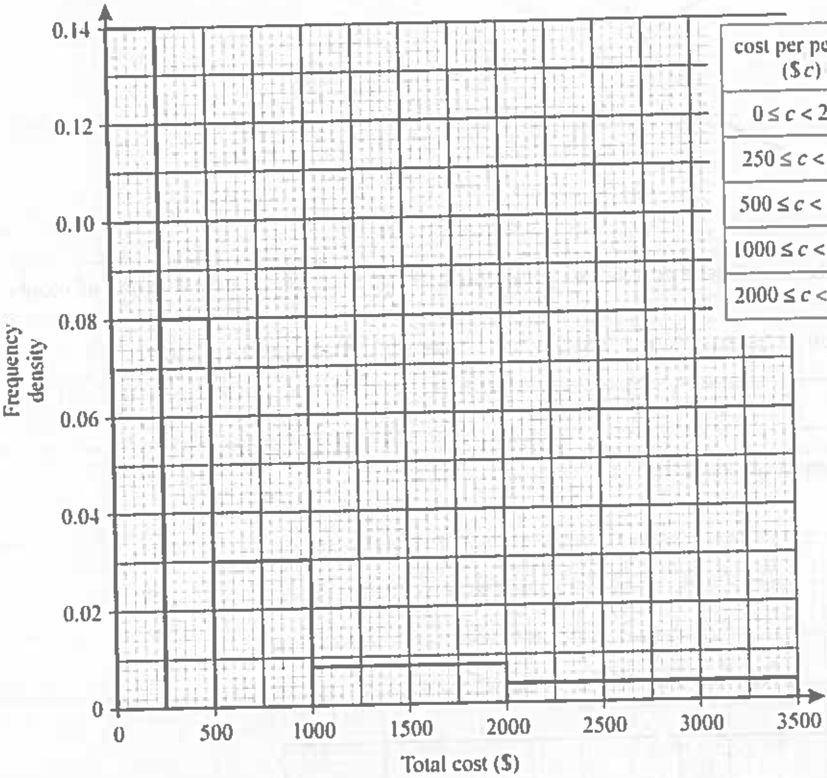
Thinking Process

(b) (i) To draw the histogram find the frequency density for each interval.

(ii) By referring to histogram, calculate the number of people who spent from \$500 to \$700. Then add up the required frequencies

Solution

(b) (i)



cost per person (\$ c)	Freq.	Width	Freq. density
$0 \le c < 250$	35	250	$\frac{35}{250} = 0.14$
$250 \le c < 500$	20	250	$\frac{20}{250} = 0.08$
$500 \le c < 1000$	15	500	$\frac{15}{500} = 0.03$
$1000 \le c < 2000$	8	1000	$\frac{8}{1000} = 0.008$
$2000 \le c < 3500$	6	1500	$\frac{6}{1500} = 0.004$

(ii) From histogram, the frequency density from \$500 to \$700 = 0.03

$\Rightarrow$ frequency = f.d × width
= 0.03 × 200
= 6

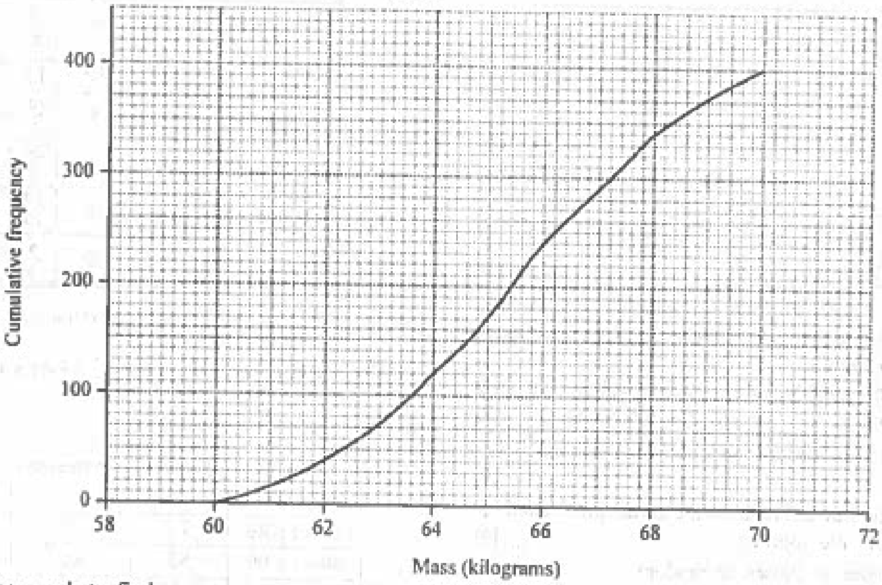
$\therefore$ number of people who spent less than \$700 = 35 + 20 + 6
= 61 **Ans.**

(iii) People who spent less than \$250 = 35

$\therefore$ Number of people who did not go on holiday $= \frac{2}{7} \times 35 = 10$ **Ans.**

34 *(N2015/P1 Q20)*

The masses of 400 goats were measured.
The results are shown in the cumulative frequency graph.



(a) Use the graph to find
  (i) the median, [1]
  (ii) the 30th percentile, [1]
  (iii) the number of goats whose mass is more than 66 kg. [1]

(b) It was noticed later that the scales used were faulty and that the true readings should all be 2 kg more.
On the grid above, draw the true cumulative frequency graph. [1]

Thinking Process

(a) (i) Find 50% of 400. Mark the value on the graph and read for x-values.
  (ii) Find 30% of 400. Mark the value on the graph and read the corresponding x-values.
  (iii) From graph, find the number of goats whose mass is equal to 66 kg. Subtract it from 400.

(b) To draw the true cumulative frequency graph ✎ shift the whole curve 2 units to the right along the x-axis.

Solution

(a) (i) Median = 65.4 kg **Ans.**

  (ii) 30% of 400

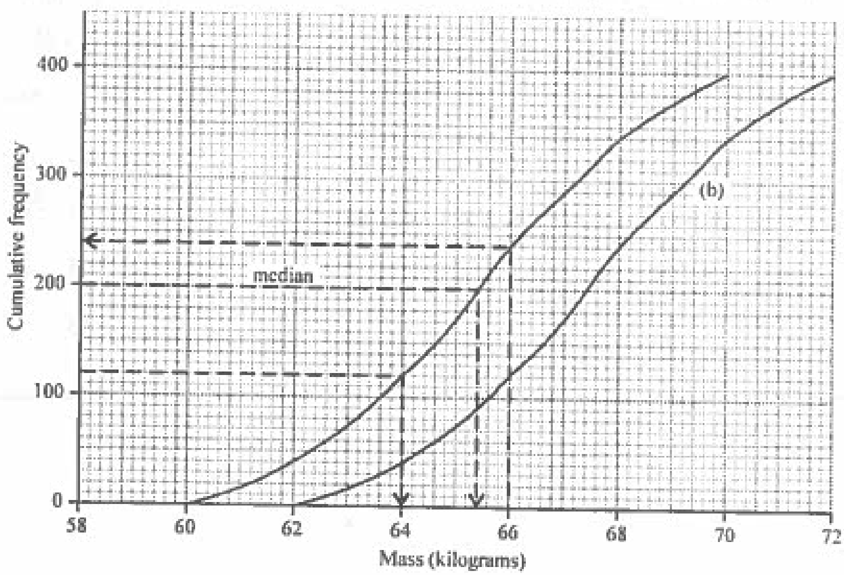
$$= \frac{30}{100} \times 400 = 120$$

∴ from graph,
30th percentile
= 64 kg **Ans.**

  (iii) From graph, at 66 kg,
number of goats = 240

∴ number of goats with mass more than 66 kg
= 400 − 240
= 160 **Ans.**

(b) Refer to graph.



35 *(N2015/P2/Q10)*

The length of time taken by 80 drivers to complete a particular journey is summarised in the table below.

Time (t minutes)	$60 < t \le 80$	$80 < t \le 90$	$90 < t \le 95$	$95 < t \le 100$	$100 < t \le 110$	$110 < t \le 130$
Number of drivers	4	10	14	20	24	8

(a) Using a scale of 2 cm to represent 10 minutes, draw a horizontal axis for times from 60 minutes to 130 minutes.
Choose a suitable scale for the vertical axis and draw a histogram to represent this information. [3]

(b) In which of the intervals does the median time lie? [1]

(c) Calculate an estimate of the mean time taken to complete the journey. [3]

(d) One driver is chosen at random.
Calculate the probability that this driver took 95 minutes or less for the journey. [1]

(e) Two of the 80 drivers are chosen at random.
(i) Calculate the probability that both took more than 100 minutes for the journey. [2]
(ii) Calculate the probability that one took 80 minutes or less and the other took more than 110 minutes. [2]

Thinking Process

(a) To draw the histogram ✐ find the frequency density for each interval.
(b) ✐ Find the median from the table.
(c) Use mean $= \frac{\sum fx}{\sum f}$ ✐ Compute the mid-point (x) of each interval.
(d) By referring to table, express the number of drivers who took 95 minutes or less, as a fraction of total drivers.
(e) (i) Note that first driver is chosen from 80 drivers, and second is chosen from the remaining 79 drivers.
(ii) ✐ Add the number of drivers who took 80 minutes or less and who took more than 100 minutes.

Solution

(a)

Time (t minutes)	Freq.	Width	Freq. density
$60 < t \le 80$	4	20	$\frac{4}{20} = 0.2$
$80 < t \le 90$	10	10	$\frac{10}{10} = 1$
$90 < t \le 95$	14	5	$\frac{14}{5} = 2.8$
$95 < t \le 100$	20	5	$\frac{20}{5} = 4$
$100 < t \le 110$	24	10	$\frac{24}{10} = 2.4$
$110 < t \le 130$	8	20	$\frac{8}{20} = 0.4$

Refer to histogram on next page.

(b) Median lies in the interval $95 < t \le 100$ **Ans.**

(c)

Time (t minutes)	Midpoint. x	Frequency f	Product. fx
$60 < t \le 80$	70	4	280
$80 < t \le 90$	85	10	850
$90 < t \le 95$	92.5	14	1295
$95 < t \le 100$	97.5	20	1950
$100 < t \le 110$	105	24	2520
$110 < t \le 130$	120	8	960
		$\sum f = 80$	$\sum fx = 7855$

$$\text{mean} = \frac{\sum fx}{\sum f} = \frac{7855}{80}$$

$$= 98.187 \approx 98.2 \text{ minutes} \quad \textbf{Ans.}$$

(d) $P(\text{driver took 95 minutes or less}) = \frac{28}{80}$

$$= \frac{7}{20} \quad \textbf{Ans.}$$

(e) (i) $P(\text{both drivers took more than 100 minutes})$

$$= \frac{32}{80} \times \frac{31}{79}$$

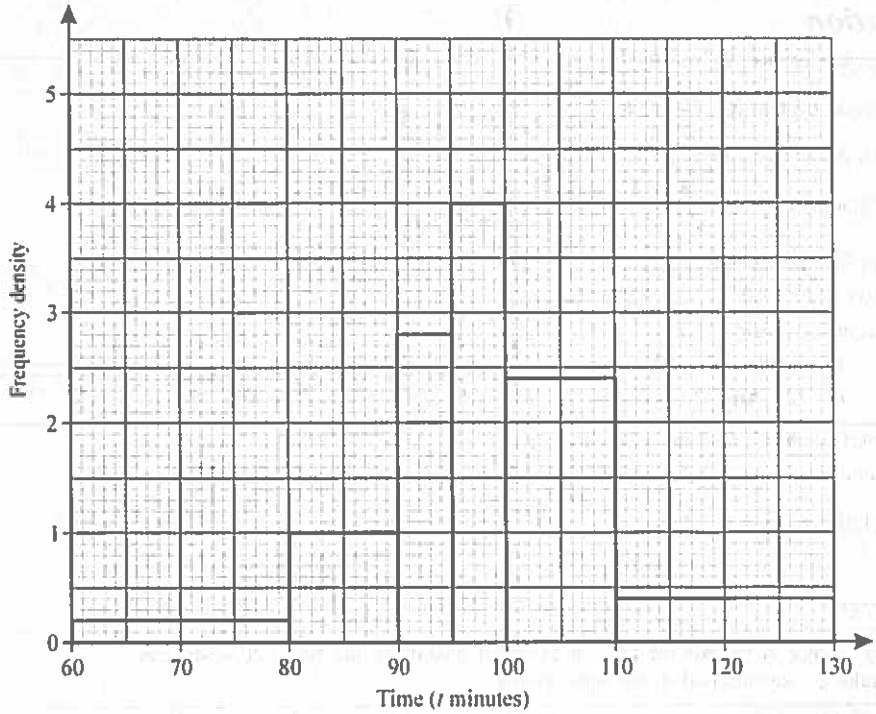
$$= \frac{62}{395} \quad \textbf{Ans.}$$

(ii) $P(\text{one took} \le 80 \text{ min. other took} > 110 \text{ min.})$

$$= \left(\frac{4}{80} \times \frac{8}{79}\right) + \left(\frac{8}{80} \times \frac{4}{79}\right)$$

$$= \frac{2}{395} + \frac{2}{395}$$

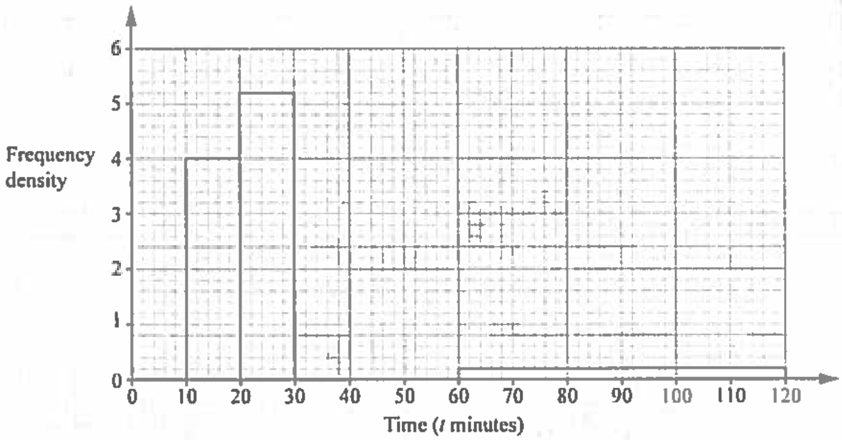
$$= \frac{4}{395} \quad \textbf{Ans.}$$



36 *(J2016/P1/Q23)*

The table and histogram show some information about the times taken by a group of students to travel to school one day.

Time (t minutes)	$0 < t \leq 10$	$10 < t \leq 20$	$20 < t \leq 30$	$30 < t \leq 60$	$60 < t \leq 120$
Frequency	28	40	52	18	m



(a) Complete the histogram. [2]

(b) Find the value of m. [1]

(c) Work out the fraction of students who took more than half an hour to travel to school. [2]

Thinking Process

(a) To complete the histogram ➶ find the frequency density of the required intervals.

(b) To find m ➶ consider the formula: Frequency = freq. density × class width

(c) From the table, express the number of students who took more than 30 minutes as a fraction of the total number of students..

Solution

(a) For the interval $0 < t \le 10$,

frequency density $= \frac{28}{10} = 2.8$

For the interval $30 < t \le 60$,

frequency density $= \frac{18}{30} = 0.6$

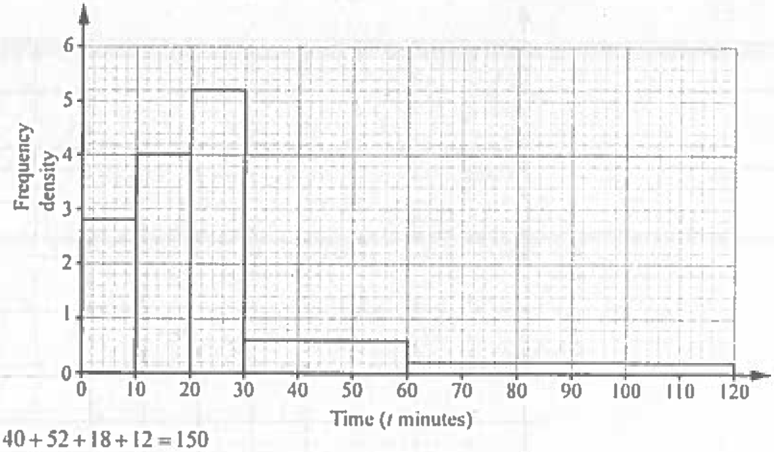
(b) For the interval $60 < t \le 120$,

class width $= 60$,

frequency density $= 0.2$

$\therefore$ Frequency $= 0.2 \times 60 = 12$

$\Rightarrow$ $m = 12$ **Ans.**



(c) Total number of students $= 28 + 40 + 52 + 18 + 12 = 150$

Number of students who took more than 30 minutes $= 18 + 12 = 30$

Fraction $= \frac{30}{150} = \frac{1}{5}$ **Ans.**

---

37 *(J2016/P2/Q7)*

One day, garage A records the amount of petrol bought by the first 120 customers. The results are summarised in the table below.

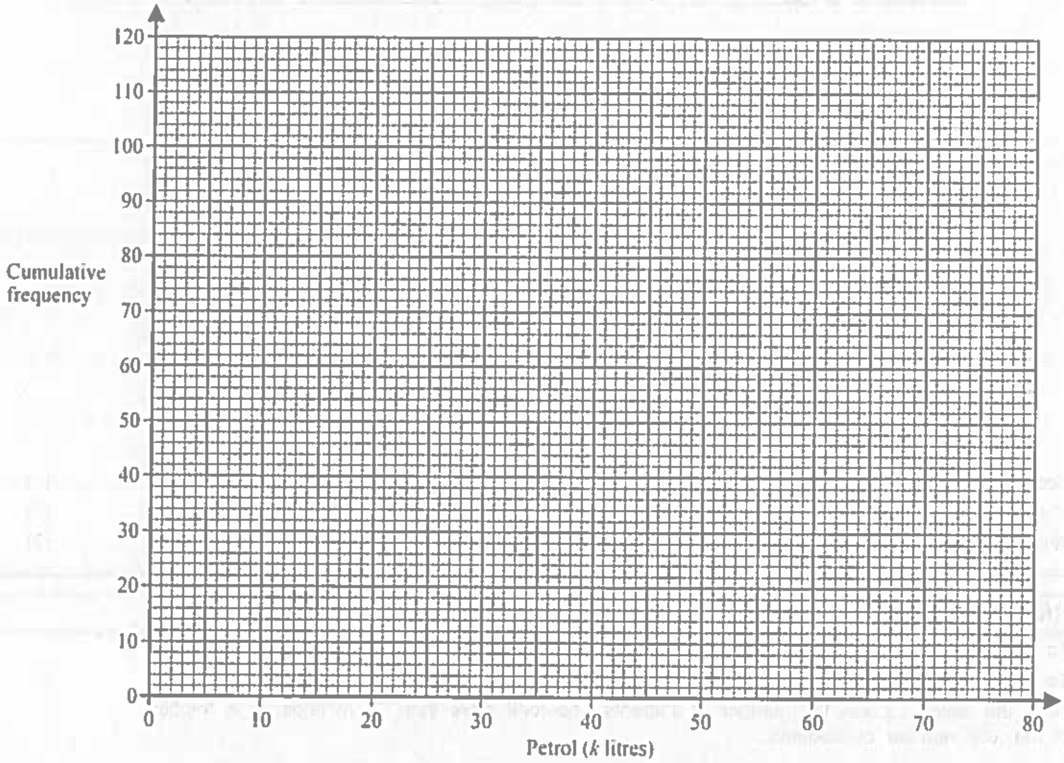
Petrol (k litres)	$0 < k \le 10$	$10 < k \le 20$	$20 < k \le 30$	$30 < k \le 40$	$40 < k \le 50$	$50 < k \le 60$	$60 < k \le 70$	$70 < k \le 80$
Number of customers	9	13	36	30	16	9	5	2

(a) Complete the cumulative frequency table below

Petrol (k litres)	$k \le 10$	$k \le 20$	$k \le 30$	$k \le 40$	$k \le 50$	$k \le 60$	$k \le 70$	$k \le 80$
Cumulative frequency	9	22						120

[1]

(b) On the grid below, draw a cumulative frequency curve to represent this data. [3]



(c) Use your graph to estimate

(i) the median, [1]

(ii) the 90th percentile of the distribution. [1]

(d) On the same day, garage B also recorded the amount of petrol bought by its first 120 customers. The results are summarised below.

6 customers bought 10 litres or less.
The most petrol bought by any customer was 60 litres.
The median amount of petrol bought was 34 litres.
The lower quartile of the distribution was 25 litres.
The interquartile range of the distribution was 19 litres.

Draw the cumulative frequency curve for garage B on the grid on the previous page. [3]

(e) Petrol is priced at $2.60 per litre at both garages.
Garage A offers a gift to customers who buy over 35 litres.
Garage B offers a gift to customers who spend over $104.
Use your graphs to estimate the number of these customers offered a gift at each garage and complete the sentence below. Show your working.

Garage .... offers a gift to .... more customers than garage .... [3]

Thinking Process

(a) Use frequency table to complete the cumulative frequency table.

(c) (i) Find 50% of total frequency and read the corresponding value for litres.

(ii) Find 90% of total frequency. Mark the value on the graph and read the corresponding value for litres.

(d) Plot the points using the information given, and draw a cumulative frequency curve on the same graph.

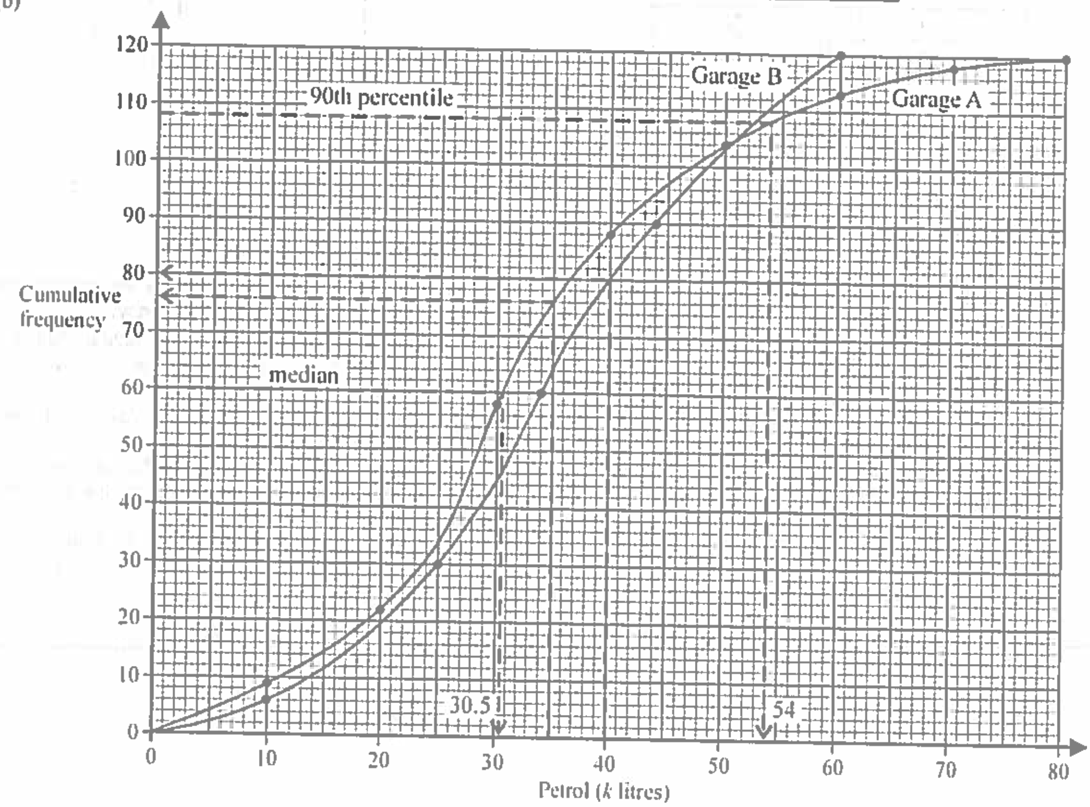
(e) For Garage A, find the number of customers who bought more than 35 litres.
For Garage B, find the number of litres over which gift is offered, then use the graph to find the number of customers who were offered gift.

Solution

(a)

Petrol (k litres)	$k \le 10$	$k \le 20$	$k \le 30$	$k \le 40$	$k \le 50$	$k \le 60$	$k \le 70$	$k \le 80$
Cumulative frequency	9	22	58	88	104	113	118	120

(b)



(c) (i) Median = 30.5 litres **Ans.**

(ii) $\frac{90}{100} \times 120 = 108$

∴ from graph, 90th percentile = 54 litres. **Ans.**

(d) Refer to graph.

(e) Garage A: From graph, number of customers who buy 35 litres = 76

∴ number of customers who were offered gift = 120 − 76 = 44

Garage B: $\frac{\$104}{\$2.60} = 40$ litres

From graph, number of customers who buy 40 litres = 80

∴ number of customers who were offered gift = 120 − 80 = 40

∴ Garage A offers a gift to 4 more customers than garage B. **Ans.**

38 *(J2016/P2 Q3)*

Steven asked 25 women how many children they have.
The results are summarised in the table below.

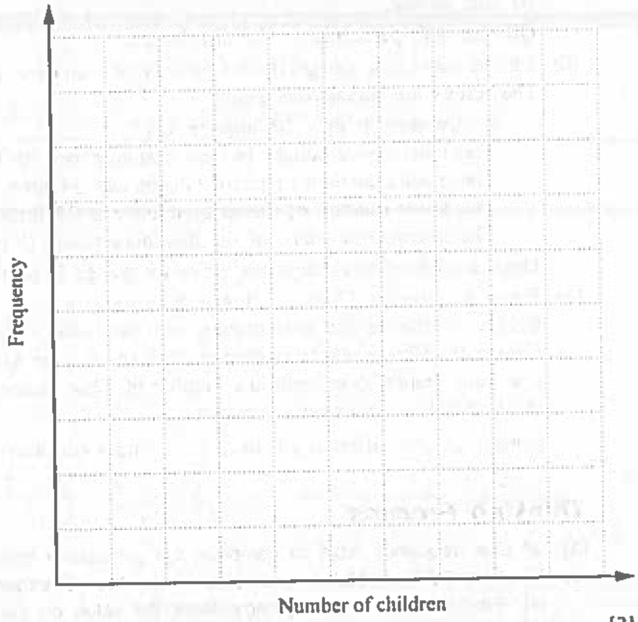
Number of children	Frequency
0	7
1	5
2	6
3	4
4	3

(a) Find
(i) the mean, [2]
(ii) the median, [1]
(iii) the mode. [1]

(b) Steven says that the mode is the average that best represents the data.
Explain why Steven is wrong. [1]

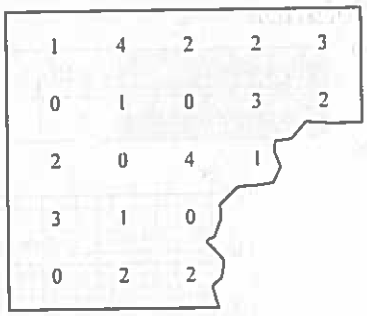
(c) Steven chooses two women at random from the group. Calculate the probability that both of them have just one child.
Give your answer as a fraction in its simplest form. [2]

(d) Draw a bar chart to represent this data.



[2]

(e) Steven shows Frank the paper on which he recorded the data from his survey.
Part of the paper has been torn.



Which five numbers are missing from the paper? [1]

Thinking Process

(a) (i) To find mean ⇒ divide the total number of children by total frequency.

(ii) Median is the data in the middle position. Since there are 25 women, the 13th woman gives the median reading.

(b) Take note that mode is the value with the highest frequency.

(c) Note that first woman is chosen from 25, and second woman is chosen from the remaining 24 women.

(e) To find the missing numbers ⇒ compare the data on paper with the data given in the table.

Solution

(a) (i) Mean

$= \frac{(0\times 7)+(1\times 5)+(2\times 6)+(3\times 4)+(4\times 3)}{25}$

$= \frac{41}{25} = 1.64$ **Ans.**

(ii) The median is at the $\frac{25+1}{2} = 13^{th}$ position.

$\therefore$ median = 2 **Ans.**

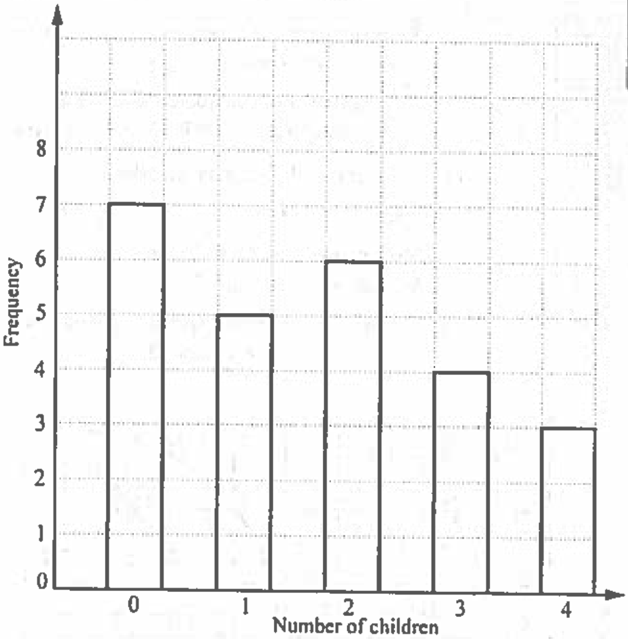
(iii) Mode = 0 **Ans.**

(b) Mode is the value that occurs most. Mode does not give the average or the central values that describes the data.

(c) $P(\text{both women have one child}) = \frac{5}{25}\times\frac{4}{24}$

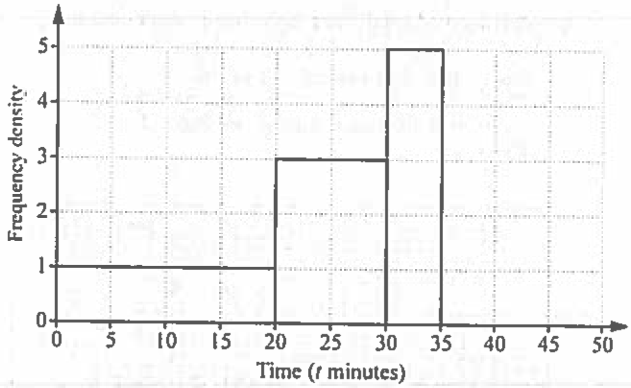
$= \frac{1}{30}$ **Ans.**

(d)



(e) Comparing the data with the given table, the missing numbers are : 0, 0, 1, 3, 4. **Ans.**

39 (N2016/P1/Q9)



The diagram shows part of the histogram which represents the distribution of times taken by some people to travel to work.

(a) Complete the table.

Time (t minutes)	$0 < t \le 20$	$20 < t \le 30$	$30 < t \le 35$	$35 < t \le 50$
Frequency		30		30

[2]

(b) Complete the histogram [1]

Thinking Process

(a) To complete the table ✎ consider the formula: Frequency = freq. density × class width.

(ii) To complete the histogram ✎ find the frequency density of the required interval.

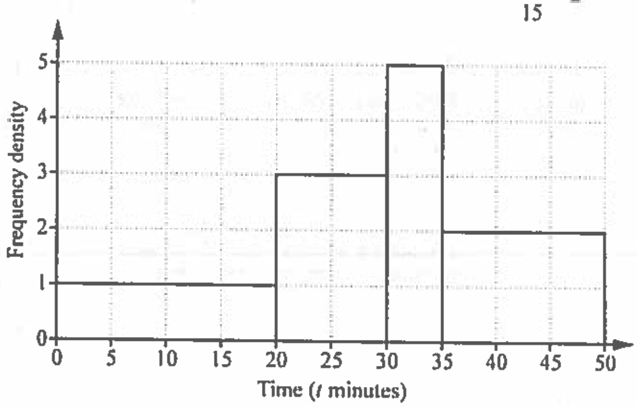
Solution

(a) For the interval $0 < t \le 20$,
class width = 20, frequency density = 1
$\therefore$ Frequency = $1\times 20 = 20$ **Ans.**

For the interval $30 < t \le 35$,
class width = 5, frequency density = 5
$\therefore$ Frequency = $5\times 5 = 25$ **Ans.**

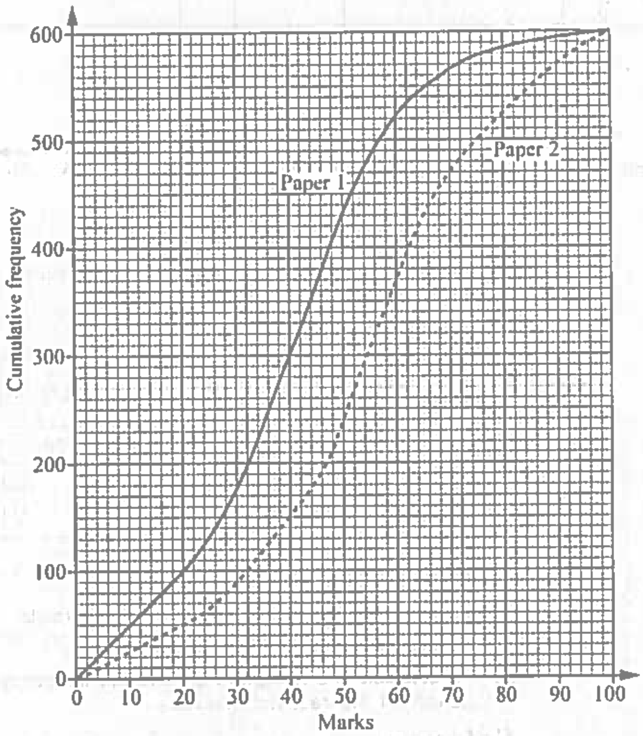
(b) Frequency density from $35 < t \le 50 = \frac{\text{frequency}}{\text{width}}$

$= \frac{30}{15} = 2$



40 *(N2016 P2 Q11 a)*

(a) Six hundred candidates took a mathematics examination which consisted of two papers.
Each paper was marked out of 100.
The diagram shows, on the same grid, the cumulative frequency curves for Paper 1 and Paper 2.



(i) Use the cumulative frequency curve for **Paper 1** to find an estimate of

(a) the median, [1]

(b) the interquartile range, [2]

(c) the number of candidates who scored more than 45. [1]

(ii) A candidate scored 60 on Paper 1. Using both graphs, estimate this candidate's mark on Paper 2. [1]

(iii) State, with a reason, which you think was the more difficult paper. [1]

Thinking Process

(a) (i) (a) Find 50% of total frequency and take reading of the corresponding value for the marks.

(b) Interquartile range = upper quartile − lower quartile.

(c) From graph, find the number of candidates who scored 45 marks. Subtract it from total frequency.

(ii) Mark the cumulative frequency that corresponds to 60 marks. Read the marks of paper 2 at this frequency.

(iii) The test with higher median mark is the easier test.

Solution

(a) (i) (a) Median marks = 40 **Ans.**

(b) Interquartile range = 52 − 27 = 25 **Ans.**

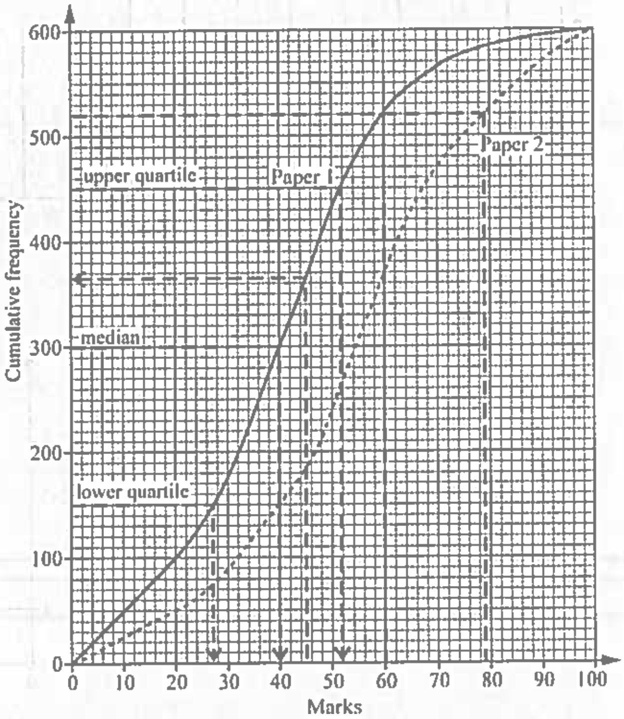
(c) From graph, the number of students who scored 45 marks = 365

∴ number of candidates who scored more than 45 = 600 − 365 = 235 **Ans.**

(ii) From graph, the candidate marks in Paper 2 = 79 **Ans.**

(iii) Median mark for Paper 1 = 40
Median mark for Paper 2 = 55

∴ Paper 1 was more difficult as the median mark is lower than paper 2.



41 *(N2016/P1/Q12)*

A school recorded the number of absent students over a 50-day period.
The results are given in the table.

Number of absent students	0	1	2	3	4	5 or more
Number of days	25	15	6	3	1	0

(a) Write down the mode. [1]
(b) Calculate the mean. [2]

Thinking Process

(a) Look for the number of absent students with highest frequency.
(b) To find the mean divide the total number of absent students by the total frequency.

Solution

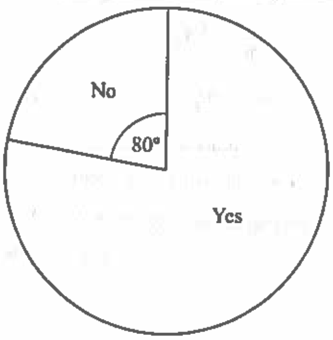
(a) Mode = 0 **Ans.**

(b) $\text{Mean} = \dfrac{(0\times 25)+(1\times 15)+(2\times 6)+(3\times 3)+(4\times 1)}{50}$

$= \dfrac{0+15+12+9+4}{50}$

$= \dfrac{40}{50} = 0.8$ **Ans.**

42 *(J2017/P1/Q4)*



A group of students were asked if they wanted a later start to the school day.
The pie chart summarises the results.
200 students said no.
Work out the number of students who said yes. [2]

Thinking Process

Find the angle that represents number of students who said yes.

Solution

Angle representing students who said yes
$= 360° - 80° = 280°$

80° represents — 200 students

280° represents — $\dfrac{200}{80} \times 280 = 700$ students

∴ 700 students said yes. **Ans.**

43 *(J2017/P1/Q10)*

(a) Here are the masses, in grams, of 8 apples.

189 175 185 192 202 161 174 196

Find the median mass. [1]

(b) A bag contains 5 carrots.
The mean mass of the carrots is 60 g.
Another carrot is added to the bag.
The mean mass of the 6 carrots is 65 g.
Work out the mass of the carrot added to the bag. [2]

Thinking Process

(a) Find the middle value. Write the given masses in increasing order.

(b) $\text{Mean} = \dfrac{\text{total mass of 6 carrots}}{\text{total number of carrots}}$.

Solution

(a) Writing the masses in increasing order.

161 174 175 185 189 192 196 202

middle positions are 4th and 5th

∴ median mass $= \dfrac{185+189}{2} = 187$ g **Ans.**

(b) Total mass of 5 carrots $= 60 \times 5 = 300$ g

Mean mass of 6 carrots $= \dfrac{\text{total mass of 6 carrots}}{\text{total number of carrots}}$

$65 = \dfrac{300 + x}{6}$

$300 + x = 390$

$x = 90$

∴ mass of 6th carrot = 90 g **Ans.**

44 *(J2017 P2 Q11)*

(a) The table below summarises the times taken by 50 athletes to run 400 m.

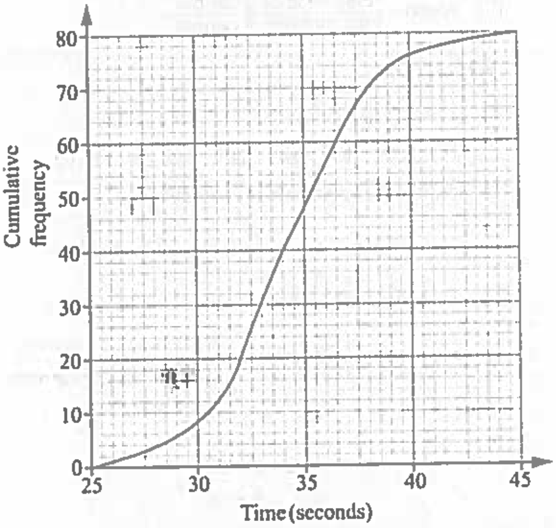
Time (t seconds)	$50 \le t < 55$	$55 \le t < 60$	$60 \le t < 65$	$65 \le t < 70$	$70 \le t < 75$
Frequency	7	16	15	11	1

(i) State the modal class. [1]

(ii) Calculate an estimate of the mean time taken by these athletes. [3]

(iii) Calculate the probability that an athlete chosen at random took less than 60 seconds to run the 400 m. [2]

(b) The cumulative frequency curve summarises the times taken by 80 boys to run 200 m.



(i) Find the median time. [1]

(ii) Find the interquartile range. [2]

(iii) 60 girls also ran 200 m.
The girl who took the longest time ran 200 m in 40 seconds.
The girl who took the shortest time ran 200 m in 28 seconds.
The lower quartile for the boys and the girls is the same.
The interquartile range for the girls is 4 seconds.
Draw the cumulative frequency curve on the grid above. [3]

Thinking Process

(a) (i) Look for the class which has the highest frequency.

(ii) Use mean $= \frac{\sum fx}{\sum f}$ ✍ Compute the mid-point (x) of each interval.

(iii) By referring to table, express the number of athletes who took less than 60 seconds as a fraction of total athletes.

(b) (i) To find median ✍ find the time when cumulative frequency = 40.

(ii) Interquartile range = upper quartile − lower quartile.

(iii) Using the given information, plot the points and draw a cumulative frequency curve on the same graph.

Solution

(a) (i) Modal class = $55 \le t < 60$ **Ans.**

(ii)

Time (t seconds)	Midpoint, x	Frequency f	Product, fx
$50 \le t < 55$	52.5	7	367.5
$55 \le t < 60$	57.5	16	920
$60 \le t < 65$	62.5	15	937.5
$65 \le t < 70$	67.5	11	742.5
$70 \le t < 75$	72.5	1	72.5
		$\sum f = 50$	$\sum fx = 3040$

Mean $= \frac{\sum fx}{\sum f} = \frac{3040}{50}$

$= 60.8$ seconds **Ans.**

(iii) No. of athletes who took less than 60 seconds $= 7 + 16 = 23$

$\therefore$ P(an athlete took less than 60 s)

$= \frac{23}{50}$ **Ans.**

(b) (i) Median time = 34 seconds. **Ans.**

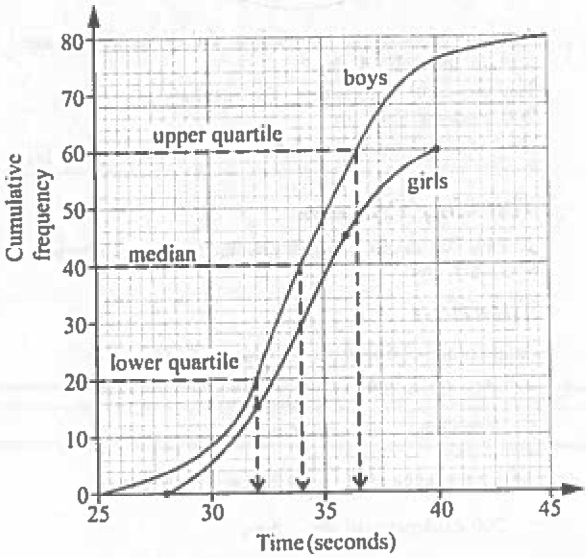
(ii) Upper quartile = 36.5 seconds

Lower quartile = 32 seconds

Interquartile range $= 36.5 - 32$

$= 4.5$ seconds. **Ans.**

(iii)



45 (N2017/P1/Q2)

The masses, in kilograms, of 20 parcels sent by a dispatch centre are given in the table.

4.2	5.3	5.1	7.8	8.2	7.5	3.2	5.7	4.1	5.9
8.4	5.6	8.0	3.2	4.8	6.9	6.2	3.2	5.4	4.7

(a) By using tally marks, or otherwise, complete the grouped frequency distribution for these masses.

Mass (m kilograms)	Tally marks	Frequency
$3 < m \leq 5$		
$5 < m \leq 7$		
$7 < m \leq 9$		

[1]

(b) The results are to be shown in a pie chart. Calculate the angle of the sector representing the group with the smallest frequency. [1]

Thinking Process

(a) Construct a tally table.
(b) 360° represents 20 parcels. Find the angle that represents least number of parcels.

Solution

(a)

Mass (m kilograms)	Tally marks	Frequency
$3 < m \leq 5$	𝍸 //	7
$5 < m \leq 7$	𝍸 ///	8
$7 < m \leq 9$	𝍸	5

(b) $\frac{5}{20} \times 360° = 90°$ **Ans.**

46 (N2017/P1/Q8)

Thirty students were asked on how many days they ate pasta last week.
The results are given in the table.

Number of days	0	1	2	3	4	5
Frequency	9	6	7	4	2	2

(a) Find the mode. [1]
(b) Find the median. [1]

Thinking Process

(a) Look for the number of days that have the highest frequency.
(b) Look for the number of days in the middle position.

Solution

(a) Mode = 0 **Ans.**

(b) Middle positions are 15th and 16th.

$\therefore$ median $= \frac{1+2}{2} = 1.5$ **Ans.**

47 (N2017/P1/Q11)

The mean age of Ali, Ben and Chris is 14 years 3 months.
Dai's age is 15 years and 3 months.
Calculate the mean age of the four people. [2]

Thinking Process

To find the mean age find the total sum of ages of four people and divide it by 4.

Solution

Mean age of Ali, Ben and Chris $= (14 \times 12) + 3$
$= 171$ months

Dai's age $= (15 \times 12) + 3 = 183$ months

$\therefore$ mean age of the four people

$= \frac{\text{sum of ages of four people}}{\text{total number of people}}$

$= \frac{3(171) + 183}{4}$

$= \frac{696}{4} = 174$ months

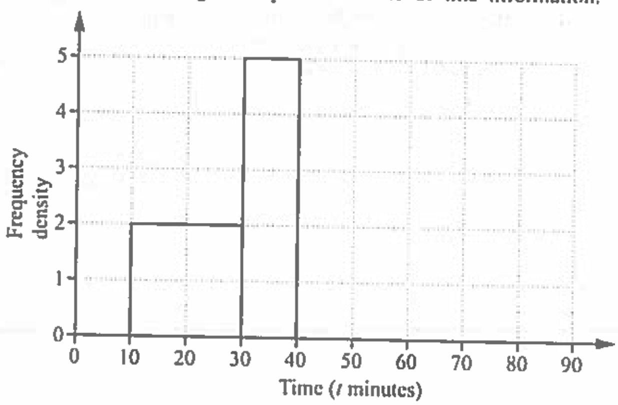
$= 14$ years 6 months **Ans.**

48 (N2017/P1/Q13)

The distribution of the lengths of time spent on the internet on a Monday by each member of a group of students is given in the table.

Time (t minutes)	$10 < t \leq 30$	$30 < t \leq 40$	$40 < t \leq 50$	$50 < t \leq 80$
Frequency	k	50	30	30

The histogram represents some of this information.

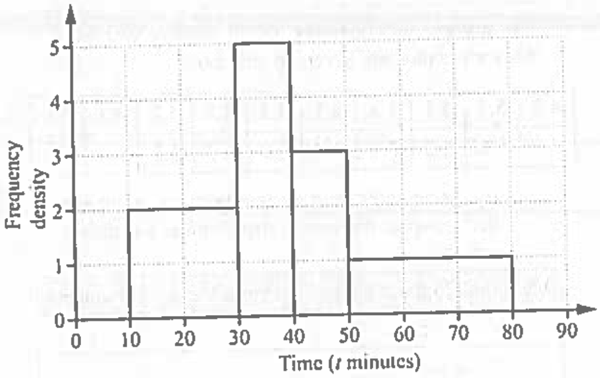


(a) Find k. [1]
(b) Complete the histogram. [2]

Thinking Process

(a) To find k ✐ consider the formula,
Frequency = freq. density × class width

(b) To complete the histogram ✐ Find the frequency densities of last two columns.



Solution

(a) From histogram, for the interval $10 < t \leq 30$,

frequency = frequency density × class width

$\Rightarrow \quad k = 2 \times 20 = 40$ **Ans.**

(b) For interval $40 < t \leq 50$, freq. density $= \dfrac{30}{10} = 3$

For interval $50 < t \leq 80$, freq. density $= \dfrac{30}{30} = 1$

49 *(N2017/P2 Q2)*

A company asked their employees how long they took to travel to work one day. The table summarises the times for 120 employees.

Time (t minutes)	$0 < t \leq 20$	$20 < t \leq 40$	$40 < t \leq 60$	$60 < t \leq 80$	$80 < t \leq 100$
Frequency	12	28	45	22	13

(a) (i) Complete the cumulative frequency table below. [1]

Time (t minutes)	$t \leq 0$	$t \leq 20$	$t \leq 40$	$t \leq 60$	$t \leq 80$	$t \leq 100$
Cumulative frequency	0					120

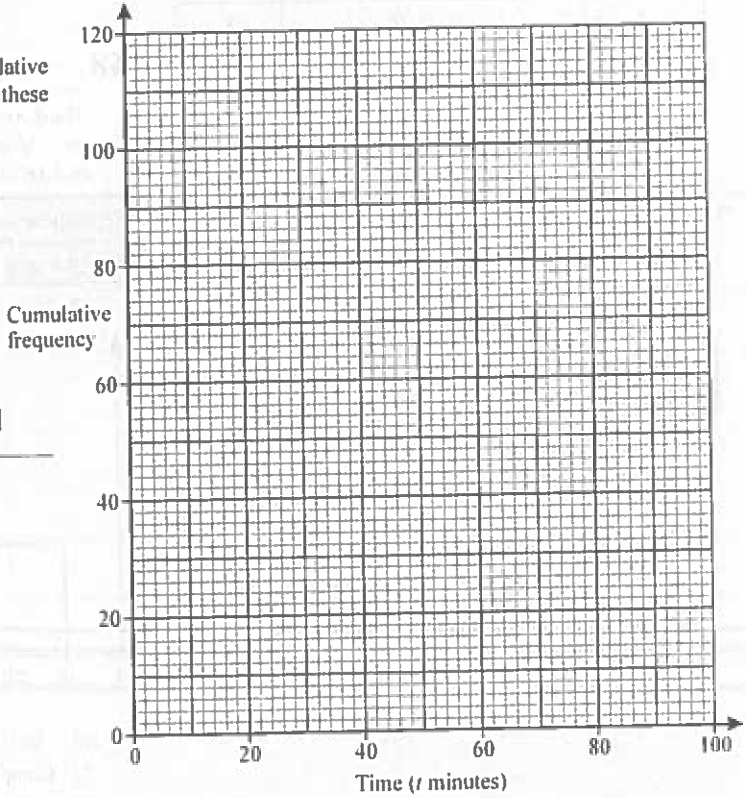
(ii) On the grid, draw a smooth cumulative frequency curve to represent these results. [2]

(b) Use your curve to estimate

(i) the median time, [1]

(ii) the interquartile range of the times. [2]

(c) Calculate an estimate of the mean time taken for the employees to travel to work. [3]



Thinking Process

(a) (i) Accumulate the values in the first table to fill in the blanks in the second table.

(ii) Draw the cumulative curve according to values in the second table.

(b) (i) Find 50% of the total frequency. Mark the value on the graph and read for x-values.

(ii) Find 25% and 75% of total frequency. Mark the two values on the graph. Read the corresponding x-values and find their difference.

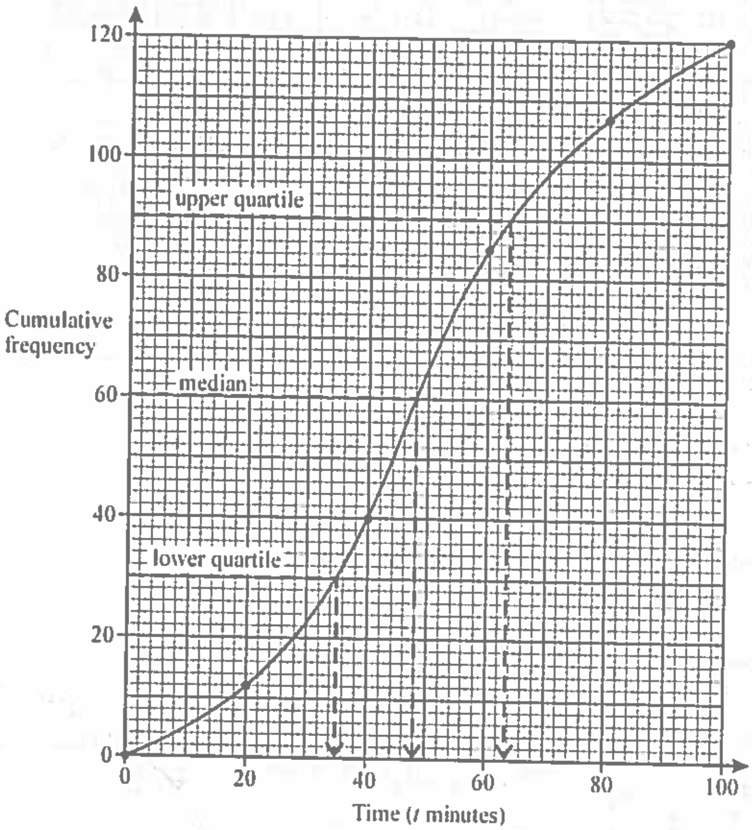
(c) Use mean $= \frac{\sum fx}{\sum f}$ ✎ Compute the mid-point (x) of each interval.

Solution

(a) (i)

Time (t minutes)	$t \le 0$	$t \le 20$	$t \le 40$	$t \le 60$	$t \le 80$	$t \le 100$
Cumulative frequency	0	12	40	85	107	120

(ii)



(b) (i) Median = 48 minutes. **Ans.**

(ii) Inter quartile range

= upper quartile − lower quartile

$= 63.5 - 35 = 28.5$ minutes. **Ans.**

(c)

Time (t minutes)	Midpoint, x	Frequency f	Product, fx
$0 < t \le 20$	10	12	120
$20 < t \le 40$	30	28	840
$40 < t \le 60$	50	45	2250
$60 < t \le 80$	70	22	1540
$80 < t \le 100$	90	13	1170
		$\sum f = 120$	$\sum fx = 5920$

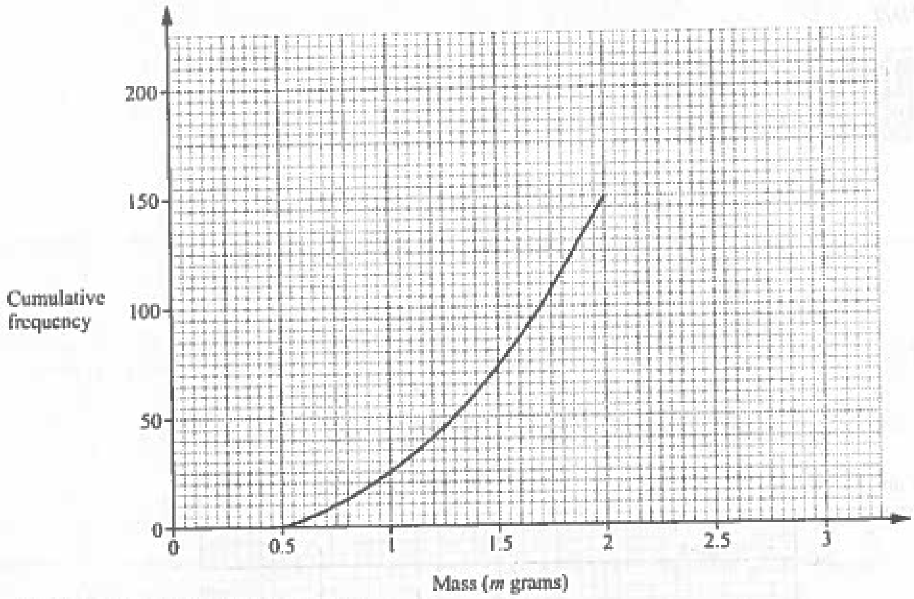
$$\text{mean} = \frac{\sum fx}{\sum f} = \frac{5920}{120}$$

$= 49.3$ minutes **Ans.**

50 *(J2018 P1 Q16)*

The masses of 200 beetles were measured.
The results are summarised in the cumulative frequency table and part of the cumulative frequency curve is drawn.

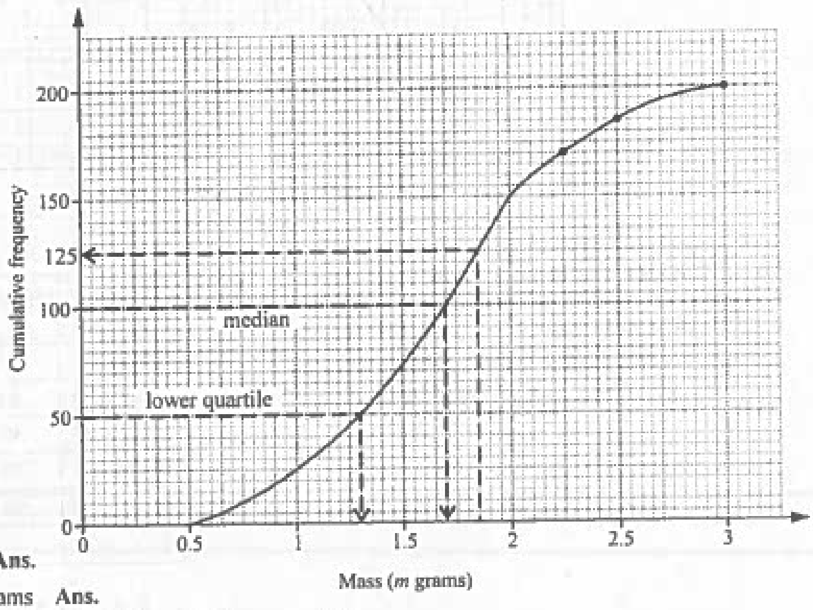
Mass (m grams)	$m \le 0.5$	$m \le 1$	$m \le 1.5$	$m \le 2$	$m \le 2.25$	$m \le 2.5$	$m \le 3$
Cumulative frequency	0	25	75	150	170	185	200



(a) Complete the cumulative frequency curve. [1]
(b) Use the curve to find an estimate for
 (i) the median, [1]
 (ii) the lower quartile, [1]
 (iii) the number of beetles that have a mass greater than 1.85 grams. [2]

Thinking Process

(a) Complete the curve using the remaining points in the given table.
(b) (i) Find 50% of total frequency and read the corresponding value for the mass.
 (ii) Find 25% of the total frequency and read the corresponding value for the mass.
 (iii) From graph, find the number of beetles that corresponds to 1.85 grams. Subtract it from 200.



Solution

(a) Refer to graph.
(b) (i) Median = 1.7 grams **Ans.**
 (ii) Lower quartile = 1.3 grams **Ans.**
 (iii) From graph, the number of beetles with mass 1.85 grams = 125
 $\therefore$ number of beetles with mass greater than 1.85 grams = 200 − 125 = 75 **Ans.**

51 (J2018/P1/Q13)

In a school of 270 children, the distance each child can swim was recorded.
The distances are summarised in the table.

Distance (d metres)	$0 \le d < 100$	$100 \le d < 200$	$200 \le d < 500$	$500 \le d < 1000$
Number of children	110	50	60	50
Frequency density				

(a) Complete the table to show the frequency densities. [2]
(b) Calculate an estimate for the number of children who could swim more than 400 metres. [1]

Thinking Process

(a) Use, Frequency density $= \dfrac{\text{frequency}}{\text{class width}}$

(b) To Calculate an estimate find the number of children in the interval $400 \le d < 500$.

Solution

(a)

Distance (d metres)	$0 \le d < 100$	$100 \le d < 200$	$200 \le d < 500$	$500 \le d < 1000$
Number of children	110	50	60	50
Frequency density	**1.1**	**0.5**	**0.2**	**0.1**

(b) For interval $400 \le d < 500$, frequency $= 0.2 \times 100 = 20$

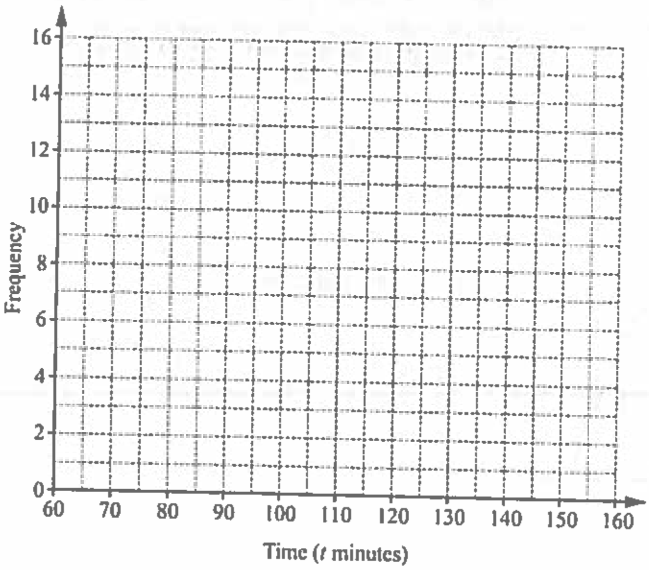
$\therefore$ Number of children who could swim more than 400 metres $= 20 + 50 = 70$ **Ans.**

52 (J2018/P2/Q2)

(a) Jenny recorded the time, in minutes, of 40 movies.
The table summarises her results.

Time (t minutes)	$60 < t \le 80$	$80 < t \le 100$	$100 < t \le 120$	$120 < t \le 140$	$140 < t \le 160$
Frequency	2	7	15	11	5

On the grid, draw a frequency polygon to represent this information. [2]



(b) Jenny asked 60 people how many movies they had each watched in the last month.
The table summarises her results.

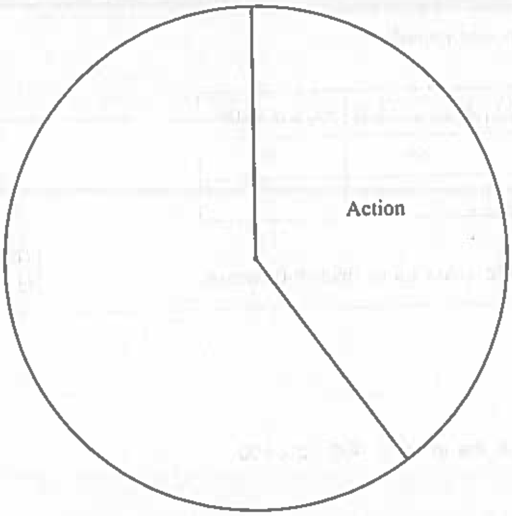
Number of movies	0	1	2	3	4	5	6
Frequency	p	14	15	7	q	5	2

The mean number of movies watched is 2.3 .
Find the value of p and the value of q. [3]

(c) Jenny also asked which type of movie each of the 60 people preferred.
The table summarises her results.

Type of movie	Action	Comedy	Drama	Horror
Frequency	24	15	9	12

(i) Complete the pie chart to represent the results.



[3]

(ii) One of the 60 people is chosen at random. Find the probability that this person preferred drama or horror movies. [1]

(iii) Two of the 60 people are chosen at random. Calculate the probability that they both preferred comedy movies. [2]

Thinking Process

(a) To draw a frequency polygon ✎ plot each frequency against the mid-value of the class interval.

(b) Equate total frequency to 60 and form an equation. Equate mean to 2.3 and form another equation. Solve for p and q.

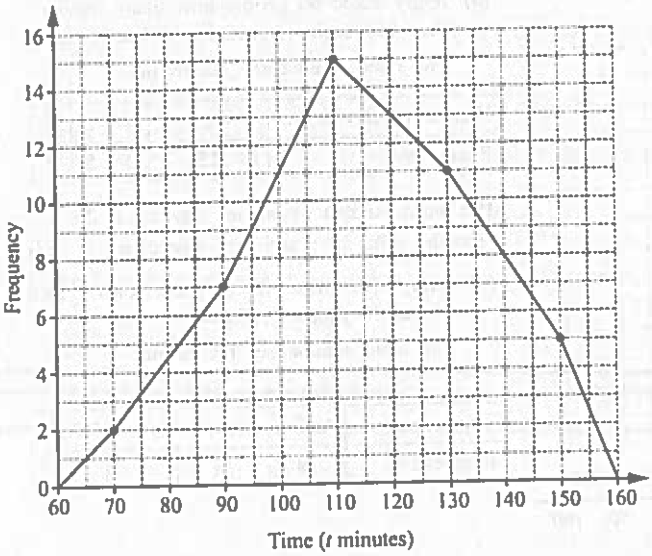
(c) (i) To complete the pie chart ✎ calculate the angle representing each type of movie.

(ii) Find P(drama) + P(horror).

(iii) Find P(movie) × P(movie).

Solution

(a)



(b) Total frequency = 60

$\Rightarrow p+14+15+7+q+5+2=60$

$p+q+43=60$

$p+q=17$ ………(1)

$\text{Mean} = \dfrac{0+14+30+21+4q+25+12}{60}$

$\Rightarrow \dfrac{0+14+30+21+4q+25+12}{60}=2.3$

$102+4q=138$

$4q=36$

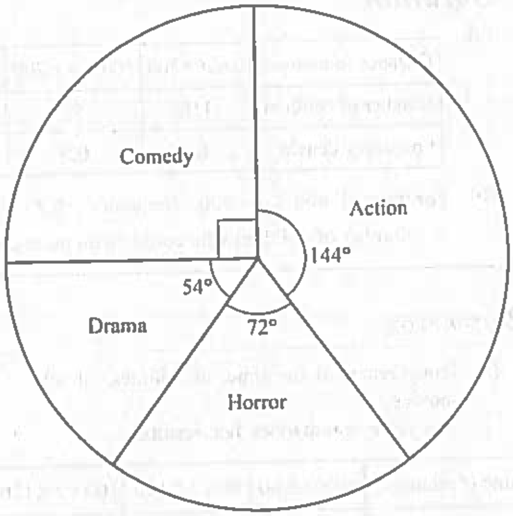
$q=9$

substitute $q=9$ into (1).

$p+9=17 \Rightarrow p=8.$

$\therefore \quad p=8, \quad q=9$ **Ans.**

(c) (i)



Angle represented by Comedy $= \dfrac{15}{60}\times 360° = 90°$

Angle represented by Drama $= \dfrac{9}{60}\times 360° = 54°$

Angle represented by Horror $= \dfrac{12}{60}\times 360° = 72°$

(ii) $P(\text{drama or horror}) = \dfrac{9}{60}+\dfrac{12}{60}$

$= \dfrac{7}{20}$ **Ans.**

(iii) $P(\text{both prefer comedy}) = \dfrac{15}{60}\times\dfrac{14}{59}$

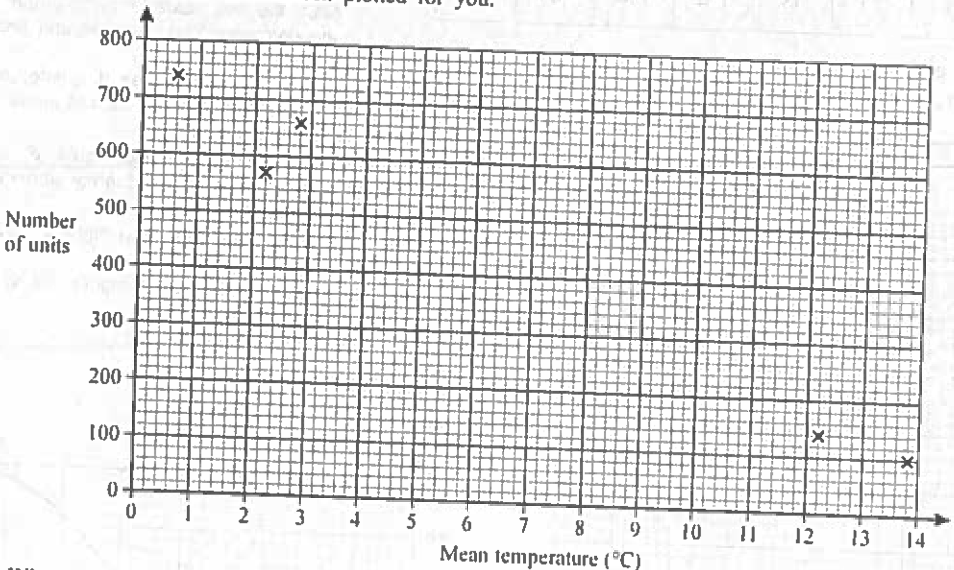
$= \dfrac{7}{118}$ **Ans.**

53 *(N2018 P1 Q15)*

Each week, Henri records the number of units of gas used in his house and the mean temperature outside. Ten of his results are shown in the table.

Mean temperature (°C)	12.2	13.8	0.6	2.2	2.8	4.4	5.6	6.8	9.0	10.6
Number of units	140	100	740	570	660	600	500	560	410	320

(a) On the grid, complete the scatter diagram.
The first five results have been plotted for you.



[2]

(b) What type of correlation does your scatter diagram show? [1]

(c) Draw a line of best fit on the grid. [1]

(d) Use your line of best fit to estimate the number of units used for one week when the mean temperature outside is 7.6 °C. [1]

Thinking Process

(a) Plot the points given in the table.
(b) Observe that the diagram shows negative correlation.
(d) From graph, find the number of units that corresponds to 7.6 °C.

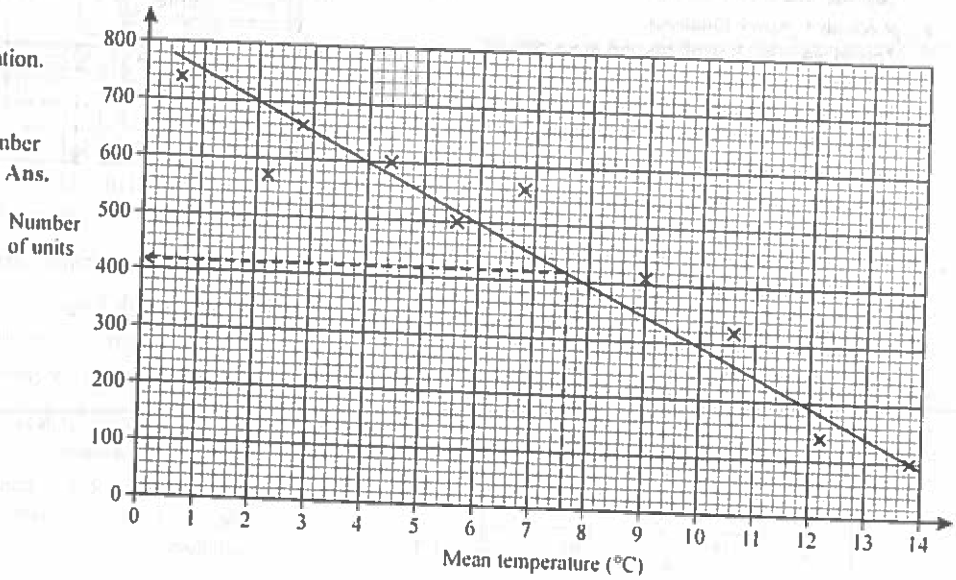
Solution

(a) Refer to graph.

(b) Negative correlation.

(c) Refer to graph.

(d) From graph, number of units = 418 Ans.

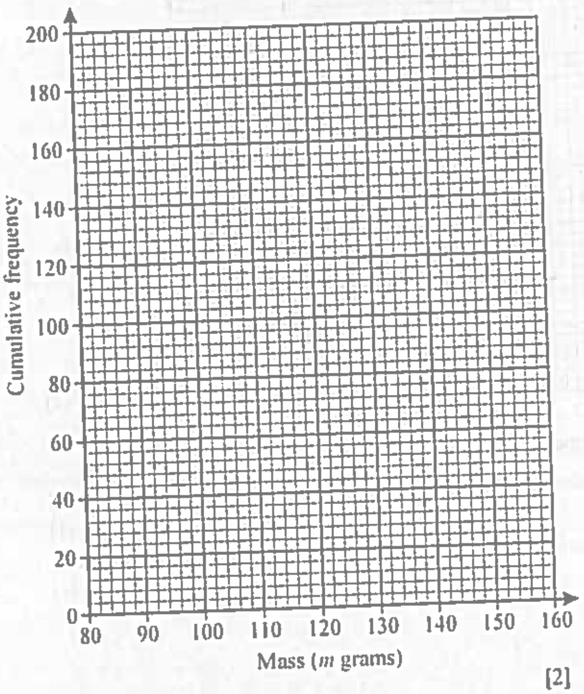


54 *(N2018/P2 Q2)*

Lim grows tomatoes. The masses, m grams, of 200 of her tomatoes are recorded.
The cumulative frequency table shows the results.

Mass (m grams)	$m \le 80$	$m \le 100$	$m \le 110$	$m \le 120$	$m \le 130$	$m \le 140$	$m \le 160$
Cumulative frequency	0	20	48	112	158	184	200

(a) On the grid, draw a cumulative frequency diagram to represent these results.



[2]

(b) Use your diagram to estimate
(i) the median, [1]
(ii) the interquartile range. [2]

(c) Ravi also grows tomatoes.
The masses of 200 of his tomatoes are also recorded.
For Ravi's tomatoes, the median mass was 124 g and the interquartile range of the masses was 12 g.
Make two comments comparing the masses of tomatoes grown by Lim with those grown by Ravi. [2]

(d) **(i)** Complete the frequency table for the masses of tomatoes grown by Lim.

Mass (m grams)	Frequency
$80 < m \le 100$	20
$100 < m \le 110$	28
$110 < m \le 120$	64
$120 < m \le 130$	
$130 < m \le 140$	
$140 < m \le 160$	16

[1]

(ii) Write down the modal class. [1]

(iii) Calculate an estimate for the mean mass of these tomatoes. [3]

Thinking Process

(b) (i) Find 50% of the total frequency. Mark the value on the graph and read for x-value.

(ii) Find 25% and 75% of total frequency. Mark the two values on the graph. Read the corresponding x-values and find their difference.

(c) Note that Ravi's tomatoes has a greater median mass and a shorter interquartile range as compared to Lim's tomatoes

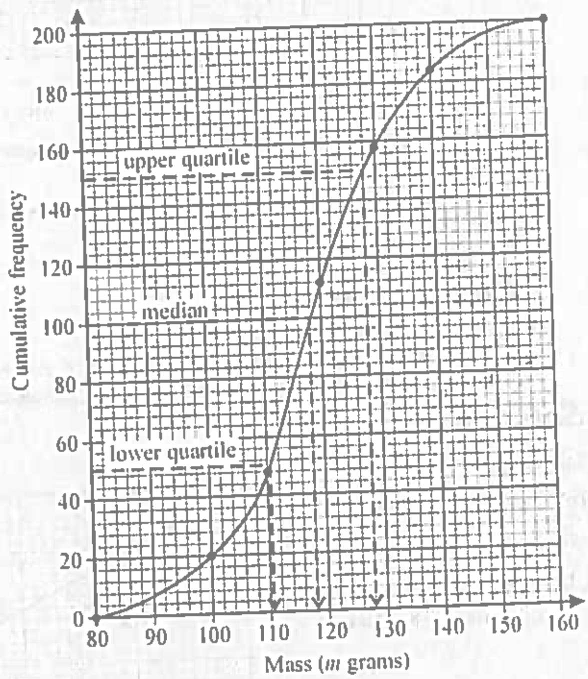
(d) (i) To complete the frequency table ✐ use the relationship between frequency and cumulative frequency.

(ii) ✐ Look for the class with highest frequency.

(iii) Use mean $= \frac{\sum fx}{\sum f}$ ✐ Compute the mid-point (x) of each interval.

Solution

(a)



(b) (i) Median = 118 grams. **Ans.**

(ii) Inter quartile range
= upper quartile − lower quartile
= 128 − 110.5 = 17.5 grams. **Ans.**

(c) 1. The average mass of Ravi's tomatoes is higher than Lim's tomatoes.

2. The masses of Ravi's tomatoes are more consistent than the masses of Lim's tomatoes.

(d) (i)

Mass (m grams)	Frequency
$80 < m \leq 100$	20
$100 < m \leq 110$	28
$110 < m \leq 120$	64
$120 < m \leq 130$	46
$130 < m \leq 140$	26
$140 < m \leq 160$	16

(ii) Modal class: $110 < m \leq 120$ **Ans.**

(iii)

Mass (m grams)	Midpoint, x	Frequency f	Product, fx
$80 < m \leq 100$	90	20	1800
$100 < m \leq 110$	105	28	2940
$110 < m \leq 120$	115	64	7360
$120 < m \leq 130$	125	46	5750
$130 < m \leq 140$	135	26	3510
$140 < m \leq 160$	150	16	2400
		$\sum f = 200$	$\sum fx = 23\,760$

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

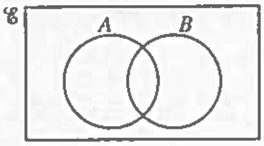
$$= \frac{23760}{200} = 118.8 \text{ grams} \quad \textbf{Ans.}$$

Topic 20

Sets and Venn Diagrams

1 *(J2007 P1/Q9)*

(a) The sets A and B are shown on the Venn Diagram in the answer space.
The element y is such that $y \in A$ and $y \notin B$.
On the diagram, write y in the correct region.



[1]

(b) $\xi = \{x : x \text{ is an integer and } 1 \leq x \leq 8\}$.

$P = \{x : x > 5\}$.

$Q = \{x : x \leq 3\}$.

(i) Find the value of $n(P \cup Q)$. [1]

(ii) List the elements of $P' \cap Q'$. [1]

Thinking Process

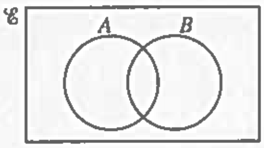
(a) To add y, consider the region that belongs to only A.

(b) (i) To find $n(P \cup Q)$ ✐ List the elements of P and Q.

(ii) Note that $(P' \cap Q') = (P \cup Q)'$.

Solution

(a)



(b) (i) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$P = \{6, 7, 8\}$

$Q = \{1, 2, 3\}$

Now, $P \cup Q = \{1, 2, 3, 6, 7, 8\}$

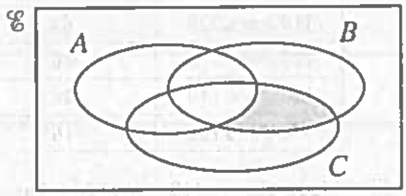
$\therefore$ $n(P \cup Q) = 6$ **Ans.**

(ii) We know that $(P' \cap Q') = (P \cup Q)'$

$\therefore$ $P' \cap Q' = \{4, 5\}$ **Ans.**

2 *(N2007 P1/Q9)*

(a) Express, in set notation, as simply as possible, the subset shaded in the Venn diagram.



[1]

(b) It is given that $n(\xi) = 40$, $n(P) = 18$,

$n(Q) = 20$ and $n(P \cap Q) = 7$.

Find

(i) $n(P \cup Q)$, [1]

(ii) $n(P' \cap Q')$. [1]

Thinking Process

(a) ✐ Note that part of $B \cap C$ is shaded. Set A is empty. The area outside sets A, B & C is also empty.

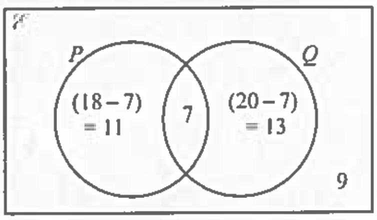
(b) (i) Construct a Venn diagram to find the answer.

(ii) Recall $n(P' \cap Q') = n(P \cup Q)'$.

Solution

(a) $A' \cap (B \cap C)$ **Ans**

(b) (i)



$\therefore$ $n(P \cup Q) = 11 + 7 + 13 = 31$ **Ans**

(ii) $n(P' \cap Q') = n(P \cup Q)'$

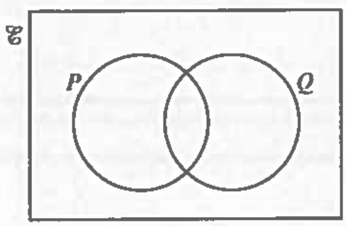
$= \xi - (P \cup Q)$

$= 40 - 31$

$= 9$ **Ans**

3 *(N2008 P1/Q9)*

(a) On the Venn diagram shown in the answer space, shade the set $P \cup Q'$.



[1]

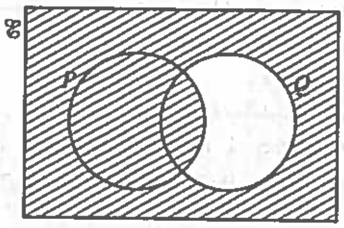
(b) There are 27 children in a class.
Of these children, 19 own a bicycle, 15 own a scooter and 3 own neither a bicycle nor a scooter.
Using a Venn diagram, or otherwise, find the number of children who own a bicycle but not a scooter. [2]

Thinking Process

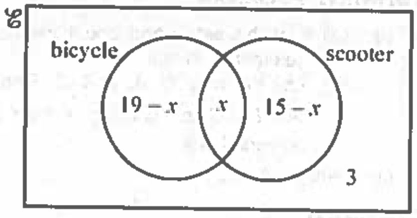
(a) Region to be shaded are set P and universal set without Q.
(b) Draw a Venn diagram and use it to solve the question.

Solution

(a)



(b)



$19 - x + x + 15 - x + 3 = 27$

$37 - x = 27$

$x = 10$

$\therefore$ children who own only bicycle $= 19 - 10$

$= 9$ **Ans.**

4 *(J2009/P1/Q18)*

(a) $\mathscr{E} = \{1, 2, 3, 4, 5\}$,
$A = \{1, 2, 3\}$,
$B = \{5\}$,
$C = \{3, 4\}$.

List the elements of

(i) $A \cup C$ [1]

(ii) $B' \cap C'$. [1]

(b) A group of 60 children attend an after school club. Of these, 35 children play football and 29 play hockey. 3 children do not play either football or hockey.
By drawing a Venn diagram, or otherwise, find the number of children who play only hockey. [2]

Thinking Process

(a) (ii) Find $B \cup C$ first and then subtract it from the universal set. Note that $B' \cap C' = (B \cup C)'$.
(b) Draw a Venn diagram and use it to solve the question.

Solution

(a) (i) $A \cup C = \{1, 2, 3\} \cup \{3, 4\}$

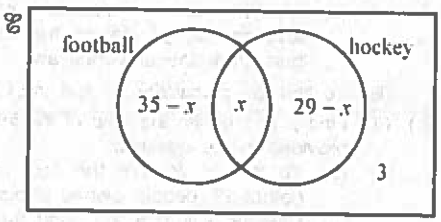
$= \{1, 2, 3, 4\}$ **Ans.**

(ii) $B' \cap C' = (B \cup C)'$

$\therefore \quad (B \cup C) = \{3, 4, 5\}$

$(B \cup C)' = \{1, 2\}$ **Ans.**

(b)



$35 - x + x + 29 - x + 3 = 60$

$67 - x = 60$

$x = 7$

$\therefore$ children who play only hockey $= 29 - 7$

$= 22$ **Ans.**

5 *(N2009/P2 Q4)*

(a) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$L = \{x : x \text{ is an odd number}\}$

$M = \{x : x \text{ is a multiple of } 3\}$

(i) Write down

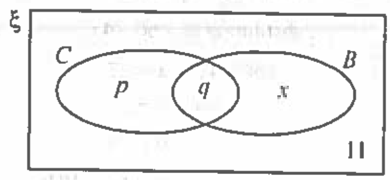
(a) $L \cap M$. [1]

(b) $L' \cap M$. [1]

(ii) A number n is chosen at random from ξ.

Find the probability that $n \in L \cup M$. [1]

(b) In a survey, a number of people were asked "Do you own a car?" and "Do you own a bicycle?". The Venn diagram shows the set C of car owners and the set B of bicycle owners.
The letters p, q and x are the numbers of people in each subset.
11 people owned neither a car nor a bicycle.



A total of 66 people owned a car.
4 times as many people owned a car only as owned a bicycle only.

(i) Write down expressions, in terms of x, for
 (a) p, [1]
 (b) q. [1]

(ii) A total of 27 people owned a bicycle. Calculate
 (a) x, [2]
 (b) the total number of people who were in the survey. [1]

Thinking Process

(a) (i) (a) Find the common elements of set L and M.
(b) List the common elements of set L' and set M. ∴ delete the odd numbers from the universal set.
(ii) To find the probability ∴ find $n(L \cup M)$.

(b) (i) Find p and q with the help of the information provided in the question.
(ii) (a) To find x ∴ use the fact that altogether 27 people owned a bicycle.
(b) Use the value of x to find the total number of people.

Solution

(a) (i) (a) $L = \{1, 3, 5, 7, 9, 11, 13, 15\}$
$M = \{3, 6, 9, 12, 15\}$
$\therefore \quad L \cap M = \{3, 9, 15\}$ **Ans.**

(b) $L' = \{2, 4, 6, 8, 10, 12, 14\}$
$M = \{3, 6, 9, 12, 15\}$
$\therefore \quad L' \cap M = \{6, 12\}$ **Ans.**

(ii) $n(\xi) = 15$
$L \cup M = \{1, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$
$\therefore \quad n(L \cup M) = 10$
$P(n \in L \cup M) = \frac{10}{15} = \frac{2}{3}$ **Ans.**

(b) (i) (a) $p = 4x$ **Ans.**

(b) $p + q = 66$
substitute $p = 4x$,
$4x + q = 66$
$q = 66 - 4x$ **Ans.**

(ii) (a) Given that, $q + x = 27$
substituting $q = 66 - 4x$,
$(66 - 4x) + x = 27$
$66 - 3x = 27$
$3x = 39$
$x = 13$ **Ans.**

(b) Total number of people
$= p + q + x + 11$
substituting the values of p and q,
$= 4x + 66 - 4x + x + 11$
$= 77 + x$ (since $x = 13$)
$= 77 + 13 = 90$ **Ans.**

6 *(J2010/P2 Q5c)*

(c) $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$
$A = \{x : x \text{ is a multiple of } 3\}$
$B = \{x : x \text{ is a factor of } 24\}$
$C = \{x : x \text{ is an odd number}\}$

(i) Find
 (a) $n(B)$, [1]
 (b) $(A \cup B \cup C)'$ [1]

(ii) A number, k, is chosen at random from $\mathscr{E}$. Find the probability that $k \in A \cap B$. [2]

Thinking Process

(c) (i) (a) List the set B and count the no. of elements in the set.
(b) List the sets of A, B & C. Find $A \cup B \cup C$ and subtract it from the universal set.
(ii) Find $n(A \cap B)$

Solution

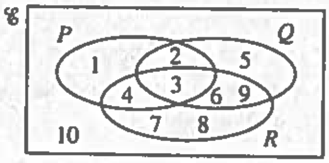
(c) (i) (a) $B = \{1, 2, 3, 4, 6, 8, 12\}$
$\therefore \quad n(B) = 7$ **Ans.**

(b) $A = \{3, 6, 9, 12, 15\}$
$B = \{1, 2, 3, 4, 6, 8, 12\}$
$C = \{1, 3, 5, 7, 9, 11, 13, 15\}$
$(A \cup B \cup C)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 15\}$
$\therefore \quad (A \cup B \cup C)' = \{10, 14, 16\}$ **Ans.**

(ii) $(A \cap B) = \{3, 6, 12\}$
$\therefore \quad n(A \cap B) = 3$
also $n(\mathscr{E}) = 16$
$\therefore \quad P(k \in A \cap B) = \frac{3}{16}$ **Ans.**

7 *(N2010/P1/Q11)*

The Venn diagram shows the sets $\mathscr{E}$, P, Q and R.
$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$



(a) Find the value of $n(Q \cup R)$. [1]

(b) List the elements of the set $P' \cap (Q \cup R)$. [1]

(c) One element is chosen at random from $\mathscr{E}$. Write down the probability that this element belongs to $(P \cap Q) \cup (P \cap R)$. [1]

Thinking Process

(a) ✎ Count the members of set Q and set R.

(b) ✎ List the common members of P' and $(Q \cup R)$.

(c) ✎ find $n[(P \cap Q) \cup (P \cap R)]$.

Solution

(a) From Venn diagram.

$(Q \cup R) = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore\ n(Q \cup R) = 8$ **Ans.**

(b) $P' = \{5, 6, 7, 8, 9\}$

$(Q \cup R) = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore\ P' \cap (Q \cup R) = \{5, 6, 7, 8, 9\}$ **Ans.**

(c) $(P \cap Q) = \{2, 3\}$. $(P \cap R) = \{3, 4\}$

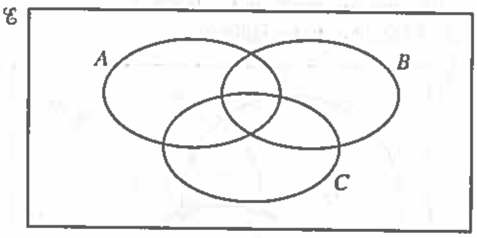
$(P \cap Q) \cup (P \cap R) = \{2, 3, 4\}$

$\Rightarrow\ n((P \cap Q) \cup (P \cap R)) = 3$

$\therefore\ P(\text{element} \in (P \cap Q) \cup (P \cap R)) = \frac{3}{10}$ **Ans.**

8 *(J2011/P1/Q12)*

(a) On the Venn diagram, shade the set $A \cap B \cap C'$.



[1]

(b) $\mathscr{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$P = \{x : x \text{ is a prime number}\}$

$Q = \{x : x \geq 5\}$

(i) Find the value of $n(P \cap Q)$. [1]

(ii) List the elements of $P \cup Q'$. [1]

Thinking Process

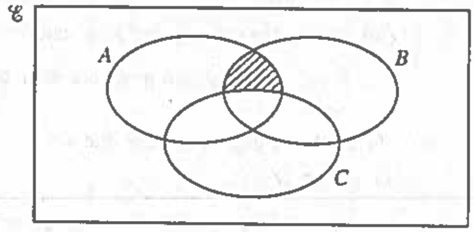
(a) ✎ Note that the required part is the common area of A and B excluding C.

(b) (i) ✎ List the common members of set P & Q.

(ii) List the members of P and Q'. Find their union.

Solution

(a)



(b) (i) $P = \{2, 3, 5, 7\}$

$Q = \{5, 6, 7, 8, 9, 10\}$

$P \cap Q = \{5, 7\}$

$\therefore\ n(P \cap Q) = 2$ **Ans.**

(ii) $P = \{2, 3, 5, 7\}$

$Q' = \{2, 3, 4\}$

$P \cup Q' = \{2, 3, 4, 5, 7\}$ **Ans.**

9 *(N2011/P1/Q10)*

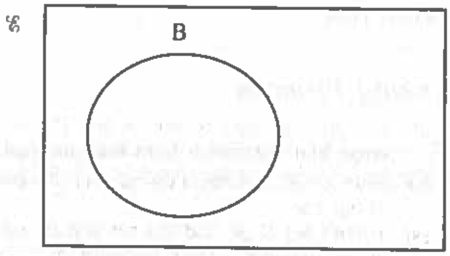
The Venn diagram shows the Universal set and the set B.

A and C are two sets such that

$A \cup B = B$, $A \cap B \neq B$, $A \cap C = \varnothing$

and $B \cap C \neq \varnothing$.

Draw the sets A and C in the Venn diagram.

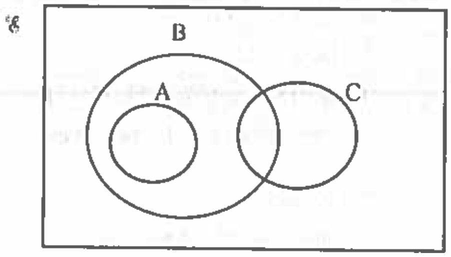


[2]

Thinking Process

✎ With the given information, add sets A and C to the venn diagram.

Solution



10 *(J2012 P2 Q6)*

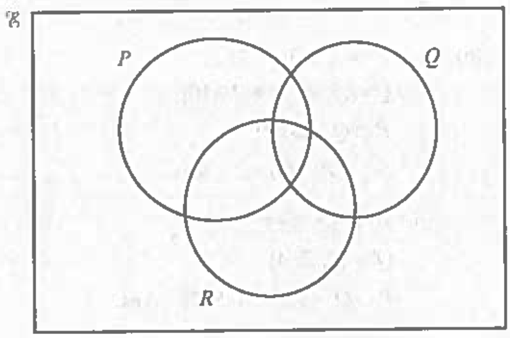
(a) $\mathscr{E} = \{ x : x \text{ is an integer, } 2 \le x \le 14 \}$
$A = \{ x : x \text{ is a prime number}\}$
$B = \{ x : x \text{ is a multiple of } 3\}$

(i) List the members of $(A \cup B)'$ [1]

(ii) Find $n(A \cap B)$. [1]

(iii) Given that $C \subset A$, $n(C) = 3$ and $B \cap C = \varnothing$, list the members of a possible set C. [1]

(b) On the Venn diagram, shade the set $(P \cup R) \cap Q'$



[1]

(c) A group of 80 people attended a recreation centre on one day.
Of these people, 48 used the gym
31 used the swimming pool
17 used neither the gym nor the swimming pool.

By drawing a Venn diagram, or otherwise, find the number of people who used both the gym and the swimming pool. [2]

Thinking Process

(a) (i) List the members of sets A and B, find their union and subtract it from the universal set.

(ii) Find $A \cap B$ and count the number of members in the set.

(iii) To find set C find the set that consists of prime numbers without multiples of 3.

(b) Region to be shaded are set P and set R, without whole of set Q.

Solution *with* TEACHER's COMMENTS

(a) (i) $\mathscr{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$A = \{2, 3, 5, 7, 11, 13\}$

$B = \{3, 6, 9, 12\}$

$A \cup B = \{2, 3, 5, 6, 7, 9, 11, 12, 13\}$

$\therefore \quad (A \cup B)' = \{4, 8, 10, 14\}$ **Ans.**

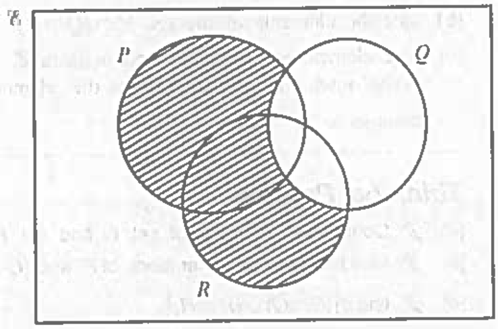
(ii) $A \cap B = \{3\}$

$\therefore \quad n(A \cap B) = 1$ **Ans.**

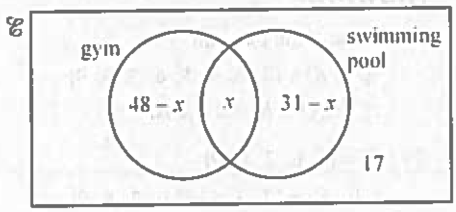
(iii) $C = \{2, 5, 7\}$ **Ans.**

$C \subset A$, $\Rightarrow$ C is a set of prime numbers between 2 and 14.
$B \cap C = \varnothing$, $\Rightarrow$ B and C are disjoint sets.
$n(C) = 3$, $\Rightarrow$ set C consists of 3 members.
$\therefore$ set C contains any 3 prime numbers from 2 to 14 excluding 3.

(b)



(c)



$48 - x + x + 31 - x + 17 = 80$

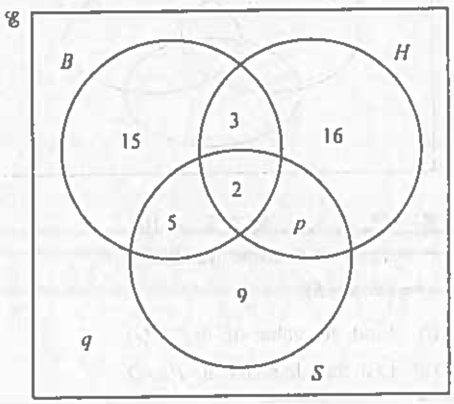
$96 - x = 80$

$x = 16$

$\therefore$ number of people who used both the gym and swimming pool is 16 **Ans.**

11 *(N2012 P1 Q14)*

In a survey, 60 students are asked which of the subjects Biology (B), History (H) and Spanish (S) they are studying.
The Venn diagram shows the results.
27 students study History.



(a) Find the values of p and q. [1]

(b) Find $n(H')$. [1]

(c) Find $n((B \cup H) \cap S')$. [1]

Thinking Process

(a) To find p ✎ equate all the History students to 27. To find q ✎ equate all the students in the venn diagram to 60.

(b) ✎ $n(H')$ = students who do not study History.

(c) $n((B \cup H) \cap S')$ = students who study Biology or History but do not study spanish.

Solution

(a) $16 + 3 + 2 + p = 27$

$21 + p = 27$

$p = 6$ **Ans.**

$27 + 15 + 5 + 9 + q = 60$

$56 + q = 60$

$q = 4$ **Ans.**

(b) $n(H') = 60 - 27$

$= 33$ **Ans.**

(c) $n((B \cup H) \cap S') = 15 + 3 + 16$

$= 34$ **Ans.**

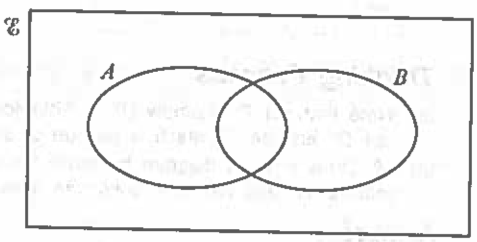
12 *(J2013/P1/Q10)*

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{\text{odd numbers}\}$

$B = \{\text{multiples of } 3\}$

(a) Complete the Venn diagram to illustrate this information.



[1]

(b) Find the value of $n(A \cup B)$. [1]

(c) List the elements of the set $A \cap B'$. [1]

Thinking Process

(a) To complete the venn diagram ✎ list the elements of set A and set B.

(b) Find $A \cup B$ and count the number of elements in the set.

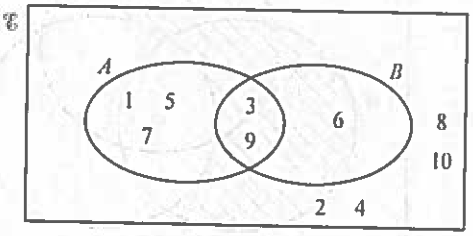
(c) ✎ List the common members of set A and B'.

Solution

(a) $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}$

$B = \{3, 6, 9\}$



(b) $(A \cup B) = \{1, 3, 5, 6, 7, 9\}$

$\therefore \quad n(A \cup B) = 6$ **Ans.**

(c) $A = \{1, 3, 5, 7, 9\}$

$B' = \{1, 2, 4, 5, 7, 8, 10\}$

$\therefore \quad A \cap B' = \{1, 5, 7\}$ **Ans.**

13 *(N2013/P2/Q5)*

(a) $\mathscr{E} = \{x : x \text{ is an integer and } 2 \leq x \leq 12\}$
$M = \{x : x \text{ is a multiple of } 3\}$
$P = \{x : x \text{ is a prime number}\}$

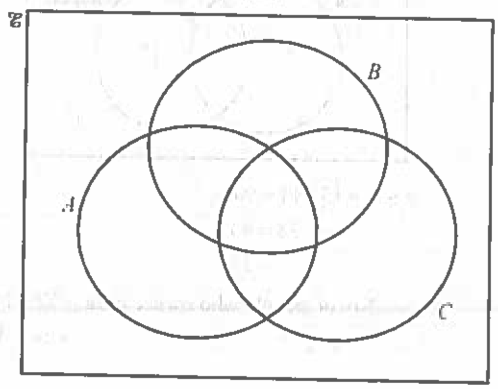
(i) $a \in M \cap P$
Find a. [1]

(ii) Find $(M \cup P)'$. [1]

(b) In a survey, 90 people were asked "Do you own a car?" and "Do you own a bicycle?".
A total of 27 people said they owned a bicycle.
Of these, 13 owned only a bicycle.
11 people owned neither a car nor a bicycle.
By drawing a Venn diagram, or otherwise, find how many people said that they owned a car. [2]

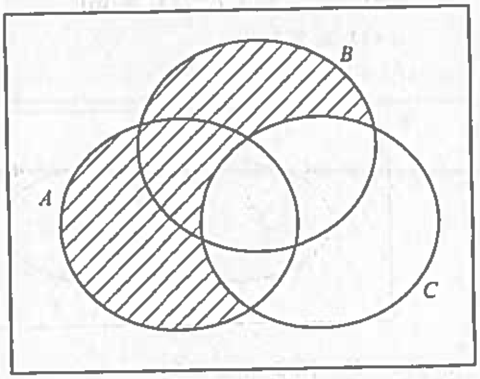
(c) The Venn diagrams show a Universal set, ℰ, and subsets A, B and C.

(i) Shade the set $(A \cup C)' \cap B$.



[1]

(ii) Express in set notation the subset shaded in this diagram.



[1]

Thinking Process

(a) (i) To find *a* list the common members of set *M* and *P*.

(ii) Find $M \cup P$ and subtract it from the universal set.

(b) Draw a Venn diagram by using the given information and use it to solve the question.

(c) (i) Region to be shaded is part of set *B* which is outside of sets *A* and *C*.

(ii) Note that set *C* is empty (C'). The area outside sets *A*, *B* and *C* is also empty.

Solution

(a) (i) $\mathscr{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$M = \{3, 6, 9, 12\}$

$P = \{2, 3, 5, 7, 11\}$

$M \cap P = \{3\}$

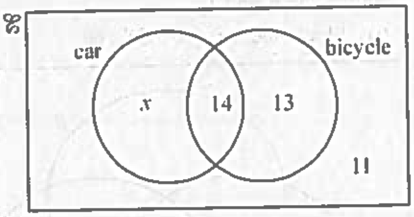
$\therefore\ a = 3$ **Ans.**

(ii) $M = \{3, 6, 9, 12\}$, $P = \{2, 3, 5, 7, 11\}$

$M \cup P = \{2, 3, 5, 6, 7, 9, 11, 12\}$

$\therefore\ (M \cup P)' = \{4, 8, 10\}$ **Ans.**

(b)



$x + 14 + 13 + 11 = 90$

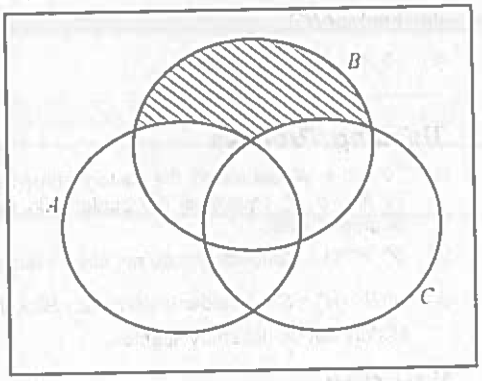
$x + 38 = 90$

$x = 52$

$\therefore$ No. of people who owned a car $= 52 + 14$

$= 66$ **Ans.**

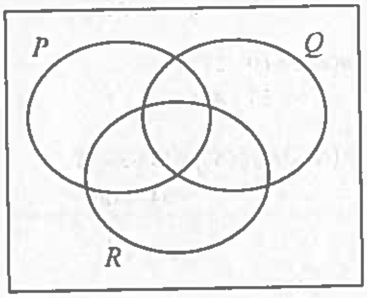
(c) (i)



(ii) $(A \cup B) \cap C'$ **Ans.**

14 *(J2014 P1 Q11)*

(a) On the Venn diagram, shade the set $P' \cap (Q \cup R)$.



[1]

(b) A group of 40 children are asked what pets they own.
Of these children, 13 own a cat, 5 own both a cat and a dog and 15 own neither a cat nor a dog.
Using a Venn diagram, or otherwise, find the number of children who own a dog, but not a cat. [2]

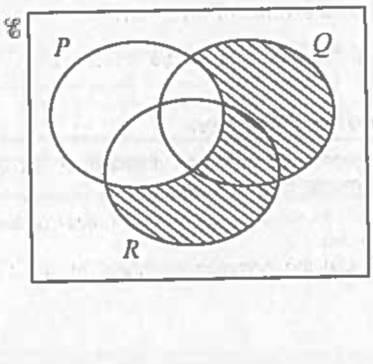
Thinking Process

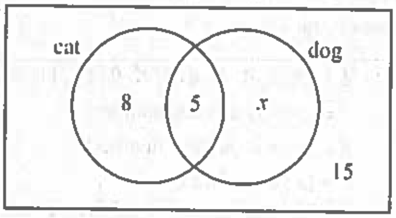
(a) Note that set *P* is empty (P'). Therefore shade set *Q* and set *R* which is outside of set *P*.

(b) Draw a Venn diagram by using the given information and use it to solve the question.

Solution

(a)



(b) 

$x+5+8+15=40$

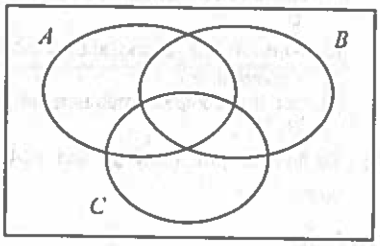
$x+28=40$

$x=12$

∴ Number of children who own a dog only $=12$ **Ans.**

15 (N2014/P1 Q6)

(a) On the Venn diagram, shade the $C'\cap(A\cup B)$.



[1]

(b) $\mathscr{E}=\{-1, 0, 1, 2, 3, 4, 5, 6\}$

$P=\{-1, 0, 1, 2\}$

$Q=\{x^2 : x\in P\}$

Find n(Q). [1]

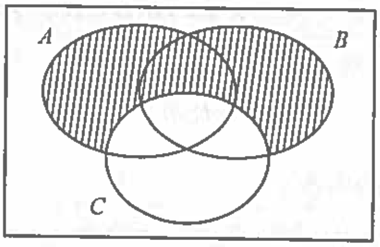
Thinking Process

(a) Note that set C is empty (C'). Therefore shade set A and set B which is outside of set C.

(b) To find n(Q) ✐ List the elements of Q.

Solution *with* TEACHER's COMMENTS

(a)



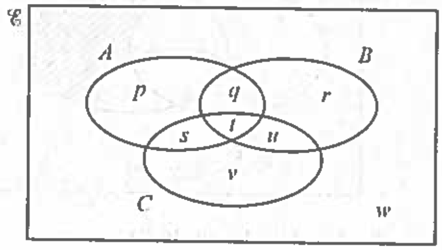
(b) $Q=\{0, 1, 4\}$

∴ n(Q) $=3$ **Ans.**

> $Q=\{x^2 : x\in P\}$
> $x\in P$ means that x is a member of set P.
> ∴ set Q is the set of squared numbers of set P.

16 (J2015/P1 Q8)

The Venn diagram shows the sets A, B and C.



List the elements of

(a) $A\cup B$. [1]

(b) $B'\cap C$. [1]

Thinking Process

(a) List all the elements of set A and set B.

(b) List the common members of set C and B'

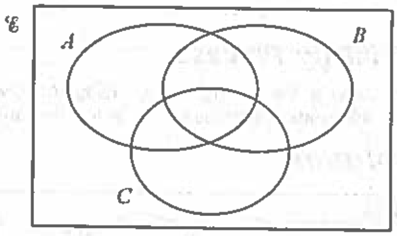
Solution

(a) $A\cup B=\{p, q, r, s, t, u\}$ **Ans.**

(b) $B'\cap C=\{s, v\}$ **Ans.**

17 (N2015/P1 Q15)

(a) On the Venn diagram, shade the set $B\cap(A\cup C)'$.



[1]

(b) $\mathscr{E}=\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$

$W=\{x : x \text{ is a multiple of } 2\}$

$H=\{x : x \text{ is a multiple of } 3\}$

(i) Find n($W\cup H$). [1]

(ii) List the members of $W\cap H'$. [1]

Thinking Process

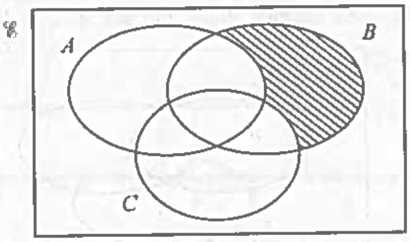
(a) ✐ Region to be shaded is part of set B that is outside of sets A and C.

(b) (i) List all the elements of set W and set H.

(ii) List the common members of set W and H'.

Solution

(a)



(b) (i) $W = \{10, 12, 14, 16, 18\}$

$H = \{12, 15, 18\}$

$W \cup H = \{10, 12, 14, 15, 16, 18\}$

$\therefore$ $n(W \cup H) = 6$ **Ans.**

(ii) $W = \{10, 12, 14, 16, 18\}$

$H' = \{10, 11, 13, 14, 16, 17, 19\}$

$\therefore$ $W \cap H' = \{10, 14, 16\}$ **Ans.**

18 *(J2016 P1 Q9)*

50 students are asked what type of movie they like to watch.
Of these students,

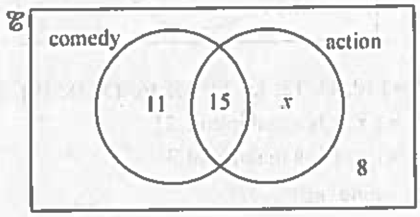
- 26 like comedy,
- 15 like both action and comedy and
- 8 like neither action nor comedy.

Using a Venn diagram, or otherwise, find the number of students who like action but not comedy. [2]

Thinking Process

Draw a Venn diagram by using the given information and use it to solve the question.

Solution



$11 + 15 + x + 8 = 50$

$x + 34 = 50$

$x = 16$

$\therefore$ Number of students who like only action $= 16$ **Ans.**

19 *(J2016 P2 Q6 a)*

(a) $\mathscr{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{x : x \text{ is a prime number}\}$

$B = \{x : x \text{ is an even number}\}$

$C = \{x : x \text{ is a multiple of } 5\}$

(i) List the members of the subsets

(a) $B \cap C$, [1]

(b) $(A \cup B \cup C)'$. [1]

(c) $A \cap B'$. [1]

(ii) A number q is chosen at random from $\mathscr{E}$.
Find the probability that $q \in A \cap B'$. [1]

Thinking Process

(a) (i) List the members of sets of A, B & C.

(a) List the common members of set B and C.

(b) Find $A \cup B \cup C$ and subtract it from the universal set.

(c) List the common members of set A and B'.

(ii) To find the probability find $n(A \cap B')$ and $n(\mathscr{E})$.

Solution

(a) $\mathscr{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{2, 3, 5, 7, 11\}$,

$B = \{2, 4, 6, 8, 10, 12\}$

$C = \{5, 10\}$

(i) (a) $B \cap C = \{10\}$ **Ans.**

(b) $A \cup B \cup C$
$= \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12\}$

$\therefore$ $(A \cup B \cup C)' = \{9\}$ **Ans.**

(c) $B' = \{3, 5, 7, 9, 11\}$

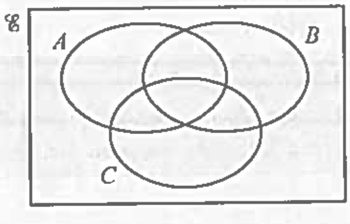
$\therefore$ $A \cap B' = \{3, 5, 7, 11\}$ **Ans.**

(ii) $n(\mathscr{E}) = 11$, $n(A \cap B') = 4$

$\therefore$ $P(q \in A \cap B') = \frac{4}{11}$ **Ans.**

20 *(N2016 P1 Q14)*

(a) In the Venn diagram, shade the region which represents the subset $(A \cap B') \cup C$.



[1]

(b) In a group of 36 students,
23 study Spanish,
17 study French,
4 study neither Spanish nor French.
By drawing a Venn diagram, or otherwise, find the number of students who study both Spanish and French. [2]

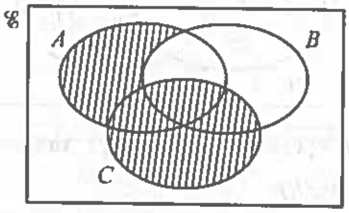
Thinking Process

(a) Set B is empty (B'). Therefore region to be shaded is set A which is outside of set B and whole of set C.

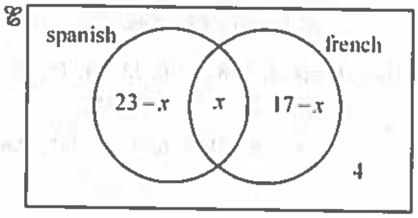
(b) Draw a Venn diagram by using the given information and use it to solve the question.

Solution

(a)



(b)

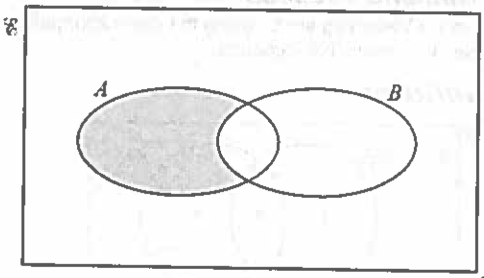


$$23 - x + x + 17 - x + 4 = 36$$
$$44 - x = 36$$
$$x = 8$$

∴ number of students who study both = 8 Ans.

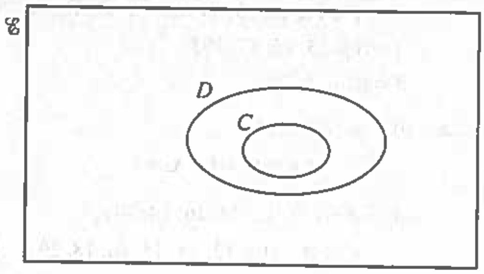
21 (J2017/P1/Q7)

(a) Use set notation to describe the shaded set in the Venn diagram.



[1]

(b) Use set notation to complete the statement about sets C and D.



C .... D [1]

Thinking Process

(a) Note that set B is empty i.e. B'.

(b) Note that set C is the subset of set D.

Solution

(a) $A \cap B'$ Ans.

(b) $C \subset B$ Ans.

22 (N2017/P2/Q6)

(a) $\mathscr{E} = \{x : x \text{ is an integer and } 10 \le x \le 20\}$

$A = \{x : x \text{ is an odd number}\}$

$B = \{x : x \text{ is a multiple of } 5\}$

(i) Find $n(A \cap B)$. [1]

(ii) Find $A' \cup B$. [1]

(iii) A number, r, is chosen at random from $\mathscr{E}$.

Find the probability that $r \in A \cup B$. [1]

(b) In a survey, 40 people were asked what they had read that day.

- A total of 10 people had read a book
- A total of 24 people had read a newspaper
- 14 people had read neither a book nor a newspaper

(i) By drawing a Venn diagram, or otherwise, find the number of people who had read both a book and a newspaper. [2]

(ii) Two of the 10 people who had read a book are selected at random.
Work out the probability that they had both read a book and a newspaper. [2]

Thinking Process

(a) (i) First list the sets of A & B, then count the number of common elements of set A and B.

(ii) List all the elements of set A' and set B.

(iii) To find the probability find $n(A \cup B)$.

(b) (i) Using the given information, draw a Venn diagram and use it to solve the question.

(ii) To find the probability use the answer found in part (b)(i).

Solution

$\mathscr{E} = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{11, 13, 15, 17, 19\}$

$B = \{10, 15, 20\}$

(a) (i) $A \cap B = \{15\}$

$\therefore$ $n(A \cap B) = 1$ **Ans.**

(ii) $A' = \{10, 12, 14, 16, 18, 20\}$

$A' \cup B = \{10, 12, 14, 15, 16, 18, 20\}$ **Ans.**

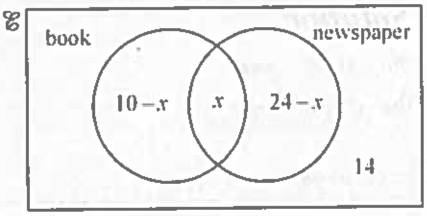
(iii) $A \cup B = \{10, 11, 13, 15, 17, 19, 20\}$

$\therefore$ $n(A \cup B) = 7$

also, $n(\mathscr{E}) = 11$

$\therefore$ $P(r \in A \cup B) = \frac{7}{11}$ **Ans.**

(b) (i)



$(10 - x) + x + (24 - x) + 14 = 40$

$48 - x = 40$

$x = 8$

$\therefore$ No. of people who read both = 8 **Ans.**

(ii) P(both read a book and a newspaper)

$= \frac{8}{10} \times \frac{7}{9}$

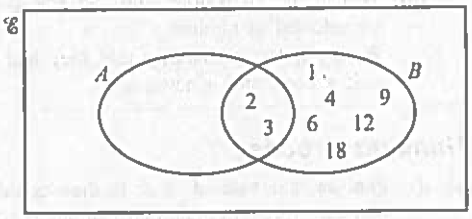
$= \frac{56}{90} = \frac{28}{45}$ **Ans.**

23 *(J2018/P2 Q4 a)*

(a) $\mathscr{E} = \{x : x \text{ is an integer } 1 \le x \le 18\}$

$A = \{x : x \text{ is a prime number}\}$

$B = \{1, 2, 3, 4, 6, 9, 12, 18\}$



(i) Complete the Venn diagram to illustrate this information. [1]

(ii) Complete the description of the set B.

$B = \{x : x$ is a factor of ....$\}$ [1]

(iii) Find $n(A \cup B)$. [1]

(iv) List the elements of $A' \cap B$. [1]

Thinking Process

(a) (i) To complete the venn diagram ∮ list the members of universal set and set A.

(ii) Observe that all members of set B are factors of 36.

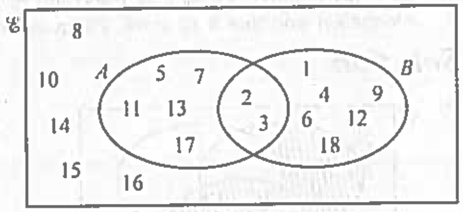
(iv) List the common members of set A' and B.

Solution

(a) (i) $\mathscr{E} = \{1, 2, 3, 4, 5, 6, \ldots\ldots\ldots\ldots 16, 17, 18\}$

$A = \{2, 3, 5, 7, 11, 13, 17\}$

$B = \{1, 2, 3, 4, 6, 9, 12, 18\}$



(ii) $B = \{x : x$ is a factor of $36\}$ **Ans.**

(iii) $(A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 17, 18\}$

$\therefore$ $n(A \cup B) = 13$ **Ans.**

(iv) $A' = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$B = \{1, 2, 3, 4, 6, 9, 12, 18\}$

$\therefore$ $A' \cap B = \{1, 4, 6, 9, 12, 18\}$ **Ans.**

24 *(N2018 P1 Q7)*

In a group of 35 people,

22 are wearing spectacles,

10 are wearing a hat,

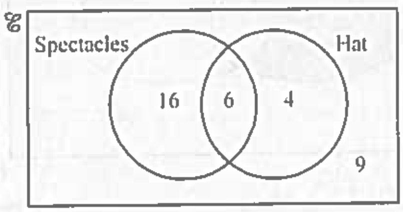
6 are wearing spectacles and a hat.

By drawing a Venn diagram, or otherwise, find the number of people who are wearing neither spectacles nor a hat. [2]

Thinking Process

Draw a Venn diagram by using the given information and use it to solve the question.

Solution



$\therefore$ Number of people wearing neither spectacles nor a hat $= 35 - (16 + 6 + 4)$

$= 35 - 26 = 9$ **Ans.**

25 (N2018 P1 Q22)

$\mathscr{E} = \{0, 1, 2, 3, 4, 5, 6\}$

$P = \{x : x = 0, 1, 2\}$

$Q = \{y : y = 0, 2\}$

(a) List the members of $P \cap Q$. [1]

(b) Find $n(P' \cup Q)$. [1]

(c) $R = \{z : z = 2x + y, x \in P, y \in Q\}$
List the members of R. [2]

Thinking Process

(a) List the common elements of set P and Q.
(b) List the members of P' and Q. Find their union.
(c) To list set R ✍ Substitute all the ordered pairs from the elements of set P and Q into $z = 2x + y$

Solution

(a) $P \cap Q = \{0, 2\}$ **Ans.**

(b) $P' = \{3, 4, 5, 6\}$

$P' \cup Q = \{0, 2, 3, 4, 5, 6\}$

$\therefore$ $n(P' \cup Q) = 6$ **Ans.**

(c) $P = \{x : x = 0, 1, 2\}$ and $Q = \{y : y = 0, 2\}$

Possible options are.

(0, 0), (0, 2), (1, 0), (1, 2), (2, 0), (2, 2)

Substitute each coordinate into $z = 2x + y$.

For (0, 0), $z = 0$, For (0, 2), $z = 2$

For (1, 0), $z = 2$, For (1, 2), $z = 4$

For (2, 0), $z = 4$, For (2, 2), $z = 6$

$\therefore$ $R = \{0, 2, 4, 6\}$ **Ans.**

Topic 21

Matrices

1 (N2008/P1/Q15)

$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -3 \\ 1 & 0 \end{pmatrix}$

Find

(a) AB. [2]

(b) B^{-1}. [2]

Thinking Process

(a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$

(b) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $A^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Solution

(a) $AB = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 4 & -3 \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 8+(-1) & -6+0 \\ 4+3 & -3+0 \end{pmatrix} = \begin{pmatrix} 7 & -6 \\ 7 & -3 \end{pmatrix}$ **Ans.**

(b) Det. $B = (4\times 0) - (-3\times 1) = 0+3 = 3$

$B^{-1} = \frac{1}{3}\begin{pmatrix} 0 & 3 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{3} & \frac{4}{3} \end{pmatrix}$ **Ans.**

2 (N2009/P2/Q5)

(a) Evaluate

(i) $3\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$. [2]

(ii) $\begin{pmatrix} 1 & 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 4 \\ 3 & 1 \\ 5 & 0 \end{pmatrix}$. [2]

(b) $A = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$

(i) Find A^{-1}. [2]

(ii) The transformation represented by the matrix A maps (h, k) onto $(10, 2)$.
Find the value of h and the value of k. [2]

Thinking Process

(a) (i) perform the required calculation.
(ii) Multiply respective row by respective column.

(b) (i) Recall: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \dfrac{1}{\text{Det. } A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

where Det. $A = ad - bc$

(ii) Pre-multiply the matrix A by (h, k) and equate to $(10, 2)$.

Solution

(a) (i) $3\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 12 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 12 \\ -6 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}$ **Ans.**

(ii) $\begin{pmatrix} 1 & 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 4 \\ 3 & 1 \\ 5 & 0 \end{pmatrix}$

$= \begin{pmatrix} 1\times 0+3\times 3+4\times 5 & 1\times 4+3\times 1+4\times 0 \end{pmatrix}$

$= \begin{pmatrix} 29 & 7 \end{pmatrix}$ **Ans.**

(b) (i) Det. $A = (2\times 1) - (-3\times 0) = 2$

$A^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ 0 & 1 \end{pmatrix}$ **Ans.**

(ii) $\begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 2h-3k \\ k \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$

$\Rightarrow \quad 2h - 3k = 10$ and $k = 2$

$2h - 3(2) = 10$

$2h = 16$

$h = 8$

$\therefore \quad h = 8$ and $k = 2$ **Ans.**

3 (J2010/P1/Q19)

$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}$

Find

(a) A – B, [1]

(b) B^{-1}. [2]

Thinking Process

(a) Subtract the corresponding numbers of the matrices.

(b) Recall: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Solution

(a) $A - B = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 0 & -1 \end{pmatrix}$ **Ans.**

(b) Det. $B = (0 \times 3) - (2 \times -1) = 2$

$$B^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -1 \\ \frac{1}{2} & 0 \end{pmatrix} \quad \textbf{Ans.}$$

4 *(J2010/P2/Q5a,b)*

(a) Bertie goes shopping and buys three different types of fruit.
The first matrix below shows the number of kilograms of each fruit bought during two different weeks.
The second matrix shows the price per kilogram, in cents, of each fruit.

$$\begin{array}{l} \\ \text{Week 1} \\ \text{Week 2} \end{array} \begin{array}{c} \text{bananas} \quad \text{apples} \quad \text{grapes} \\ \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix} \end{array} \qquad \begin{array}{c} \text{price/kg} \\ \begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix} \end{array} \begin{array}{l} \\ \text{bananas} \\ \text{apples} \\ \text{grapes} \end{array}$$

(i) $F = \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix}\begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix}$.

Find F. [2]

(ii) Explain the meaning of the information given by the matrix F. [1]

(iii) Find the total amount of money, in dollars, that Bertie spent on fruit during the two weeks. [1]

(b) The matrix M satisfies the equation

$$8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} + 5M = M.$$

Find M. [2]

Thinking Process

(a) (i) Perform the matrix multiplication.
(ii) To understand the meaning ✐ consider the intermediate steps in the calculation of the product.
(iii) To find the total amount spent ✐ use your answer to (a)(ii).

Solution

(a) (i) $F = \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix}\begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix}$

$$= \begin{pmatrix} (1 \times 290) + (2 \times 160) + (0.5 \times 640) \\ (1.5 \times 290) + (1 \times 160) + (1 \times 640) \end{pmatrix}$$

$$= \begin{pmatrix} 290 + 320 + 320 \\ 435 + 160 + 640 \end{pmatrix}$$

$$= \begin{pmatrix} 930 \\ 1235 \end{pmatrix} \quad \textbf{Ans.}$$

(ii) Matrix F represents the total cost in cents spent on fruit during week 1 and week 2.

(iii) Total amount spent during the two weeks
$= 930 + 1235$
$= 2165$ cents
$= \$21.65$ **Ans.**

(b) $8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} + 5M = M$

$$8\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} = -4M$$

$$M = -\frac{8}{4}\begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix}$$

$$M = \begin{pmatrix} -6 & 0 \\ 2 & -4 \end{pmatrix} \quad \textbf{Ans.}$$

5 *(N2010/P1/Q16)*

$$A = \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & -4 \\ -2 & 2 \end{pmatrix}$$

Find
(a) $2A - B$, [1]
(b) A^{-1}. [2]

Thinking Process

(a) Perform the required calculation.

(b) Recall: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Solution

(a) $2A - B = 2\begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 5 & -4 \\ -2 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 4 & -6 \\ -2 & 0 \end{pmatrix} - \begin{pmatrix} 5 & -4 \\ -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 \\ 0 & -2 \end{pmatrix} \quad \textbf{Ans.}$$

(b) Det. $A = (2 \times 0) - (-3 \times -1) = -3$

$$A^{-1} = -\frac{1}{3}\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ -\frac{1}{3} & -\frac{2}{3} \end{pmatrix} \quad \textbf{Ans.}$$

6 *(N2010/P1/Q17)*

A shop sells bunches of flowers.
One bunch contains 3 roses, 4 carnations and 5 freesias.
Another bunch contains 6 roses and 4 carnations.
Each rose costs 60 cents, each carnation costs 40 cents and each freesia costs 30 cents.

This information can be represented by the matrices P and Q below.

$$P=\begin{pmatrix}3 & 4 & 5\\ 6 & 4 & 0\end{pmatrix} \qquad Q=\begin{pmatrix}60\\ 40\\ 30\end{pmatrix}$$

(a) Find PQ. [2]

(b) Explain what the numbers in your answer represent. [1]

Thinking Process

(a) Perform the matrix multiplication.

(b) To understand what it represent ✐ consider what P and Q stand for.

Solution

(a) $PQ=\begin{pmatrix}3 & 4 & 5\\ 6 & 4 & 0\end{pmatrix}\begin{pmatrix}60\\ 40\\ 30\end{pmatrix}$

$=\begin{pmatrix}(3\times 60)+(4\times 40)+(5\times 30)\\ (6\times 60)+(4\times 40)+(0\times 30)\end{pmatrix}$

$=\begin{pmatrix}180+160+150\\ 360+160+0\end{pmatrix}$

$=\begin{pmatrix}490\\ 520\end{pmatrix}$ **Ans.**

(b) First row represents the total cost of one bunch of flowers while the second row represents total cost of second bunch of flowers.

7 *(J2011 P2 Q8 a)*

(a) $A=\begin{pmatrix}4 & 3\\ -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}5 & 4\\ -3 & -2\end{pmatrix}$

Find

(i) $2A-B$, [2]

(ii) B^{-1}. [2]

Thinking Process

(a) (i) Multiply matrix A by 2. Subtract.

(ii) Recall, if $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

$\Rightarrow A^{-1}=\dfrac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$

Solution

(a) (i) $2A-B=2\begin{pmatrix}4 & 3\\ -1 & 1\end{pmatrix}-\begin{pmatrix}5 & 4\\ -3 & -2\end{pmatrix}$

$=2\begin{pmatrix}8 & 6\\ -2 & 2\end{pmatrix}-\begin{pmatrix}5 & 4\\ -3 & -2\end{pmatrix}$

$=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$ **Ans.**

(ii) Det. $B=(5\times -2)-(4\times -3)$

$=-10+12=2$

$B^{-1}=\frac{1}{2}\begin{pmatrix}-2 & -4\\ 3 & 5\end{pmatrix}=\begin{pmatrix}-1 & -2\\ \frac{3}{2} & \frac{5}{2}\end{pmatrix}$ **Ans.**

8 *(N2011 P1 Q22)*

$$A=\begin{pmatrix}4 & -2\\ -1 & 1\end{pmatrix} \qquad B=\begin{pmatrix}-3 & 2\\ -1 & 4\end{pmatrix}$$

(a) Find $2A-B$. [2]

(b) Find A^{-1}. [2]

Thinking Process

(a) Subtract the elements in B from the corresponding elements in A ✐ multiply each element of A by 2.

(b) If $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix} \Rightarrow A^{-1}=\dfrac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$.

Solution

(a) $2A-B=2\begin{pmatrix}4 & -2\\ -1 & 1\end{pmatrix}-\begin{pmatrix}-3 & 2\\ -1 & 4\end{pmatrix}$

$=\begin{pmatrix}8 & -4\\ -2 & 2\end{pmatrix}-\begin{pmatrix}-3 & 2\\ -1 & 4\end{pmatrix}$

$=\begin{pmatrix}11 & -6\\ -1 & -2\end{pmatrix}$ **Ans.**

(b) Det. $A=(4\times 1)-(-2\times -1)$

$=4-2=2$

$A^{-1}=\frac{1}{2}\begin{pmatrix}1 & 2\\ 1 & 4\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & 1\\ \frac{1}{2} & 2\end{pmatrix}$ **Ans.**

9 *(J2012 P1/Q12)*

$$m=\begin{pmatrix}3\\ -2\end{pmatrix} \quad n=\begin{pmatrix}-1\\ 4\end{pmatrix}$$

(a) Calculate $m-2n$. [1]

(b) Given that $sm+3n=\begin{pmatrix}12\\ t\end{pmatrix}$, calculate s and t. [2]

Thinking Process

(a) ✐ Perform the required calculation.

(b) ✐ Substitute the matrices m and n into the equation and equate it to $\begin{pmatrix}12\\ t\end{pmatrix}$.

Solution

(a) $m-2n=\begin{pmatrix}3\\ -2\end{pmatrix}-2\begin{pmatrix}-1\\ 4\end{pmatrix}$

$=\begin{pmatrix}3\\ -2\end{pmatrix}-\begin{pmatrix}-2\\ 8\end{pmatrix}=\begin{pmatrix}5\\ -10\end{pmatrix}$ **Ans.**

(b) $sm + 3n = \begin{pmatrix}12\\t\end{pmatrix}$

$s\begin{pmatrix}3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\4\end{pmatrix}=\begin{pmatrix}12\\t\end{pmatrix}$

$\begin{pmatrix}3s\\-2s\end{pmatrix}+\begin{pmatrix}-3\\12\end{pmatrix}=\begin{pmatrix}12\\t\end{pmatrix}$

$\begin{pmatrix}3s-3\\-2s+12\end{pmatrix}=\begin{pmatrix}12\\t\end{pmatrix}$

$\Rightarrow \quad 3s-3=12$

$3s=15 \quad \Rightarrow \quad s=5$ **Ans.**

also, $-2s+12=t$

$\Rightarrow \quad -2(5)+12=t \quad \Rightarrow \quad t=2$ **Ans.**

10 *(J2012/P2/Q9 a,b)*

(a) $A=\begin{pmatrix}1&2\\-3&0\end{pmatrix}$ $\quad B=\begin{pmatrix}3&1\\-2&-1\end{pmatrix}$

(i) Find $A-2B$. [1]

(ii) Find A^{-1} [2]

(b) Zara is going to put carpet and underlay in three rooms, *A*, *B* and *C*, of her house.
The cost per square metre for the carpet in *A* is \$18, in *B* is \$22 and in *C* is \$25.
The cost per square metre for the underlay is \$6 in *A* and \$8 in the other two rooms. This information is represented by matrix P below.

$$P=\begin{pmatrix}18&22&25\\6&8&8\end{pmatrix}$$

The amount of carpet and underlay required for *A*, *B* and *C* is $8\,m^2$, $15\,m^2$ and $20\,m^2$ respectively. This information is represented by matrix Q below.

$$Q=\begin{pmatrix}8\\15\\20\end{pmatrix}$$

(i) Find PQ. [2]

(ii) Explain what the matrix PQ represents. [1]

Thinking Process

(a) (ii) If $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$, then $A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$

(b) (i) Multiply respective row by respective column.
(ii) Note that the rows in matrix P show the cost of carpet and underlay per m^2. Matrix Q represents the area of each room in m^2.

Solution

(a) (i) $A-2B=\begin{pmatrix}1&2\\-3&0\end{pmatrix}-2\begin{pmatrix}3&1\\-2&-1\end{pmatrix}$

$=\begin{pmatrix}1&2\\-3&0\end{pmatrix}-\begin{pmatrix}6&2\\-4&-2\end{pmatrix}$

$=\begin{pmatrix}-5&0\\1&2\end{pmatrix}$ **Ans.**

(ii) Det. $A=(1\times 0)-(2\times -3)=6$

$A^{-1}=\frac{1}{6}\begin{pmatrix}0&-2\\3&1\end{pmatrix}$

$=\begin{pmatrix}0&-\frac{1}{3}\\\frac{1}{2}&\frac{1}{6}\end{pmatrix}$ **Ans.**

(b) (i) $PQ=\begin{pmatrix}18&22&25\\6&8&8\end{pmatrix}\begin{pmatrix}8\\15\\20\end{pmatrix}$

$=\begin{pmatrix}(18\times 8)+(22\times 15)+(25\times 20)\\(6\times 8)+(8\times 15)+(8\times 20)\end{pmatrix}$

$=\begin{pmatrix}144+330+500\\48+120+160\end{pmatrix}=\begin{pmatrix}974\\328\end{pmatrix}$ **Ans.**

(ii) PQ represents total cost of carpet and total cost of underlay.

11 *(N2012/P1/Q19)*

$$M=\begin{pmatrix}1&1\\-1&3\end{pmatrix}$$

(a) Express as a single matrix $\begin{pmatrix}-4&2\\-4&0\end{pmatrix}-2M$. [2]

(b) Find M^{-1} [2]

Thinking Process

(b) Recall: If $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$, then $A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$.

Solution

(a) $\begin{pmatrix}-4&2\\-4&0\end{pmatrix}-2M$

$=\begin{pmatrix}-4&2\\-4&0\end{pmatrix}-2\begin{pmatrix}1&1\\-1&3\end{pmatrix}$

$=\begin{pmatrix}-4&2\\-4&0\end{pmatrix}-\begin{pmatrix}2&2\\-2&6\end{pmatrix}$

$=\begin{pmatrix}-6&0\\-2&-6\end{pmatrix}$ **Ans.**

(b) Det. $M=(1\times 3)-(1\times -1)=3+1=4$

$M^{-1}=\frac{1}{4}\begin{pmatrix}3&-1\\1&1\end{pmatrix}=\begin{pmatrix}\frac{3}{4}&-\frac{1}{4}\\\frac{1}{4}&\frac{1}{4}\end{pmatrix}$ **Ans.**

12 *(J2013/P1/Q13)*

$A=\begin{pmatrix}2&3\\-2&0\end{pmatrix}$ $\quad B=\begin{pmatrix}-2&4\\-3&1\end{pmatrix}$

(a) Find $A-B$. [1]

(b) Find A^{-1}. [2]

Thinking Process

(a) Perform the required calculation.

(b) Recall: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Solution

(a) $A - B = \begin{pmatrix} 2 & 3 \\ -2 & 0 \end{pmatrix} - \begin{pmatrix} -2 & 4 \\ -3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 4 & -1 \\ 1 & -1 \end{pmatrix}$ **Ans.**

(b) Det. $A = (2\times 0) - (3\times -2) = 6$

$A^{-1} = \frac{1}{6}\begin{pmatrix} 0 & -3 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ **Ans.**

13 *(N2013/P2/Q7)*

(a) Express as a single matrix $5\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 4\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$. [2]

(b) Express as a single matrix $\begin{pmatrix} 7 & -1 & 3 \\ 2 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$. [2]

(c) $A = \begin{pmatrix} 1 & 0 \\ -2 & 4 \end{pmatrix}$

(i) Find A^{-1}. [2]

(ii) $B + 3I = A$ where I is the 2×2 identity matrix. Find B. [2]

Thinking Process

(b) Multiply respective row by respective column.

(c) (i) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $A^{-1} = \frac{1}{\text{Det. } A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

(ii) ✐ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Solution

(a) $5\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 4\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ -5 \\ 15 \end{pmatrix} - \begin{pmatrix} 4 \\ -12 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \\ 15 \end{pmatrix}$ **Ans.**

(b) $\begin{pmatrix} 7 & -1 & 3 \\ 2 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} (7\times 1)+(-1\times 0)+(3\times 2) \\ (2\times 1)+(0\times 0)+(4\times 2) \end{pmatrix}$

$= \begin{pmatrix} 7+0+6 \\ 2+0+8 \end{pmatrix} = \begin{pmatrix} 13 \\ 10 \end{pmatrix}$ **Ans.**

(c) (i) $A = \begin{pmatrix} 1 & 0 \\ -2 & 4 \end{pmatrix}$

Det. $A = (1\times 4) - (0\times -2) = 4$

$A^{-1} = \frac{1}{4}\begin{pmatrix} 4 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{4} \end{pmatrix}$ **Ans.**

(ii) $B + 3I = A$

$B = A - 3I$

$= \begin{pmatrix} 1 & 0 \\ -2 & 4 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ -2 & 4 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$= \begin{pmatrix} -2 & 0 \\ -2 & 1 \end{pmatrix}$ **Ans.**

14 *(J2014/P1 Q12)*

A café sells hot drinks.
On Monday it sells 80 teas, 60 coffees and 40 hot chocolates.
On Tuesday it sells 70 teas, 90 coffees and 50 hot chocolates.
A cup of tea costs \$0.80, a cup of coffee costs \$1 and a cup of hot chocolate costs \$1.20.

This information can be represented by the matrices M and N below.

$M = \begin{pmatrix} 80 & 60 & 40 \\ 70 & 90 & 50 \end{pmatrix} \qquad N = \begin{pmatrix} 0.8 \\ 1 \\ 1.2 \end{pmatrix}$

(a) Work out MN. [2]

(b) Explain what the numbers in your answer represent. [1]

Thinking Process

(a) ✐ To find MN ✐ perform matrix multiplication.

(b) To understand what the numbers represent ✐ consider the intermediate steps in the calculation of the product MN.

Solution

(a) $MN = \begin{pmatrix} 80 & 60 & 40 \\ 70 & 90 & 50 \end{pmatrix}\begin{pmatrix} 0.8 \\ 1 \\ 1.2 \end{pmatrix}$

$= \begin{pmatrix} (80\times 0.8)+(60\times 1)+(40\times 1.2) \\ (70\times 0.8)+(90\times 1)+(50\times 1.2) \end{pmatrix}$

$= \begin{pmatrix} 64+60+48 \\ 56+90+60 \end{pmatrix}$

$= \begin{pmatrix} 172 \\ 206 \end{pmatrix}$ **Ans.**

(b) First row represents total amount earned on Monday while the second row represents total amount earned on Tuesday.

15 *(N2014/P1/Q26)*

$$A=\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix} \quad A^{-1}=k\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}$$

(a) Find the value of k. [1]

(b) Find the matrix X, where $2A+X=\begin{pmatrix}5 & -2\\ 0 & 4\end{pmatrix}$ [2]

(c) Find the matrix Y, where $YA=(6 \quad 2)$. [2]

Thinking Process

(a) Note that k represents $\frac{1}{\text{Det. A}}$.

(c) To find matrix Y write $Y=(6 \quad 2)A^{-1}$.

Solution

(a) $k=\frac{1}{\text{determinant of A}}$

$=\frac{1}{(3\times 2)-(1\times(-1))}=\frac{1}{6+1}=\frac{1}{7}$ Ans.

(b) $2A+X=\begin{pmatrix}5 & -2\\ 0 & 4\end{pmatrix}$

$X=\begin{pmatrix}5 & -2\\ 0 & 4\end{pmatrix}-2A$

$=\begin{pmatrix}5 & -2\\ 0 & 4\end{pmatrix}-2\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}$

$=\begin{pmatrix}5 & -2\\ 0 & 4\end{pmatrix}-\begin{pmatrix}6 & 2\\ -2 & 4\end{pmatrix}=\begin{pmatrix}-1 & -4\\ 2 & 0\end{pmatrix}$ Ans.

(c) $YA=(6 \quad 2)$

$\Rightarrow Y=(6 \quad 2)A^{-1}$

$\Rightarrow Y=(6 \quad 2)\left[\frac{1}{7}\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}\right]$

$=\frac{1}{7}(6 \quad 2)\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}$

$=\frac{1}{7}(12+2 \quad -6+6)$

$=\frac{1}{7}(14 \quad 0)=(2 \quad 0)$ Ans.

16 *(J2015/P1/Q21)*

(a) Express as a single matrix $3\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}-\begin{pmatrix}4 & 0\\ -1 & 2\end{pmatrix}$. [2]

(b) $A=\begin{pmatrix}3 & -2\\ p & -1\end{pmatrix}$

The determinant of A is 2 .

(i) Find p. [1]

(ii) Find A^{-1}. [1]

Thinking Process

(a) Perform the required calculation.

(b) (i) If $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, then $\text{Det}(A)=ad-bc$

(ii) If $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, then $A^{-1}=\frac{1}{\text{Det}(A)}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$

Solution

(a) $3\begin{pmatrix}1 & 3\\ -2 & 5\end{pmatrix}-\begin{pmatrix}4 & 0\\ -1 & 2\end{pmatrix}$

$=\begin{pmatrix}3 & 9\\ -6 & 15\end{pmatrix}-\begin{pmatrix}4 & 0\\ -1 & 2\end{pmatrix}$

$=\begin{pmatrix}-1 & 9\\ -5 & 13\end{pmatrix}$ Ans.

(b) (i) $A=\begin{pmatrix}3 & -2\\ p & -1\end{pmatrix}$

determinant of A = 2

$\Rightarrow (3\times -1)-(-2\times p)=2$

$-3+2p=2$

$2p=5$

$p=\frac{5}{2}=2.5$ Ans.

(ii) $A^{-1}=\frac{1}{2}\begin{pmatrix}-1 & 2\\ -\frac{5}{2} & 3\end{pmatrix}$

$=\begin{pmatrix}-\frac{1}{2} & 1\\ -\frac{5}{4} & \frac{3}{2}\end{pmatrix}$ Ans.

17 *(N2015/P1/Q6)*

Evaluate $3\begin{pmatrix}0 & 3\\ -3 & 1\end{pmatrix}-2\begin{pmatrix}1 & 5\\ -4 & -1\end{pmatrix}$ [2]

Solution

$3\begin{pmatrix}0 & 3\\ -3 & 1\end{pmatrix}-2\begin{pmatrix}1 & 5\\ -4 & -1\end{pmatrix}$

$=\begin{pmatrix}0 & 9\\ -9 & 3\end{pmatrix}-\begin{pmatrix}2 & 10\\ -8 & -2\end{pmatrix}=\begin{pmatrix}-2 & -1\\ -1 & 5\end{pmatrix}$ Ans.

18 *(J2016/P2/Q6 b)*

(b) $X=\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}$ $Y=\begin{pmatrix}2 & 2\\ -1 & 1\end{pmatrix}$

Find

(i) $2X+Y$. [2]

(ii) Y^{-1}. [2]

Thinking Process

(b) (ii) If $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, then $A^{-1}=\frac{1}{\text{Det. }A}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$.

Solution

(b) (i) $2X+Y=2\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}+\begin{pmatrix}2 & 2\\ -1 & 1\end{pmatrix}$

$=\begin{pmatrix}6 & -2\\ 4 & 0\end{pmatrix}+\begin{pmatrix}2 & 2\\ -1 & 1\end{pmatrix}$

$=\begin{pmatrix}8 & 0\\ 3 & 1\end{pmatrix}$ **Ans.**

(ii) $Y=\begin{pmatrix}2 & 2\\ -1 & 1\end{pmatrix}$

Det. $Y=(2\times 1)-(2\times -1)=4$

$Y^{-1}=\frac{1}{4}\begin{pmatrix}1 & -2\\ 1 & 2\end{pmatrix}=\begin{pmatrix}\frac{1}{4} & -\frac{1}{2}\\ \frac{1}{4} & \frac{1}{2}\end{pmatrix}$ **Ans.**

19 *(N2016/P2/Q6)*

$A=\begin{pmatrix}2 & 0\\ 3 & 1\end{pmatrix}$ $B=\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$

(a) Find A + 2B. [2]

(b) Find AB. [2]

(c) $A\begin{pmatrix}x\\ 2\end{pmatrix}=\begin{pmatrix}8\\ 2y\end{pmatrix}$

Find x and y. [2]

(d) Find B^{-1}. [2]

Thinking Process

(b) Multiply respective row by respective column.

(c) Pre-multiply the matrix A by $\begin{pmatrix}x\\ 2\end{pmatrix}$ and equate to $\begin{pmatrix}8\\ 2y\end{pmatrix}$.

(d) If $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, then $A^{-1}=\frac{1}{\text{Det. }A}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$.

Solution

(a) $A+2B=\begin{pmatrix}2 & 0\\ 3 & 1\end{pmatrix}+2\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$

$=\begin{pmatrix}2 & 0\\ 3 & 1\end{pmatrix}+\begin{pmatrix}2 & 4\\ -2 & 6\end{pmatrix}$

$=\begin{pmatrix}4 & 4\\ 1 & 7\end{pmatrix}$ **Ans.**

(b) $AB=\begin{pmatrix}2 & 0\\ 3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$

$=\begin{pmatrix}2+0 & 4+0\\ 3-1 & 6+3\end{pmatrix}=\begin{pmatrix}2 & 4\\ 2 & 9\end{pmatrix}$ **Ans.**

(c) $A\begin{pmatrix}x\\ 2\end{pmatrix}=\begin{pmatrix}8\\ 2y\end{pmatrix}$

$\begin{pmatrix}2 & 0\\ 3 & 1\end{pmatrix}\begin{pmatrix}x\\ 2\end{pmatrix}=\begin{pmatrix}8\\ 2y\end{pmatrix}$

$\begin{pmatrix}2x+0\\ 3x+2\end{pmatrix}=\begin{pmatrix}8\\ 2y\end{pmatrix}$

$\Rightarrow\ 2x=8\ \Rightarrow\ x=4$

also, $3x+2=2y$

$\Rightarrow\ 3(4)+2=2y$

$14=2y\ \Rightarrow\ y=7$

$\therefore\ x=4,\ y=7$ **Ans.**

(d) $B=\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}$

Det. $B=(1\times 3)-(2\times -1)=5$

$B^{-1}=\frac{1}{5}\begin{pmatrix}3 & -2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}\frac{3}{5} & -\frac{2}{5}\\ \frac{1}{5} & \frac{1}{5}\end{pmatrix}$ **Ans.**

20 *(J2017/P2/Q4)*

$A=\begin{pmatrix}3 & 2\\ -4 & -2\end{pmatrix}$ $B=\begin{pmatrix}5 & 3\\ -2 & 1\end{pmatrix}$ $C=\begin{pmatrix}-2\\ 1\end{pmatrix}$

(a) Calculate 2B – 3A. [2]

(b) Calculate BC. [2]

(c) Calculate $A^{-1}+A$. [3]

Thinking Process

(b) Multiply respective row by respective column.

(c) Recall: $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}\Rightarrow A^{-1}=\frac{1}{\text{Det. }A}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$

Solution

(a) $2B-3A=2\begin{pmatrix}5 & 3\\ -2 & 1\end{pmatrix}-3\begin{pmatrix}3 & 2\\ -4 & -2\end{pmatrix}$

$=\begin{pmatrix}10 & 6\\ -4 & 2\end{pmatrix}-\begin{pmatrix}9 & 6\\ -12 & -6\end{pmatrix}$

$=\begin{pmatrix}1 & 0\\ 8 & 8\end{pmatrix}$ **Ans.**

(b) $BC=\begin{pmatrix}5 & 3\\ -2 & 1\end{pmatrix}\begin{pmatrix}-2\\ 1\end{pmatrix}$

$=\begin{pmatrix}(5\times -2)+(3\times 1)\\ (-2\times -2)+(1\times 1)\end{pmatrix}$

$=\begin{pmatrix}-10+3\\ 4+1\end{pmatrix}=\begin{pmatrix}-7\\ 5\end{pmatrix}$ **Ans.**

(c) Det. A $= (3\times -2) - (2\times -4) = 2$

$$A^{-1} + A = \frac{1}{2}\begin{pmatrix} -2 & -2 \\ 4 & 3 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ -4 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 \\ 2 & \frac{3}{2} \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ -4 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ -2 & -\frac{1}{2} \end{pmatrix} \text{ Ans.}$$

21 *(N2017/P1/Q21)*

(a) Express $3\begin{pmatrix} 3 & 1 \\ -5 & -4 \end{pmatrix} - 2\begin{pmatrix} 1 & -3 \\ 0 & 2 \end{pmatrix}$ as a single matrix. [2]

(b) Find the inverse of $\begin{pmatrix} 3 & 1 \\ -5 & -4 \end{pmatrix}$. [2]

Thinking Process

(b) Recall: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Solution

(a) $3\begin{pmatrix} 3 & 1 \\ -5 & -4 \end{pmatrix} - 2\begin{pmatrix} 1 & -3 \\ 0 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 9 & 3 \\ -15 & -12 \end{pmatrix} - \begin{pmatrix} 2 & -6 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ -15 & -16 \end{pmatrix} \text{ Ans.}$$

(b) Inverse $= \frac{1}{-12-(-5)}\begin{pmatrix} -4 & -1 \\ 5 & 3 \end{pmatrix}$

$$= \frac{1}{-7}\begin{pmatrix} -4 & -1 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} \frac{4}{7} & \frac{1}{7} \\ -\frac{5}{7} & -\frac{3}{7} \end{pmatrix} \text{ Ans.}$$

22 *(N2017/P2/Q3)*

Anya makes T-shirts.
The matrix, M, shows the number of T-shirts of different types she makes in one week.

$$M = \begin{matrix} & \text{Small} & \text{Medium} & \text{Large} & \\ & \left(\begin{matrix} 10 \\ 20 \end{matrix}\right. & \begin{matrix} 25 \\ 40 \end{matrix} & \left.\begin{matrix} 30 \\ 25 \end{matrix}\right) & \begin{matrix} \text{Men} \\ \text{Women} \end{matrix} \end{matrix}$$

(a) Anya sells all of these T-shirts to a shop.
She charges \$5 for each small T-shirt, \$6 for each medium T-shirt and \$8 for each large T-shirt.
Represent these amounts in a 3×1 column matrix N. [1]

(b) (i) Work out P = MN. [2]

(ii) Explain what the elements in matrix P represent. [1]

(c) The shopkeeper sells all sizes of men's T-shirts at \$10 each.
He sells all sizes of women's T-shirts at \$9.50 each.
He sells all of these T-shirts.

(i) Work out $(10 \quad 9.50)\begin{pmatrix} 10 & 25 & 30 \\ 20 & 40 & 25 \end{pmatrix}$. [2]

(ii) Work out the percentage profit the shopkeeper makes when he sells all of the T-shirts. [3]

Thinking Process

(b) (i) Multiply respective row by respective column.

(ii) To understand what it represent ✐ consider what M and N stand for.

(c) (i) Order of first matrix is (1×2) and the order of second matrix is (2×3). So the answer matrix should be of order (1×3), i.e. one row and three columns.

(ii) Using the answers to parts (b)(i) and (c)(i), find the total cost price and total selling price. Find the profit and express the amount as a percentage of the original cost.

Solution

(a) $N = \begin{pmatrix} 5 \\ 6 \\ 8 \end{pmatrix}$ Ans.

(b) (i) P = MN

$$= \begin{pmatrix} 10 & 25 & 30 \\ 20 & 40 & 25 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 50+150+240 \\ 100+240+200 \end{pmatrix} = \begin{pmatrix} 440 \\ 540 \end{pmatrix}$$

(ii) First row represents the total amount Anya makes from Mens T-shirt while the second row represents the total amount she makes from Women's T-shirt.

(c) (i) $(10 \quad 9.50)\begin{pmatrix} 10 & 25 & 30 \\ 20 & 40 & 25 \end{pmatrix}$

$= (100+190 \quad 250+380 \quad 300+237.5)$

$= (290 \quad 630 \quad 537.5)$ Ans.

(ii) Total cost price $= \$440 + \$540 = \$980$

Total amount earned $= \$290 + \$630 + \$537.5$

$= \$\,1457.5$

profit $= \$\,1457.5 - \$980 = \$477.5$

percentage profit $= \frac{477.5}{980} \times 100$

$= 48.72 \approx 48.7\%$ Ans.

23 (J2018/P1/Q15)

During two weeks, a shopkeeper records the number of packets of two different types of tea he sells and the profit he makes from them.

Week 1

- Type A tea, 30 packets sold, profit of \$1.20 on each packet
- Type B tea, 20 packets sold, profit of \$2 on each packet

Week 2

- Type A tea, 40 packets sold, loss of \$0.50 on each packet
- Type B tea, 30 packets sold, profit of \$3 on each packet

This information can be represented by these matrices.

$$\begin{pmatrix}30 & 20\end{pmatrix} \quad \begin{pmatrix}40 & 30\end{pmatrix} \quad \begin{pmatrix}1.2\\2\end{pmatrix} \quad \begin{pmatrix}-0.5\\3\end{pmatrix}$$

(a) Work out $\begin{pmatrix}30 & 20\end{pmatrix}\begin{pmatrix}1.2\\2\end{pmatrix}-\begin{pmatrix}40 & 30\end{pmatrix}\begin{pmatrix}-0.5\\3\end{pmatrix}$. [2]

(b) Explain the meaning of your answer to part (a). [1]

Thinking Process

(a) Perform the required calculation.

(b) To understand what the answer represents ☞ consider the intermediate steps in the calculation of the matrices.

Solution

(a) $\begin{pmatrix}30 & 20\end{pmatrix}\begin{pmatrix}1.2\\2\end{pmatrix}-\begin{pmatrix}40 & 30\end{pmatrix}\begin{pmatrix}-0.5\\3\end{pmatrix}$

$=(36+40)-(-20+90)$

$=76-70=6$ **Ans.**

(b) It represents the difference in total profit earned during week 1 and week 2.

24 (J2018/P1/Q23)

$$A=\begin{pmatrix}4 & -1\\2 & 0\end{pmatrix} \qquad B=\begin{pmatrix}6 & -3\\0 & -2\end{pmatrix}$$

(a) Find the matrix X, such that $2A+X=B$. [2]

(b) Find the matrix Y, such that $AY=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$. [3]

Thinking Process

(a) Perform the required calculation.

(b) To find matrix Y ☞ write $Y=A^{-1}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$.

Solution

(a) $2A+X=B$

$\Rightarrow X=B-2A$

$=\begin{pmatrix}6 & -3\\0 & -2\end{pmatrix}-2\begin{pmatrix}4 & -1\\2 & 0\end{pmatrix}$

$=\begin{pmatrix}6 & -3\\0 & -2\end{pmatrix}-\begin{pmatrix}8 & -2\\4 & 0\end{pmatrix}$

$=\begin{pmatrix}-2 & -1\\-4 & -2\end{pmatrix}$ **Ans.**

(b) $A=\begin{pmatrix}4 & -1\\2 & 0\end{pmatrix}$. Det. $A=(4\times 0)-(-1\times 2)=2$

$\Rightarrow A^{-1}=\frac{1}{2}\begin{pmatrix}0 & 1\\-2 & 4\end{pmatrix}$

Now, $AY=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\Rightarrow Y=A^{-1}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$=\frac{1}{2}\begin{pmatrix}0 & 1\\-2 & 4\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$=\frac{1}{2}\begin{pmatrix}0 & 1\\-2 & 4\end{pmatrix}=\begin{pmatrix}0 & \frac{1}{2}\\-1 & 2\end{pmatrix}$ **Ans.**

25 (N2018/P1/Q23)

(a) Express $\begin{pmatrix}2\\1\end{pmatrix}-3\begin{pmatrix}-1\\2\end{pmatrix}+2\begin{pmatrix}0\\-2\end{pmatrix}$ as a single vector. [2]

(b) Find $\begin{pmatrix}2 & -1\end{pmatrix}\begin{pmatrix}0 & -1 & 2\\3 & 1 & -3\end{pmatrix}$. [2]

Thinking Process

(b) Multiply row by respective column.

Solution

(a) $\begin{pmatrix}2\\1\end{pmatrix}-3\begin{pmatrix}-1\\2\end{pmatrix}+2\begin{pmatrix}0\\-2\end{pmatrix}$

$=\begin{pmatrix}2\\1\end{pmatrix}-\begin{pmatrix}-3\\6\end{pmatrix}+\begin{pmatrix}0\\-4\end{pmatrix}$

$=\begin{pmatrix}2+3+0\\1-6-4\end{pmatrix}=\begin{pmatrix}5\\-9\end{pmatrix}$ **Ans.**

(b) $\begin{pmatrix}2 & -1\end{pmatrix}\begin{pmatrix}0 & -1 & 2\\3 & 1 & -3\end{pmatrix}$

$=\begin{pmatrix}0-3 & -2-1 & 4+3\end{pmatrix}$

$=\begin{pmatrix}-3 & -3 & 7\end{pmatrix}$ **Ans.**

Topic 22

Functions

1 *(J2007/P1 Q16)*

Given that $f(x)=\frac{5x-4}{3}$, find

(a) $f(1\frac{1}{5})$, [1]

(b) $f^{-1}(x)$. [2]

Thinking Process

(a) Write $1\frac{1}{5}$ as improper fraction.

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $1\frac{1}{5}$ can be written as $\frac{6}{5}$

$$\therefore f(\tfrac{6}{5})=\frac{5\left(\frac{6}{5}\right)-4}{3}$$
$$=\frac{6-4}{3}$$
$$=\frac{2}{3} \quad \text{Ans.}$$

(b) $$f(x)=\frac{5x-4}{3}$$

Let $y=\frac{5x-4}{3}$

$\Rightarrow 3y=5x-4$

$\Rightarrow 5x=3y+4$

$\Rightarrow x=\frac{3y+4}{5}$

$\therefore f^{-1}(x)=\frac{3x+4}{5}$ Ans.

2 *(N2007/P1 Q6)*

It is given that $f(x)=\frac{3-x}{2}$.

Find

(a) $f(-9)$, [1]

(b) $f^{-1}(x)$. [1]

Thinking Process

(a) To find $f(-9)$ ✎ substitute $x = -9$.

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(-9)=\frac{3-(-9)}{2}=\frac{3+9}{2}=\frac{12}{2}=6$ Ans.

(b) Let $y=\frac{3-x}{2}$ $\quad$ $\boxed{y=f(x)}$

$\Rightarrow 2y=3-x$

$\Rightarrow x=3-2y$ $\quad$ $\boxed{x=f^{-1}(y)}$

i.e. $f^{-1}(y)=3-2y$

$\therefore f^{-1}(x)=3-2x$ Ans.

3 *(J2008/P1 Q3)*

It is given that $f(x)=5x+2$.

Find

(a) $f(-2)$, [1]

(b) $f^{-1}(x)$. [1]

Thinking Process

(a) To find $f(-2)$ ✎ substitute $x = -2$

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(x)=5x+2$.

$\therefore f(-2)=5(-2)+2$
$=-10+2$
$=-8$ Ans.

(b) $f(x)=5x+2$

Let $y=f(x)$

$\therefore y=5x+2$

$\Rightarrow 5x=y-2$

$\Rightarrow x=\frac{y-2}{5}$ $\quad$ $\boxed{x=f^{-1}(y)}$

i.e. $f^{-1}(y)=\frac{y-2}{5}$

$\therefore f^{-1}(x)=\frac{x-2}{5}$ Ans.

4 *(N2008/P1/Q12)*

Given that $f(x)=\frac{4x+3}{2x}$, find

(a) f(3), [1]

(b) $f^{-1}(x)$. [2]

Thinking Process

(a) Substitute $x = 3$

(b) Let $y = f(x)$. Make x the subject of the formula.

Solution

(a) $f(x)=\frac{4x+3}{2x}$

$\therefore f(3)=\frac{4(3)+3}{2(3)}$

$=\frac{15}{6}$

$=\frac{5}{2}=2\frac{1}{2}$ **Ans.**

(b) $f(x)=\frac{4x+3}{2x}$

Let $y=f(x)$

$\therefore y=\frac{4x+3}{2x}$

$\Rightarrow 2xy=4x+3$

$2xy-4x=3$

$x(2y-4)=3$

$x=\frac{3}{2y-4}$

i.e. $f^{-1}(y)=\frac{3}{2y-4}$ $\because x=f^{-1}(y)$

$\therefore f^{-1}(x)=\frac{3}{2x-4}$ **Ans.**

5 *(J2009/P1/Q16)*

It is given that $f(x) = 12 - 5x$. Find

(a) f(4), [1]

(b) the value of x for which $f(x) = 17$, [1]

(c) $f^{-1}(x)$. [2]

Thinking Process

(a) To find f(4) substitute $x = 4$

(b) substitute the given value and solve for x.

(c) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(x)=12-5x$

$\therefore f(4)=12-5(4)$

$=12-20=-8$ **Ans.**

(b) $f(x)=17$

$\Rightarrow 17=12-5x$

$\therefore 5x=12-17$

$5x=-5$

$x=-1$ **Ans.**

6 *(N2009/P1/Q12)*

Given that $f(x)=4x-7$, find

(a) $f\left(\frac{1}{2}\right)$, [1]

(b) the value of p when $f(p)=p$. [2]

Thinking Process

(a) Substitute $x=\frac{1}{2}$.

(b) Substitute $x = p$. Solve the given equation for p.

Solution

(a) $f(x)=4x-7$

$\therefore f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)-7$

$=2-7=-5$ **Ans.**

(b) $f(p)=p$

$4p-7=p$

$3p=7$

$p=\frac{7}{3}=2\frac{1}{3}$ **Ans.**

7 *(J2010/P1/Q11)*

Given that $f(x)=\frac{5-2x}{3x}$, find

(a) f(–2), [1]

(b) $f^{-1}(x)$. [2]

Thinking Process

(a) To find f(–2) substitute $x=-2$ into the expression of f(x).

(b) To express f^{-1} in x solve $f(x) = y$.

Solution

(a) $f(-2)=\frac{5-2(-2)}{3(-2)}$

$=\frac{5+4}{-6}$

$=-\frac{9}{6}$

$=-\frac{3}{2}=-1\frac{1}{2}$ **Ans.**

(b) Let $y = f(x)$

$\Rightarrow \quad y = \frac{5-2x}{3x}$

$3xy = 5 - 2x$

$3xy + 2x = 5$

$x(3y+2) = 5$

$x = \frac{5}{3y+2}$ $\boxed{\because x = f^{-1}(y)}$

i.e. $f^{-1}(y) = \frac{5}{3y+2}$

$\therefore \quad f^{-1}(x) = \frac{5}{3x+2}$ **Ans.**

8 *(N2010/P1/Q12)*

$f(x) = 6 - \frac{x}{2}$

(a) Find f(5). [1]

(b) Find $f^{-1}(x)$. [2]

Thinking Process

(a) To find f(5) substitute $x = 5$ into the expression of $f(x)$.

(b) Let $f(x) = y$. Make x the subject of the formula.

Solution

(a) $f(5) = 6 - \frac{5}{2}$

$= \frac{7}{2} = 3\frac{1}{2}$ **Ans.**

(b) Let $y = f(x)$

$\Rightarrow \quad y = 6 - \frac{x}{2}$

$\frac{x}{2} = 6 - y$

$x = 12 - 2y$ $\boxed{\because x = f^{-1}(y)}$

$\Rightarrow \quad f^{-1}(y) = 12 - 2y$

$\therefore \quad f^{-1}(x) = 12 - 2x$ **Ans.**

9 *(J2011/P2/Q1 b)*

(b) A function is defined by $f(x) = \frac{2x-3}{4}$.

(i) Find f(2). [1]

(ii) Given that $f^{-1}(x) = cx + d$, find the values of c and d. [2]

(iii) Given that $f(g) = -g$, find the value of g. [2]

Thinking Process

(b) (i) Substitute $x = 2$ in $f(x)$.

(ii) Find $f^{-1}(x)$, then find c and d by comparision.

(iii) Substitute $x = g$ in $f(x)$ and solve for g.

Solution

(b) (i) $f(2) = \frac{2(2)-3}{4}$

$= \frac{4-3}{4} = \frac{1}{4}$ **Ans.**

(ii) Let $y = f(x)$

$\Rightarrow \quad y = \frac{2x-3}{4}$

$4y = 2x - 3$

$2x = 4y + 3$

$x = \frac{4y+3}{2}$ $\boxed{\because x = f^{-1}(y)}$

i.e. $f^{-1}(y) = \frac{4y+3}{2}$

$\therefore \quad f^{-1}(x) = \frac{4x+3}{2}$

$= 2x + \frac{3}{2}$

comparing it by $f^{-1}(x) = cx + d$

$c = 2, \quad d = \frac{3}{2}$ **Ans.**

(iii) $f(g) = -g$

$\Rightarrow \quad \frac{2(g)-3}{4} = -g$

$2g - 3 = -4g$

$6g = 3$

$g = \frac{3}{6} = \frac{1}{2}$ **Ans.**

10 *(N2011/P1/Q4)*

It is given that $f(x) = \frac{3+x}{2}$.

(a) Find f(−3). [1]

(b) Find $f^{-1}(x)$. [1]

Thinking Process

(a) replace x by −3

(b) To express f^{-1} in x solve $f(x) = y$.

Solution

(a) $f(-3) = \frac{3+(-3)}{2}$

$= \frac{0}{2} = 0$ **Ans.**

(b) $f(x) = \frac{3+x}{2}$

Let $y = f(x)$

$\Rightarrow \quad y = \frac{3+x}{2}$

$2y = 3 + x$

$x = 2y - 3$ $\boxed{\because x = f^{-1}(y)}$

i.e. $f^{-1}(y) = 2y - 3$

$\therefore \quad f^{-1}(x) = 2x - 3$ **Ans.**

11 *(J2012/P1/Q19)*

(a) $f(x) = x^3 - 4$

Find

(i) $f(-2)$. [1]

(ii) $f^{-1}(x)$. [1]

(b) $g(y) = y^2 - 3y + 1$

Write down and simplify an expression for $g(a-2)$. [2]

Thinking Process

(a) (i) Replace x by -2.

(ii) Let $y = f(x)$. Make x the subject of formula.

(b) Replace y by $a-2$ and simplify.

Solution

(a) (i) $f(-2) = (-2)^3 - 4$

$= -8 - 4$

$= -12$ **Ans.**

(ii) $f(x) = x^3 - 4$

Let $y = f(x)$

$\Rightarrow \quad y = x^3 - 4$

$x^3 = y + 4$

$x = (y+4)^{\frac{1}{3}}$ $\quad \boxed{\because x = f^{-1}(y)}$

$\Rightarrow \quad f^{-1}(y) = (y+4)^{\frac{1}{3}}$

$\therefore \quad f^{-1}(x) = \sqrt[3]{(x+4)}$ **Ans.**

(b) $g(y) = y^2 - 3y + 1$

$g(a-2) = (a-2)^2 - 3(a-2) + 1$

$= a^2 - 4a + 4 - 3a + 6 + 1$

$= a^2 - 7a + 11$ **Ans.**

12 *(N2012/P1/Q4)*

$f(x) = 5 + 3x$

(a) Evaluate $f\left(-\frac{1}{2}\right)$. [1]

(b) Find $f^{-1}(x)$. [1]

Thinking Process

(a) Replace x by $-\frac{1}{2}$.

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(x) = 5 + 3x$

$f\left(-\frac{1}{2}\right) = 5 + 3\left(-\frac{1}{2}\right)$

$= 5 - \frac{3}{2}$

$= \frac{7}{2} = 3\frac{1}{2}$ **Ans.**

(b) $f(x) = 5 + 3x$

Let $y = f(x)$

$\Rightarrow \quad y = 5 + 3x$

$x = \frac{y-5}{3}$ $\quad \boxed{\because x = f^{-1}(y)}$

$\Rightarrow \quad f^{-1}(y) = \frac{y-5}{3}$

$\therefore \quad f^{-1}(x) = \frac{x-5}{3}$ **Ans.**

13 *(J2013/P2/Q9 b)*

(b) $f(x) = \frac{3x+2}{5}$

Find

(i) $f(-4)$ [1]

(ii) the value of g such that $f(g) = 7$, [2]

(iii) $f^{-1}(x)$. [2]

Thinking Process

(b) (i) Substitute $x = 2$ in $f(x)$

(ii) Substitute $x = g$ in $f(x)$ and solve for g.

(iii) Let $f(x) = y$. Make x the subject of the formula.

Solution *with* TEACHER's COMMENTS

(b) (i) $f(x) = \frac{3x+2}{5}$

$\therefore \quad f(-4) = \frac{3(-4)+2}{5}$

$= \frac{-10}{5} = -2$ **Ans.**

(ii) $f(g) = 7$

$\Rightarrow \quad \frac{3g+2}{5} = 7$

$3g + 2 = 35$

$3g = 33$

$g = 11$ **Ans.**

(iii) Let $y = f(x)$

$\Rightarrow y = \frac{3x+2}{5}$

$5y = 3x + 2$

$3x = 5y - 2$

$x = \frac{5y-2}{3}$ $\because x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{5y-2}{3}$

$\therefore f^{-1}(x) = \frac{5x-2}{3}$ **Ans.**

14 *(N2013/P1/Q3)*

$$f(x) = 2x - 6$$

(a) Evaluate $f\left(-\frac{1}{2}\right)$. [1]

(b) Find $f^{-1}(x)$. [1]

Thinking Process

(a) Replace x by $-\frac{1}{2}$.

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(x) = 2x - 6$

$f(-\frac{1}{2}) = 2(-\frac{1}{2}) - 6$

$= -1 - 6 = -7$ **Ans.**

(b) Let $y = f(x)$

$\Rightarrow y = 2x - 6$

$2x = y + 6$

$x = \frac{y+6}{2}$ $\because x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{y+6}{2}$

$\therefore f^{-1}(x) = \frac{x+6}{2}$ **Ans.**

15 *(J2014/P1/Q13)*

$f(x) = 2 - 3x$

Find

(a) $f(-5)$. [1]

(b) $f^{-1}(x)$. [2]

Thinking Process

(a) Replace x by -5.

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(-5) = 2 - 3(-5)$

$= 2 + 15 = 17$ **Ans.**

(b) $f(x) = 2 - 3x$

Let $y = f(x)$

$\Rightarrow y = 2 - 3x$

$3x = 2 - y$

$x = \frac{2-y}{3}$ $\because x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{2-y}{3}$

$\therefore f^{-1}(x) = \frac{2-x}{3}$ **Ans.**

16 *(N2014/P1/Q4)*

$$f(x) = 2(x - 3)$$

(a) Evaluate $f\left(\frac{1}{2}\right)$. [1]

(b) Find $f^{-1}(x)$. [1]

Thinking Process

(a) Replace x by $\frac{1}{2}$.

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(\frac{1}{2}) = 2(\frac{1}{2} - 3)$

$= 2(-\frac{5}{2}) = -5$ **Ans.**

(b) $f(x) = 2(x - 3)$

Let $y = f(x)$

$\Rightarrow y = 2(x - 3)$

$y = 2x - 6$

$x = \frac{y+6}{2}$ $\because x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{y+6}{2}$

$\therefore f^{-1}(x) = \frac{x+6}{2}$ **Ans.**

17 *(J2015/P2/Q7 a)*

(a) $f(x) = \frac{2x+7}{3}$

(i) Find $f^{-1}(x)$. [2]

(ii) Given that $f(m) = \frac{m}{2}$, find m. [2]

Thinking Process

(a) (i) Let $y = f(x)$. Make x the subject of formula.

(ii) Substitute $x = m$ in $f(x)$ and equate it with $\frac{m}{2}$.

Solution

(a) (i) $f(x)=\frac{2x+7}{3}$

Let $y=f(x)$

$\Rightarrow \quad y=\frac{2x+7}{3}$

$3y=2x+7$

$2x=3y-7$

$x=\frac{3y-7}{2}$ $\quad \boxed{\because x=f^{-1}(y)}$

$\Rightarrow \quad f^{-1}(y)=\frac{3y-7}{2}$

$\therefore \quad f^{-1}(x)=\frac{3x-7}{2}$ **Ans.**

(ii) $f(m)=\frac{m}{2}$

$\frac{2m+7}{3}=\frac{m}{2}$

$4m+14=3m$

$m=-14$ **Ans.**

18 *(N2015/P1/Q4)*

$f(x)=1+4x$

(a) Find $f\left(-\frac{2}{5}\right)$. [1]

(b) Find $f^{-1}(x)$. [1]

Thinking Process

(a) Replace x by $-\frac{2}{5}$.

(b) Let $y=f(x)$. Make x the subject of formula.

Solution

(a) $f(x)=1+4x$

$f\left(-\frac{2}{5}\right)=1+4(-\frac{2}{5})$

$=1-\frac{8}{5}$

$=-\frac{3}{5}$ **Ans.**

(b) $f(x)=1+4x$

Let $y=f(x)$

$\Rightarrow \quad y=1+4x$

$4x=y-1$

$x=\frac{y-1}{4}$ $\quad \boxed{\because x=f^{-1}(y)}$

$\Rightarrow \quad f^{-1}(y)=\frac{y-1}{4}$

$\therefore \quad f^{-1}(x)=\frac{x-1}{4}$ **Ans.**

19 *(J2016/P1/Q21)*

(a) The table shows the values of the function $f(x)$ for some values of x.

x	1	2	3	4	5
$f(x)$	5	7	9	11	13

Express the function $f(x)$ in terms of x. [1]

(b) $g(x)=\frac{8-3x}{2}$

(i) Evaluate $g(-2)$. [1]

(ii) Find $g^{-1}(x)$. [2]

Thinking Process

(a) Using two pairs of values of x and $f(x)$ given in the table, form two equations and solve them simultaneously to find the function $f(x)$

(b) (i) Replace x by -2.

(ii) Let $y=g(x)$. Make x the subject of formula.

Solution

(a) Let $f(x)=ax+b$

from given table, $f(x)=5$ when $x=1$,

$\Rightarrow \quad 5=a+b \cdots\cdots\cdots(1)$

also from table, $f(x)=7$ when $x=2$,

$\Rightarrow \quad 7=2a+b \cdots\cdots\cdots(2)$

from eq. (1), $a=5-b$, put in eq. (2)

$\Rightarrow \quad 7=2(5-b)+b$

$7=10-2b+b$

$b=3$

substitute $b=3$ into (1),

$\Rightarrow \quad 5=a+3$

$a=2$

$\therefore \quad f(x)=2x+3$ **Ans.**

(b) (i) $g(-2)=\frac{8-3(-2)}{2}$

$=\frac{8+6}{2}$

$=\frac{14}{2}=7$ **Ans.**

(ii) $g(x)=\frac{8-3x}{2}$

Let $y=g(x)$

$\Rightarrow \quad y=\frac{8-3x}{2}$

$2y=8-3x$

$3x=8-2y$

$x=\frac{8-2y}{3}$ $\quad \boxed{\because x=g^{-1}(y)}$

$\Rightarrow \quad g^{-1}(y)=\frac{8-2y}{3}$

$\therefore \quad g^{-1}(x)=\frac{8-2x}{3}$ **Ans.**

20 (N2016/P1/Q10)

$$f(x) = 4 + 3x$$

(a) Find $f\left(-2\frac{1}{2}\right)$. [1]

(b) Find $f^{-1}(5)$. [2]

Thinking Process

(a) Substitute $x = -\frac{5}{2}$ in f(x)

(b) Let $y = f^{-1}(5) \Rightarrow f(y) = 5$. Solve for y

Solution

(a) $f(-2\frac{1}{2}) = f(-\frac{5}{2})$

$= 4 + 3(-\frac{5}{2})$

$= 4 - \frac{15}{2}$

$= -\frac{7}{2} = -3\frac{1}{2}$ **Ans.**

(b) Let $y = f^{-1}(5)$

$\Rightarrow f(y) = 5$

$\Rightarrow 4 + 3y = 5$

$\Rightarrow 3y = 1 \Rightarrow y = \frac{1}{3}$

$\therefore f^{-1}(5) = \frac{1}{3}$ **Ans.**

21 (J2017/P1/Q14)

$$f(x) = \frac{3x - k}{4}$$

(a) Given that $f(11) = 7$, find the value of k. [2]

(b) Find $f^{-1}(x)$. [2]

Thinking Process

(a) Find f(11) by replacing x by 11. Equate the expression to 7

(b) Let $f(x) = y$. Make x the subject of the formula.

Solution

(a) $f(11) = \frac{3(11) - k}{4} = \frac{33 - k}{4}$

given that, $f(11) = 7$

$\therefore 7 = \frac{33 - k}{4}$

$28 = 33 - k$

$k = 5$ **Ans.**

(b) $f(x) = \frac{3x - 5}{4}$

Let $y = f(x)$

$\Rightarrow y = \frac{3x - 5}{4}$

$3x - 5 = 4y$

$3x = 4y + 5$

$x = \frac{4y + 5}{3}$ $\quad \because x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = \frac{4y + 5}{3}$

$\therefore f^{-1}(x) = \frac{4x + 5}{3}$ **Ans.**

22 (N2017/P1/Q4)

$$f(x) = \frac{x}{4}$$

(a) Find $f\left(\frac{1}{2}\right)$. [1]

(b) Find $f^{-1}(x)$. [1]

Thinking Process

(a) Substitute $x = \frac{1}{2}$

(b) Let $f(x) = y$. Make x the subject of the formula.

Solution

(a) $f(\frac{1}{2}) = \frac{\frac{1}{2}}{4}$

$= \frac{1}{8}$ **Ans.**

(b) $f(x) = \frac{x}{4}$

Let $y = f(x)$

$\Rightarrow y = \frac{x}{4}$

$x = 4y$ $\quad \because x = f^{-1}(y)$

$\Rightarrow f^{-1}(y) = 4y$

$\therefore f^{-1}(x) = 4x$ **Ans.**

23 (J2018/P1/Q11)

$$f(x) = \frac{1}{3x + 2}$$

(a) Find $f(-2)$. [1]

(b) Find $f^{-1}(x)$. [2]

Thinking Process

(a) Replace x by -2.

(b) Let $y = f(x)$. Make x the subject of formula.

Solution

(a) $f(-2)=\dfrac{1}{3(-2)+2}=-\dfrac{1}{4}$ **Ans.**

(b) $f(x)=\dfrac{1}{3x+2}$

Let $y=f(x)$

$\Rightarrow\ y=\dfrac{1}{3x+2}$

$\Rightarrow\ 3x+2=\dfrac{1}{y}$

$\Rightarrow\ 3x=\dfrac{1}{y}-2\ \Rightarrow\ x=\dfrac{1-2y}{3y}\quad \because x=f^{-1}(y)$

$\Rightarrow\ f^{-1}(y)=\dfrac{1-2y}{3y}$

$\therefore\ f^{-1}(x)=\dfrac{1-2x}{3x}$ **Ans.**

24 *(N2018/P1 Q6)*

$$f(x)=\frac{2x+5}{3x}$$

(a) Find $f(-2)$ [1]

(b) Find $f^{-1}(x)$. [3]

Thinking Process

(a) Replace x by -2.

(b) Let $y=f(x)$. Make x the subject of formula.

Solution

(a) $f(-2)=\dfrac{2(-2)+5}{3(-2)}$

$=\dfrac{-4+5}{-6}=-\dfrac{1}{6}$ **Ans.**

(b) $f(x)=\dfrac{2x+5}{3x}$

Let $y=f(x)$

$\Rightarrow\ y=\dfrac{2x+5}{3x}$

$\Rightarrow\ 3xy=2x+5$

$\Rightarrow\ 3xy-2x=5$

$\Rightarrow\ x(3y-2)=5$

$\Rightarrow\ x=\dfrac{5}{3y-2}\quad \because x=f^{-1}(y)$

$\Rightarrow\ f^{-1}(y)=\dfrac{5}{3y-2}$

$\therefore\ f^{-1}(x)=\dfrac{5}{3x-2}$ **Ans.**

Topic 23

Problem-Solving and Patterns

1 *(J2009/P2/Q6)*

The diagram shows the first four rows of a pattern of numbers.

Row 1	1	2	1						
Row 2	2	3	2	3	2				
Row 3	3	4	3	4	3	4	3		
Row 4	4	5	4	5	4	5	4	5	4

The table shows some results obtained from this pattern.

Row number	1	2	3	4	5		n
Number of numbers in the row	3	5	7	9	p		x
Product of the first two numbers in the row	2	6	12	20	q		y
Sum of all the numbers in the row	4	12	24	40	r		z
Middle number in the row	2	2	4	4	s		

(a) Find the values of p, q, r and s. [2]

(b) Find expressions, in terms of n, for x, y and z. [3]

(c) Write down the middle number in Row 101. [1]

Thinking Process

(a) p : Each subsequent number is 2 more than the previous one.

q : Consider the increasing difference between consecutive terms.

r : Observe that the difference between consecutive terms is increasing by 4.

s : Observe the pattern, 2,2,4,4,.......

(b) x : x is always 1 more than $2n$.

y : Compare the sequence 1, 2, 3, 4.......with 2, 6, 12, 20.

z : Observe that rows y and z follow similar pattern i.e. the difference in their consecutive terms is increasing. Therefore the expression for z must be similar to the expression for y.

Solution

(a) $p = 11$ **Ans.**

$q = 30$ **Ans.**

$r = 60$ **Ans.**

$s = 6$ **Ans.**

(b) $x = 2n + 1$ **Ans.**

$y = n^2 + n$ **Ans.**

$z = 2n^2 + 2n$ **Ans.**

(c) Middle number in row $101 = 102$ **Ans.**

2 *(N2009/P1/Q22)*

The diagrams below show small black, grey and white triangles forming a pattern.

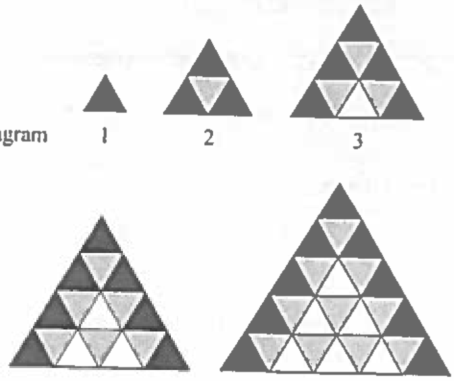


Diagram 1, 2, 3, 4, 5

The table below shows the number of triangles in each diagram.

Diagram (n)	1	2	3	4	5	6
Small triangles	1	4	9	16	25	
Black triangles	1	3	5	7	9	
Grey triangles	0	1	3	6	10	
White triangles	0	0	1	3	6	10

(a) Complete the column for Diagram 6. [2]

(b) Write an expression, in terms of n, for the number of

(i) small triangles in Diagram n, [1]

(ii) black triangles in Diagram n. [1]

Thinking Process

(a) Observe that numbers in row two are perfect squares. Numbers in row three are odd numbers. In row four, consider the increasing difference.

(b) (i) Small triangles corresponds to perfect square of numbers.

(ii) Black triangles corresponds to odd numbers.

Solution

(a)

Diagram (n)	1	2	3	4	5	6
Small triangles	1	4	9	16	25	36
Black triangles	1	3	5	7	9	11
Grey triangles	0	1	3	6	10	15
White triangles	0	0	1	3	6	10

(b) (i) Small triangles $= n^2$ **Ans.**

(ii) Black triangles $= 2n - 1$ **Ans.**

3 *(J2010/P2/Q10)*

```
++++        +++++
+OO+        +OOO+
++++        +OOO+
            +++++
```

Flowerbed 1   Flowerbed 2

```
++++++      +++++++
+OOOO+      +OOOOO+
+OOOO+      +OOOOO+
+OOOO+      +OOOOO+
++++++      +OOOOO+
            +++++++
```

Flowerbed 3   Flowerbed 4

The diagrams above show the first four flowerbeds in a sequence.
Each flowerbed contains two types of plant, pansies (+) and primroses (O).
The table shows the number of plants in the first three flowerbeds.

Flowerbed number (n)	1	2	3	4	5
Number of pansies	10	14	18		
Number of primroses	2	6	12		
Total number of plants	12	20	30		

(a) Copy and complete the columns for flowerbeds 4 and 5. [2]

(b) Find an expression, in terms of n, for

(i) the number of pansies in flowerbed n, [1]

(ii) the number of primroses in flowerbed n. [1]

(c) Hence show that the total number of plants in flowerbed n can be expressed in the form

$$(n+2)(n+3).$$ [2]

(d) Calculate the total number of plants in flowerbed 10. [1]

(e) There are 306 plants in flowerbed k.

(i) Show that k satisfies the equation

$$k^2 + 5k - 300 = 0.$$ [2]

(ii) Solve the equation $k^2 + 5k - 300 = 0$. [2]

(iii) Hence find the number of pansies in flowerbed k. [1]

Thinking Process

(a) Row 2: Each subsequent number is always 4 more than the previous one.
Row 3: The difference between numbers is increasing by 2.
Row 4: It is the sum of row 2 and row 3.

(b) (i) & (ii) Observe the pattern and deduce the nth term.

(c) Add up the answers to part (b) (i) & (ii) and simplify.

(d) Substitute n = 10 into the expression given in part (c).

(e) (i) Substitute $n = k$ into the expression given in part (c). Equate it to 306.

(ii) Solve by quadratic formula.

(iii) Replace n by k in (b) (i). Substitute the value of k found in (iii) and simplify.

Solution *with* TEACHER's COMMENTS

(a)

Flowerbed number (n)	1	2	3	4	5
Number of pansies	10	14	18	22	26
Number of primroses	2	6	12	20	30
Total number of plants	12	20	30	42	56

(b) (i) No. of pansies in flowerbed $n = 6 + 4n$ **Ans.**

> The row starts from 10 and then 4 is added each time to get the next term.
> $6 + 4 = 10$, $6 + (4 \times 2) = 14$, $6 + (4 \times 3) = 18$,
> .... $6 + (4 \times n) = 6 + 4n$

(ii) No. of primroses in flowerbed $n = n^2 + n$ **Ans.**

> When $n = 1$, no. of Primroses = 2 $(1^2 + 1)$
> When $n = 2$, no. of Primroses = 6 $(2^2 + 2)$
> When $n = 3$, no. of Primroses = 12 $(3^2 + 3)$
> $\therefore$ When $n = n$, no. of Primroses $= n^2 + n$

(c) Total number of plants in flowerbed n
$= $ number of pansies in flowerbed n + number of primroses in flowerbed n
$= 6 + 4n + n^2 + n$
$= n^2 + 5n + 6$
$= n^2 + 3n + 2n + 6$
$= n(n+3) + 2(n+3)$
$= (n+2)(n+3)$ **Shown.**

(d) Total number of plants in flowerbed 10
$= (10+2)(10+3)$
$= (12)(13) = 156$ **Ans.**

(e) (i) Substituting $n = k$ in (c),
$(k+2)(k+3) = 306$
$k^2 + 5k + 6 = 306$
$k^2 + 5k - 300 = 0$ **Shown.**

(ii) $k^2 + 5k - 300 = 0$
By formula,
$$k = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-300)}}{2(1)}$$
$$= \frac{-5 \pm \sqrt{1225}}{2}$$
$\Rightarrow k = \frac{-5+35}{2}$ or $k = \frac{-5-35}{2}$
$\Rightarrow k = 15$ or $k = -20$ **Ans.**

(iii) Substituting $n = k$ in (b)(i),
the number of pansies in flowerbed k
$= 6 + 4k$
$= 6 + 4(15)$
$= 66$ **Ans.**

4 *(J2011/P2/Q4)*

u_n is the nth term of the sequence 4, 7, 10, 13, ...

(a) (i) Write down an expression, in terms of n, for u_n. [1]
(ii) Hence find the 20th term of the sequence. [1]

(b) v_n is the nth term of the sequence 15, 13, 11, 9, ...
(i) Write down an expression, in terms of n, for v_n. [1]
(ii) w_n is the nth term of another sequence that is obtained by multiplying u_n by v_n. Given that $w_n = 17 + kn - 6n^2$, find k. [1]

Thinking Process

(a) (i) Observe that each subsequent number is 3 more than the previous one.
(ii) Substitute n = 20 into the expression found in (i).

(b) (i) Observe that the numbers in the sequence have a common difference of −2.
(ii) Calculate w_n and compare it with the given expression to find k.

Solution *with* TEACHER's COMMENTS

(a) (i) $u_n = 3n + 1$ **Ans.**

> The sequence has a common difference of 3 and first term is 4
> $(3\times1)+1 = 4,\ (3\times2)+1 = 7,\ (3\times3)+1 = 10.$
> $.........(3\times n)+1 = 3n+1$

(ii) $u_{20} = 3(20) + 1$
$= 61$ **Ans.**

(b) (i) $v_n = 17 - 2n$ **Ans.**

> The sequence has a common difference of −2 and first term is 15
> $(-2\times1)+17 = 15,\ (-2\times2)+17 = 13,$
> $(-2\times3)+17 = 11,\ ....(-2\times n)+17 = -2n+17$

(ii) $w_n = u_n \times v_n$
$= (3n+1)(17-2n)$
$= 51n + 17 - 6n^2 - 2n$
$= 17 + 49n - 6n^2$
given that,
$w_n = 17 + kn - 6n^2$
$\therefore$ by comparision, $k = 49$ **Ans.**

5 *(N2011/P1/Q8)*

The first four terms of a sequence are 55, 53, 49, 41.
The nth term of this sequence is $57 - 2^n$.
(a) Calculate the fifth term. [1]
(b) Write down the nth term of the sequence 56, 55, 52, 45 ... . [1]

Thinking Process

(a) To find the 5th term substitute $n = 5$.
(b) Consider the relationship of the sequence with the sequence given in part (a).

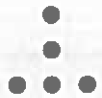
Solution

(a) $T_5 = 57 - 2^5$
$= 57 - 32 = 25$ **Ans.**

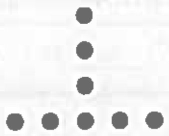
(b) $T_n = 57 + n - 2^n$ **Ans.**

6 (J2012 P1 Q22)

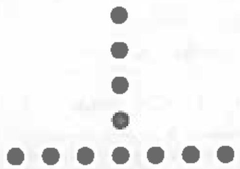
The diagrams below show the first three patterns in a sequence.



Pattern 1



Pattern 2



Pattern 3

(a) Complete the table.

Pattern number	1	2	3	4	5
Number of dots	5	8			

[1]

(b) Find an expression, in terms of n, for the number of dots in Pattern n. [1]

(c) In this sequence, Pattern p has 83 dots. Find the value of p. [2]

Thinking Process

(a) Count the number of dots in pattern 3 and follow the sequence to complete the table.
(b) Observe that the number of dots in each pattern are increasing by 3.
(c) Substitute 83 in part (b).

Solution

(a)

Pattern number	1	2	3	4	5
Number of dots	5	8	11	14	17

(b) Number of dots in pattern n.

$T_n = 3n + 2$ **Ans.**

(c) For pattern p,

$T_p = 3p + 2$

$83 = 3p + 2$

$3p = 81 \Rightarrow p = 27$ **Ans.**

7 (N2012 P1 Q26)

The nth term of a sequence is $9n + 4$.

(a) Calculate the value of the term that is closest to 2012. [2]

(b) Calculate the difference between the 10th term and the 6th term. [1]

(c) (i) Find an expression, in terms of x and y, for the difference between the xth term and the yth term. [1]

(ii) Hence explain why it is not possible for any two terms of this sequence to differ by 123. [1]

Thinking Process

(a) Equate $9n+4$ to 2012.
(b) Substitute $n=10$ and $n=6$ into the expression respectively. Find their difference.
(c) (i) To find xth term substitute $n = x$.
To find yth term substitute $n = y$.
Find their difference.
(ii) Note that the difference of any two terms is a multiple of 9.

Solution

(a) $9n + 4 = 2012$

$9n = 2008$

$n = 223.11 \approx 223$

$\therefore$ 223rd term is closest to 2012

value of 223rd term is,

$T_{223} = 9(223) + 4$

$= 2011$ **Ans.**

(b) $T_{10} = 9(10) + 4 = 94$

$T_6 = 9(6) + 4 = 58$

$T_{10} - T_6 = 94 - 58 = 36$ **Ans.**

(c) (i) xth term, $T_x = 9(x) + 4 = 9x + 4$

yth term, $T_y = 9(y) + 4 = 9y + 4$

$T_y - T_x = 9y + 4 - (9x + 4)$

$= 9y + 4 - 9x - 4$

$= 9y - 9x$

$= 9(y - x)$ **Ans.**

(ii) From (c) (i), we see that the difference of two terms is a multiple of 9. Hence any two terms of this sequence cannot differ by 123 because 123 is not a multiple of 9.

8 *(J2013/P2/Q2)*

Small triangles are formed by placing rods between dots as shown in the diagrams.

Diagram 1

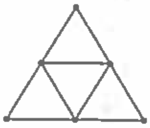


Diagram 2

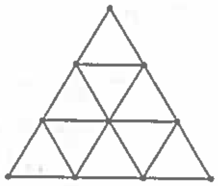


Diagram 3

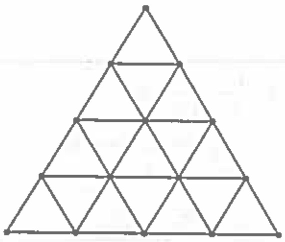


Diagram 4

(a) Complete the table.

Diagram n	1	2	3	4	5
Number of small triangles (T)	1	4	9	16	
Number of dots (D)	3	6	10	15	
Number of rods (R)	3	9	18	30	

[2]

(b) Find an expression, in terms of n, for the number of small triangles (T) formed in Diagram n. [1]

(c) Given that $R = D + T - 1$, find the value of n when $D = 561$ and $R = 1584$. [2]

(d) 1, 3, 6, 10, 15, ...

The nth term of the above sequence is $\frac{1}{2}n(n+1)$.

Hence find an expression for R in terms of n. [1]

(e) How many rods are there in Diagram 15? [1]

(f) Find an expression for D in terms of n. [2]

Thinking Process

(a) Row 2: The number are perfect squares.
Row 3: Consider the increasing difference.
Row 4: Consider the increasing difference

(b) T corresponds to perfect square of numbers.

(c) To find n substitute the given values and the value of T found in (b).

(d) Compare the given sequence with row R.

(e) Substitute n=15 into expression of R found in (d).

(f) Substitute the expressions found in (b) and (d) into the formula given in (c) and simplify.

Solution

(a)

Diagram n	1	2	3	4	5
Number of small triangles (T)	1	4	9	16	25
Number of dots (D)	3	6	10	15	21
Number of rods (R)	3	9	18	30	45

(b) $T_n = n^2$ **Ans.**

(c) $R = D + T - 1$

$\Rightarrow 1584 = 561 + n^2 - 1$

$\Rightarrow n^2 = 1584 - 561 + 1$

$\Rightarrow n^2 = 1024$

$\Rightarrow n = 32$ **Ans.**

(d) $R = 3\left(\frac{1}{2}n(n+1)\right)$

$= \frac{3}{2}n(n+1)$ **Ans.**

(e) $R = \frac{3}{2}(15)(15+1)$

$= \frac{3}{2}(15)(16) = 360$

$\therefore$ number of rods in diagram 15 = 360 **Ans.**

(f) $R = D + T - 1$

$\Rightarrow D = R - T + 1$

$\Rightarrow D = \frac{3}{2}n(n+1) - n^2 + 1$

$\Rightarrow D = \frac{3n(n+1) - 2n^2 + 2}{2}$

$\Rightarrow D = \frac{3n^2 + 3n - 2n^2 + 2}{2}$

$\Rightarrow D = \frac{n^2 + 3n + 2}{2}$

$\Rightarrow D = \frac{n^2 + 2n + n + 2}{2}$

$\Rightarrow D = \frac{n(n+2) + 1(n+2)}{2}$

$\Rightarrow D = \frac{1}{2}(n+2)(n+1)$ **Ans.**

9 *(N2013/P1/Q23)*

The first four lines of a pattern of numbers are shown below.

1st line	3^2	$-$	1^2	$=$	8×1
2nd line	5^2	$-$	1^2	$=$	$8 \times (1+2)$
3rd line	7^2	$-$	1^2	$=$	$8 \times (1+2+3)$
4th line	9^2	$-$	1^2	$=$	$8 \times (1+2+3+4)$

(a) Write down the 7th line of the pattern. [1]

(b) Write down an expression, in terms of n, to complete the nth line of the pattern. [1]

(c) Using the nth line of the pattern, show that $1+2+3+4+...+n=\frac{n(n+1)}{2}$. [2]

Thinking Process

(a) To write the 7th line ✐ carefully observe the pattern and follow the given technique.

(b) Note that the left hand side of the pattern is of the form $(2n^2+1)^2$.

(c) Expand and simplify the left hand side of the expression found in (b).

Solution

(a) $15^2 - 1^2 = 8\times(1+2+3+4+5+6+7)$ **Ans.**

(b) $(2n+1)^2 - 1^2 = 8\times(1+2+3+4+...+n)$

(c) $(2n+1)^2 - 1^2 = 8\times(1+2+3+4+...+n)$

$\Rightarrow \big((2n+1)+1\big)\big((2n+1)-1\big) = 8\times(1+2+3+4+...+n)$

$\Rightarrow (2n+2)(2n) = 8\times(1+2+3+4+...+n)$

$\Rightarrow \frac{4n(n+1)}{8} = 1+2+3+4+...+n$

$\therefore\ 1+2+3+4+...+n=\frac{n(n+1)}{2}$ **Shown.**

10 *(J2014/P2/Q6)*

(a) The first five terms of a sequence are

17, 11, 5, −1, −7.

Find, in terms of n, an expression for the nth term of this sequence. [2]

(b) The nth term, S_n, of a different sequence is found using the formula $S_n = n^2 + 3n$.

(i) Work out the first four terms of this sequence. [2]

(ii) The nth term, T_n, of another sequence is found using the formula $T_n = 5n - 12$. There are two values of n for which $\frac{S_n}{T_n} = 6$.

Form and solve an equation in n to find these two values. [4]

Thinking Process

(a) To find the nth term ✐ note that the difference between consecutive terms is decreasing by 6.

(b) (i) ✐ Substitute $n = 1, 2, 3, 4$ into the given formula.

(ii) Substitute the formulas T_n and S_n into the given equation and solve for n.

Solution

(a) $T_n = 23 - 6n$ **Ans.**

(b) (i) $S_n = n^2 + 3n$

$S_1 = (1)^2 + 3(1) = 4$

$S_2 = (2)^2 + 3(2) = 10$

$S_3 = (3)^2 + 3(3) = 18$

$S_4 = (4)^2 + 3(4) = 28$

$\therefore$ first four terms are: 4, 10, 18, 28 **Ans.**

(ii) $\frac{S_n}{T_n} = 6$

$\Rightarrow \frac{n^2+3n}{5n-12} = 6$

$\Rightarrow n^2 + 3n = 6(5n-12)$

$\Rightarrow n^2 + 3n = 30n - 72$

$\Rightarrow n^2 - 27n + 72 = 0$

$\Rightarrow n^2 - 3n - 24n + 72 = 0$

$\Rightarrow n(n-3) - 24(n-3) = 0$

$\Rightarrow (n-3)(n-24) = 0$

$\Rightarrow n-3 = 0$ or $n - 24 = 0$

$\Rightarrow n = 3$ or $n = 24$

$\therefore\ n = 3$ and 24 **Ans.**

11 *(N2014/P1/Q11)*

The sequence of diagrams below shows small black and small white squares in an arrangement to form large squares.

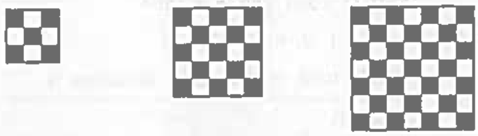


Diagram 1 Diagram 2 Diagram 3

The table below shows the numbers of black and white squares in each diagram.

Diagram (n)	1	2	3	4
Black squares	5	13	25	
White squares	4	12	24	
Total number of black and white squares	9	25	49	

(a) For each diagram, how many more black squares are there than white squares? [1]

(b) On the table, complete the column for Diagram 4. [1]

(c) Write down an expression, in terms of n, for the total number of black and white squares in Diagram n. [1]

Thinking Process

(a) ✎ Observe the sequence of black and white squares in each diagram.

(b) 2nd row: Consider the increasing difference between consecutive terms.
3rd row: Notice that white squares are one less than black squares in each diagram.
4th row: Observe that it is the sum of black and white squares in each diagram.

(c) To form an expression ✎ note that the 4th row corresponds to perfect squares of numbers.

Solution

(a) One. **Ans.**

(b)

Diagram (n)	1	2	3	4
Black squares	5	13	25	41
White squares	4	12	24	40
Total number of black and white squares	9	25	49	81

(c) Total number of black and white squares in Diagram n $= (2n+1)^2$ **Ans.**

12 *(J2015/P1/Q26)*

(a) The first four terms of a sequence, S, are 89, 83, 77, 71.

(i) Find an expression for S_n, the nth term of this sequence. [2]

(ii) Find the smallest value of n for which $S_n < 0$. [1]

(b) The nth term of a different sequence, T, is given by $T_n = n^2 - 4n$.

(i) Find and simplify an expression for $T_{n+1} - T_n$. [2]

(ii) The difference between T_{p+1} and T_p is 75. Find the value of p. [1]

Thinking Process

(a) (i) To find the nth term ✎ note that all the terms in the sequence differ by 6.

(ii) Substitute the expression of S_n in the inequality and simplify to find smallest n.

(b) (i) To find $T_{n+1} - T_n$ ✎ substitute $n+1$ into the expression of T_n.

(ii) Replace n by p in the expression found in (b) (i). Equate it to 75.

Solution

(a) (i) $S_n = 95 - 6n$ **Ans.**

(ii) $S_n < 0$

$95 - 6n < 0$

$-6n < -95$

$n > \frac{-95}{-6}$

$n > 15\frac{5}{6}$

$\therefore$ smallest value of $n = 16$ **Ans.**

(b) (i) $T_{n+1} - T_n$

$= \left((n+1)^2 - 4(n+1)\right) - (n^2 - 4n)$

$= (n^2 + 2n + 1 - 4n - 4) - (n^2 - 4n)$

$= n^2 - 2n - 3 - n^2 + 4n$

$= 2n - 3$ **Ans.**

(ii) From (b) (i), $T_{n+1} - T_n = 2n - 3$

replacing n by p, we have,

$T_{p+1} - T_p = 2p - 3$

given that, $T_{p+1} - T_p = 75$

$\Rightarrow 2p - 3 = 75$

$2p = 78$

$p = 39$ **Ans.**

13 *(N2015/P1/Q24)*

The first term of a sequence is 13.
The following terms are found by alternately adding 4 and 6 to the previous term.
The first six terms are

13 17 23 27 33 37

(a) Write down the next two terms of the sequence. [1]

(b) Write down the value of the term that is closest to 999. [1]

(c) Write down the difference between the values of the 91st and 93rd terms. [1]

(d) Find the 80th term. [1]

(e) The nth term is 203. Find n. [1]

Thinking Process

(b) Observe the pattern and figure out the term closest to 999.

(c) Notice that the difference between odd terms of the sequence is 10.

(d) To find 80th term ✎ find the 40th even term.

(e) To find n ✎ note that 203 is an odd term.

Solution

Splitting the sequence into two subsequences by taking alternate terms, we have,

Odd terms: 13, 23, 33 ......

Even terms: 17, 27, 37 ......

In each case the common difference between successive terms is 10.

(a) 43, 47 **Ans.**

(b) 997 **Ans.**

(c) The difference between 91st and 93rd terms = 10 **Ans.**

(d) Consider the subsequence of even terms,

17, 27, 37 ......

The formula for the nth term is: $T_n = 7 + 10n$

now, the 80th term of the sequence will be the 40th even term.

$\Rightarrow$ $T_{40} = 7 + 10(40)$

$= 7 + 400 = 407$

$\therefore$ 80th term of the sequence = 407 **Ans.**

(e) Consider the subsequence of odd terms,

13, 23, 33 ......

The formula for the nth term is: $T_n = 3 + 10n$

$\Rightarrow$ $203 = 3 + 10n$

$200 = 10n$

$n = 20$

the 20th odd term is the 39th term of the sequence

$\therefore$ $n = 39$ **Ans.**

14 *(J2016 P1 Q25)*

(a) The nth term of a sequence is given by $n^2 - 5n$.

(i) Find the 2nd term in the sequence. [1]

(ii) The pth term in the sequence is 150. Find the value of p. [2]

(b) The nth term of another sequence is given by $3n^2 - kn$. The 5th term in this sequence is 55. Find the value of k. [2]

Thinking Process

(a) (ii) Substitute $n = p$ and equate to 150.

(b) Substitute $n = 5$ and equate to 55.

Solution

(a) (i) $T_2 = (2)^2 - 5(2)$

$= 4 - 10 = -6$ **Ans.**

(ii) $T_p = p^2 - 5p$

$\Rightarrow$ $150 = p^2 - 5p$

$\Rightarrow$ $p^2 - 5p - 150 = 0$

$\Rightarrow$ $p^2 - 15p + 10p - 150 = 0$

$\Rightarrow$ $p(p - 15) + 10(p - 15) = 0$

$\Rightarrow$ $(p - 15)(p + 10) = 0$

$\Rightarrow$ $p - 15 = 0$ or $p + 10 = 0$

$\Rightarrow$ $p = 15$ $\quad p = -10$ (rejected)

$\therefore$ $p = 15$ **Ans.**

(b) $T_5 = 3(5)^2 - k(5)$

Given that, $T_5 = 55$

$\Rightarrow$ $55 = 75 - 5k$

$5k = 20$

$k = 4$ **Ans.**

15 *(N2016 P1 Q26)*

Two sequences have 1, 3, 5 as their first three terms.

(a) In the first sequence, each term is 2 more than the term before it.

(i) Find an expression, in terms of n, for the nth term. [1]

(ii) The kth term of this sequence is 841. Find the value of k. [1]

(b) The nth term of the second sequence is

$$2^{n-1} - \frac{(n-1)(n-4)}{2}$$

(i) Find the fourth term of this sequence. [1]

(ii) Find the fifth term of this sequence. [1]

Thinking Process

(a) (i) Find the nth term as per given information.

(ii) Substitute $n = k$ into the expression found in (i) to find the kth term. Equate it to 841.

(b) (i) Substitute $n = 4$. Simplify.

(ii) Substitute $n = 5$. Simplify.

Solution

(a) (i) $T_n = 2n - 1$ **Ans.**

(ii) kth term, $T_k = 2k - 1$

given that, $T_k = 841$

$\Rightarrow$ $2k - 1 = 841$

$2k = 842$

$k = 421$ **Ans.**

(b) (i) $T_n = 2^{n-1} - \frac{(n-1)(n-4)}{2}$

$T_4 = 2^{4-1} - \frac{(4-1)(4-4)}{2}$

$= 2^3 - 0 = 8$ **Ans.**

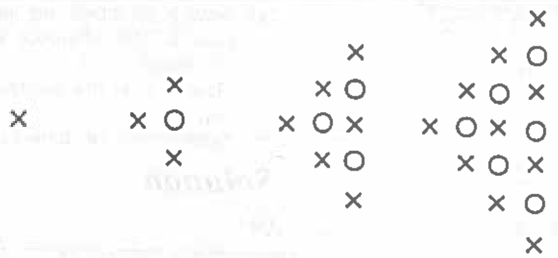
(ii) $T_5 = 2^{5-1} - \frac{(5-1)(5-4)}{2}$

$= 2^4 - \frac{(4)(1)}{2}$

$= 16 - 2$

$= 14$ **Ans.**

16 *(J2017/P2 Q7)*



Pattern 1 Pattern 2 Pattern 3 Pattern 4 Pattern 5

The diagrams show patterns made from crosses (×) and circles (○).

(a) Draw pattern 5 above. [1]

The table shows the number of crosses and circles in each pattern.

Pattern number (n)	1	2	3	4	5	6
Number of crosses	1	3	6	10		
Number of circles	0	1	3	6		
Total number of crosses and circles	1	4	9	16	25	36

(b) Complete the table. [2]

(c) Find an expression, in terms of n, for the total number of crosses and circles in pattern n. [1]

(d) An expression, in terms of n, for the number of crosses in pattern n is $\frac{1}{2}n^2+\frac{1}{2}n$.

How many crosses are there in pattern 30? [1]

(e) Show that the number of circles in pattern n is $\frac{1}{2}n^2-\frac{1}{2}n$. [1]

(f) The number of crosses in pattern m is equal to $5m$.
Find m. [3]

Thinking Process

(a) To Draw pattern 5 ✎ observe the sequence of crosses and circles in each pattern.

(b) To complete the table ✎ consider the increasing difference between consecutive terms in each row.

(c) Observe that row 4 corresponds to perfect square of numbers.

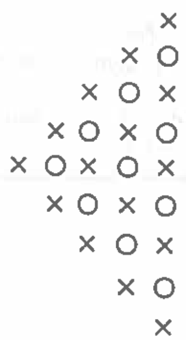
(d) ✎ Substitute $n=30$ into the given expression and simplify.

(e) ✎ Subtract the expression given in part (d) from the expression found in part (c).

(f) ✎ Substitute $n=m$ into the expression given in part (d). Equate it to $5m$.

Solution

(a)



(b)

Pattern number (n)	1	2	3	4	5	6
Number of crosses	1	3	6	10	15	21
Number of circles	0	1	3	6	10	15
Total number of crosses and circles	1	4	9	16	25	36

(c) Total number of crosses and circles in pattern n are, $T_n = n^2$ **Ans.**

(d) Number of crosses in pattern 30 are,

$$T_{30}=\frac{1}{2}(30)^2+\frac{1}{2}(30)$$

$$=450+15=465 \quad \textbf{Ans.}$$

(e) Number of circles in pattern n

= number of crosses and circles in pattern n − number of crosses in pattern n

$$=n^2-\left(\frac{1}{2}n^2+\frac{1}{2}n\right)$$

$$=\frac{1}{2}n^2-\frac{1}{2}n \quad \textbf{Shown.}$$

(f) Substitute $n = m$ into expression given in (d).

number of crosses in pattern $m = \frac{1}{2}m^2 + \frac{1}{2}m$

$\Rightarrow \frac{1}{2}m^2 + \frac{1}{2}m = 5m$

$m^2 + m = 10m$

$m^2 - 9m = 0$

$m(m-9) = 0$

$\Rightarrow m = 0$ (rejected) or $m = 9$

$\therefore m = 9$ **Ans.**

17 *(N2017/P1/Q26)*

The sequence of diagrams shows patterns made from some black beads and some white beads.
Each diagram has two rows more than the previous diagram.

Diagram 1 Diagram 2

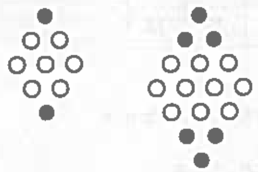
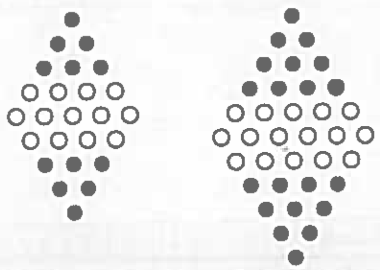


Diagram 3 Diagram 4



(a) Complete the table for Diagram 5.

Diagram number	1	2	3	4	5
Total number of beads	9	16	25	36	
Number of white beads	7	10	13	16	
Number of black beads	2	6	12	20	

[1]

(b) Write down an expression, in terms of n, for

(i) the number of white beads in Diagram n, [1]

(ii) the total number of beads in Diagram n. [1]

(c) Find an expression, in terms of n, for the number of black beads in Diagram n.
Give your answer in its simplest form. [2]

Thinking Process

(a) Row 2: Numbers are in perfect squares
Row 3: The difference between numbers is increasing by 3.
Row 4: It is the difference between row 2 and row 3.

(c) Subtract the nth term found in (b) (i) from (b) (ii).

Solution

(a)

Diagram number	1	2	3	4	5
Total number of beads	9	16	25	36	49
Number of white beads	7	10	13	16	19
Number of black beads	2	6	12	20	30

(b) (i) $T_n = 4 + 3n$ **Ans.**

(ii) $T_n = (n+2)^2$ **Ans.**

(c) Number of black beads in Diagram n

$= $ Total No. of beads − No. of white beads

$= (n+2)^2 - (4+3n)$

$= n^2 + 4n + 4 - 4 - 3n$

$= n^2 + n$ **Ans.**

18 *(J2018/P1/Q21)*

The first four terms, u_1, u_2, u_3 and u_4, in a sequence of numbers are given below.

$u_1 = 1 \times 3 + 2^2 = 7$

$u_2 = 2 \times 4 + 3^2 = 17$

$u_3 = 3 \times 5 + 4^2 = 31$

$u_4 = 4 \times 6 + 5^2 = 49$

(a) Evaluate u_5. [1]

(b) The nth term of the sequence, u_n, is of the form $n(n+p) + (n+q)^2$.
Write down the value of p and the value of q. [1]

(c) u_n can also be written in the form $An^2 + Bn + C$.
Find the values of A, B and C. [2]

Thinking Process

(a) Observe the sequence to find u_5

(b) To find u_n, observe the pattern and deduce the nth term.

(c) Simplify the expression of u_n found in part (b) to find the values of A, B and C

Solution

(a) $u_5 = 5 \times 7 + 6^2$

$= 35 + 36 = 71$ **Ans.**

(b) $u_n = n \times (n+2) + (n+1)^2$

$= n(n+2) + (n+1)^2$

$\therefore \quad p = 2, \quad q = 1$ **Ans.**

(c) From (b), $u_n = n(n+2) + (n+1)^2$

$= (n^2 + 2n) + (n^2 + 2n + 1)$

$= 2n^2 + 4n + 1$

$\therefore \quad A = 2, \quad B = 4, \quad C = 1$ **Ans.**

19 *(N2018/P1/Q11)*

Here are the first five terms of a sequence.

$$\frac{3}{4} \quad \frac{7}{8} \quad \frac{11}{12} \quad \frac{15}{16} \quad \frac{19}{20}$$

(a) Write down the next two terms. [1]

(b) The kth term is $\frac{1199}{1200}$.

Find k. [1]

(c) Find an expression, in terms of n, for the nth term. [2]

Thinking Process

(a) Observe the pattern and deduce the next 2 terms.

(b) To find k ✎ Note that the denominators of all the terms are multiples of 4.

(c) To find the nth term ✎ observe that the numerator in each term is increasing by 4 and denominators are multiples of 4.

Solution

(a) $\frac{23}{24} \quad \frac{27}{28}$ **Ans.**

(b) $k = 300$ **Ans.**

> The denomenators in the sequence are multiples of 4.
> 1st term: $4 \times 1 = 4$, 2nd term: $4 \times 2 = 8$,
> 3rd term: $4 \times 3 = 12$.
> The denomenator in kth term is 1200
> $\therefore \quad 4 \times k = 1200 \Rightarrow k = 300$

(c) nth term, $T_n = \frac{4n-1}{4n}$ **Ans.**